Appendices for the paper: 'Impact of correlation between interest rates and mortality rates on the valuation of various life insurance products'

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Appendix A: Proof of Proposition I

In order to compute

$$P_{r,\mu}(t,s) = 1_{\{\tau(x)>t\}}\mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-\int_t^s r_u\,du\right)\exp\left(-\int_t^s \mu_{x+u}\,du\right)\middle|\,\mathcal{G}_t\right], \tag{1}$$

we will proceed in three steps.

Step 1

Applying Ito's lemma to the function $r_t e^{\lambda t}$ provides us the following unique explicit solution of (3.2a)

$$r_s = r_t e^{-\lambda(s-t)} + \lambda e^{-\lambda s}\int_t^s \overline{r}_u e^{\lambda u}du + \eta e^{-\lambda s}\int_t^s e^{\lambda u}d\widetilde{W}_u^r. \tag{2}$$

By integrating (2.17) between time t and time s, we can deduce

$$\int_t^s r_u\,du = \frac{r_t}{\lambda} - \frac{r_s}{\lambda} + \int_t^s \overline{r}_u\,du + \frac{\eta}{\lambda}\int_t^s d\widetilde{W}_u^r. \tag{3}$$

If we insert (2) in (3), we obtain

$$\int_t^s r_u\,du = r_t\left(\frac{1-e^{-\lambda(s-t)}}{\lambda}\right) + \int_t^s \overline{r}_u\left(\frac{1-e^{-\lambda(s-u)}}{\lambda}\right)du + \eta\int_t^s\left(\frac{1-e^{-\lambda(s-u)}}{\lambda}\right)d\widetilde{W}_u^r, \tag{4}$$

which in virtue of (2.24) can be rewritten

$$\int_t^s r_u\,du = r_t B(\lambda,t,s) + \int_t^s \overline{r}_u\,B(\lambda,u,s)\,du + \eta\int_t^s B(\lambda,u,s)d\widetilde{W}_u^r. \tag{5}$$

Step 2

Applying Ito's lemma to the function $\mu_t^x e^{\omega t}$ provides us the following unique explicit solution of (3.2b)

$$\mu_{x+s} = \mu_{x+t}e^{-\omega(s-t)} + \omega e^{-\omega s}\int_t^s \overline{\mu}_{x+u}e^{\omega u}du + \varepsilon\rho^{r,\mu}e^{-\omega s}\int_t^s e^{\omega u}d\widetilde{W}_u^r + \varepsilon\sqrt{1-(\rho^{r,\mu})^2}e^{-\omega s}\int_t^s e^{\omega u}d\widetilde{W}_u^\mu. \tag{6}$$

By integrating (6) between time t and time s, we can deduce

$$\int_t^s \mu_{x+u}\,du = \frac{\mu_{x+t}}{\omega} - \frac{\mu_{x+s}}{\omega} + \int_t^s \overline{\mu}_{x+u}\,du + \frac{\varepsilon\rho^{r,\mu}}{\omega}\int_t^s d\widetilde{W}_u^r + \frac{\varepsilon\sqrt{1-(\rho^{r,\mu})^2}}{\omega}\int_t^s d\widetilde{W}_u^\mu. \tag{7}$$

If we insert (6) in (7), we obtain

$$\int_t^s \mu_{x+u}\,du = \mu_{x+t}\left(\frac{1-e^{-\omega(s-t)}}{\omega}\right) + \int_t^s \overline{\mu}_{x+u}\left(\frac{1-e^{-\omega(s-u)}}{\omega}\right)du + \varepsilon\rho^{r,\mu}\int_t^s\left(\frac{1-e^{-\omega(s-u)}}{\omega}\right)d\widetilde{W}_u^r$$
$$+\,\varepsilon\sqrt{1-(\rho^{r,\mu})^2}\int_t^s\left(\frac{1-e^{-\omega(s-u)}}{\omega}\right)d\widetilde{W}_u^\mu, \tag{8}$$

which in virtue of (2.24) can be rewritten

$$\int_t^s \mu_{x+u}\,du = \mu_{x+t}B(\omega,t,s) + \int_t^s \overline{\mu}_{x+u}B(\omega,u,s)du + \varepsilon\rho^{r,\mu}\int_t^s B(\omega,u,s)d\widetilde{W}_u^r$$
$$+\,\varepsilon\sqrt{1-(\rho^{r,\mu})^2}\int_t^s B(\omega,u,s)d\widetilde{W}_u^\mu. \tag{9}$$

Step 3

Let us define

$$I(t,s) = \int_t^s \left(r_u + \mu_{x+u}\right)\,du. \tag{10}$$

Inserting (5) and (9) in (10) provides

$$I(t,s) = r_tB(\lambda,t,s) + \int_t^s \overline{r}_uB(\lambda,u,s)\,du + \mu_{x+t}B(\omega,t,s) + \int_t^s \overline{\mu}_{x+u}B(\omega,u,s)\,du$$
$$+\,\eta\int_t^s B(\lambda,u,s)\,d\widetilde{W}_u^r + \varepsilon\rho^{r,\mu}\int_t^s B(\omega,u,s)\,d\widetilde{W}_u^r + \varepsilon\sqrt{1-(\rho^{r,\mu})^2}\int_t^s B(\omega,u,s)\,d\widetilde{W}_u^\mu. \tag{11}$$

We can deduce that the distribution of $I(t,s)$ conditionally to $\mathcal{F}_t$ is normal with the following conditional moments:

- Conditional expectation of I(t,s):

$$\mathbb{E}_{\mathbb{Q}}[I(t,s)|\mathcal{F}_t] = r_tB(\lambda,t,s) + \int_t^s \overline{r}_uB(\lambda,u,s)\,du + \mu_{x+t}B(\omega,t,s) + \int_t^s \overline{\mu}_{x+u}B(\omega,u,s)\,du. \tag{12}$$

