

**Cross-ownership and strategic environmental corporate social
responsibility under price competition**

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Online Appendix

We examine a unilateral cross-shareholding case where only firm 1 holds k ($0 < k < 0.5$) shares of firm 2, while the reverse is not true. The objective functions of owners are given by:

$\Pi_0 = \pi_0$, $\Pi_1 = \pi_1 + k\pi_2$ and $\Pi_2 = (1-k)\pi_2$. Other assumptions are the same as the basic model.

From the first-order conditions of stage 1, we obtain the following firm reaction functions:

$$h_0 = R(h_1, h_2) = \frac{1}{A_{0u}} r^2(r+1)(rk+6r+4)[r(r+1)(3r+2)(1-k)h_1 + r(r+1)(rk+3r+2)h_2 + (1-r)(3r+2)^2(1-k)w] \quad (\text{A1})$$

$$h_1 = R(h_0, h_2) = \frac{r(r+1)}{(3r+2)(1-k)A_{1u}} \left(\begin{array}{l} (r+1)\{r^2(1-k)(5rk+2k+6r+4)h_0 + [(3r^3+2r^2)k^2 + (7r^3-6r^2-20r) \\ -8)k+4r^2+6r^3]h_2\} + r(1-r)(3r+2)(1-k)(5rk+2k+6r+4)w \end{array} \right) \quad (\text{A2})$$

$$h_2 = R(h_0, h_1) = \frac{1}{A_{2u}} r^2(r+1)(1-k)(k+2)[r(r+1)h_0 + r(r+1)h_1 + (1-r)(3r+2)w] \quad (\text{A3})$$

where $A_{0u} = (1-k)[-r^4(2r+1)(9r^3+r^2-12r-6)k^2 + 2r^2(3r+2)(35r^4+20r^3-47r^2-48r-12)k - (3r+2)^2(2r^5+74r^4+42r^3-94r^2-96r-24)]$

and $A_{1u} = r^4(2r+1)(1-r)k^2 + 2r^2(11r^3-r^2-13r-5)k - 2r^5 - 74r^4 - 42r^3 + 94r^2 + 96r + 24$.

In the comparative analysis, we find the following Lemma.

Lemma 1'. (i) h_0 increases with h_1 and h_2 ; (ii) h_1 increases with h_0 , and h_1 increases (decreases) with h_2 if k is small (large); (iii) h_2 increases with h_0 and h_1 .

Proof. We set $M_u = r(r+1)^2[(3r^3+2r^2)k^2 + (7r^3-6r^2-20r-8)k+4r^2+6r^3]$ and can prove $M_u > (<)0$ if

$k < (>)k_u(r)$ (where $k_u(r) = \frac{-7r^3+6r^2+20r+8-\sqrt{-23r^6-180r^5-276r^4+128r^3+496r^2+320r+64}}{2r^2(3r+2)} \in (0,0.5)$). In

addition, $A_{0u} > 0$ and $A_{1u} > 0$. Thus, we have: (i) $\frac{\partial h_0}{\partial h_1} = \frac{\partial R(h_1, h_2)}{\partial h_1} = \frac{r^3(r+1)^2(rk+6r+4)(3r+2)(1-k)}{A_{0u}} > 0$

and $\frac{\partial h_0}{\partial h_2} = \frac{\partial R(h_1, h_2)}{\partial h_2} = \frac{r^3(r+1)^2(rk+6r+4)(rk+3r+2)}{A_{0u}} > 0$; (ii) $\frac{\partial h_1}{\partial h_0} = \frac{\partial R(h_0, h_2)}{\partial h_0} = \frac{r^3(r+1)^2(1-k)(5rk+2k+6r+4)}{(3r+2)(1-k)A_{1u}} > 0$,

and $\frac{\partial h_1}{\partial h_2} = \frac{\partial R(h_0, h_2)}{\partial h_2} = \frac{M_u}{(3r+2)(1-k)A_{1u}} > (<)0$ if $k < (>)k_u(r)$; (iii) $\frac{\partial h_2}{\partial h_0} = \frac{\partial R(h_0, h_1)}{\partial h_0} = \frac{r^3(r+1)^2(1-k)(k+2)}{A_{2u}} > 0$

and $\frac{\partial h_2}{\partial h_1} = \frac{\partial R(h_0, h_1)}{\partial h_1} = \frac{r^3(r+1)^2(1-k)(k+2)}{A_u} > 0$.

