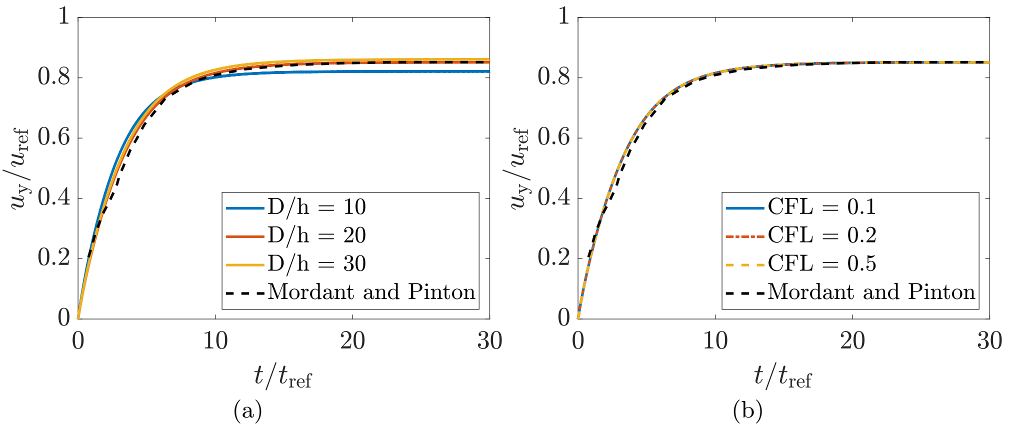

FIGURE 1. Validation results for the settling velocity of a single sphere, simulated for (a) CFL $= u\Delta t/h = 0.1$ and varying spatial step sizes h, and (b) spatial discretization $D/h = 20$ and varying CFL numbers. The simulation results are compared to the experimental data of Mordant & Pinton (2000) for $Ga = 49.26$ and $\rho' = 2.56$.

Supplement 1: single settling sphere validation

To confirm the accuracy of our numerical method, and to estimate the spatial grid size required for convergence, we include here the case of a single particle settling in fluid at rest. We consider an experimental case from the literature, of a spherical particle settling in water (Mordant & Pinton 2000). We choose to compare our numerical model against their experiment 1, with a particle diameter $D = 5 \times 10^{-4}$ m, kinematic viscosity $\nu = 0.89 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$, particle density $\rho_p = 2560\,\mathrm{kg\,m^{-3}}$, and fluid density of water at 25°C, $\rho_f = 997.05\,\mathrm{kg\,m^{-3}}$. In our notation this corresponds to a sphere settling at $Ga = 49.26$, which is on the same order as the values of the Galileo number considered in the present experiments. We vary both the grid resolution and the time step, to assess their influence on the settling velocity.

Figure 1(a) shows that we find a good quantitative agreement between the numerical values and the experimental values obtained for the particle velocity. We also find that increasing the resolution from $D/h = 10$ to $D/h = 20$ improves the accuracy of the velocity obtained in the numerical model. However, there does not appear to be a significant increase in accuracy if the grid is refined beyond $D/h = 20$.

To determine the fluid time step size Δt, as a stability condition, we define the Courant–Friedrichs–Lewy (CFL) number

$$\mathrm{CFL} = \frac{u\,\Delta t}{h}\,, \tag{0.1}$$

where u indicates the maximum vertical velocity value within the system at any given time step. Figure 1(b) demonstrates that CFL values below 0.5 give stable and accurate results for the single sphere case.

In figure 2 we compare the results of simulations performed with $D/h = 20$ for a single settling particle to our own experimental results. We also compare the results to the predicted terminal velocity u_{term}/u_{ref}, where u_{term} is the dimensional terminal velocity obtained via balancing the buoyancy and drag forces

$$u_{term} = \sqrt{\frac{4g\,(\rho' - 1)\,D}{3C_D}}\,. \tag{0.2}$$

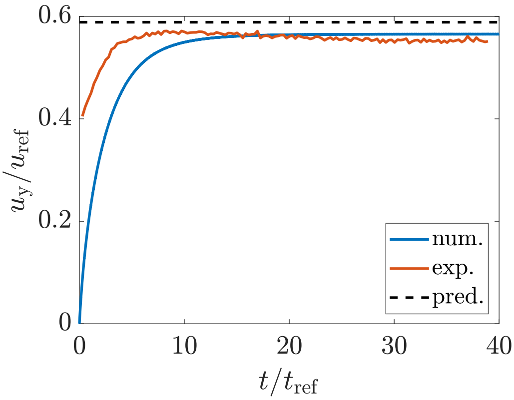

FIGURE 2. Comparison of experimental and simulation data for the time-dependent settling velocity of a single sphere, for $D/h = 20$. The predicted velocity value shown by the horizontal dashed line is obtained from equation (0.2), for a particle of size $D = 11.1\,\mathrm{mm}$ and density $\rho_{\mathrm{p}} = 1135\,\mathrm{kg\,m^{-3}}$ in a fluid of dynamic viscosity $\mu_{\mathrm{f}} = 0.0942\,\mathrm{Pa\,s}$ and density $\rho_{\mathrm{f}} = 864\,\mathrm{kg\,m^{-3}}$, with $Ga = 18.81$.

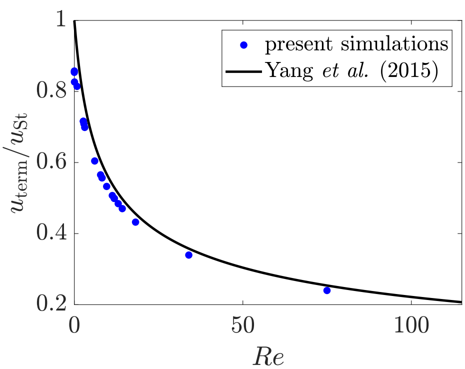

FIGURE 3. Terminal settling velocity of a single sphere obtained numerically compared to the corresponding data from Yang *et al.* (2015) (equation (42) in their paper), obtained with with $D/h = 20$ and CFL $= 0.1$. The terminal settling velocity is normalized by the Stokes settling velocity u_{St}. A good quantitative agreement is observed across the entire range of Re.

Here C_D is the drag coefficient as defined in equation (2.2) in the main text. The simulation results match the experimental data well with regard to the terminal settling velocity. The observed discrepancy during the initial, transient phase reflects the difficulties experimentally in starting the aggregate fully at rest.

Due to the potential for stability issues due to the discretization of the viscous term in the governing equations for low Reynolds (and Galileo) numbers, we additionally consider the behavior of the simulations at $Re < 1$. Figure 3 presents a comparison of our simulation results for the terminal settling velocity with corresponding results by Yang *et al.* (2015) (equations (33)-(42) in their paper), across the range $Re \in [0.01, 75]$ of interest here. In the figure the settling velocity is normalized by the Stokes settling velocity

$$u_{\mathrm{St}} = \frac{2}{9}\frac{(\rho_{\mathrm{p}} - \rho_{\mathrm{f}})}{\mu} g \left(\frac{D}{2}\right)^2 . \tag{0.3}$$

Figure 3 demonstrates that good agreement is observed across the entire range of

Reynolds numbers, showing the accuracy of the numerical method used here for a single settling sphere.

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Yang, H., Fan, M., Liu, A. & Dong, L. 2015 General formulas for drag coefficient and settling velocity of sphere based on theoretical law. *International Journal of Mining Science and Technology* **25** (2), 219–223.