- Conditional variance of I(t,s):

$$\mathbb{V}\mathrm{ar}_{\mathbb{Q}}[I(t,s)|\mathcal{F}_t] = \sigma_r^2 + \sigma_m^2 + \rho_{rm} \tag{13}$$

  where

$$\begin{cases} \sigma_r^2 = \dfrac{\eta^2}{\lambda^2}\left[(s-t) - \dfrac{\lambda}{2}B^2(\lambda,t,s) - B(\lambda,t,s)\right] \\ \sigma_m^2 = \dfrac{\varepsilon^2}{\omega^2}\left[(s-t) - \dfrac{\omega}{2}B^2(\omega,t,s) - B(\omega,t,s)\right] \\ \rho_{rm} = \dfrac{2\eta\varepsilon\rho^{r,\mu}}{\lambda\omega}\left[(s-t) - B(\lambda,t,s) - B(\omega,t,s) + B(\lambda+\omega,t,s)\right] \end{cases}. \tag{14}$$

Given (10), (1) takes the form

$$P_{r,\mu}(t,s) = 1_{\{\tau(x)>t\}}\mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-I(t,s)\right)\middle|\,\mathcal{G}_t\right]. \tag{15}$$

Using the formula of the expected value of a lognormal distribution, we can conclude that the price of the zero-coupon survival bond is given by Proposition I.

Appendix B: Analysis on the direction of the impact of the price of correlation

The price of correlation takes the form

$$P_{\rho^{r,\mu}}(t,s) = \exp\left(\frac{\eta\varepsilon\rho^{r,\mu}}{\lambda\omega}\left[T + \left(\frac{1-e^{-(\lambda+\omega)T}}{\lambda+\omega}\right) - \left(\frac{1-e^{-\lambda T}}{\lambda}\right) - \left(\frac{1-e^{-\omega T}}{\omega}\right)\right]\right). \tag{16}$$

As η , ε , λ and ω are strictly positive constants, the direction of impact of the price of correlation, namely upwards when $P_{\rho^{r,\mu}}(t,s) > 1$ and downwards when $P_{\rho^{r,\mu}}(t,s) < 1$, is directly determined by the sign of the correlation coefficient $\rho^{r,\mu}$ and the sign of the function

$$f(T) = T + \left(\frac{1-e^{-(\lambda+\omega)T}}{\lambda+\omega}\right) - \left(\frac{1-e^{-\lambda T}}{\lambda}\right) - \left(\frac{1-e^{-\omega T}}{\omega}\right). \tag{17}$$

The latter is always positive since f(0)=0 and function f is increasing. Indeed,

$$f'(T) = 1 + e^{-(\lambda+\omega)T} - e^{-\lambda T} - e^{-\omega T} \geq 0 \;\; \forall\, (\lambda,\omega,T) \in \mathbb{R}_+^3, \tag{18}$$

given that

$$1 - e^{-\lambda T} \geq e^{-\omega T} - e^{-(\lambda+\omega)T} \;\; \forall\, (\lambda,\omega,T) \in \mathbb{R}_+^3 \tag{19}$$

since the negative exponential is a convex function. Besides, it is interesting to note that function f is convex since

$$\begin{aligned} f''(T) =& -(\lambda+\omega)e^{-(\lambda+\omega)T} + \lambda e^{-\lambda T} + \omega e^{-\omega T} \\ \Leftrightarrow f''(T) =& \lambda \underbrace{\left(e^{-\lambda T} - e^{-(\lambda+\omega)T}\right)}_{\geq 0} + \omega \underbrace{\left(e^{-\omega T} - e^{-(\lambda+\omega)T}\right)}_{\geq 0} \geq 0 \;\; \forall\, (\lambda,\omega,T) \in \mathbb{R}_+^3. \end{aligned} \tag{20}$$

Hence, as $f(T) \geq 0$, the direction of the impact of the price of of correlation is entirely driven by the sign of the correlation coefficient $\rho^{r,\mu}$.

Appendix C: Proof of Proposition II

The aim is to compute the following quantity

$$D_{r,\mu}(t,u) = 1_{\{\tau(x)>t\}}\mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-\int_t^u r_v\,dv\right)\cdot\exp\left(-\int_t^u \mu_{x+v}\,dv\right)\mu_{x+u}|\mathcal{G}_t\right]. \tag{21}$$

Let us switch from the measure $\tilde{\mathbb{Q}}$ to a measure $\tilde{\mathbb{Q}}^{u,\mu} \sim \tilde{\mathbb{Q}}$ defined by the following Radon–Nikodym derivative of $\tilde{\mathbb{Q}}^{u,\mu}$ with respect to $\tilde{\mathbb{Q}}$:

$$\frac{d\tilde{\mathbb{Q}}^{u,\mu}}{d\tilde{\mathbb{Q}}} = \frac{\overbrace{\exp\left(-\int_0^u (r_v + \mu_{x+v})\,dv\right)}^{N(u)}}{\underbrace{\mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-\int_0^u (r_v + \mu_{x+v})\,dv\right)\right]}_{M(u)}}. \tag{22}$$

Let us compute $N(u)$ and $M(u)$ using (10) and (11).