Lemma 1' represents that the choices of ECSR under unilateral cross-ownership are strategic complements in most cases. However, if k is sufficiently large, h_1 decreases with h_2 (i.e., their relationship is as strategic substitutes).

Similar to the basic model, we obtain the following optimal ECSR levels:

$$h_0^U = \frac{r^2(1-r^2)(kr+4+6r)\Phi_u w}{\mathcal{G}_u} \quad (\text{A4})$$

$$h_1^U = \frac{r^2(1-r^2)\psi_u w}{\mathcal{G}_u} \quad (\text{A5})$$

$$h_2^U = \frac{r^2(k+2)(1-k)(1-r^2)\rho_u w}{\mathcal{G}_u} \quad (\text{A6})$$

Note that Φ_u , ψ_u , ρ_u and \mathcal{G}_u are given in appendix C. Using (A4)~(A6), we obtain equilibrium prices, abatement levels and outputs: $p_i^U = p_i(h_0^U, h_1^U, h_2^U)$, $y_i^U = y_i(h_i^U)$ and $q_i^U = q_i(h_0^U, h_1^U, h_2^U)$ ($i=0,1,2$). Then, we can obtain the environmental damage and social welfare in equilibrium: $D^U = D(h_0^U, h_1^U, h_2^U)$ and $SW^U = SW(h_0^U, h_1^U, h_2^U)$.¹ Using (A4)~(A6), we can prove Propositions 1' and 2'.

Proposition 1'. (i) $h_i^U > 0$ ($i=0,1,2$); (ii) $h_2^U < h_0^U < h_1^U$; (iii) $h_1^U - h_0^U < h_0^U - h_2^U$.

Proof: (i) Because $\mathcal{G}_u > 0$, $r^2(1-r^2)(kr+4+6r)\Phi_u > 0$, $r^2(1-r^2)\psi_u > 0$ and $r^2(k+2)(1-k)(1-r^2)\rho_u > 0$ for $0 < k < 0.5$ and $0 < r < 1$, $h_0^U > 0$, $h_1^U > 0$ and $h_2^U > 0$; (ii) Because

¹ Note that to ensure $SW^U \geq 0$, d cannot be too high and satisfies $0 < d \leq \frac{G_u}{Z_u^2}$ (G_u and Z_u are given in appendix C).

$$h_0^U - h_2^U = \frac{r^2(1-r^2)[(kr+4+6r)\Phi_u - (k+2)(1-k)\rho_u]w}{g_u} > 0 \quad \text{and} \quad h_1^U - h_0^U = \frac{r^2(1-r^2)[\psi_u - (kr+4+6r)\Phi_u]w}{g_u} > 0; \quad (\text{iii})$$

$$\text{Because } (h_1^U - h_0^U) - (h_0^U - h_2^U) = \frac{r^2(1-r^2)[\psi_u + (k+2)(1-k)\rho_u - 2(kr+4+6r)\Phi_u]w}{g_u} < 0, \quad h_1^U - h_0^U < h_0^U - h_2^U.$$

Proposition 2'. (i) $\frac{\partial h_0^U}{\partial k} > 0$; (ii) $\frac{\partial h_1^U}{\partial k} > 0$; (iii) $\frac{\partial h_2^U}{\partial k} < 0$.

$$\text{Proof: (i) } \frac{\partial h_0^U}{\partial k} = \frac{r^2(1-r^2)[r\Phi_u g_u + (kr+4+6r)(\Phi_u' g_u - \Phi_u g_u')]w}{g_u^2} > 0; \quad (\text{ii) } \frac{\partial h_1^U}{\partial k} = \frac{r^2(1-r^2)(\psi_u' g_u - \psi_u g_u')w}{g_u^2} > 0;$$

$$(\text{iii) } \frac{\partial h_2^U}{\partial k} = \frac{r^2(1-r^2)[-(2k+1)\rho_u g_u + (k+2)(1-k)(\rho_u' g_u - \rho_u g_u')]w}{g_u^2} < 0.$$

Using the equilibrium prices, outputs, abatement levels, environmental damage and social welfare, we can prove Lemma 2' and Proposition 3'.

