Can abatement efforts in a common pool resource promote overexploitation of the resource?

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A. Appendix

A1. Comparisons between 2nd-period appropriations (benchmark vs. SO)

Table A1. Comparisons between 2nd-period appropriations (benchmark vs. SO)

	w^*_{k2}	w^{SO}_{k2}	$w^*_{k2} - w^{SO}_{k2}$	$W^*_k - W^{SO}_k$
$\delta = 0.5$	0.52192	0.41233	0.10959	1.43196
$\delta = 0.75$	0.52246	0.41337	0.10909	1.42214
$\delta = 1$	0.52300	0.41439	0.10861	1.41239
$\beta = 0.5$	0.52192	0.41233	0.10959	1.43196
$\beta = 0.75$	0.52192	0.41206	0.10986	1.43162
$\beta = 1$	0.52192	0.41193	0.10999	1.43146
$r = 0.75$	0.52192	0.41233	0.10959	1.43196
$r = 0.5$	0.34770	0.27441	0.07329	1.43999
$r = 0.25$	0.17373	0.13696	0.03677	1.44878
$\theta = 5$	0.52192	0.41233	0.10959	1.43196
$\theta = 6$	0.62630	0.49499	0.13132	1.71859
$\theta = 7$	0.73069	0.57771	0.15298	2.00531
$c = 0.5$	0.52192	0.41233	0.10959	1.43196
$c = 0.75$	0.37069	0.28697	0.08372	0.99455
$c = 0.9$	0.31526	0.24261	0.07265	0.84003
$d = 0.5$	0.52192	0.41233	0.10959	1.43196
$d = 0.75$	0.47119	0.38568	0.08551	1.34318
$d = 0.9$	0.44102	0.37018	0.07084	1.29069

A2. Comparisons between 2nd-period appropriations (competitive abatement vs. SO)

Table A2. Comparisons between 2nd-period appropriations (competitive abatement vs. SO)

	$\tilde{w_{k2}}$	w_{k2}^{SO}	$\tilde{w_{k2}} - w_{k2}^{SO}$	$\tilde{W_k} - W_k^{SO}$
$\delta = 0.5$	0.52279	0.41233	0.11047	1.43291
$\delta = 0.75$	0.52376	0.41337	0.11038	1.42360
$\delta = 1$	0.52470	0.41439	0.11031	1.41438
$\beta = 0.5$	0.52279	0.41233	0.11047	1.43291
$\beta = 0.75$	0.52250	0.41206	0.11044	1.43226
$\beta = 1$	0.52236	0.41193	0.11043	1.43193
$r = 0.75$	0.52279	0.41233	0.11047	1.43291
$r = 0.5$	0.34809	0.27441	0.07369	1.44042
$r = 0.25$	0.17383	0.13696	0.03687	1.44888
$\theta = 5$	0.52279	0.41233	0.11047	1.43291
$\theta = 6$	0.62756	0.49499	0.13258	1.71997
$\theta = 7$	0.73240	0.57771	0.15469	2.00718
$c = 0.5$	0.52279	0.41233	0.11047	1.43291
$c = 0.75$	0.37081	0.28697	0.08384	0.99467
$c = 0.9$	0.31531	0.24261	0.07269	0.84008
$d = 0.5$	0.52279	0.41233	0.11047	1.43291
$d = 0.75$	0.47313	0.38568	0.08745	1.34529
$d = 0.9$	0.44378	0.37018	0.07361	1.29371

A3. Lemma 1

Each firm i maximizes the following profit function in period 2:

$$\pi_{i2} = \sqrt{w_{i2}}\sqrt{x_{i2}} - cx_{i2} - \frac{w_{i2}\left(w_{i2} + w_{j2}\right)}{r\left(-dw_{i1} - dw_{j1} + \theta\right)}$$

Taking first order conditions with respect to w_{i2} and x_{i2}, we obtain

$$\frac{\sqrt{x_{i2}}}{2\sqrt{w_{i2}}} - \frac{2w_{i2} + w_{j2}}{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right)} = 0$$

$$\frac{\sqrt{w_{i2}}}{2\sqrt{x_{i2}}} - c = 0.$$

Simultaneously solving the above equations we obtain the following best response functions,

$$w_{i2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right) - 4cw_{j2}}{8c}$$

$$x_{i2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right) - 4cw_{j2}}{32c^3}$$

