

Supplementary material: Potential to recover a record of Holocene climate and sea ice from Müller Ice Cap, Canada

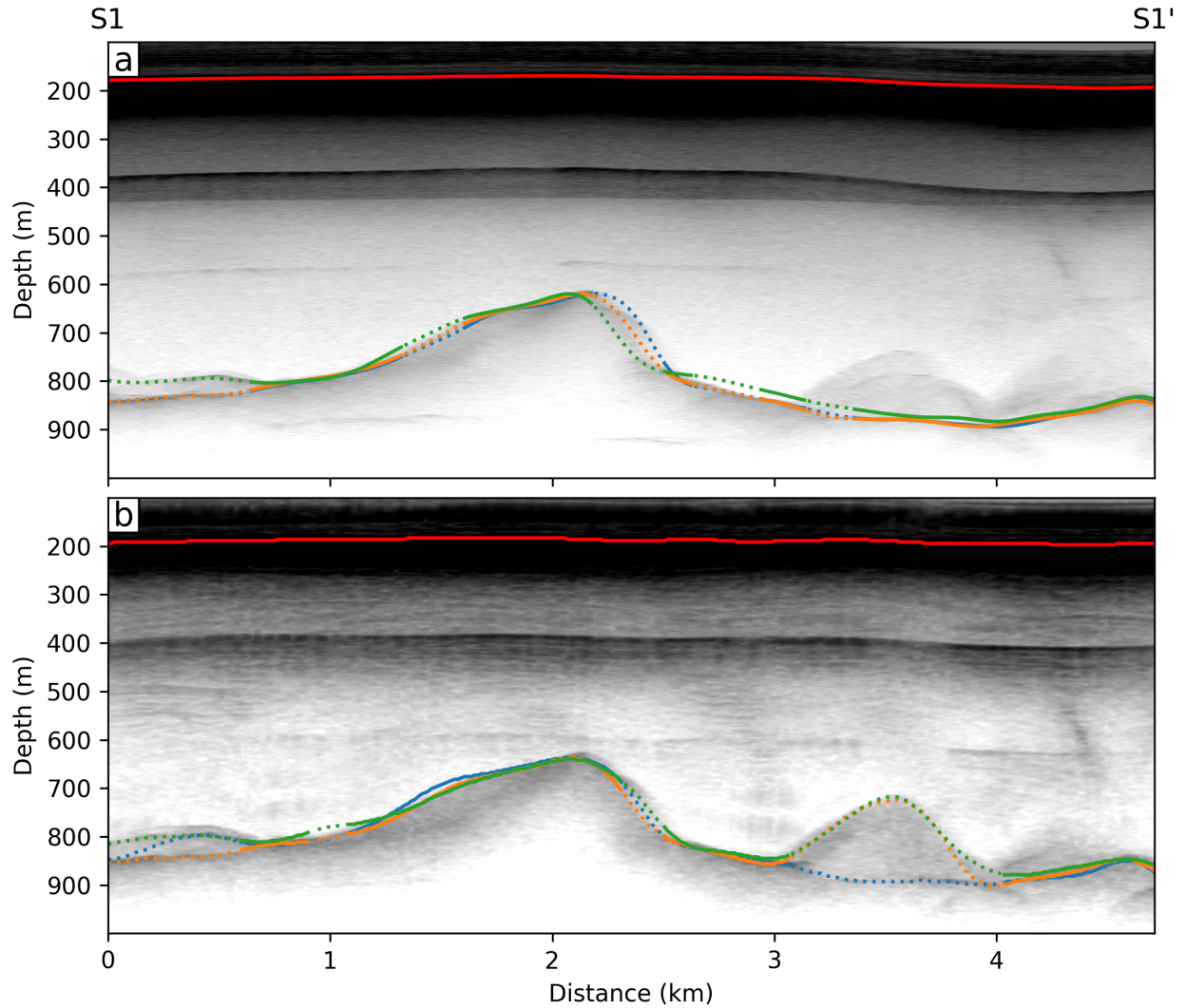
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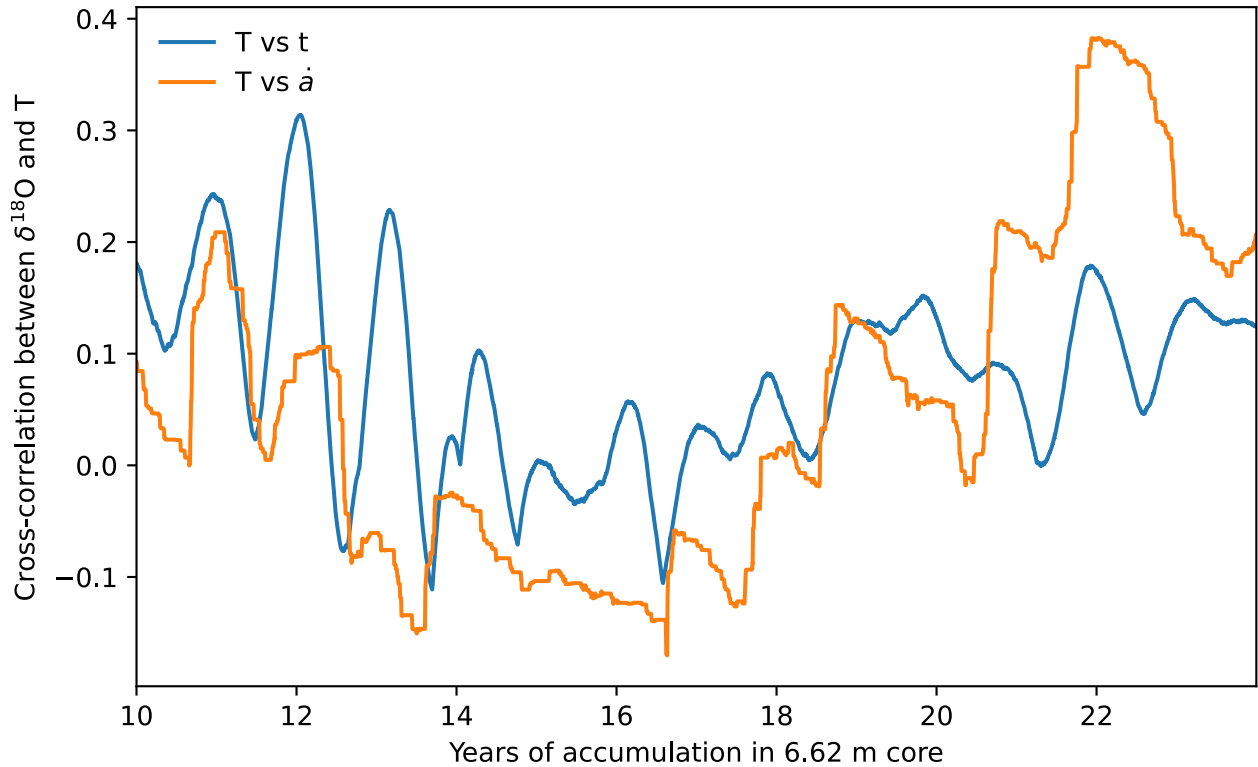
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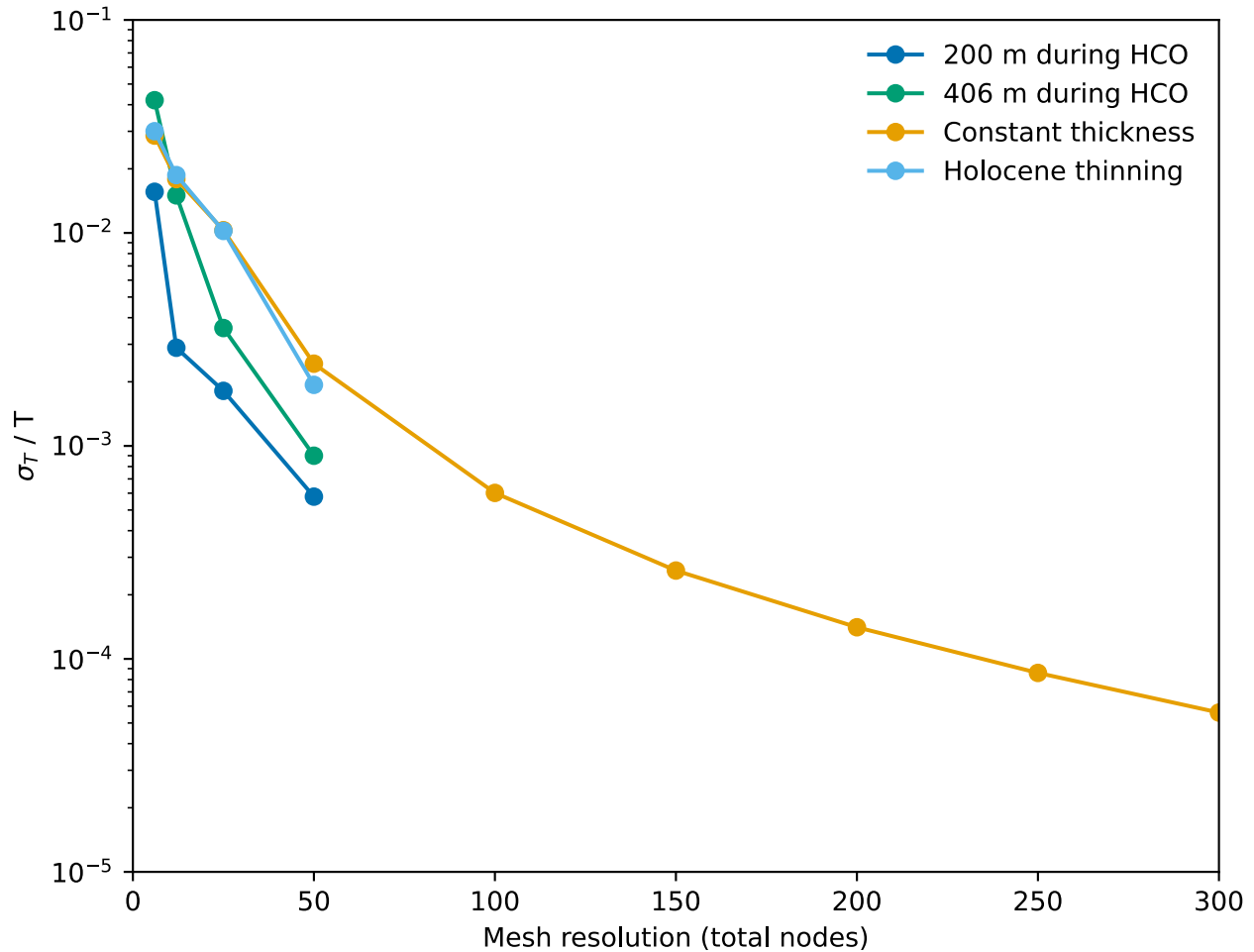
- Supplementary figures 1–4



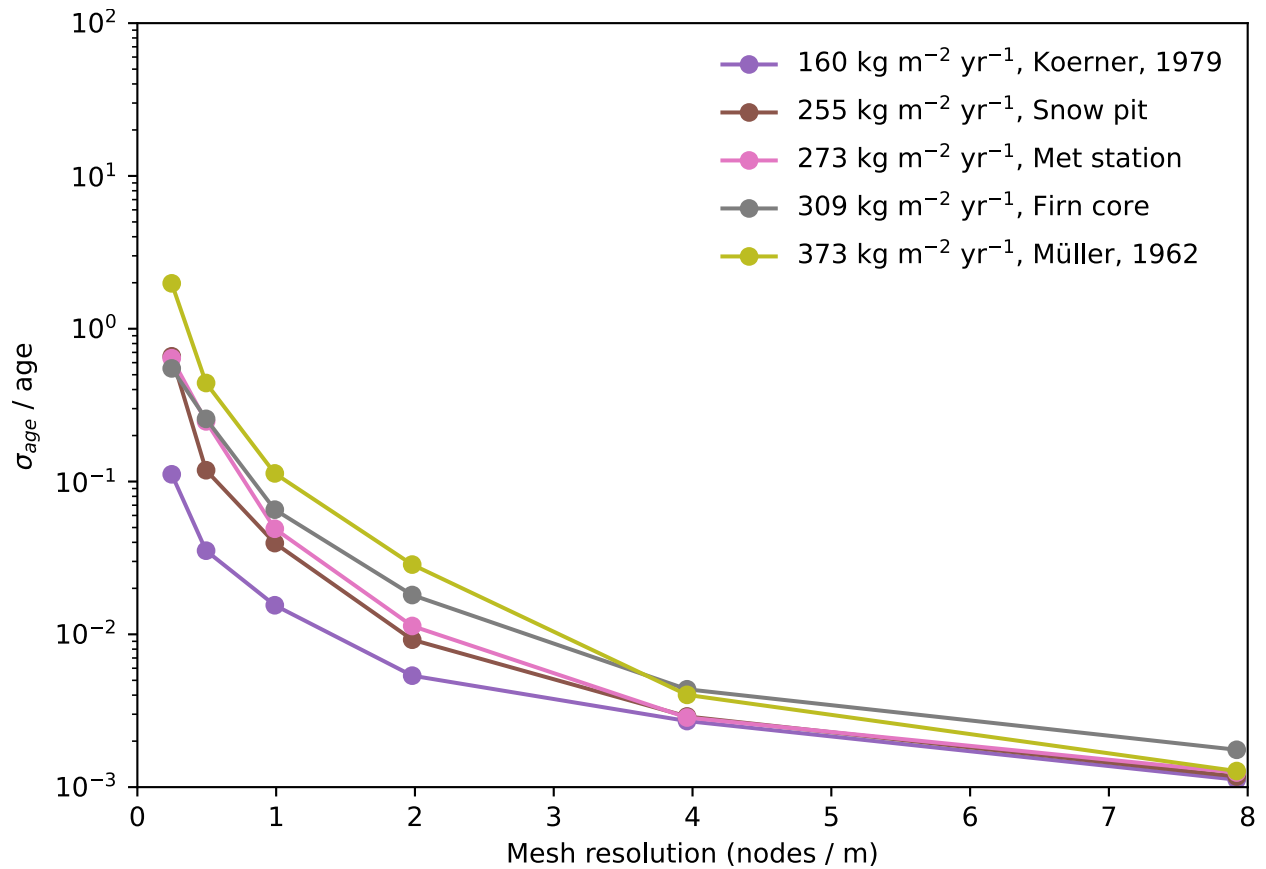
Supplementary Figure 1. Example radargrams from airborne radar survey flown by AWI's Polar 5 aircraft, from profile S1–S1' in Fig. 5 of the main text. **a.