$$\text{Lemma 2'.$$
 (i) $\frac{\partial p_i^U}{\partial k} > 0 \quad (i=0,1,2)$; (ii) $\frac{\partial Q^U}{\partial k} < 0$ where $Q^U = \sum_{i=0}^2 q_i^U$; (iii) $\frac{\partial y_i^U}{\partial k} > 0 \quad (i=0,1,2)$; (iv)

$$\frac{\partial(E^U)}{\partial k} < 0 \quad \text{where} \quad E^U = \sum_{i=0}^2 e_i^U.$$

Proof: We can obtain $p_i^U = \frac{(1-r)\beta_{iu}w}{g_u}$, $Q^U = \frac{\lambda_u w}{(2r+1)g_u}$, $y_i^U = \frac{r^2(1-r^2)\alpha_{iu}w}{g_u}$ and $E^U = \frac{\varsigma_u w}{(2r+1)g_u}$, where

β_{iu} , α_{iu} , λ_u and ς_u ($i=0,1,2$) are given in appendix C. Then, we can prove: (i)

$$\frac{\partial p_i^U}{\partial k} = \frac{(1-r)(\beta_{iu}' g_u - \beta_{iu} g_u')w}{g_u^2} > 0; \quad (\text{ii) } \frac{\partial Q^U}{\partial k} = \frac{(\lambda_u' g_u - \lambda_u g_u')w}{(2r+1)g_u^2} < 0; \quad (\text{iii) } \frac{\partial y_i^U}{\partial k} = \frac{r^2(1-r^2)(\alpha_{iu}' g_u - \alpha_{iu} g_u')w}{g_u^2} > 0; \quad (\text{iv) }$$

$$\frac{\partial(E^U)}{\partial k} = \frac{(\varsigma_u' g_u - \varsigma_u g_u')w}{(2r+1)g_u^2} < 0.$$

Proposition 3'. (i) $\frac{\partial D^U}{\partial k} < 0$; (ii) There exists $\hat{d}(r,k)$ ($\hat{d}(r,k) > 0$) making $\frac{\partial SW^U}{\partial k} < (>) 0$ if

$$d < (>) \hat{d}(r,k).$$

Proof: (i) We can obtain $D^U = \frac{Z_u^2 d w^2}{2(2r+1)^2 g_u^2}$ where Z_u is given in appendix C. Then, we obtain

$$\frac{\partial D^U}{\partial k} = \frac{Z_u(Z_u' g_u - Z_u g_u') d w^2}{(2r+1)^2 g_u^3} < 0; \quad (\text{ii) We obtain } SW^U = \frac{(G_u - Z_u^2 d) w^2}{2(2r+1)^2 g_u^2}$$
 where G_u is given in appendix C.

Then, we obtain $\frac{\partial SW^U}{\partial k} = \frac{[2(Z_u^2 g_u' - Z_u Z_u' g_u) d - (2G_u g_u' - G_u' g_u)] w^2}{8(2r+1)^2 g_u^3}$. We can prove that $g_u > 0$,

$Z_u^2 g_u' - Z_u Z_u' g_u > 0$ and $2G_u g_u' - G_u' g_u > 0$. We set $\hat{d}(r, k) = \frac{2G_u g_u' - G_u' g_u}{2(Z_u^2 g_u' - Z_u Z_u' g_u)}$ and thus obtain

$$\frac{\partial SW^B}{\partial k} < (>) 0 \text{ if } d < (>) \hat{d}(r, k).$$

Proposition 3' (i) implies that cross ownership can decrease environmental damage and improve environmental quality, similar to Proposition 3(i) under bilateral cross-shareholding. As a result, Proposition 3' (ii) implies that cross ownership decreases social welfare when environmental damage is minimal; however, it increases social welfare when environmental damage is severe, similar to the bilateral cross-shareholding case.

