Supplementary Material for 'Contact Angle Hysteresis on Rough Surfaces: Mechanical Energy Balance Framework' by Dalton J.E. Harvie

1. Additional Fakir wetting data

Additional data for Fakir droplets taken from other publications and analysed using the theory of section 4.2.4 (main text) is given in Table 1. As previously ϕ is the surface area covered by the poles while h/s is the height of each pole (h) divided by the span distance between adjacent poles (s). The parameters ϕ and h/s are important as if either is too low then the fluid/fluid interface has the potential to either a) not be adequately supported on the poles, or b) alternatively touch the base surface of the solid when advancing, subsequently destabilising the Fakir wetting state. On this basis some data from Priest *et al.* (2009) that had $\phi < 0.04$ has been omitted from the table. θ_e is the inherent (Youngs) contact angle of the underlying surface (calculated as the average of the inherent advancing and receding angles) and $\mathit{\Delta}\theta_\mathrm{e}$ the underlying inherent CAH (being the difference of the inherent advancing and receding angles). As noted in the main text, the inherent CAH of the surfaces prepared by Öner & McCarthy (2000) is considerably smaller than that of the other datasets, making this dataset the most relevant when comparing against the present theory. θ_a and θ_r are the measured advancing and receding angles.

The following calculated parameters are also presented in Table 1; the equilibrium Cassie-Baxter angle θ_CB (main text equation (4.11), the dissipation geometry factors α and β (back-calculated from main text equations (4.13) and (4.12), respectively) and the advancing, receding and total non-dimensional dissipation energies D'_a, D'_r and D'_t, respectively (also calculated from main text equations (4.1) and (4.11)).

In general the advancing geometry factor β is around 1 for most of the Öner & McCarthy data, irrespective of the cross-sectional geometry of the poles, whereas for the other datasets $\beta \approx 0.7$, irrespective of the cross-section, inherent angle or area fraction. One difference between the two cases is that the latter sets have poles arranged in a regular pattern, whereas the Öner & McCarthy surfaces all use a staggered arrangement. Hence a possibility is that for an interface advancing over a line of poles concurrently (as in a regular pattern), the whole of the next pole area may not be wetted simultaneously as multiple 'slip' motions happen concurrently. Contrastingly, the receding geometry factors α corresponding to the Öner & McCarthy (2000) data show a strong dependence on pole cross-section, with the more complex shapes (such as the star) providing more points on which to pin the receding interface, and hence correspondingly higher α. The regular array data for α does not show a significant difference between the square and circular pole cross-sections, with $\alpha \approx 1$ describing most of the data that has $\phi \lessapprox 0.3$. Above this area fraction the Priest *et al.* data suggests a slight decrease in α with increasing ϕ, presumably because the deformation of the fluid/fluid interface during the recede (as normalised by the pole top surface area) becomes more limited by the spacing between the poles (s). Note also that Jiang *et al.* (2019) observed droplets of fluid remaining on the tops of poles after the recede dissipation event for $\phi = 0.13$, highlighting that any simple fakir theory will break down as ϕ becomes non-dilute, and the deformation during the dissipation events becomes more complex.

Dissipation event energies are also shown in Table 1. Across all datasets both the advancing D'_a and receding D'_r event dissipations are significant contributors to the total dissipation D'_t. $D'_\mathrm{r,Ji}$ and $D'_\mathrm{r,Rs}$ are the equivalent receding event dissipation energies as suggested by Jiang *et al.* (2019) and Reyssat & Quéré (2009), respectively. Jiang *et al.*'s correlation predicts D'_r reasonably well for surfaces having circular cross-section poles in regular patterns with $\phi \lessapprox 0.1$. The Reyssat & Quéré correlation for D'_r consistently predicts

