# Comparison of nonlinear functions to describe lactation curves for cumulative milk yield in buffalo

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SUPPLEMENTARY FILE

**Table S1.** Milk yield traits characterising the buffalo lactation curves used in the study.

| Trait [1] | Data source | | | | | |
|---|---|---|---|---|---|---|
| | Sahoo *et al*. (2014) | | Sharma *et al*. (2017) | | | |
| | | | 1991-1995 | 1996-2000 | 2001-2005 | 2006-2010 |
| $m_{y5}$ (kg/d) | 3.93 | | 2.52 | 3.32 | 2.62 | 2.89 |
| $m_{y305}$ (kg/d) | 2.35 | | 4.42 | 4.43 | 3.93 | 4.63 |
| $m_y^+$ (kg/d) | 7.92 | | 5.51 | 6.36 | 5.97 | 6.26 |
| $t^+$ (d) | 50 | | 125 | 125 | 125 | 125 |
| $y_{305}$ (kg) | 1891 | | 1347 | 1655 | 1422 | 1541 |

[1] Daily milk yield at 5 or 305 days in milk ($m_{y5}$ *and* $m_{y305}$, respectively), maximum daily milk yield ($m_y^+$) and days in milk when $m_y^+$ was recorded ($t^+$) and cumulative milk production at 305 d lactation ($y_{305}$)

**Table S2.** Mathematical functions fit to buffalo cumulative milk production curves.

| **Function** | **Functional form**[1] | **Time at inflexion point** | **Yield at inflexion point** |
|---|---|---|---|
| Linear | $y = y_0 + kt$ | Not applicable | Not applicable |
| Gompertz | $y = y_0 \times \exp\left[\ln\left(\frac{y_f}{y_0}\right) \times (1 - e^{-ct})\right]$ | $\frac{1}{c}\ln\left[\ln\left(\frac{y_f}{y_0}\right)\right]$ | $\frac{y_f}{e}$ |
| Schumacher | $y = y_f \times \exp\left(\frac{-Kt_0t_0}{t + t_0}\right)$ | $t_0\left(\frac{Kt_0}{2} - 1\right)$ | $\frac{y_f}{e^2}$ |
| Richards | $y = \frac{y_f}{\left[1 + n \times \exp\left(-c(t - t^*)\right)\right]^{\frac{1}{n}}}$ | $\frac{1}{c}\ln\left(\frac{y_f^n - y_0^n}{ny_0^n}\right)$ | $\frac{y_f}{(n+1)^{\frac{1}{n}}}$ |
| Sinusoidal | $y = y^* + a \times \sin\left(\frac{2\pi t}{b} + \theta\right)$ | $\frac{b}{2\pi}\left(2\pi - \theta\right)$ | $y^*$ |

[1] $y$ = cumulative milk yield, $y_0$ = initial yield, $y_f$ = asymptotic yield, $t$ = days in milk; $k$ is the slope; $c$, $K$ and $n$ are parameters that define the position, scale and shape of the cumulative lactation curve. For the sinusoidal, $a$ is the amplitude, $y^*$ is the vertical shift and $\theta$ is the phase shift. This sinusoidal equation is periodic with period $b$

**Table S3.** Statistics used to assess goodness-of-fit for model comparison.

| Statistic | Calculation |
|---|---|
| Adjusted coefficient of determination ( $R^2_{\text{adj}}$ ) | $R^2_{\text{adj}} = 1 - \left[ \frac{(n-1)}{(n-p)} \right] \left( \frac{\text{RSS}}{\text{TSS}} \right)$ |
| Root mean square error (RMSE) | $\text{RMSE} = \sqrt{\frac{\text{RSS}}{n-p-1}}$ |
| Akaike's information criterion (AIC) | $\text{AIC} = n \times \ln\left( \frac{\text{RSS}}{n} \right) + 2p$ |
| Bayesian information criterion (BIC) | $\text{BIC} = n \times \ln\left( \frac{\text{RSS}}{n} \right) + p \ln(n)$ |

*n* is the number of observations (data points), *p* is the number of parameters in the equation, RSS is residual sum of squares and TSS is total sum of squares.

**Table S4.** Parameters for different data profiles estimated using the different functions.

| | | Data source | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | Sharma *et al*. (2017) | | | |
| **Function** | **Parameter**[1] | **Sahoo** ***et al*****. (2014)** | | **1991-1995** | **1996-2000** | **2001-2005** | **2006-2010** |
| Linear | $K$ | 6.491 | | 4.727 | 5.712 | 5.305 | 5.536 |
| | $y_0$ | 102.49 | | -20.59 | 2.35 | -6.23 | 2.57 |
| Gompertz | $C$ | 0.0103 | | 0.0089 | 0.0092 | 0.0095 | 0.0092 |
| | $y_0$ | 111.9 | | 57.8 | 76.1 | 62.9 | 74.5 |
| | $y_f$ | 2169 | | 1712 | 2015 | 1833 | 1953 |
| Schumacher | $K$ | 0.0728 | | 0.0671 | 0.0681 | 0.0782 | 0.0684 |
| | $t_0$ | 59.5 | | 73.8 | 70.1 | 63.9 | 69.8 |
| | $y_f$ | 3957 | | 3625 | 4094 | 3681 | 3953 |
| Richards | $C$ | 0.0059 | | 0.0018 | 0.0027 | 0.0035 | 0.0025 |
| | $n$ | -0.5890 | | -0.8869 | -0.8170 | -0.7499 | -0.8414 |
| | $t^*$ | 73.0 | | 75.1 | 75.3 | 83.7 | 70.6 |
| | $y_f$ | 2552 | | 3815 | 3407 | 2746 | 3445 |
| Sinusoidal | $a$ | 1439 | | 1296 | 1452 | 1158 | 1575 |
| | $b$ | 1164 | | 1641 | 1477 | 1251 | 1656 |
| | $\theta$ | 5.901 | | 5.829 | 5.871 | 5.727 | 5.972 |
| | $y^*$ | 549.3 | | 544.0 | 559.7 | 588.0 | 457.0 |
| | $y_f$ | 1989 | | 1804 | 2036 | 1746 | 2032 |