By symmetry, firm j has similar best response functions. That is,

$$w_{j2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right) - 4cw_{i2}}{8c}$$

$$x_{j2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right) - 4cw_{i2}}{32c^3}$$

Simultaneously solving them we obtain period-two appropriations,

$$w_{i2} = w_{j2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right)}{12c}$$

$$x_{i2} = x_{j2} = \frac{r\left(\theta - d\left(w_{i1} + w_{j1}\right)\right)}{48c^3}$$

which are positive if $d < \dfrac{\theta}{w_{i1} + w_{j1}}$.

A4. Proposition 1

Second-period profits are obtained by plugging results from Lemma 1 into period 2 profit function,

$$\frac{r\left(-dw_{i1} - dw_{j1} + \theta\right)}{144c^2}.$$

Hence first-period aggregated profits are

$$\sqrt{w_{i1}}\sqrt{x_{i1}} - cx_{i1} - \frac{w_{i1}\left(w_{i1} + w_{j1}\right)}{\theta} + \frac{\delta\left(r\left(-dw_{i1} - dw_{j1} + \theta\right)\right)}{144c^2}.$$

Taking first order conditions with respect to w_{i1} and x_{i1} we obtain

$$-\frac{d\delta r}{144c^2} - \frac{w_{i1} + w_{j1}}{\theta} - \frac{w_{i1}}{\theta} + \frac{\sqrt{x_{i1}}}{2\sqrt{w_{i1}}} = 0$$

$$\frac{\sqrt{w_{i1}}}{2\sqrt{x_{ti1}}} - c = 0$$

Simultaneously solving the above equations we obtain firm i's best response

$$w_{i1} = \frac{\theta(36c - d\delta r)}{288c^2} - \frac{w_{j1}}{2}.$$

Similarly, the best response function for firm j is

$$w_{j1} = \frac{\theta(36c - d\delta r)}{288c^2} - \frac{w_{i1}}{2}.$$

Simultaneously solving the above best response functions, we obtain first-period equilibrium appropriations for each firm i,

$$w_{i1}^* = \frac{\theta(36c - d\delta r)}{432c^2} \; and \; x_{i1}^* = \frac{\theta(36c - d\delta r)}{1728c^4}$$

which are positive if $r < \dfrac{36c}{\delta d}$.

Finally period 2 equilibrium appropriations are obtained by substituting the above results into Lemma 1. We find

$$w_{i2}^* = \frac{\theta r\left(36c(6c - d) + \delta d^2 r\right)}{2592c^3} \; and \; x_{i2}^* = \frac{\theta r\left(36c(6c - d) + \delta d^2 r\right)}{10368c^5}$$

By symmetry, firm j equilibrium appropriations are the same than those for firm i. Comparative statics period 1:

$\dfrac{\partial w_{i1}^*}{\partial \theta} = \dfrac{(36c - d\delta r)}{432c^2}$, which is positive for all admissible values of r.

Comparative statics period 2:

$\dfrac{\partial w_{i2}^*}{\partial r} = \dfrac{\theta\left(18c(6c - d) + \delta d^2 r\right)}{1296c^3}$, which is positive if $r > \dfrac{-18c(6c - d)}{d^2\delta}$, which is satisfied for all admissible values of r.

$$\frac{\partial w_{i2}^*}{\partial c} = -\frac{\theta r\left(72c^2 - 24cd + d^2\delta r\right)}{864c^4}, \text{ which is positive if } d < \frac{6c\left(\sqrt{2}\sqrt{2-\delta r}+2\right)}{\delta r}$$

The condition that guarantees $w_{i1}^* > 0$ is $r < \dfrac{36c}{d\delta}$ and that for $w_{i2}^* > 0$ is $r \geq \dfrac{36c(6c-d)}{d^2\delta}$.
Thus, combining these two conditions we obtain that $r \in [\dfrac{36(6c-d)}{d^2\delta}, \dfrac{36c}{d\delta}]$.
Comparing both cutoff we have that,

$$\frac{36c(6c-d)}{d^2\delta} < \frac{36c}{d\delta}$$

$$3c < d$$

Hence, equilibrium appropriations in both periods are positive if $r \in [\dfrac{36(6c-d)}{d^2\delta}, \dfrac{36c}{d\delta}]$ and $c < \dfrac{d}{3}$.

