Generalized Processing Tree Models: Supplementary Material

1 Further Simulation Details and Results

Figure 1 shows the probability density and cumulative density functions of the shifted-Wald distributions used for generating data in the second simulation. Whereas the shift and shape parameters were fixed at $\tau = 500$ and $\lambda = 1,500$, respectively, the mean parameter differed for the three components ($\mu_{do} = 300$, $\mu_{dn} = 400$, and $\mu_g = 500$). Note that the mean parameter of the shifted-Wald distribution also affects the standard deviation, thereby inducing a linear relation between mean and standard deviation, a property often found in empirical response-time data (Wagenmakers & Brown, 2007).

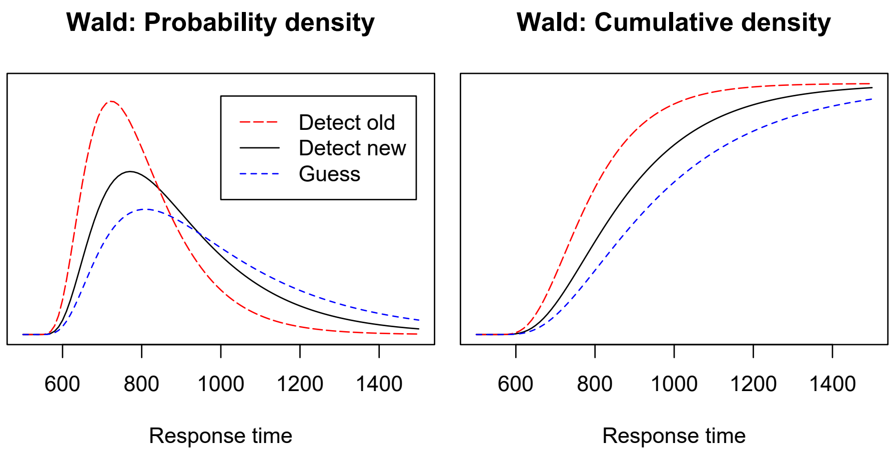

Figure 1: Three shifted-Wald distributions with different mean parameters were used as data-generating component distributions.

Besides assessing the statistical power and robustness of GPT models, the simulation also served as a validation for the absolute goodness-of-fit test based on the Dzhaparidze-Nikulin statistic Z^2 (Dzhaparidze & Nikulin, 1974; Voinov, Nikulin, & Balakrishnan, 2013). As boundaries for categorization, we computed the model-implied $1/B$-quantiles conditional for each discrete category, where B is the number of bins per category.

Figure 2 shows QQ-plots for the simulated p-values based on 600 responses under the assumption that the component distributions differ (i.e., for different mean parameters of the shifted-Wald). All models assumed the correct structure of conditional probabilities and latent components, but differed in the parametric assumptions about the distributional family (e.g., the shifted-Wald, ex-Gaussian, shifted-lognormal, shifted-gamma, or Gaussian). Note that the other three right-skewed distributions all have a parameter that both affects the mean and the standard deviation, similar as the Wald distribution (Wagenmakers & Brown, 2007).

The results show that the simulated p-values closely matched the uniform distribution when the data-generating shifted-Wald distribution was fitted. However, the test had low statistical power (reflected by the deviation between the actual QQ-plot and the diagonal) to detect GPT versions with other right-skewed component distributions, but very high power when symmetric Gaussian distributions were fitted (in which case components were assumed to have different means but equal variances).

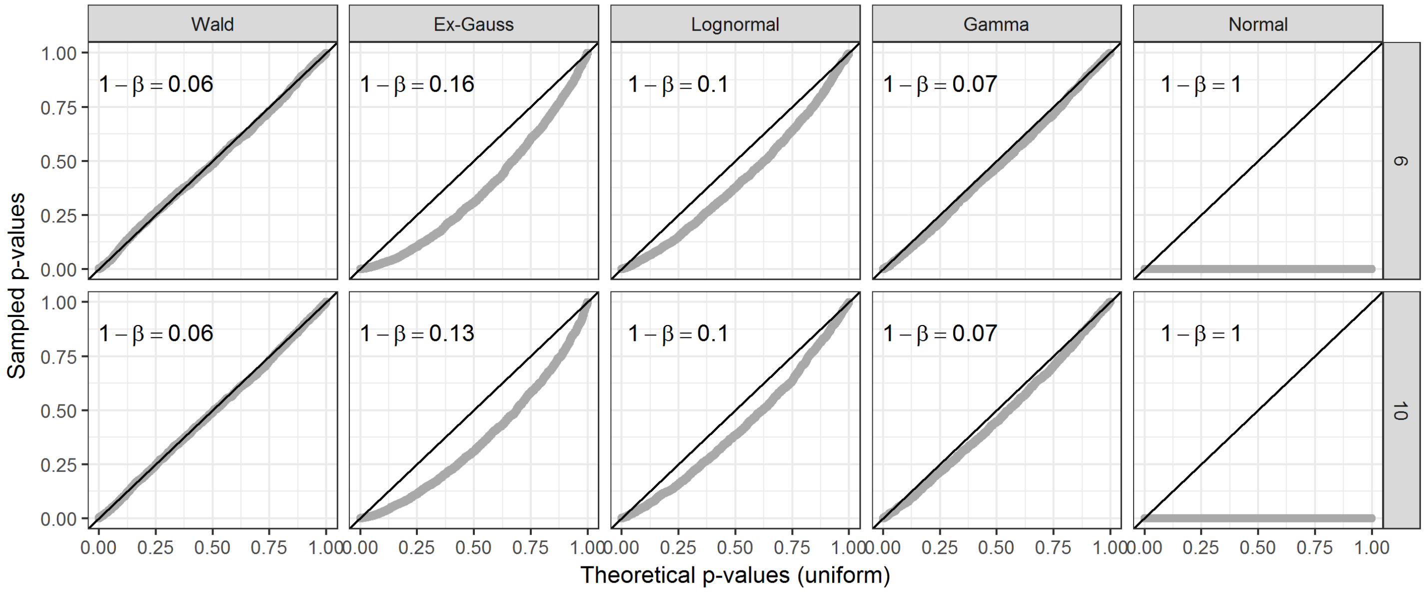

Figure 2: QQ-plots of the simulated vs. theoretical uniform distribution of p-values of the Dzhaparidze-Nikulin statistic when using 6 or 10 bins for categorization (first and second row, respectively) for five fitted distributions (columns). The simulation was based on 2,000 replications with 600 responses generated by a 2HTM version with the shifted-Wald distribution.

References

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Voinov, V., Nikulin, M. S., & Balakrishnan, N. (201325). *Chi-squared goodness of fit tests with applications*. Waltham, MA: Academic Press.

Wagenmakers, E.-J. & Brown, S. (2007). On the linear relation between the mean and the standard deviation of a response time distribution. *Psychological Review*, *114*, 830–841. doi:10.1037/0033-295x.114.3.830