- Calculation of $N(u)$:

$$\begin{aligned} N(u) =& \exp\left(-\int_0^u (r_v + \mu_{x+v})\,dv\right) \\ \Leftrightarrow N(u) =& \exp\left(-I(0,u)\right) \\ \Leftrightarrow N(u) =& \exp\Bigg(-r_0 B(\lambda,0,u) - \int_0^u \overline{r}_v B(\lambda,v,u)\,dv - \mu_x B(\omega,0,u) - \int_0^u \overline{\mu}_{x+v} B(\omega,v,u)\,dv \\ & - \int_0^u \left(\eta B(\lambda,v,u) + \varepsilon\rho^{r,\mu} B(\omega,v,u)\right)\,d\widetilde{W}_v^r - \varepsilon\sqrt{1-(\rho^{r,\mu})^2}\int_0^u B(\omega,v,u)\,d\widetilde{W}_v^\mu\Bigg). \end{aligned} \tag{23}$$

- Calculation of $M(u)$:

$$
\begin{aligned}
& M(u) = \mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-\int_0^u (r_v + \mu_{x+v})\, dv\right)\right] \\
\Leftrightarrow\ & M(u) = \mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-I(0,u)\right)\right] \\
\Leftrightarrow\ & M(u) = \exp\left(\mathbb{E}_{\tilde{\mathbb{Q}}}\left[-I(0,u)\right] + \frac{\mathbb{V}ar_{\tilde{\mathbb{Q}}}\left[I(0,u)\right]}{2}\right) \\
\Leftrightarrow\ & M(u) = \exp\left(-r_0 B(\lambda,0,u) - \int_0^u \overline{r}_v B(\lambda,v,u)\, dv - \mu_x B(\omega,0,u) - \int_0^u \overline{\mu}_{x+v} B(\omega,v,u)\, dv \right. \\
& \qquad \left. + \frac{1}{2}\int_0^u \left(\eta B(\lambda,v,u) + \varepsilon\rho^{r,\mu} B(\omega,v,u)\right)^2 dv + \frac{(\varepsilon\sqrt{1-(\rho^{r,\mu})^2})^2}{2}\int_0^u B^2(\omega,v,u)\, dv\right).
\end{aligned}
\tag{24}
$$

Inserting (23) and (24) in (22) provides

$$
\begin{aligned}
& \frac{d\tilde{\mathbb{Q}}^{u,\mu}}{d\tilde{\mathbb{Q}}} = \frac{N(u)}{M(u)} \\
\Leftrightarrow & \frac{d\tilde{\mathbb{Q}}^{u,\mu}}{d\tilde{\mathbb{Q}}} = \exp\left(-\int_0^u \left[\eta B(\lambda,v,u) + \varepsilon\rho^{r,\mu}B(\omega,v,u)\right] d\widetilde{W}_v^r - \int_0^u \left[\varepsilon\sqrt{1-(\rho^{r,\mu})^2}B(\omega,v,u)\right] d\widetilde{W}_v^\mu \right. \\
& \qquad \left. -\frac{1}{2}\int_0^u \left[\eta B(\lambda,v,u) + \varepsilon\rho^{r,\mu}B(\omega,v,u)\right]^2 dv - \frac{1}{2}\int_0^u \left[\varepsilon\sqrt{1-(\rho^{r,\mu})^2}B(\omega,v,u)\right]^2 dv\right).
\end{aligned}
\tag{25}
$$

The change of measure defined by the Radon–Nikodym derivative (25) allows to rewrite (21) as

$$
D_{r,\mu}(t,u) = 1_{\{\tau(x)>t\}} P_{r,\mu}(t,u)\mathbb{E}_{\tilde{\mathbb{Q}}^{u,\mu}}\left[\mu_{x+u}|\mathcal{F}_t\right]. \tag{26}
$$

The form of (25) implies, according to the multidimensional Girsanov theorem, that the Brownian motions $\widehat{W}_t^r$ and $\widehat{W}_t^\mu$, defined as

$$
\left\{
\begin{aligned}
\widehat{W}_t^r &= \widetilde{W}_t^r + \textstyle\int_0^t \left[\eta B(\lambda,v,u) + \varepsilon\rho^{r,\mu}B(\omega,v,u)\right] dv \\
\widehat{W}_t^\mu &= \widetilde{W}_t^\mu + \varepsilon\sqrt{1-(\rho^{r,\mu})^2}\textstyle\int_0^t B(\omega,v,u)\, dv
\end{aligned}
\right. , \tag{27}
$$

are standard Brownian motions under $\tilde{\mathbb{Q}}^{u,\mu}$.

Inserting (27) in (3.3b) provides the following expression of μ_{x+u} under measure $\tilde{\mathbb{Q}}^{u,\mu}$:

$$
\begin{aligned}
\mu_{x+u} =& \mu_{x+t}e^{-\omega(u-t)} + \omega e^{-\omega u}\int_t^u \overline{\mu}_{x+v}e^{\omega v}dv + \varepsilon\rho^{r,\mu}e^{-\omega u}\int_t^u e^{\omega v}d\widehat{W}_v^r \overbrace{- \eta\varepsilon\rho^{r,\mu}e^{-\omega u}\int_t^u e^{\omega v}B(\lambda,v,u)\, dv}^{*} \\
& + \varepsilon\sqrt{1-(\rho^{r,\mu})^2}e^{-\omega u}\int_t^u e^{\omega v}d\widehat{W}_v^\mu \underbrace{- \varepsilon^2 e^{-\omega u}\int_t^u e^{\omega v}B(\omega,v,u)\, dv}_{**}.
\end{aligned}
\tag{28}
$$

After some calculations, we obtain:

$$
* = \frac{\eta\varepsilon\rho^{r,\mu}}{\lambda}B(\omega,t,u) - \frac{\eta\varepsilon\rho^{r,\mu}}{\lambda}B(\lambda+\omega,t,u) \tag{29}
$$

and

$$
** = \frac{\varepsilon^2}{2}B^2(\omega,t,u). \tag{30}
$$

Inserting (29) and (30) into (28) gives

$$\mu_{x+u} = \mu_{x+t} e^{-\omega(u-t)} + \omega e^{-\omega u} \int_t^u \overline{\mu}_{x+v} e^{\omega v} dv + \varepsilon \rho^{r,\mu} e^{-\omega u} \int_t^u e^{\omega v} d\widehat{W}_v^r - \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\omega, t, u)$$
$$+ \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\lambda + \omega, t, u) + \varepsilon \sqrt{1 - (\rho^{r,\mu})^2} e^{-\omega u} \int_t^u e^{\omega v} d\widehat{W}_v^\mu - \frac{\varepsilon^2}{2} B^2(\omega, t, u), \quad (31)$$

which implies

$$\mathbb{E}_{\tilde{\mathbb{Q}}^{u,\mu}} [\mu_{x+u} | \mathcal{F}_t] = \mu_{x+t} e^{-\omega(u-t)} + \omega e^{-\omega u} \int_t^u \overline{\mu}_{x+v} e^{\omega v} dv - \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\omega, t, u) + \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\lambda + \omega, t, u)$$
$$- \frac{\varepsilon^2}{2} B^2(\omega, t, u). \quad (32)$$