A5. Proposition 2

Each firm i maximizes following profit function in the second-period,

$$\pi_{i2} = \sqrt{w_{i2}x_{i2}} - -cx_{i2} - \frac{w_{i2}\left(w_{i2}+w_{j2}\right)}{r\left(-dw_{i1}\left(1-z_{i1}\right) - dw_{j1}\left(1-z_{j1}\right)+\theta\right)}.$$

Taking first order conditions with respect to w_{i2} and x_{i2}, we obtain

$$\frac{\sqrt{x_{i2}}}{2\sqrt{w_{i2}}} - \frac{2w_{i2}+w_{j2}}{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)} = 0$$

$$\frac{\sqrt{w_{i2}}}{2\sqrt{x_{i2}}} - c = 0.$$

Simultaneously solving the above results yields the following best response functions,

$$w_{i2} = \frac{\theta r + drw_{i1}\left(z_{i1}-1\right) + drw_{j1}\left(z_{j1}-1\right) - 4cw_{j2}}{8c}$$

$$x_{i2} = \frac{\theta r + drw_{i1}\left(z_{i1}-1\right) + drw_{j1}\left(z_{j1}-1\right) - 4cw_{j2}}{32c^3}.$$

Similarly, best response functions for firm j are

$$w_{j2} = \frac{\theta r + drw_{i1}\left(z_{i1}-1\right) + drw_{j1}\left(z_{j1}-1\right) - 4cw_{i2}}{8c}$$

$$x_{j2} = \frac{\theta r + drw_{i1}\left(z_{i1}-1\right) + drw_{j1}\left(z_{j1}-1\right) - 4cw_{i2}}{32c^3}.$$

Simultaneously solving the above best-response functions we obtain optimal period-two appropriations,

$$w_{i2} = w_{j2} = \frac{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)}{12c}$$

$$x_{i2} = x_{j2} = \frac{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)}{48c^3}.$$

Thus, second-period profits can be obtained by plugging in the above results into period-two profit function, which yields,

$$\frac{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)}{144c^2}.$$

Therefore first-period aggregated profits are,

$$\sqrt{w_{i1}}\sqrt{x_{i1}}+\frac{\delta\left(r\left(-dw_{i1}\left(1-z_{i1}\right)-dw_{j1}\left(1-z_{j1}\right)+\theta\right)\right)}{144c^2}-\frac{w_{i1}\left(w_{i1}+w_{j1}\right)}{\theta}-cx_{i1}-\frac{1}{2}\beta z_{i1}^2$$

and taking first-order condition with respect to w_{i1}, x_{i1} and z_{i1}, we obtain

$$\frac{\sqrt{x_{i1}}}{2\sqrt{w_{i1}}}-\frac{d\delta r\left(1-z_{i1}\right)}{144c^2}-\frac{w_{i1}+w_{j1}}{\theta}-\frac{w_{i1}}{\theta}=0$$

$$\frac{\sqrt{w_{i1}}}{2\sqrt{x_{i1}}}-c=0$$

$$\frac{d\delta r w_{i1}}{144c^2}-\beta z_{i1}=0.$$

Simultaneously solving the above results gives us the following best response functions,

$$w_{i1} = \frac{5184c^3\theta\beta-144c^c\beta d\delta\theta r-20736c^4\beta w_{j1}}{41472c^4\beta-d^2\delta^2\theta r^2}$$

$$x_{i1} = \frac{1296\beta c\theta-36\beta d\delta\theta r-5184\beta c^2 w_{j1}}{41472c^4\beta-d^2\delta^2\theta r^2}$$

$$z_{i1} = \frac{d^2\delta^2\theta r^2-36c\theta d\delta r+144c^2 d\delta r w_{j1}}{d^2\delta^2\theta r^2-41472c^4\beta}.$$