** Ultra-wide-band echogram (150–520 MHz, flown May 8th, 2023). Red line shows surface. Blue, green, and orange lines show the bed picks of three independent tracers, with dashed lines indicating areas of disagreement. **b.** As in a, but for narrow-band data on approximately the same profile (180–210 MHz, flown May 9th, 2023).



Supplementary Figure 2. Cross-correlation between $\delta^{18}\text{O}$ and temperature at Eureka. The blue line shows the cross-correlation with the temperature at Eureka through time (i.e. with a linear time axis for the Eureka temperature). Orange shows the cross-correlation with the temperature at Eureka as a function of accumulation (i.e. assuming that the temperature is recorded with snowfall). Note the loosely bimodal distribution (with peaks around 12 and 22 years) depending on assumptions.



Supplementary Figure 3. Relative error in modeled temperature (calculated as the relative 1- σ relative deviation compared to the finest-resolution simulation) vs mesh resolution. Different colored curves show different assumed thickness histories as in Figs. 8–10 of the main text. Each curve assumed $255 \text{ kg m}^{-2} \text{ yr}^{-1}$ of modern accumulation. Due to computational expense, the highest resolution simulations used to estimate the true values had 351 nodes for the constant thickness simulation and 101 nodes for all others. These curves indicate that all temperature simulations used in the main text have numerical error of less than 0.1%.



Supplementary Figure 4. Relative error in modeled age (calculated as the relative 1- σ relative deviation compared to the finest-resolution simulation) vs mesh resolution. All simulations assumed constant ice thickness throughout the run. Different colored curves show different assumed accumulation histories, matching colors in Figure 8b of the main text. These curves indicate that all age simulations used in the main text have numerical error of less than 0.5%.