Inserting the expression (32) into (26) yields

$$D_{r,\mu}(t,s) = 1_{\{\tau(x) > t\}} P_{r,\mu}(t,s) \cdot M_{r,\mu}(t,s) \quad (33)$$

where $P_{r,\mu}(t,s)$ is given by (3.10) and $M_{r,\mu}(t,s)$ is defined by

$$M_{r;\mu}(t,s) := \mu_{x+t} e^{-\omega(u-t)} + \omega e^{-\omega u} \int_t^u \overline{\mu}_{x+v} e^{\omega v} dv - \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\omega, t, u) + \frac{\eta \varepsilon \rho^{r,\mu}}{\lambda} B(\lambda + \omega, t, u) - \frac{\varepsilon^2}{2} B^2(\omega, t, u). \quad (34)$$

This proves Proposition II.

Appendix D: Inclusion of jumps

In this appendix, we illustrate in details that the results and analysis made using a traditional affine continuous diffusion setup can be generalized to affine jump diffusions by computing the price of the zero-coupon survival bond in the presence of jumps. For completeness, we incorporate the motivation and text of section 6 within this appendix.

Let us recall that the Hull and White model[2] (2.16) can be written as a state vector $\mathbf{X}_t = (r_t \;\; \mu_{x+t})^T$, which follows the stochastic differential form:

$$d\mathbf{X}_t = \mu(t, \mathbf{X}_t)\, dt + \sigma(t, \mathbf{X}_t)\, d\widetilde{\mathbf{W}}_t, \quad (35)$$

where

$$\begin{cases} \boldsymbol{\mu}(t, \mathbf{X}_t) = \begin{pmatrix} \lambda\,(\overline{r}_t - r_t) \\ \omega\,(\overline{\mu}_{x+t} - \mu_{x+t}) \end{pmatrix}, \\ \\ \boldsymbol{\sigma}(t, \mathbf{X}_t) \boldsymbol{\sigma}(t, \mathbf{X}_t)^T = \begin{pmatrix} \eta^2 & \eta \varepsilon \rho^{r,\mu} \\ \eta \varepsilon \rho^{r,\mu} & \varepsilon^2 \end{pmatrix}. \end{cases} \quad (36)$$

We propose to improve the modelling of the interest rate and mortality intensity processes by including three jump components respectively denoted $\mathbf{Z}_t^r$, $\mathbf{Z}_t^\mu$ and $\mathbf{Z}_t^{r,\mu}$. The model (35) hence becomes

$$d\mathbf{X}_t = \mu(t, \mathbf{X}_t)\, dt + \sigma(t, \mathbf{X}_t)\, d\widetilde{\mathbf{W}}_t + d\mathbf{Z}_t^r + d\mathbf{Z}_t^\mu + d\mathbf{Z}_t^{r,\mu}. \quad (37)$$

Let us describe each of the three jump components.
Firstly, an univariate jump component in r_t with its own rhythm and intensity, accounting for shocks only affecting the interest rates. It is defined as

$$\mathbf{Z}_t^r := \begin{pmatrix} \sum_{i=1}^{M_t} J_{1,i}^r \\ \\ 0 \end{pmatrix}, \quad (38)$$

where $(M_t)_{t\in[0\ S]}$ is a Poisson process with arrival intensity $\delta^r > 0$ and where $\{\mathbf{J}_i^r = (J_{1,i}^r, 0)\}_{i\in\mathbb{N}}$ is a sequence of independent and identically distributed (i.i.d.) bivariate random vectors representing the jump sizes. Since the second jump component is zero, these can be considered as univariate jumps concerning r_t.

Secondly, an univariate jump component in μ_{x+t} with its own rhythm and intensity, accounting for shocks only affecting the mortality rates. It is defined as

$$\mathbf{Z}_t^{\mu} := \begin{pmatrix} 0 \\ \\ \sum_{i=1}^{N_t} J_{2,i}^{\mu} \end{pmatrix}, \tag{39}$$

where $(N_t)_{t\in[0\ S]}$ is a Poisson process with arrival intensity $\delta^{\mu} > 0$ and where $\{\mathbf{J}_i^{\mu} = (0, J_{2,i}^{\mu})\}_{i\in\mathbb{N}}$ is a sequence of i.i.d. bivariate random vectors representing the jump sizes. Since the first jump component is zero, these are indeed only jumps on μ_{x+t}.

Thirdly, a bivariate jump component with its own rhythm and intensities of marginals, accounting for simultaneous correlated shocks in r_t and μ_{x+t}. It is defined as

$$\mathbf{Z}_t^{r,\mu} := \begin{pmatrix} \sum_{i=1}^{O_t} J_{1,i}^{r,\mu} \\ \\ \sum_{i=1}^{O_t} J_{2,i}^{r,\mu} \end{pmatrix}, \tag{40}$$

where $(O_t)_{t\in[0\ S]}$ is a Poisson process with arrival intensity $\delta^{r,\mu} > 0$ and where $\{\mathbf{J}_i^{\mu} = (J_{1,i}^{r,\mu}, J_{2,i}^{r,\mu})\}_{i\in\mathbb{N}}$ is a sequence of i.i.d. bivariate random vectors. This bivariate jump size distribution setup in case of simulatenous shocks allows for different jump magnitudes distribution for each process (marginals) while allowing a correlation between the two jumps sizes through a correlation coefficient ρ^j.