By symmetry, best response functions for firm j coincide with those for firm i. Simultaneously solving the above functions we obtain the following first-period appropriations and abatement efforts in equilibrium.

$$\tilde{w}_{i1} = \frac{144\beta c^2\theta(36c-d\delta r)}{62208\beta c^4-d^2\delta^2\theta r^2}$$

$$\tilde{x}_{i1} = \frac{36\beta\theta(36c-d\delta r)}{62208\beta c^4-d^2\delta^2\theta r^2}$$

$$\tilde{z}_{i1} = \frac{d\delta\theta r(d\delta r - 36c)}{d^2\delta^2\theta r^2 - 62208\beta c^4}.$$

Hence first period appropriation levels are positive if $r < \frac{36c}{d\delta}$. Plugging the above results into the second-period appropriation we obtain the following,

$$\tilde{w}_{i2} = \frac{\theta r\left(17915904a^2c^6\left(36c(6c-d) + \delta d^2 r\right) - 10368ac^3d^2\delta\theta r(12c(\delta r - 3) + d\delta r) + d^4\delta^4\theta^2 r^4\right)}{12c\left(d^2\delta^2\theta r^2 - 62208ac^4\right)^2}$$

$$\tilde{x}_{i2} = \frac{\theta r\left(17915904\beta^2c^6\left(36c(6c-d) + \delta d^2 r\right) - 10368\beta c^3d^2\delta\theta r(12c(\delta r - 3) + d\delta r) + d^4\delta^4\theta^2 r^4\right)}{48c^3\left(d^2\delta^2\theta r^2 - 62208\beta c^4\right)^2}.$$

By symmetry firm j's equilibrium appropriations are the same than those for firm i. Comparative statics period 1:

$\frac{\partial\tilde{w}_{i1}}{\partial\theta} = \frac{8957952a^2c^6(36c - d\delta r)}{\left(d^2\delta^2\theta r^2 - 62208ac^4\right)^2}$, which is positive for all admissible values of r.

$\frac{\partial\tilde{w}_{i1}}{\partial r} = -\frac{144ac^2d\delta\theta\left(62208ac^4 + d\delta\theta r(d\delta r - 72c)\right)}{\left(d^2\delta^2\theta r^2 - 62208ac^4\right)^2}$, which is negative for all admissible values of r.

$\frac{\partial\tilde{w}_{i1}}{\partial\beta} = \frac{144c^2d^2\delta^2\theta^2r^2(d\delta r - 36c)}{\left(d^2\delta^2\theta r^2 - 62208ac^4\right)^2}$, which is negative for all admissible values of r.

Comparative statics period 1:

$\frac{\partial\tilde{z}_{i1}}{\partial\beta} = \frac{62208c^4d\delta\theta r(d\delta r - 36c)}{\left(d^2\delta^2\theta r^2 - 62208ac^4\right)^2}$, which negative for all admissible values of r.

A6. Lemma 2

Let us consider the following difference in appropriations, for each firm k where $k = i, j$,

$$w^*_{k1} - \tilde{w}_{k1} = \frac{d^2\delta^2\theta^2r^2(36c - d\delta r)}{432c^2\left(62208\beta c^4 - d^2\delta^2\theta r^2\right)}.$$

The above result is positive if

$$\frac{d^2\delta^2\theta^2r^2(36c - d\delta r)}{432c^2\left(62208\beta c^4 - d^2\delta^2\theta r^2\right)} > 0$$

and solving for r we obtain

$$r < \frac{36c}{\delta d} \equiv r^*.$$

A7. Lemma 3

Let us consider the following difference in appropriations, for each firm k where $k = i, j$,

$$\tilde{w}_{k2} - w_{k2}^{*} = \frac{d^2\delta\theta^2 r^2(36c - d\delta r)\left(124416ac^4(18c - d\delta r) + d^3\delta^3\theta r^3\right)}{2592c^3\left(d^2\delta^2\theta r^2 - 62208\beta c^4\right)^2}.$$

From the above difference we observe that the denominator is always positive. Hence we focus on the numerator. It is positive if $36 - d\delta r > 0$ or $r < \dfrac{36c}{d\delta} \equiv r^{*}$ and $18c - d\delta r > 0$ or $r < \dfrac{18c}{\delta d} \equiv \dfrac{r^{*}}{2}$. Since the second condition is more demanding, we have that $\tilde{w_{i2}} - w_{i2}^{*}$ is unambiguously positive if $r < \dfrac{r^{*}}{2}$.

