Supplementary Appendix to "The Real Effects of Financing and Trading Frictions"

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SA.1 Alternative Assumptions about the Firm Financing

In this section, I consider alternative assumptions about the firm's financing. I keep the same assumptions about secondary market transactions and liquidity provision as in the main text, meaning that the endogenous bid-ask spread is pinned down following steps similar to those reported in Section III of the paper. This is done to preserve comparability and fairly assess robustness to alternative assumptions about the firm financing.

SA.1.1 Debt Financing as Bank Credit Lines

Small firms typically do not have access to the corporate bond or commercial paper markets, and are more likely to tap debt financing by drawing funds from bank lines of credit. A credit line is a source of funding that the firm can access at any time up to a pre-established limit L. Whenever the credit limit is finite ($L < \infty$), the firm has a positive demand for cash.[1] In this section, I assess the model results in the presence of this additional source of financing.

I follow Bolton et al. (2011) and assume that the firm pays a constant spread, β, over the risk-free rate on the amount of credit used. Because of this cost, it is optimal for the firm to tap the credit line when cash reserves are exhausted. The firm then uses cash as the marginal source of financing if $c \in [0, C_V(L)]$ (the cash region), where $C_V(L)$ denotes the target cash level in this environment. Conversely, the firm draws funds from the credit line when $c \in [-L, 0]$ (the credit line region). Firm value satisfies equation (5) in the cash region, whereas it satisfies

$$\text{(SA.1)} \quad \rho V(c;\eta) = [(\rho+\beta)c+\mu]\, V'(c;\eta) + \frac{\sigma^2}{2} V''(c;\eta) + \lambda\, [V(C_V;\eta) - V(c;\eta) - C_V + c] \\ + \delta \left[(1-\min[\eta(c);\chi])V(c;\eta) - V(c;\eta)\right]$$

in the credit-line region. On top of the smooth-pasting and super-contact conditions at $C_V(L)$ similar to (9) and (10), the system of ODEs (5)–(SA.1) is solved subject to the following

[1] As shown by Bolton, Chen, and Wang (2011), this is true for exogenous or endogenous (value-maximizing) L. Empirically, firms often face credit supply frictions that prevent them from taking the value-maximizing limit L. Endogenizing L is an interesting extension to understand the relation between stock liquidity and the firm's willingness to access bank credit, and I leave it for future research.

boundary conditions. The first condition, $V(-L) = \max[\ell - L, 0]$, means that shareholders are residual claimants on the firm's liquidation value.[2] Moreover, the conditions $\lim_{c\uparrow 0} V(0) = \lim_{c\downarrow 0} V(0)$ and $\lim_{c\uparrow 0} V'(0) = \lim_{c\downarrow 0} V'(0)$ guarantee continuity and smoothness at the point where the cash and the credit line regions are pasted together.

[Figure SA.1]

Figure SA.1 studies the impact of the cost of liquidity provision when allowing for credit line availability. I use the parametrization in Table 1 in the main text and additionally set $L = 0.08$ and $\beta = 1.5\%$ (see Sufi (2009)). The figure shows that the effects of ν on corporate outcomes are similar irrespective of the firm's access to bank credit.[3] Access to credit relaxes the precautionary demand for cash, leading to a lower target cash level. Still, the target cash level continues to be decreasing with the order-processing cost ν borne by liquidity providers. Moreover, the probability of liquidation and of payout continue to be increasing with ν. Finally, credit line availability does not affect much the zero-NPV cost—i.e., the maximum amount that the firm is willing to pay to exercise the growth option—which continues to be decreasing with ν as in the baseline model with no access to bank credit.

SA.1.2 Modeling Financing Frictions as Issuance Costs

Consistent with the difficulties faced by small firms in raising external funds, the baseline model developed in the paper features financing frictions as capital supply uncertainty. This section alternatively models such frictions as issuance costs, as in Décamps, Mariotti, Rochet, and Villeneuve (2011) or Bolton et al. (2011). In this setup, the firm chooses the timing of equity issuances rather than waiting for stochastic financing opportunities.

Namely, I assume that external financing entails proportional and fixed costs, denoted by ϵ and ψ, respectively. These costs prompt the firm to keep precautionary cash reserves, and I

[2] If if $\ell \geq L$, the credit line is fully secured.