Each of the Poisson processes mentioned above is supposed to be independent of the other Poisson processes and of the different jump size processes. The jump sizes $\mathbf{J}^r$, $\mathbf{J}^{\mu}$ and $\mathbf{J}^{r,\mu}$ respectively defined in (38), (39) and (40) are assumed to be independent of the Brownian process $\widetilde{\mathbf{W}}_t$. The choice of both univariate and the bivariate jump size distributions will be discussed below.

Before the inclusion of jumps, the only tool available to introduce dependence between the interest rates and the mortality rates was through the introduction of correlation between the two Brownian motions captured by the linear correlation coefficient $\rho^{r,\mu}$. In the model including jumps, we have on top of that, the concomitance of jumps, and the jump correlation coefficient ρ^j.

According to (Duffie, Pan and Singleton, 2000)), a process $\mathbf{X}_t$ obeying a stochastic differential equation of form (37) belongs to the affine jump diffusion (AJD) class if the drift term $\boldsymbol{\mu}(t, \mathbf{X}_t)$, the variance-covariance matrix $\boldsymbol{\sigma}(t, \mathbf{X}_t)\boldsymbol{\sigma}(t, \mathbf{X}_t)^T$, the arrival intensities of the jumps $\mathbf{Z}_t^r$, $\mathbf{Z}_t^r$ and $\mathbf{Z}_t^{r,\mu}$ respectively denoted $\xi_1(t, \mathbf{X}_t)$, $\xi_2(t, \mathbf{X}_t)$ and $\xi_3(t, \mathbf{X}_t)$, and finally the discounting component $R(t, \mathbf{X}_t)$ can be written in affine form. In our case, we can write

$$
\begin{cases}
\boldsymbol{\mu}(t,\mathbf{X}_t) = \begin{pmatrix} \lambda\,\overline{r}_t \\ \omega\,\overline{\mu}_{x+t} \end{pmatrix} + \begin{pmatrix} -\lambda & 0 \\ 0 & -\omega \end{pmatrix} \mathbf{X}_t, \\[2ex]
\boldsymbol{\sigma}(t,\mathbf{X}_t)\boldsymbol{\sigma}(t,\mathbf{X}_t)^T = \begin{pmatrix} \eta^2 & \eta\varepsilon\rho^{r,\mu} \\ \eta\varepsilon\rho^{r,\mu} & \varepsilon^2 \end{pmatrix} + \mathbf{X}_t^T \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}, \\[2ex]
\xi_1(t,\mathbf{X}_t) = \delta^r + \mathbf{X}_t^T \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad , \quad \xi_2(t,\mathbf{X}_t) = \delta^\mu + \mathbf{X}_t^T \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{and} \quad \xi_3(t,\mathbf{X}_t) = \delta^{r,\mu} + \mathbf{X}_t^T \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\[2ex]
R(t,\mathbf{X}_t) = 0 + \mathbf{X}_t^T \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\end{cases} , \tag{41}
$$

which shows that the process $\mathbf{X}_t$ belongs to the affine jump diffusion class.
(Duffie, Pan and Singleton, 2000) have shown that for processes in that class, under technical regularity conditions, a closed form solution of the discounted characteristic function defined as

$$
\phi(\mathbf{X}_t, t, s, \mathbf{u}) := \mathbb{E}_{\tilde{\mathbb{Q}}}\left[\exp\left(-\int_t^s R(\mathbf{X}_v)\,dv\right)\exp\left(i\mathbf{u}\mathbf{X}_T\right)|\mathcal{G}_t\right], \tag{42}
$$

where $\mathbf{u} \in \mathbb{R}^2$, exists and is given by

$$
\phi(\mathbf{X}_t, t, s, \mathbf{u}) = \exp\left(A(t,s,\mathbf{u}) + \mathbf{B}(t,s,\mathbf{u})^T \mathbf{X}_t\right), \tag{43}
$$

where the coefficients $A(t,s,\mathbf{u})$ and $\mathbf{B}(t,s,\mathbf{u})$ satisfy the following system of ODEs

$$
\begin{cases}
\dfrac{d}{dt}A(t,s,\mathbf{u}) = \rho_0 - K_0^T\,\mathbf{B}(t,s,\mathbf{u}) - \dfrac{1}{2}\mathbf{B}^T(t,s,\mathbf{u})\,H_0\,\mathbf{B}(t,s,\mathbf{u}) - \sum_{i=1}^{3} l_0^i\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}_i\cdot\mathbf{B}(t,s,\mathbf{u})} - 1\right], \\[2ex]
A(s,s,\mathbf{u}) = 0, \\[2ex]
\dfrac{d}{dt}\mathbf{B}(t,s,\mathbf{u}) = \rho_1 - K_1^T\,\mathbf{B}(t,s,\mathbf{u}) - \dfrac{1}{2}\mathbf{B}^T(t,s,\mathbf{u})H_1\mathbf{B}(t,s,\mathbf{u}) - \sum_{i=1}^{3} l_1^i\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}_i\cdot\mathbf{B}(t,s,\mathbf{u})} - 1\right], \\[2ex]
\mathbf{B}(s,s,\mathbf{u}) = \mathbf{u},
\end{cases} \tag{44}
$$