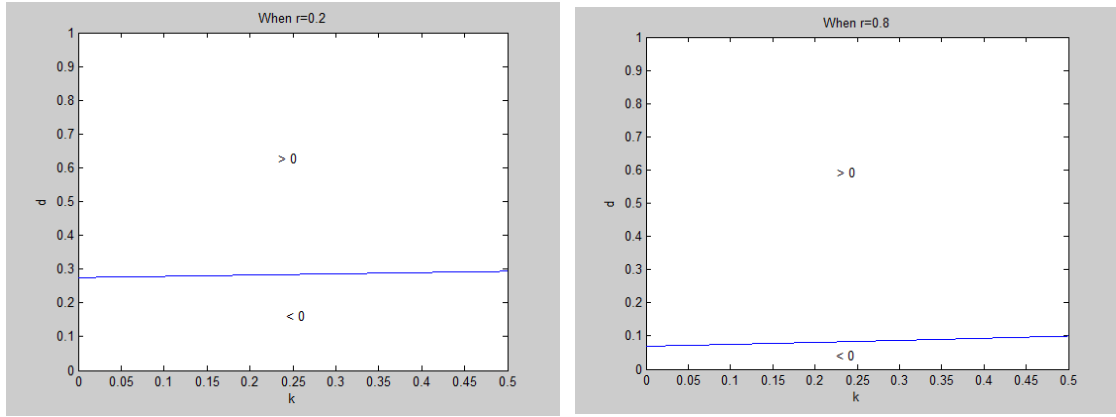


Figure A1. The sign of $\frac{\partial SW^U}{\partial k}$ under unilateral cross ownership.

Appendix C

$$\Phi_u = [r^2(r-1)(2+3r)(2r+1)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12][r^2(6r^3 - 7r - 3)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12],$$

$$\Psi_u = \left(\begin{array}{l} [r^2(2r+1)(9r^3 + r^2 - 12r - 6)k - (2+3r)(35r^4 + 20r^3 - 47r^2 - 48r - 12)][r(10r^4 - 4r^3 - 15r^2)] \\ -9r - 2)k^2 + (12r^5 - 61r^4 - 42r^3 + 59r^2 + 56r + 12)k - 70r^4 - 40r^3 + 94r^2 + 96r + 24 \end{array} \right),$$

$$\rho_u = [r^2(r-1)(2+3r)(2r+1)k + 12 - 35r^4 - 20r^3 + 47r^2 + 48r][r^2(2r+1)(9r^3 + r^2 - 12r - 6)k - (2+3r)(35r^4 + 20r^3 - 47r^2 - 48r - 12)],$$

$$g_u = \begin{pmatrix} -r^8(9r^3 + r^2 - 12r - 6)(1-r)^2(2r+1)^3k^4 + 2r^6(2r+1)(390r^8 - 210r^7 - 1201r^6 + 116r^5 + 1419r^4 + 430r^3 - \\ 555r^2 - 384r - 69)k^3 - r^4(216r^{11} + 13802r^{10} + 5271r^9 - 52381r^8 - 44668r^7 + 56358r^6 + 86007r^5 + 9775r^4 - \\ 43066r^3 - 31930r^2 - 9360r - 1032)k^2 + 2r^2(-12 - 48r - 47r^2 + 20r^3 + 35r^4)(36r^8 + 842r^7 + 327r^6 - 2184r^5 - \\ -1950r^4 + 950r^3 + 1895r^2 + 864r + 132)k - 2(2+3r)(r^4 + 13r^3 - r^2 - 15r - 6)(35r^4 + 20r^3 - 47r^2 - 48r - 12)^2 \end{pmatrix},$$

$$\beta_{0u} = \begin{pmatrix} [(6r^5 - 7r^3 - 3r^2)k + 12 + 48r + 49r^2 - 16r^3 - 33r^4][(6r^5 + r^4 - 5r^3 - 2r^2)k - 35r^4] \\ - 20r^3 + 47r^2 + 48r + 12][(6r^5 - 7r^3 - 3r^2)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12] \end{pmatrix},$$

$$\beta_{1u} = \begin{pmatrix} [(-24r^3 + 18r^6 - 6r^2 - 23r^4 + 11r^5)k + 24 - 105r^5 + 132r + 101r^3 + 238r^2 - 130r^4][r^5(2r+1)^2(r \\ - 1)^2k^3 + r^3(-44r + 12r^6 - 9r^2 - 48r^5 + 74r^3 + 12r^4 - 13)k^2 - r(-127r^2 - 244r^5 + 224r^3 - 156r \\ - 115r^6 + 136r^7 - 36 + 206r^4)k + (11r^3 - 2r^2 - 15r - 6)(35r^4 + 20r^3 - 47r^2 - 48r - 12)] \end{pmatrix},$$