A8. Lemma 4

The social planner maximizes joint second-period profits as follows,

$$\pi_{i2} + \pi_{j2} =$$

$$\sqrt{w_{i2}x_{i2}} - cx_{i2} - \frac{w_{i2}\left(w_{i2} + w_{j2}\right)}{r\left(-dw_{i1}\left(1 - z_{i1}\right) - dw_{j1}\left(1 - z_{j1}\right) + \theta\right)} + \sqrt{w_{j2}x_{j2}} - cx_{j2} - \frac{w_{j2}\left(w_{i2} + w_{j2}\right)}{r\left(-dw_{i1}\left(1 - z_{i1}\right) - dw_{j1}\left(1 - z_{j1}\right)\right.}$$

Taking first order condition with respect to w_{i2} and x_{i2}, we obtain,

$$\frac{\sqrt{x_{i2}}}{2\sqrt{w_{i2}}} - \frac{2\left(w_{i2} + w_{j2}\right)}{r\left(dw_{i1}\left(z_{i1} - 1\right) + dw_{j1}\left(z_{j1} - 1\right) + \theta\right)} = 0$$

$$\frac{\sqrt{w_{i2}}}{2\sqrt{x_{i2}}} - c = 0.$$

Simultaneously solving the above equations we obtain the following best-response functions,

$$w_{i2} = \frac{\theta r + drw_{i1}z_{i1} - drw_{i1} + drw_{j1}z_{j1} - drw_{j1} - 8cw_{j2}}{8c}$$

$$x_{i2} = \frac{\theta r + drw_{i1}\left(z_{i1} - 1\right) + drw_{j1}\left(z_{j1} - 1\right) - 8cw_{j2}}{32c^3}.$$

Similarly, best response functions for firm j is,

$$w_{j2} = \frac{\theta r + drw_{i1}z_{i1} - drw_{i1} + drw_{j1}z_{j1} - drw_{j1} - 8cw_{i2}}{8c}$$

$$x_{j2} = \frac{\theta r + drw_{i1}z_{i1} - drw_{i1} + drw_{j1}z_{j1} - drw_{j1} - 8cw_{i2}}{32c^3}.$$

Simultaneously solving the above best-response function we have that,

$$w_{i2} = w_{j2} = \frac{r\left(dw_{i1}\left(z_{i1} - 1\right) + dw_{j1}\left(z_{j1} - 1\right) + \theta\right)}{16c}$$

$$x_{i2} = x_{j2} = \frac{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)}{64c^3}.$$

Hence, second-period aggregate profits are,

$$\frac{r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)}{64c^2}.$$

In addition,

$$\frac{\partial w_{i2}}{\partial d} = \frac{r[w_{i1}(z_{i1}-1)+w_{j1}(z_{j1}-1)]}{16c} < 0 \; if \;\; z_{k1} < 1.$$
$$\frac{\partial w_{i2}}{\partial w_{i1}} = \frac{dr(z_i 1-1)}{16c} < 0 \; if \;\; z_{i1} < 1.$$

A9. Proposition 3

The first period profits can be expressed as

$$\sqrt{w_{i1}}\sqrt{x_{i1}} - cx_{i1} - \frac{w_{i1}\left(w_{i1}+w_{j1}\right)}{\theta} - \frac{1}{2}z_{i1}^2\beta + \sqrt{w_{j1}}\sqrt{x_{j1}} - cx_{j1} - \frac{w_{j1}\left(w_{i1}+w_{j1}\right)}{\theta} - \frac{1}{2}z_{j1}^2\beta$$
$$+\frac{\delta\left(r\left(dw_{i1}\left(z_{i1}-1\right)+dw_{j1}\left(z_{j1}-1\right)+\theta\right)\right)}{64c^2}.$$