where

$$
\begin{cases}
K_0 = \begin{pmatrix} \lambda\,\overline{r}_t \\ \omega\,\overline{\mu}_{x+t} \end{pmatrix} \quad , \quad K_1 = \begin{pmatrix} -\lambda & 0 \\ 0 & -\omega \end{pmatrix}, \\[2ex]
H_0 = \begin{pmatrix} \eta^2 & \eta\varepsilon\rho^{r,\mu} \\ \eta\varepsilon\rho^{r,\mu} & \varepsilon^2 \end{pmatrix} \quad , \quad H_1 = \begin{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{pmatrix}, \\[2ex]
l_0^1 = \delta^r \quad , \quad l_1^1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad , \quad l_0^2 = \delta^\mu \quad , \quad l_1^2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad , \quad l_0^3 = \delta^{r,\mu} \quad , \quad l_1^3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\[2ex]
\rho_0 = 0 \quad , \quad \rho_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\end{cases} \tag{45}
$$

and where the expectations $\mathbb{E}_{\tilde{\mathbb{Q}}}[.]$ in (44) are taken conditionnally on $\mathcal{G}_t$, which will not be mentioned in this section for notational convenience. After injecting (45) into (44) and choosing $\mathbf{u} = (0,0)$ and denoting

$A(t,s,\mathbf{0}) =: A(t,s)$ and $B(t,s,\mathbf{0}) =: B(t,s)$, we obtain

$$\begin{cases} \dfrac{d}{dt}A(t,s) = -\lambda\,\overline{r}_t\,B_1(t,s) - \omega\,\overline{\mu}_{x+t}\,B_2(t,s) - \dfrac{1}{2}\eta^2 B_1^2(t,s) - \dfrac{1}{2}\varepsilon^2 B_2^2(t,s) - \eta\varepsilon\rho B_1(t,s)B_2(t,s) \\ \qquad\qquad -\delta^r\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J^r}\cdot\mathbf{B}(t,s)}-1\right] - \delta^\mu\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^\mu\cdot\mathbf{B}(t,s)}-1\right] - \delta^{r,\mu}\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^{\mathbf{r},\mu}\cdot\mathbf{B}(t,s)}-1\right], \\ A(s,s)=0, \\ \left.\begin{array}{l} \dfrac{d}{dt}B_1(t,s) = 1+\lambda B_1(t,s) \\ B_1(s,s)=0 \end{array}\right\} B_1(t,s) = \left(\dfrac{e^{-\lambda(s-t)}-1}{\lambda}\right) = -B(\lambda,t,s) \quad \text{by (2.24)}, \\ \left.\begin{array}{l} \dfrac{d}{dt}B_2(t,s) = 1+\omega B_2(t,s) \\ B_2(s,s)=0 \end{array}\right\} B_2(t,s) = \left(\dfrac{e^{-\omega(s-t)}-1}{\omega}\right) = -B(\omega,t,s) \quad \text{by (2.24)}. \end{cases} \tag{46}$$

If we remove the terms that account for the jumps, i.e. $-\delta^r\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J^r}\cdot\mathbf{B}(t,s)}-1\right]$, $-\delta^\mu\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^\mu\cdot\mathbf{B}(t,s)}-1\right]$ and $-\delta^{r,\mu}\,\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^{\mathbf{r},\mu}\cdot\mathbf{B}(t,s)}-1\right]$, the system (46) is identical to the one that would have been obtained in a context without jumps. Therefore, all we have to do is solve the equation

$$\overline{A}(t,s) := \delta^r \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J^r}\cdot\mathbf{B}(q,s)}-1\right]dq + \delta^\mu \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^\mu\cdot\mathbf{B}(q,s)}-1\right]dq + \delta^{r,\mu} \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^{\mathbf{r},\mu}\cdot\mathbf{B}(q,s)}-1\right]dq$$

and from this deduce a result generalising Proposition I to the case with jumps. It gives

$$P_{r,\mu}(t,s) = 1_{\{\tau(x)>t\}}\cdot P_r(t,s)\cdot P_\mu(t,s)\cdot P_{\rho^{r,\mu}}(t,s)\cdot P_{\mathbf{J}^r}(t,s)\cdot P_{\mathbf{J}^\mu}(t,s)\cdot P_{\mathbf{J}^{r,\mu}}(t,s), \tag{47}$$

where $P_r(t,s)$, $P_\mu(t,s)$ and $P_{\rho^{r,\mu}}(t,s)$ are identical to Proposition I and

$$\begin{cases} P_{\mathbf{J}^r}(t,s) := \exp\left(\delta^r \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J^r}\cdot\mathbf{B}(q,s)}\right]dq - \delta^r(s-t)\right), \\ P_{\mathbf{J}^\mu}(t,s) := \exp\left(\delta^\mu \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^\mu\cdot\mathbf{B}(q,s)}\right]dq - \delta^\mu(s-t)\right), \\ P_{\mathbf{J}^{r,\mu}}(t,s) := \exp\left(\delta^{r,\mu} \int_t^s \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^{\mathbf{r},\mu}\cdot\mathbf{B}(q,s)}\right]dq - \delta^{r,\mu}(s-t)\right). \end{cases} \tag{48}$$

The terms $P_{\mathbf{J}^r}(t,s)$, $P_{\mathbf{J}^\mu}(t,s)$ and $P_{\mathbf{J}^{r,\mu}}(t,s)$ defined in (48) depend on the choice of the distributions of $\mathbf{J^r}$, $\mathbf{J}^\mu$ and $\mathbf{J}^{\mathbf{r},\mu}$. According to these choices, they may or may not be explicitly calculable.

