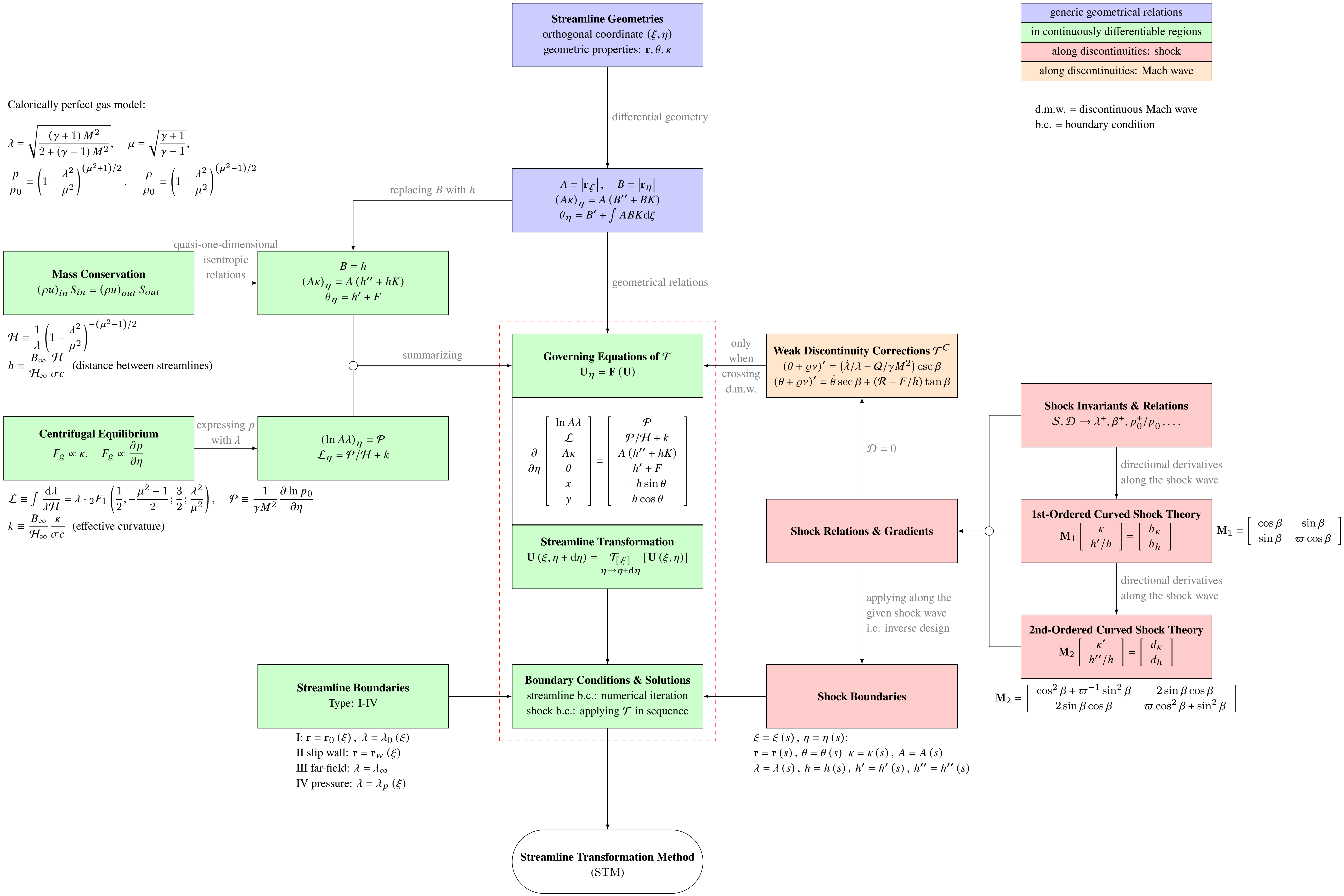

Streamline Geometries
orthogonal coordinate (ξ, η)
geometric properties: $\mathbf{r}, \theta, \kappa$
differential geometry
$A = |\mathbf{r}_\xi|, \quad B = |\mathbf{r}_\eta|$
$(A\kappa)_\eta = A(B'' + BK)$
$\theta_\eta = B' + \int ABK \mathrm{d}\xi$
replacing B with h
geometrical relations
generic geometrical relations
in continuously differentiable regions
along discontinuities: shock
along discontinuities: Mach wave
d.m.w. = discontinuous Mach wave
b.c. = boundary condition
Calorically perfect gas model:
$\lambda = \sqrt{\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}}, \quad \mu = \sqrt{\frac{\gamma+1}{\gamma-1}},$
$\frac{p}{p_0} = \left(1 - \frac{\lambda^2}{\mu^2}\right)^{(\mu^2+1)/2}, \quad \frac{\rho}{\rho_0} = \left(1 - \frac{\lambda^2}{\mu^2}\right)^{(\mu^2-1)/2}$
Mass Conservation
$(\rho u)_{in} S_{in} = (\rho u)_{out} S_{out}$
quasi-one-dimensional isentropic relations
$B = h$
$(A\kappa)_\eta = A(h'' + hK)$
$\theta_\eta = h' + F$
$\mathcal{H} \equiv \frac{1}{\lambda}\left(1 - \frac{\lambda^2}{\mu^2}\right)^{-(\mu^2-1)/2}$
$h \equiv \frac{B_\infty}{\mathcal{H}_\infty}\frac{\mathcal{H}}{\sigma c}$ (distance between streamlines)
summarizing
Governing Equations of $\mathcal{T}$
$\mathbf{U}_\eta = \mathbf{F}(\mathbf{U})$
$\frac{\partial}{\partial\eta}\begin{bmatrix} \ln A\lambda \\ \mathcal{L} \\ A\kappa \\ \theta \\ x \\ y \end{bmatrix} = \begin{bmatrix} \mathcal{P} \\ \mathcal{P}/\mathcal{H} + k \\ A(h'' + hK) \\ h' + F \\ -h\sin\theta \\ h\cos\theta \end{bmatrix}$
only when crossing d.m.w.