$$\beta_{2u} = \begin{pmatrix} [(-r^2 - r^3 + 2r^4)k + 15r + 2r^2 - 11r^3 + 6][(6r^5 + r^4 - 5r^3 - 2r^2)k - 35r^4 - 20r^3 + 47r^2 + 48r \\ + 12][(-24r^3 + 18r^6 - 6r^2 - 23r^4 + 11r^5)k + 24 - 105r^5 + 132r + 101r^3 + 238r^2 - 130r^4] \end{pmatrix},$$

$$\alpha_{0u} = (kr + 4 + 6r)[(6r^5 - 7r^3 - 3r^2)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12][(6r^5 + r^4 - 5r^3 - 2r^2)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12],$$

$$\alpha_{1u} = \begin{pmatrix} [(-24r^3 + 18r^6 - 6r^2 - 23r^4 + 11r^5)k + 24 - 105r^5 + 132r + 101r^3 + 238r^2 - 130r^4][(-9r^2 - 2r + \\ 10r^5 - 4r^4 - 15r^3)k^2 + (56r + 12 - 42r^3 - 61r^4 + 59r^2 + 12r^5)k + 96r - 70r^4 + 24 - 40r^3 + 94r^2] \end{pmatrix},$$

$$\alpha_{2u} = \begin{pmatrix} (k + 2)[(6r^5 + r^4 - 5r^3 - 2r^2)k - 35r^4 - 20r^3 + 47r^2 + 48r + 12][(-24r^3 + \\ 18r^6 - 6r^2 - 23r^4 + 11r^5)k + 24 - 105r^5 + 132r + 101r^3 + 238r^2 - 130r^4] \end{pmatrix},$$

$$\lambda_u = \begin{pmatrix} -r^7(-6 - 12r + r^2 + 9r^3)(r-1)^2(2r+1)^4k^4 + r^5(162r^9 + 582r^8 - 903r^7 - 1970r^6 + 902r^5 + 2548r^4 \\ + 381r^3 - 1098r^2 - 630r - 102)(2r+1)^2k^3 - r^3(2r+1)(5886r^{11} + 12737r^{10} - 18838r^9 - 57181r^8 \\ - 5126r^7 + 78079r^6 + 60306r^5 - 15375r^4 - 42140r^3 - 23188r^2 - 5640r - 528)k^2 + r(2034r^9 + \\ 3960r^8 - 3191r^7 - 12032r^6 - 6058r^5 + 6434r^4 + 9019r^3 + 4310r^2 + 924r + 72)(35r^4 + 20r^3 - \\ 47r^2 - 48r - 12)k - (9r + 6)(r+1)(13r^3 - 15r - 6)(35r^4 + 20r^3 - 47r^2 - 48r - 12)^2 \end{pmatrix},$$

$$\zeta_u = \begin{pmatrix} -r^7(-6 - 12r + r^2 + 9r^3)(r-1)^2(2r+1)^4k^4 + r^5(324r^9 + 565r^8 - 1473r^7 - 2134r^6 + 1556r^5 + 2985r^4 + \\ 225r^3 - 1342r^2 - 720r - 114)(2r+1)^2k^3 + r^3(2r+1)(648r^{12} - 9234r^{11} - 18715r^{10} + 29376r^9 + 77615r^8 \\ + 506r^7 - 103597r^6 - 72428r^5 + 22121r^4 + 51020r^3 + 26808r^2 + 6312r + 576)k^2 - r(35r^4 + 20r^3 - \\ 47r^2 - 48r - 12)(432r^{10} - 2160r^9 - 5731r^8 + 2803r^7 + 14446r^6 + 7676r^5 - 7121r^4 - 10075r^3 - 4698r^2 \\ - 972r - 72)k + (9r + 6)(r+1)(r^2 - 3r - 2)(4r^2 - 3r - 3)(35r^4 + 20r^3 - 47r^2 - 48r - 12)^2 \end{pmatrix},$$