In the literature, when a jump component is added to the Black and Scholes option-pricing framework, two popular choices of jump size distributions are either a normal distribution, as introduced by Merton in (Merton, 1976), or a double exponential distribution, as introduced by Kou in (Kou, 2002). In (Wu and al., 2018), the authors consider mixed-exponential jumps whereas in (Li and al., 2023), in the bivariate context of an affine jump-diffusion model to describe the joint dynamics of interest rate and excess mortality, they employ a bivariate normal distribution for the jump sizes. Inspired by (Li and al., 2023), let us consider that:

• $\mathbf{J}^r$ follows a bivariate normal distribution with marginal means m_r^j and 0, standard deviations σ_r^j and 0, denoted by

$$\mathbf{J}^r \sim \mathcal{N}(m_r^j, 0; \sigma_r^j, 0); \tag{49}$$

• $\mathbf{J}^\mu$ follows a bivariate normal distribution with marginal means 0 and m_μ^j, standard deviations 0 and σ_μ^j, denoted by

$$\mathbf{J}^\mu \sim \mathcal{N}(0, m_\mu^j; 0, \sigma_\mu^j); \tag{50}$$

• $\mathbf{J}^{r,\mu}$ follows a bivariate normal distribution with marginal means $m_{r;\mu}^j$ and $m_{\mu;r}^j$, standard deviations $\sigma_{r;\mu}^j$ and $\sigma_{\mu;r}^j$, and jump size correlation coefficient ρ^j, denoted by

$$\mathbf{J}^{r,\mu} \sim N(m_{r;\mu}^j, m_{\mu;r}^j; \sigma_{r;\mu}^j, \sigma_{\mu;r}^j; \rho^j). \tag{51}$$

Using the moment generating function of a bivariate normal distribution, we can write

$$\begin{cases} \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^r\cdot\mathbf{B}(t,s)}\right] = \exp\left(-B(\lambda,t,s)m_r^j + \dfrac{B^2(\lambda,t,s)(\sigma_r^j)^2}{2}\right), \\ \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^\mu\cdot\mathbf{B}(t,s)}\right] = \exp\left(-B(\omega,t,s)m_\mu^j + \dfrac{B^2(\omega,t,s)(\sigma_\mu^j)^2}{2}\right), \\ \mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{\mathbf{J}^{r,\mu}\cdot\mathbf{B}(t,s)}\right] = \exp\left(I(t,s)\right), \end{cases} \tag{52}$$

with

$$\begin{aligned} I(t,s) = &- B(\lambda,t,s)m_{r;\mu}^j - B(\omega,t,s)m_{\mu;r}^j + \frac{B^2(\lambda,t,s)(\sigma_{r;\mu}^j)^2}{2} + \frac{B^2(\omega,t,s)(\sigma_{\mu;r}^j)^2}{2} \\ &+ \rho^j B(\lambda,t,s)B(\omega,t,s)\sigma_{r;\mu}^j\sigma_{\mu;r}^j. \end{aligned} \tag{53}$$

By injecting (52) in (48), we obtain

$$P_{\mathbf{J}^r}(t,s) = \exp\left(\delta^r\int_t^s \exp\left(-B(\lambda,q,s)m_r^j + \frac{B^2(\lambda,q,s)(\sigma_r^j)^2}{2}\right)dq - \delta^r(s-t)\right), \tag{54a}$$

$$P_{\mathbf{J}^\mu}(t,s) = \exp\left(\delta^\mu\int_t^s \exp\left(-B(\omega,q,s)m_\mu^j + \frac{B^2(\omega,q,s)(\sigma_\mu^j)^2}{2}\right)dq - \delta^\mu(s-t)\right), \tag{54b}$$

$$P_{\mathbf{J}^{r,\mu}}(t,s) = \exp\left(\delta^{r,\mu}\int_t^s \exp\left(I(q,s)\right)dq - \delta^{r,\mu}(s-t)\right). \tag{54c}$$

The expressions $P_{\mathbf{J}^r}(t,s)$, $P_{\mathbf{J}^\mu}(t,s)$ and $P_{\mathbf{J}^{r,\mu}}(t,s)$ cannot be calculated explicitly, but can be easily found numerically, for example by a Runge-Kutta method.

After injecting expressions (54) in (47), we can rearrange (47) to arrive at a multiplicative structure which contains the following terms.

• A term denoted $P_{r'}(t,s)$ which accounts for the diffusion related factor $(P_r(t,s))$ and the jump effects $(P_{\mathbf{J}^r}(t,s)$ and $P_{\mathbf{J}_r^{r,\mu}}(t,s))$ of the interest rate process.

Let us define

$$P_{r'}(t,s) := P_r(t,s)\cdot P_{\mathbf{J}^r}(t,s)\cdot P_{\mathbf{J}_r^{r,\mu}}(t,s), \tag{55}$$

where $P_r(t,s)$ is given by (2.22) and (2.23), $P_{\mathbf{J}^r}(t,s)$ by (54a) and

$$P_{\mathbf{J}_r^{r,\mu}}(t,s) = \exp\left(\delta^{r,\mu}\int_t^s \exp\left(-B(\lambda,q,s)m_{r;\mu}^j + \frac{B^2(\lambda,q,s)(\sigma_{r;\mu}^j)^2}{2}\right)dq - \delta^{r,\mu}(s-t)\right). \tag{56}$$

• A term denoted $P_{\mu'}(t,s)$ which accounts for the diffusion effects $(P_\mu(t,s))$ and the jump effects $(P_{\mathbf{J}^\mu}(t,s)$ and $P_{\mathbf{J}_\mu^{r,\mu}}(t,s))$ of the mortality intensity process.

We define

$$P_{\mu'}(t,s) := P_\mu(t,s)\cdot P_{\mathbf{J}^\mu}(t,s)\cdot P_{\mathbf{J}_\mu^{r,\mu}}(t,s), \tag{57}$$

where $P_\mu(t,s)$ is given by (2.30) and (2.31), $P_{\mathbf{J}^\mu}(t,s)$ by (54b) and

$$P_{\mathbf{J}_\mu^{r,\mu}}(t,s) = \exp\left(\delta^{r,\mu}\int_t^s \exp\left(-B(\omega,q,s)m_{\mu;r}^j + \frac{B^2(\omega,q,s)(\sigma_{\mu;r}^j)^2}{2}\right)dq - \delta^{r,\mu}(s-t)\right). \tag{58}$$

• A term denoted $P_{\rho^{r,\mu}}(t,s)$, which is identical to (3.11) in Proposition I, which accounts for the correlation impact generated by the interest rate and the mortality intensity Brownian motions.