Weak Discontinuity Corrections $\mathcal{T}^C$
$(\theta + \varrho\nu)' = (\dot{\lambda}/\lambda - Q/\gamma M^2)\csc\beta$
$(\theta + \varrho\nu)' = \dot{\theta}\sec\beta + (\mathcal{R} - F/h)\tan\beta$
Shock Invariants & Relations
$\mathcal{S}, \mathcal{D} \to \lambda^\mp, \beta^\mp, p_0^+/p_0^-, \ldots$
directional derivatives along the shock wave
Centrifugal Equilibrium
$F_g \propto \kappa, \quad F_g \propto \frac{\partial p}{\partial \eta}$
expressing p with λ
$(\ln A\lambda)_\eta = \mathcal{P}$
$\mathcal{L}_\eta = \mathcal{P}/\mathcal{H} + k$
$\mathcal{D} = 0$
$\mathcal{L} \equiv \int \frac{\mathrm{d}\lambda}{\lambda\mathcal{H}} = \lambda \cdot {}_2F_1\left(\frac{1}{2}, -\frac{\mu^2-1}{2}; \frac{3}{2}; \frac{\lambda^2}{\mu^2}\right), \quad \mathcal{P} \equiv \frac{1}{\gamma M^2}\frac{\partial \ln p_0}{\partial \eta}$
$k \equiv \frac{B_\infty}{\mathcal{H}_\infty}\frac{\kappa}{\sigma c}$ (effective curvature)
Streamline Transformation
$\mathbf{U}(\xi, \eta + \mathrm{d}\eta) = \underset{\eta \to \eta + \mathrm{d}\eta}{\mathcal{T}_{[\xi]}} [\mathbf{U}(\xi, \eta)]$
Shock Relations & Gradients
1st-Ordered Curved Shock Theory
$\mathbf{M}_1 \begin{bmatrix} \kappa \\ h'/h \end{bmatrix} = \begin{bmatrix} b_\kappa \\ b_h \end{bmatrix}$
$\mathbf{M}_1 = \begin{bmatrix} \cos\beta & \sin\beta \\ \sin\beta & \varpi\cos\beta \end{bmatrix}$
directional derivatives along the shock wave
applying along the given shock wave i.e. inverse design
2nd-Ordered Curved Shock Theory
$\mathbf{M}_2 \begin{bmatrix} \kappa' \\ h''/h \end{bmatrix} = \begin{bmatrix} d_\kappa \\ d_h \end{bmatrix}$
$\mathbf{M}_2 = \begin{bmatrix} \cos^2\beta + \varpi^{-1}\sin^2\beta & 2\sin\beta\cos\beta \\ 2\sin\beta\cos\beta & \varpi\cos^2\beta + \sin^2\beta \end{bmatrix}$
Streamline Boundaries
Type: I-IV
Boundary Conditions & Solutions
streamline b.c.: numerical iteration
shock b.c.: applying $\mathcal{T}$ in sequence
Shock Boundaries
I: $\mathbf{r} = \mathbf{r}_0(\xi), \ \lambda = \lambda_0(\xi)$
II slip wall: $\mathbf{r} = \mathbf{r}_w(\xi)$
III far-field: $\lambda = \lambda_\infty$
IV pressure: $\lambda = \lambda_p(\xi)$
$\xi = \xi(s), \ \eta = \eta(s)$:
$\mathbf{r} = \mathbf{r}(s), \ \theta = \theta(s) \ \kappa = \kappa(s), \ A = A(s)$
$\lambda = \lambda(s), \ h = h(s), \ h' = h'(s), \ h'' = h''(s)$
Streamline Transformation Method
(STM)