$$Z_u = \begin{pmatrix} -r^7(9r^3 + r^2 - 12r - 6)(1-r)^2(2r+1)^4k^4 + r^5(2r+1)^2(324r^9 + 565r^8 - 1473r^7 - 2134r^6 + 1556r^5 + 2985r^4 + 225r^3 - 1342r^2 \\ - 720r - 114)k^3 + r^3(2r+1)(648r^{12} - 9234r^{11} - 18715r^{10} + 29376r^9 + 77615r^8 + 506r^7 - 103597r^6 - 72428r^5 + 22121r^4 + \\ 51020r^3 + 26808r^2 + 6312r + 576)k^2 - r(35r^4 + 20r^3 - 47r^2 - 48r - 12)(432r^{10} - 2160r^9 - 5731r^8 + 2803r^7 + 14446r^6 + \\ 7676r^5 - 7121r^4 - 10075r^3 - 4698r^2 - 972r - 72)k + 3(3r + 2)(r+1)(4r^2 - 3r - 3)(r^2 - 3r - 2)(35r^4 + 20r^3 - 47r^2 - 48r - 12)^2 \end{pmatrix} \quad \text{and}$$

$$G_u = (2r+1) \left(\begin{aligned} & r^{14}(2r-1)(r-1)^4(2r+1)^7(9r^3+r^2-12r-6)^2k^8 - 2r^{12}(r-1)^2(2r+1)^5(9r^3+r^2-12r-6)(54r^{10}+1320r^9-1504r^8-3773r^7+2471r^6 \\ & +4698r^5-820r^4-2577r^3-447r^2+348r+102)k^7 - r^{10}(2r+1)^3(+17496r^{19}-185328r^{18}-1550628r^{17}+3376963r^{16}+8424132r^{15}- \\ & 12448517r^{14}-23543530r^{13}+19318572r^{12}+39953496r^{11}-10965942r^{10}-40501692r^9-5506489r^8+21923232r^7+10448935r^6 \\ & -4148542r^5-4391914r^4-737988r^3+335940r^2+148536r+16884)k^6 + 2r^8(2r+1)^2(565704r^{21}-3244968r^{20}-19571574r^{19}+ \\ & 28372986r^{18}+129630757r^{17}-61236857r^{16}-405959310r^{15}-51105532r^{14}+680962024r^{13}+403006748r^{12}-563227388r^{11}- \\ & 651023966r^{10}+88681765r^9+439656951r^8+181293426r^7-79258032r^6-95058314r^5-28874218r^4+1247670r^3+2954280r^2 \\ & +719208r+59616)k^5 - r^6(2r+1)(139968r^{24}+31218696r^{23}-112946076r^{22}-759964382r^{21}+467113423r^{20}+4998381772r^{19} \\ & +1700624430r^{18}-14609799124r^{17}-14326332121r^{16}+18777959426r^{15}+35381771786r^{14}-835543402r^{13}-38683141955r^{12}- \\ & 24675749216r^{11}+12699021338r^{10}+23477430128r^9+8645610845r^8-4223507878r^7-5530025046r^6-2343196436r^5- \\ & 377845552r^4+58808688r^3+38585472r^2+6908544r+457344)k^4 + 2r^4(35r^4+20r^3-47r^2-48r-12)(93312r^{22}+6797520r^{21} \\ & -18233964r^{20}-127308714r^{19}+20259166r^{18}+667003990r^{17}+503202983r^{16}-1372437504r^{15}-2140701783r^{14}+594759834r^{13} \\ & +3293173206r^{12}+1828131494r^{11}-1549879480r^{10}-2449892286r^9-862137253r^8+543563516r^7+709389777r^6+337478350r^5 \\ & +79471700r^4+4930104r^3-2079456r^2-518400r-38016)k^3 - r^2(35r^4+20r^3-47r^2-48r-12)^2(46656r^{19}+1716660r^{18}- \\ & 4154118r^{17}-25297949r^{16}+2052128r^{15}+104262705r^{14}+84400528r^{13}-148459766r^{12}-255011732r^{11}-15606650r^{10}+238593422r^9 \\ & +193678671r^8-817020r^7-93433211r^6-68413896r^5-24273724r^4-4369056r^3-226224r^2+42624r+5184)k^2 + 2r(3r+2)(35r^4 \\ & +20r^3-47r^2-48r-12)^3(864r^{14}+19314r^{13}-55772r^{12}-201489r^{11}+109489r^{10}+642387r^9+247737r^8-699283r^7-733621r^6+ \\ & 2823r^5+393735r^4+269016r^3+80976r^2+10944r+432)k - 3(r+1)(3r+2)^2(r^2-3r-2)(35r^4+20r^3-47r^2-48r-12)^4(8r^6+ \\ & 137r^5-92r^4-276r^3+30r^2+171r+54) \end{aligned} \right).$$