• A term denoted $P_{\rho^j}(t,s)$ which accounts for the part of the correlation induced by the presence of jumps attributable to the correlation between the interest rate jump size and the mortality intensity jump size.

Let us define

$$P_{\rho^j}(t,s) := \frac{P_{\mathbf{J}^{r,\mu}}(t,s)}{P_{\mathbf{J}_0^{r,\mu}}(t,s)}, \tag{59}$$

where $P_{\mathbf{J}^{r,\mu}}(t,s)$ is given by (54c) and

$$P_{\mathbf{J}_0^{r,\mu}}(t,s) := \exp\left(\delta^{r,\mu}\int_t^s \exp\left(-B(\lambda,q,s)m^j_{r;\mu} - B(\omega,q,s)m^j_{\mu;r} + \frac{B^2(\lambda,q,s)(\sigma^j_{r;\mu})^2}{2} + \frac{B^2(\omega,q,s)(\sigma^j_{\mu;r})^2}{2}\right)dq - \delta^{r,\mu}(s-t)\right), \tag{60}$$

which has been obtained by putting $\rho^j = 0$ in (54c).
Echoing the three scenarios on the value of $\rho^{r,\mu}$ studied above, we can analyse the following three scenarios depending on the value of ρ^j. When the jump size correlation coefficient ρ^j is zero, as expected we have $P_{\rho^j}(t,s) = 1$. In case of strictly positive correlation between the two jump sizes ($\rho^j > 0$), the term $P_{\rho^j}(t,s)$ is strictly higher than 1, which implies that the price of the zero-coupon survival bond is higher than when assuming independence between the jump sizes ($\rho^j = 0$). This confirms the fact already observed in the diffusion model without jumps (section 3) where the introduction of a positive correlation between interest rates and mortality rates generates an increase in terms of price of the zero-coupon survival bond . The same effect appears here in case of positive correlation between the jump sizes on interest rates and mortality rates. Ignoring these correlations when positive, leads once again to an underestimation of prices. In case of strictly negative correlation between the two jump sizes ($\rho^j < 0$), the term $P_{\rho^j}(t,s)$ is strictly lower than 1, which implies that the price of the zero-coupon survival bond is lower than when assuming independence between the jump sizes ($\rho^j = 0$).

• A term denoted $P_{con}(t,s)$ which accounts for the part of the correlation induced by the presence of jumps attributable to the concomitance of the interest rate jumps and the mortality intensity jumps.

Let us define

$$P_{con}(t,s) := \frac{P_{\mathbf{J}_0^{r,\mu}}(t,s)}{P_{\mathbf{J}_r^{r,\mu}}(t,s)P_{\mathbf{J}_\mu^{r,\mu}}(t,s)}, \tag{61}$$

where $P_{\mathbf{J}_0^{r,\mu}}(t,s)$, $P_{\mathbf{J}_r^{r,\mu}}(t,s)$ and $P_{\mathbf{J}_\mu^{r,\mu}}(t,s)$ are respectively given (60), (56) and (58).

Proposition III

Considering the Hull and White[2] model with jumps (37) with explicit expressions of the moving targets (2.18) and (2.27) and the jump size distributions (49), (50) and (51), the price at time t of a zero-coupon survival bond of maturity time s, for an individual initially aged x at time 0, is given by

$$P_{r,\mu}(t,s) = 1_{\{\tau(x)>t\}} \cdot P_{r'}(t,s) \cdot P_{\mu'}(t,s) \cdot P_{\rho'}(t,s), \tag{62}$$

where

- $P_{r'}(t,s)$ is a term encompassing all purely interest rate impacts:

$$P_{r'}(t,s) = P_r(t,s) \cdot P_{\mathbf{J}^r}(t,s) \cdot P_{\mathbf{J}_r^{r,\mu}}(t,s), \tag{63}$$

  where
  ▷ $P_r(t,s)$ accounts for the diffusion part. It is given by (2.22) and (2.23).
  ▷ $P_{\mathbf{J}^r}(t,s)$ accounts for the pure interest rates jumps. It is given by (54a).
  ▷ $P_{\mathbf{J}_r^{r,\mu}}(t,s)$ accounts for the interest rate component of the common jumps. It is given by (56).

- $P_{\mu'}(t,s)$ is a term encompassing all purely mortality intensity impacts:

$$P_{\mu'}(t,s) = P_{\mu}(t,s) \cdot P_{\mathbf{J}^{\mu}}(t,s) \cdot P_{\mathbf{J}_{\mu}^{r,\mu}}(t,s), \tag{64}$$

  where
  ▷ $P_{\mu}(t,s)$ accounts for the diffusion part. It is given by (2.30) and (2.31).
  ▷ $P_{\mathbf{J}^{\mu}}(t,s)$ accounts for the pure mortality intensity jumps. It is given by (54b).
  ▷ $P_{\mathbf{J}_{\mu}^{r,\mu}}(t,s)$ accounts for the mortality intensity component of the common jumps. It is given by (58).

- $P_{\rho'}(t,s)$ is a term encompassing all the correlation impacts:

$$P_{\rho'}(t,s) = P_{\rho^{r,\mu}}(t,s) \cdot P_{\rho^{j}}(t,s) \cdot P_{con}(t,s), \tag{65}$$

  where
  ▷ $P_{\rho^{r,\mu}}(t,s)$ accounts for the diffusion correlation. It is given by (3.11).
  ▷ $P_{\rho^{j}}(t,s)$ accounts for the jump size correlation of common jumps. It is given by (59), (54c) and (60).
  ▷ $P_{con}(t,s)$ accounts for the concomitance of the common jumps. It is given by (61), (60), (56) and (58).

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