A. Expressions of coefficients for the vertical velocity

The detailed expressions of $w_{k,1}$ and $w_{k,2}$ in equation (2.19) are shown below, which are in 1DH version for brevity. The superscripts, e.g. (1,0), denote the order of partial differentiation with respect to space (x) and time (t). Moreover, h' represents the spatial derivative of the sea bottom.

$$w_{k,1} = \frac{h'u_{k+1}}{c_k - c_{k+1}} - \frac{h'u_k}{c_k - c_{k+1}} + \frac{c_{k+1}Hu_k{}^{(1,0)}}{c_k - c_{k+1}} - \frac{c_kHu_{k+1}{}^{(1,0)}}{c_k - c_{k+1}} \tag{A.1}$$

$$w_{k,2} = \frac{H^{(1,0)}u_k}{2\left(c_k - c_{k+1}\right)} - \frac{H^{(1,0)}u_{k+1}}{2\left(c_k - c_{k+1}\right)} + \frac{Hu_{k+1}{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} - \frac{Hu_k{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} \tag{A.2}$$

B. Expressions of coefficients for the pressure field

The detailed expressions of $p_{k,1}$ to $p_{k,4}$ in equation (2.22) are shown below, which are in 1DH version for brevity.

$$\begin{aligned} p_{k,1} = & \frac{c_kh'u_{k+1}w_{k,1}}{\left(c_k - c_{k+1}\right)H} - \frac{c_{k+1}h'u_kw_{k,1}}{\left(c_k - c_{k+1}\right)H} + \frac{c_ku_{k+1}w_{k,0}{}^{(1,0)}}{c_k - c_{k+1}} - \frac{c_{k+1}u_kw_{k,0}{}^{(1,0)}}{c_k - c_{k+1}} \\ & + \frac{w_{k,0}w_{k,1}}{H} + w_{k,0}{}^{(0,1)} \end{aligned} \tag{B.1}$$

$$\begin{aligned} p_{k,2} = & \frac{h'u_kw_{k,1}}{2\left(c_k - c_{k+1}\right)H} - \frac{h'u_{k+1}w_{k,1}}{2\left(c_k - c_{k+1}\right)H} + \frac{c_kh'u_{k+1}w_{k,2}}{\left(c_k - c_{k+1}\right)H} - \frac{c_{k+1}h'u_kw_{k,2}}{\left(c_k - c_{k+1}\right)H} \\ & + \frac{c_{k+1}H^{(1,0)}u_kw_{k,1}}{2\left(c_k - c_{k+1}\right)H} - \frac{c_kH^{(1,0)}u_{k+1}w_{k,1}}{2\left(c_k - c_{k+1}\right)H} + \frac{u_kw_{k,0}{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} + \frac{c_ku_{k+1}w_{k,1}{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} \\ & - \frac{u_{k+1}w_{k,0}{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} - \frac{c_{k+1}u_kw_{k,1}{}^{(1,0)}}{2\left(c_k - c_{k+1}\right)} - \frac{H^{(0,1)}w_{k,1}}{2H} + \frac{w_{k,1}{}^2}{2H} + \frac{w_{k,0}w_{k,2}}{H} + \frac{1}{2}w_{k,1}{}^{(0,1)} \end{aligned} \tag{B.2}$$

$$\begin{aligned} p_{k,3} = & \frac{2h'u_kw_{k,2}}{3\left(c_k - c_{k+1}\right)H} - \frac{2h'u_{k+1}w_{k,2}}{3\left(c_k - c_{k+1}\right)H} + \frac{2c_{k+1}H^{(1,0)}u_kw_{k,2}}{3\left(c_k - c_{k+1}\right)H} - \frac{2c_kH^{(1,0)}u_{k+1}w_{k,2}}{3\left(c_k - c_{k+1}\right)H} \\ & + \frac{H^{(1,0)}u_{k+1}w_{k,1}}{3\left(c_k - c_{k+1}\right)H} - \frac{H^{(1,0)}u_kw_{k,1}}{3\left(c_k - c_{k+1}\right)H} + \frac{u_kw_{k,1}{}^{(1,0)}}{3\left(c_k - c_{k+1}\right)} + \frac{c_ku_{k+1}w_{k,2}{}^{(1,0)}}{3\left(c_k - c_{k+1}\right)} \\ & - \frac{u_{k+1}w_{k,1}{}^{(1,0)}}{3\left(c_k - c_{k+1}\right)} - \frac{c_{k+1}u_kw_{k,2}{}^{(1,0)}}{3\left(c_k - c_{k+1}\right)} - \frac{2H^{(0,1)}w_{k,2}}{3H} + \frac{w_{k,1}w_{k,2}}{H} + \frac{1}{3}w_{k,2}{}^{(0,1)} \end{aligned} \tag{B.3}$$

$$p_{k,4} = \frac{H^{(1,0)}u_{k+1}w_{k,2}}{2\left(c_k - c_{k+1}\right)H} - \frac{H^{(1,0)}u_kw_{k,2}}{2\left(c_k - c_{k+1}\right)H} + \frac{u_kw_{k,2}{}^{(1,0)}}{4\left(c_k - c_{k+1}\right)} - \frac{u_{k+1}w_{k,2}{}^{(1,0)}}{4\left(c_k - c_{k+1}\right)} + \frac{w_{k,2}{}^2}{2H} \tag{B.4}$$