[1] $y_0$ = initial yield, $y_f$ = asymptotic yield, $t$ = days in milk; $k$ is the slope; $c$, $K$, $t_0$, $t^*$ and $n$ are parameters that define the position of the inflexion point, the scale and the shape of the cumulative lactation curve. For the sinusoidal, $a$ is the amplitude, $y^*$ is the vertical shift and $\theta$ is the phase shift. $y_f$ for the sinusoidal is not an estimated parameter. It was calculated from the fitted equations as: $y_f = y^* + a$

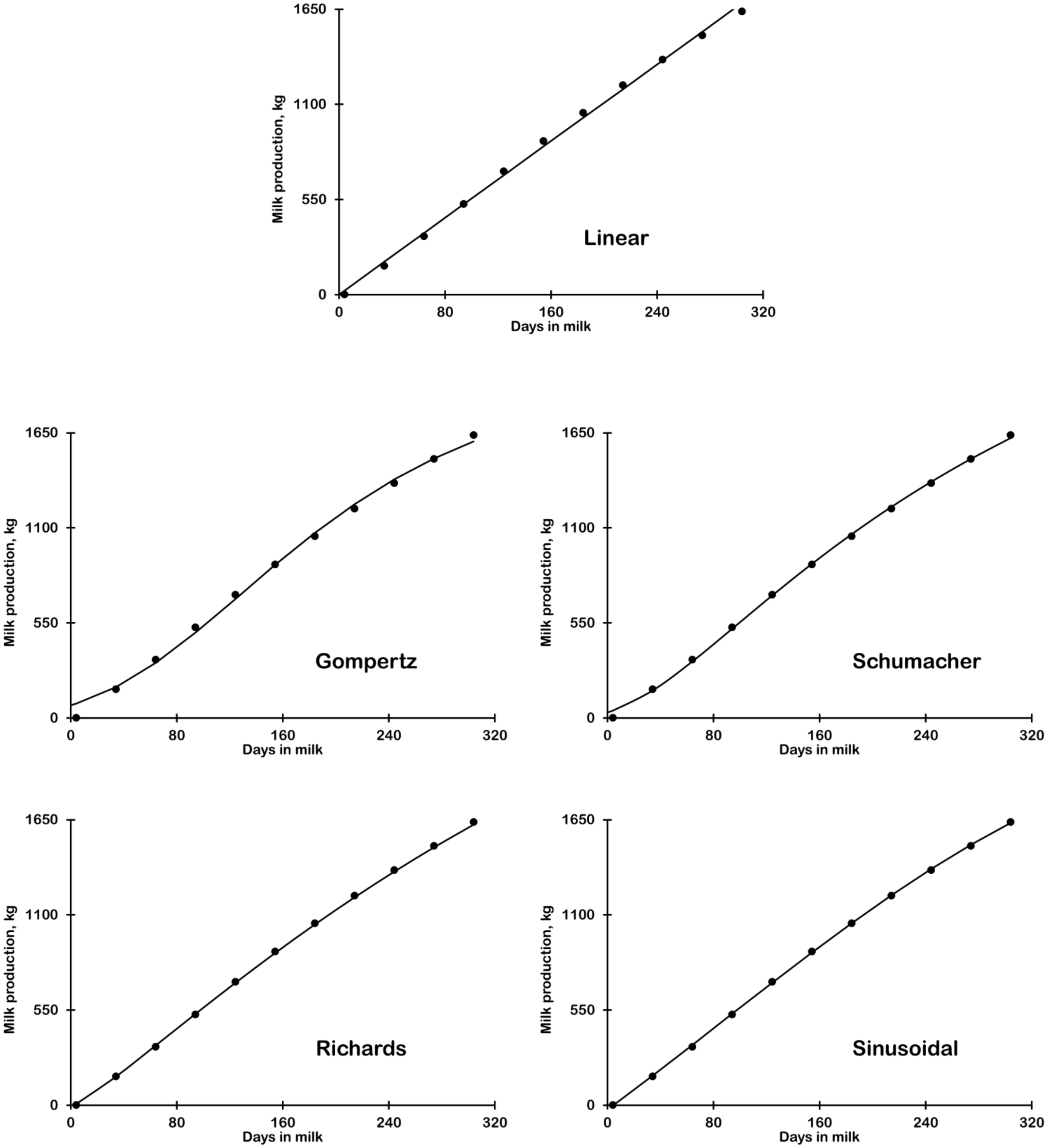


**Figure S1.** Plots of cumulative milk production (kg) against days in milk showing the fit of different functions to the data of Sharma *et al*. (2017) during the period 2006-2010 (dots are the observed values and the solid line is the fitted curve for each function)