ϕ	h/s	θ_e	$\Delta\theta_\mathrm{e}$	θ_a	θ_r	θ_CB	α	β	D'_a	D'_r	D'_t	$D'_\mathrm{r,Ji}$	$D'_\mathrm{r,Rs}$
Öner & McCarthy (2000, Table 1), square cross-section, staggered layout, average of 2–16 μm widths													
0.25	2.5	98	8	173.7	138.0	141.7	0.17	0.98	0.209	0.042	0.251	0.162	0.601
0.25	2.5	104.5	5	173.3	139.7	144.4	0.20	0.97	0.181	0.050	0.231	0.105	0.601
0.25	2.5	114.5	9	169.3	143.7	148.6	0.19	0.93	0.129	0.048	0.177	0.050	0.601
Öner & McCarthy (2000, Table 1), indented square 'X' cross-section, staggered layout													
0.2	5	98	8	173.0	140.0	145.9	0.31	0.96	0.165	0.062	0.227	0.151	0.523
0.2	5	104.5	5	175.0	143.0	148.2	0.26	0.98	0.146	0.051	0.198	0.095	0.523
0.2	5	114.5	9	169.0	146.0	152.0	0.27	0.91	0.099	0.054	0.153	0.041	0.523
Öner & McCarthy (2000, Table 1), rhombus cross-section, staggered layout													
0.0625	5	98	8	174.0	155.0	161.1	0.64	0.91	0.048	0.040	0.088	0.097	0.233
0.0625	5	104.5	5	176.0	156.0	162.4	0.63	0.96	0.044	0.040	0.084	0.056	0.233
0.0625	5	114.5	9	168.0	153.0	164.5	1.16	0.65	0.015	0.072	0.087	0.015	0.233
Öner & McCarthy (2000, Table 1), four-armed star cross-section, staggered layout													
0.083	5	98	8	174.0	147.0	158.2	1.08	0.93	0.066	0.090	0.156	0.109	0.287
0.083	5	104.5	5	175.0	149.0	159.7	0.97	0.95	0.058	0.081	0.139	0.064	0.287
0.083	5	114.5	9	170.0	148.0	162.1	1.25	0.82	0.033	0.103	0.137	0.020	0.287
Jiang *et al.* (2019, Table S1), circular cross-section, regular layout													
0.05	2	108	20	163.0	154.0	164.9	1.33	0.13	-0.009	0.067	0.058	0.069	0.197
0.1	1.2	108	20	165.0	147.0	158.6	0.92	0.66	0.035	0.092	0.127	0.094	0.327
0.13	0.64	108	20	163.0	141.0	155.5	1.02	0.66	0.046	0.133	0.179	0.104	0.393
0.17	0.28	108	20	165.0	137.0	151.9	0.89	0.80	0.083	0.151	0.235	0.116	0.471
Bico *et al.* (1999, Table 1), circular cross-section, regular layout													
0.05	0.22	109	18	170.0	155.0	165.1	1.20	0.70	0.019	0.060	0.078	0.060	0.197
Priest *et al.* (2009, Table S1), square cross-section, regular layout, $\phi > 0.04$ only													
0.09	0.64	105.5	21	163.0	146.0	159.1	1.17	0.51	0.022	0.105	0.127	0.107	0.304
0.09	0.64	105.5	21	165.0	143.0	159.1	1.50	0.62	0.032	0.135	0.167	0.107	0.304
0.09	0.64	105.5	21	167.0	142.0	159.1	1.62	0.72	0.040	0.146	0.186	0.107	0.304
0.14	0.89	105.5	21	163.0	137.0	153.8	1.19	0.69	0.059	0.166	0.225	0.127	0.414
0.15	0.94	105.5	21	165.0	133.0	152.9	1.39	0.77	0.076	0.208	0.284	0.130	0.433
0.22	1.3	105.5	21	160.0	128.0	147.0	1.01	0.73	0.101	0.223	0.324	0.149	0.556
0.22	1.3	105.5	21	161.0	127.0	147.0	1.08	0.75	0.107	0.237	0.344	0.149	0.556
0.23	1.3	105.5	21	166.0	124.0	146.2	1.18	0.87	0.139	0.272	0.411	0.151	0.571
0.24	1.4	105.5	21	163.0	126.0	145.5	0.98	0.82	0.132	0.236	0.369	0.153	0.586
0.33	2.0	105.5	21	165.0	122.0	139.3	0.69	0.90	0.208	0.228	0.436	0.168	0.706
0.34	2.1	105.5	21	162.0	124.0	138.7	0.56	0.86	0.200	0.192	0.392	0.170	0.718
0.36	2.3	105.5	21	163.0	117.0	137.4	0.78	0.88	0.220	0.282	0.502	0.172	0.741
0.4	2.6	105.5	21	165.0	111.0	135.0	0.87	0.91	0.259	0.349	0.608	0.177	0.783
0.52	3.9	105.5	21	168.0	105.0	128.2	0.69	0.96	0.359	0.360	0.719	0.187	0.889
0.58	4.8	105.5	21	164.0	105.0	125.1	0.55	0.93	0.386	0.316	0.702	0.190	0.931
0.59	5.0	105.5	21	161.0	93.0	124.6	0.87	0.91	0.378	0.515	0.893	0.191	0.937
0.66	6.5	105.5	21	148.0	97.0	121.1	0.60	0.77	0.332	0.395	0.726	0.194	0.978
0.67	6.8	105.5	21	159.0	96.0	120.6	0.60	0.90	0.425	0.405	0.829	0.194	0.984

Table 1: Contact angles measured in various publications for water droplets exhibiting Fakir wetting on lithography based pole surfaces.

dissipations that are larger than even D'_t. Notably this correlation is independent of θ_e, having been developed using data from only one liquid/surface combination.

REFERENCES

Bico, J., Marzolin, C. & Quéré, D. 1999 Pearl drops. *Europhysics Letters* **47** (2), 220–226.

JIANG, YOUHUA, XU, WEI, SARSHAR, MOHAMMAD AMIN & CHOI, CHANG-HWAN 2019 Generalized models for advancing and receding contact angles of fakir droplets on pillared and pored surfaces. *Journal of Colloid and Interface Science* **552**, 359–371.

ÖNER, DIDEM & MCCARTHY, THOMAS J. 2000 Ultrahydrophobic surfaces, effects of topography length scales on wettability. *Langmuir* **16**, 7777–7782.

PRIEST, CRAIG, ALBRECHT, TRENT W. J., SEDEV, ROSSEN & RALSTON, JOHN 2009 Asymmetric wetting hysteresis on hydrophobic microstructured surfaces. *Langmuir* **25** (10), 5655–5660.

REYSSAT, MATHILDE & QUÉRÉ, DAVID 2009 Contact Angle Hysteresis Generated by Strong Dilute Defects. *The Journal of Physical Chemistry B* **113** (12), 3906–3909.