[3] While, for the sake of brevity, the charts showcase the impact of the order-processing cost ν on corporate decisions, similar results continue to hold when instead investigating the impact of the other core parameters determining the illiquidity of the firm's stock. Such results are available upon request.

continue to denote the target cash level by C_V. For any $c < C_V$, firm value satisfies:

$$\text{(SA.2)} \qquad \rho V(c;\eta) = (rc + \mu)\, V'(c;\eta) + \frac{\sigma^2}{2} V''(c;\eta) + \delta \Big[(1 - \min[\eta;\chi]) V(c;\eta) - V(c;\eta) \Big] .$$

This equation differs from equation (5) in the main text as the firm does not face stochastic financing opportunities. Rather, the firm sets financing decisions to minimize issuance costs. Namely, to economize on the fixed cost, the firm raises funds in a lumpy fashion when cash reserves are depleted. Denote the optimal issue size by C_*. The following condition then holds:

$$V(0) = V(C_*) - (1 + \epsilon)C_* - \psi$$

which implies that firm value at $c = 0$ (the left-hand side of this equation) equals the firm's continuation value net of issuance cost (the right-hand side). Notably, it is optimal for the firm to raise external financing if the following inequality $V(C_*) - (1 + \epsilon)C_* - \psi > \ell$ holds, which guarantees that the firm's continuation value (the left-hand side) is larger than the liquidation value (the right-hand side). The optimal issue size C_* satisfies $V'(C_*) = 1 + \epsilon$, which warrants that the marginal benefit (the left-hand side of this equation) and cost of external financing (the right-hand side) are equalized at C_*. Lastly, Equation (SA.2) is subject to boundary conditions at the target cash level that are similar to (9) and (10) in the baseline version of the model.

[Table SA.1]

Table SA.1 shows the effect of the cost of liquidity provision on the target cash level, the optimal issuance size, and the zero-NPV cost. I use the baseline parameters in Table 1 and, in addition, consider two sets of values for the issuance costs. In the top panel, I set $\epsilon = 0.06$ and $\psi = 0.01$ as in Bolton et al. (2011), whereas in the bottom panel I set $\epsilon = 0.10$ and $\psi = 0.03$, which account for the heterogeneity in financing costs documented by Hennessy and Whited (2007). Table SA.1 confirms that the target cash level decreases with ν, as so does the zero-NPV cost, consistent with the results in the baseline model. Furthermore, this setup allows to investigate how the cost of liquidity provision affects the optimal size of equity issues (i.e., the endogenous quantity C_*). Indeed, Table SA.1 illustrates that C_* decreases with ν—i.e., the larger the cost of liquidity provision, the lower the size of refinancing events. Overall, this analysis shows that the primitive determinants of stock illiquidity continue to affect the firm's financial and investment decisions irrespective of the way financing frictions are modeled—i.e., being costs or uncertainty in raising new funds.

SA.2 Benchmark with No Financing Frictions

In this Appendix, I consider the case in which the firm faces no financing frictions as it has a frictionless access to external financing—in the model, this is the case by assuming $\lambda \to \infty$. Under this assumption, as soon as the firm looks for external financing, it is able to find it with no delay. In this case, the firm is never liquidated and has no incentives to keep cash reserves. That is, because the firm can raise external financing at no delay nor costs, hoarding cash does not bring any benefit to shareholders (whereas cash has an opportunity cost). Given the absence of financing frictions, the Modigliani-Miller logic applies and the firm has many degrees of freedom in designing dividend and financial policies—yet, their impact on firm value is trivial.[4]

In this environment, I denote firm value by V^* and the bid-ask spread by η^*. Standard arguments imply that firm value satisfies the following equation:

(SA.3) $$\rho V^* = \mu + \delta \left[(1 - \min[\eta^*; \chi]) V^* - V^* \right].$$

The left-hand side of this equation is the return required by risk-neutral investors. The first term on the right-hand side is the expected cash flow on each time interval (μ), whereas the second term is the impact of liquidity shocks on firm value (a term that admits an interpretation similar to that provided in the main text). In this setup, stock illiquidity is still determined via Nash bargaining between shocked shareholders and liquidity providers, which gives the following bid price:

(SA.4) $$b^* = \theta(1-\chi)V^* + (1-\theta)\left[(1-\nu)V^* - \omega\right]$$

[4] This result is similar to Décamps et al. (2011), see their Section II.

and the following bid-ask spread:

$$\eta^* = \theta\chi + (1-\theta)\nu + \frac{(1-\theta)\omega}{V^*}. \tag{SA.5}$$

Equations (SA.5) and (SA.3) together imply that, in this corner case too, costs borne by liquidity providers continue to be passed on the firm's investors, as firm value satisfies:

$$V^* = \frac{\mu - \delta(1-\theta)\omega}{\rho + \delta\nu(1-\theta) + \delta\theta\chi}. \tag{SA.6}$$

Notably, the parameters that directly affect stock illiquidity continue to affect firm value. Namely, they lead to a decrease in firm value, as in the case with financing frictions. Also, trading frictions continue to affect investment, as the zero-NPV cost satisfies:[5]

$$I^* = V^*(\eta^*, \mu_1) - V^*(\eta^*, \mu) = \frac{\mu_1 - \mu}{\rho + \delta\nu(1-\theta) + \delta\chi\theta}. \tag{SA.7}$$

Because financial policies (financing, payout, and cash holdings) are trivial absent financing frictions, the impact of trading frictions on such policies is trivial too. Overall, this analysis concludes that the case with financing frictions offers a more comprehensive analysis of the effect of illiquidity on corporate policies, which is particularly relevant in light of the empirical observation that firms with illiquid stocks are typically financially constrained, as discussed in the introduction of the paper.