C. Expressions of coefficients for the residual

The detailed expressions of $R_{k,0}$ to $R_{k,4}$ in equation (2.25) are shown below, which are shown in 1DH version for brevity.

$$R_{k,0} = -\frac{c_k h' u_k u_{k+1}}{(c_k - c_{k+1})(c_{k+1} - c_k) H} - \frac{c_k h' {u_{k+1}}^2}{(c_{k+1} - c_k)^2 H} - \frac{c_{k+1} h' {u_k}^2}{(c_k - c_{k+1})^2 H}$$
$$- \frac{c_{k+1} h' u_k u_{k+1}}{(c_k - c_{k+1})(c_{k+1} - c_k) H} + \frac{u_k w_{k,0}}{(c_k - c_{k+1}) H} + \frac{u_{k+1} w_{k,0}}{(c_{k+1} - c_k) H}$$
$$+ \frac{c_k^2 u_{k+1} {u_{k+1}}^{(1,0)}}{(c_{k+1} - c_k)^2} + \frac{c_{k+1} c_k u_{k+1} {u_k}^{(1,0)}}{(c_k - c_{k+1})(c_{k+1} - c_k)} + \frac{c_{k+1} c_k u_k {u_{k+1}}^{(1,0)}}{(c_k - c_{k+1})(c_{k+1} - c_k)}$$
$$- \frac{c_k {u_{k+1}}^{(0,1)}}{c_{k+1} - c_k} + \frac{c_{k+1}^2 u_k {u_k}^{(1,0)}}{(c_k - c_{k+1})^2} - \frac{c_{k+1} {u_k}^{(0,1)}}{c_k - c_{k+1}} + h' p_{k,1} + H^{(1,0)} p_{k,0} + H {p_{k,0}}^{(1,0)} \tag{C.1}$$

$$R_{k,1} = \frac{h' {u_k}^2}{(c_k - c_{k+1})^2 H} + \frac{2h' u_{k+1} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k) H} + \frac{h' {u_{k+1}}^2}{(c_{k+1} - c_k)^2 H}$$
$$+ \frac{c_{k+1} H^{(1,0)} {u_k}^2}{(c_k - c_{k+1})^2 H} + \frac{c_k H^{(1,0)} u_{k+1} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k) H} + \frac{c_{k+1} H^{(1,0)} u_{k+1} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k) H}$$
$$- \frac{H^{(0,1)} u_k}{(c_k - c_{k+1}) H} + \frac{c_k H^{(1,0)} {u_{k+1}}^2}{(c_{k+1} - c_k)^2 H} - \frac{H^{(0,1)} u_{k+1}}{(c_{k+1} - c_k) H} + \frac{u_k w_{k,1}}{(c_k - c_{k+1}) H}$$
$$+ \frac{u_{k+1} w_{k,1}}{(c_{k+1} - c_k) H} - \frac{2c_{k+1} {u_k}^{(1,0)} u_k}{(c_k - c_{k+1})^2} - \frac{c_k {u_{k+1}}^{(1,0)} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k)}$$
$$- \frac{c_{k+1} {u_{k+1}}^{(1,0)} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k)} + \frac{{u_k}^{(0,1)}}{c_k - c_{k+1}} + \frac{{u_{k+1}}^{(0,1)}}{c_{k+1} - c_k} - \frac{c_k u_{k+1} {u_k}^{(1,0)}}{(c_k - c_{k+1})(c_{k+1} - c_k)}$$
$$- \frac{c_{k+1} u_{k+1} {u_k}^{(1,0)}}{(c_k - c_{k+1})(c_{k+1} - c_k)} - \frac{2c_k u_{k+1} {u_{k+1}}^{(1,0)}}{(c_{k+1} - c_k)^2} + 2h' p_{k,2} + H {p_{k,1}}^{(1,0)} \tag{C.2}$$

$$R_{k,2} = -\frac{H^{(1,0)} {u_k}^2}{(c_k - c_{k+1})^2 H} - \frac{2H^{(1,0)} u_{k+1} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k) H} - \frac{H^{(1,0)} {u_{k+1}}^2}{(c_{k+1} - c_k)^2 H}$$
$$+ \frac{u_k w_{k,2}}{(c_k - c_{k+1}) H} + \frac{u_{k+1} w_{k,2}}{(c_{k+1} - c_k) H} + \frac{{u_k}^{(1,0)} u_k}{(c_k - c_{k+1})^2} + \frac{{u_{k+1}}^{(1,0)} u_k}{(c_k - c_{k+1})(c_{k+1} - c_k)}$$
$$+ \frac{u_{k+1} {u_k}^{(1,0)}}{(c_k - c_{k+1})(c_{k+1} - c_k)} + \frac{u_{k+1} {u_{k+1}}^{(1,0)}}{(c_{k+1} - c_k)^2} + 3h' p_{k,3} - H^{(1,0)} p_{k,2} + H {p_{k,2}}^{(1,0)} \tag{C.3}$$

$$R_{k,3} = 4h' p_{k,4} - 2H^{(1,0)} p_{k,3} + H {p_{k,3}}^{(1,0)} \tag{C.4}$$

$$R_{k,4} = H {p_{k,4}}^{(1,0)} - 3H^{(1,0)} p_{k,4} \tag{C.5}$$