[5] Interestingly, equation (SA.7) shows that ω does not affect the zero-NPV cost when there are no financing frictions. The reason is that ω enters the numerator of firm value (as highlighted by (SA.6))—i.e., it is akin to a flow cost. Because its magnitude does not change before and after the investment, then it does not impact the zero-NPV cost. In turn, the channel through which ω impacts the zero-NPV cost in the full model with financing frictions is through the target cash level (which is zero when $\lambda = \infty$), which indeed changes after investment.

References

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FIGURE SA.1: Allowing for Credit Line Availability.

The figure shows the target level of cash reserves, the probability of liquidation, the probability of payout, and the zero-NPV cost as a function of the order-processing cost ν borne by liquidity providers. The solid blue lines refer to a firm with no access to bank credit, whereas the red dashed lines refer to a firm having access to bank credit.

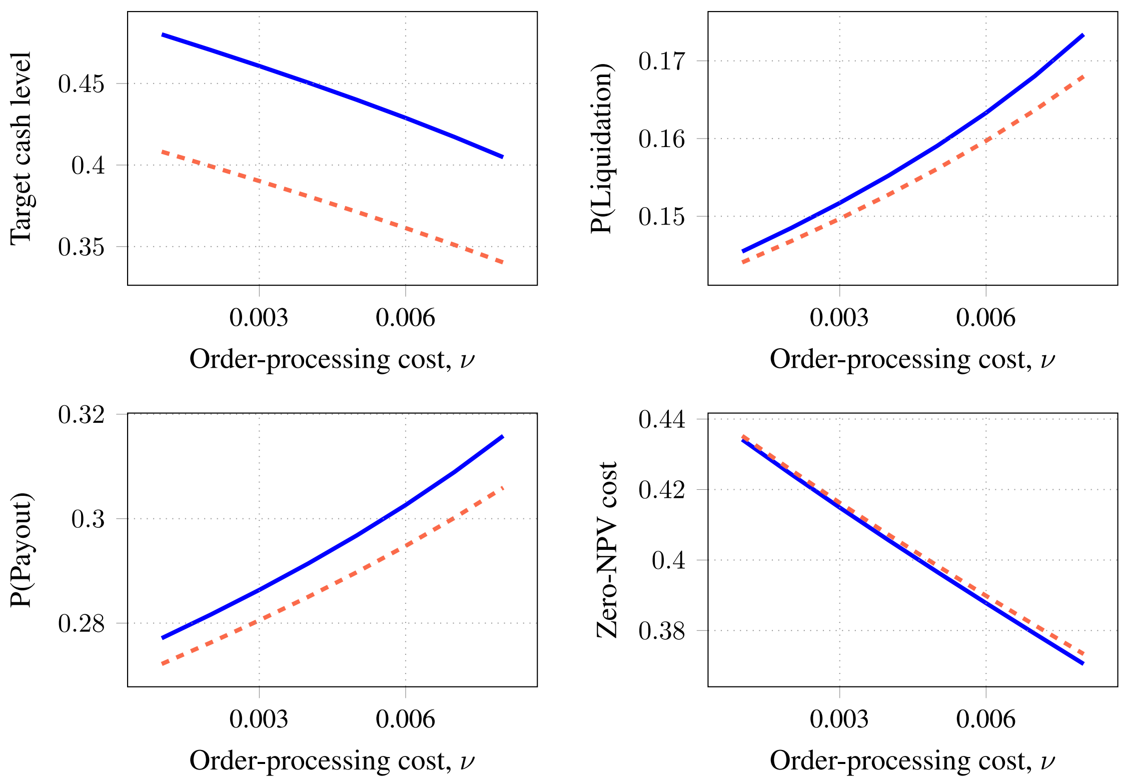

TABLE SA.1: Modeling Financing Frictions as Issuance Costs.

The table reports the target cash level, the size of equity issuances, and the zero-NPV investment cost when varying the order-processing cost ν. Proportional and fixed financing costs are respectively equal to $\epsilon = 0.06$ and $\psi = 0.01$ in the top panel, and equal to $\epsilon = 0.1$ and $\psi = 0.03$ in the bottom panel.

	Target Cash Level	Issuance Size	Zero-NPV investment cost
$\epsilon = 0.06$, $\psi = 0.01$			
$\nu = 0.001$	0.325	0.129	0.443
$\nu = 0.003$	0.318	0.127	0.425
$\nu = 0.005$	0.312	0.126	0.409
$\nu = 0.007$	0.307	0.125	0.394
$\epsilon = 0.1$, $\psi = 0.03$			
$\nu = 0.001$	0.410	0.173	0.448
$\nu = 0.003$	0.402	0.171	0.430
$\nu = 0.005$	0.395	0.170	0.413
$\nu = 0.007$	0.389	0.169	0.398