Taking first order condition with respect to w_{i1} , x_{i1} and z_{i1}, we obtain,

$$\frac{d\delta r\left(z_{i1}-1\right)}{64c^2} - \frac{w_{i1}+w_{j1}}{\theta} - \frac{w_{i1}}{\theta} + \frac{\sqrt{x_{i1}}}{2\sqrt{w_{i1}}} - \frac{w_{j1}}{\theta} = 0$$

$$\frac{\sqrt{w_{i1}}}{2\sqrt{x_{i1}}} - c = 0$$

$$\frac{d\delta r w_{i1}}{64c^2} - \beta z_{i1} = 0.$$

Simultaneously solving the above equations we obtain the following best-response functions

$$w_{i1} = \frac{1024c^3\beta\theta - 64c^2\beta d\delta\theta r - 8192c^4\beta w_{j1}}{8192c^4\beta - d^2\delta^2\theta r^2}$$

$$x_{i1} = \frac{256\beta\theta c - 16\beta d\delta\theta r - 2048\beta c^4 w_{j1}}{8192c^4\beta - d^2\delta^2\theta r^2}$$

$$z_{i1} = \frac{16c\theta d\delta r - d^2\delta^2\theta r^2 - 128c^2 d\delta r w_{j1}}{8192c^4\beta - d^2\delta^2\theta r^2}.$$

Similarly, best response functions for firm j are,

$$w_{j1} = \frac{1024c^3\beta\theta - 64c^2\beta d\delta\theta r - 8192c^4\beta w_{i1}}{8192c^4\beta - d^2\delta^2\theta r^2}$$

$$x_{j1} = \frac{256\beta\theta c - 16\beta d\delta\theta r - 2048\beta c^4 w_{i1}}{8192c^4\beta - d^2\delta^2\theta r^2}$$

$$z_{j1} = \frac{16c\theta d\delta r - d^2\delta^2\theta r^2 - 128c^2 d\delta r w_{i1}}{8192c^4\beta - d^2\delta^2\theta r^2}.$$

Simultaneously solving the above best-response functions we obtain that socially optimal equilibrium appropriation levels are,

$$w_{i1}^{SO} = \frac{64ac^2\theta(16c - d\delta r)}{16384\beta c^4 - d^2\delta^2\theta r^2}$$

$$z_{i1}^{SO} = \frac{d\delta\theta r(d\delta r - 16c)}{d^2\delta^2\theta r^2 - 16384\beta c^4}$$

$$w_{i2} = \frac{\theta r}{16c} - \frac{128\beta c^2 d\theta r(16c - d\delta r)\left(1024\beta c^3 - d\delta\theta r\right)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$$

$$x_{i2} = \frac{\theta r}{64c^3} - \frac{32\beta d\theta r(d\delta r - 16c)\left(d\delta\theta r - 1024\beta c^3\right)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}.$$

The above results are positive if $\frac{16c}{\delta r} > d$.

Comparative statics: period-one appropriations

$\frac{\partial w_{i1}^{SO}}{\partial \theta} = \frac{1048576\beta^2 c^6(16c - d\delta r)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is positive for all admissible values of r.

$\frac{\partial w_{i1}^{SO}}{\partial r} = -\frac{64\beta c^2 d\delta\theta\left(16384\beta c^4 + d\delta\theta r(d\delta r - 32c)\right)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is negative if $\beta < \frac{d\delta\theta r(32c - d\delta r)}{16384c^4}$

$\frac{\partial w_{i1}^{SO}}{\partial \delta} = -\frac{64\beta c^2 d\delta\theta\left(16384\beta c^4 + d\delta\theta r(d\delta r - 32c)\right)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is negative if $\beta < \frac{d\delta\theta r(32c - d\delta r)}{16384c^4}$

$\frac{\partial w_{i1}^{SO}}{\partial d} = -\frac{64\beta c^2 d\delta\theta\left(16384\beta c^4 + d\delta\theta r(d\delta r - 32c)\right)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is negative if $\beta < \frac{d\delta\theta r(32c - d\delta r)}{16384c^4}$.

Comparative statics: period-one abatement effort

$\frac{\partial z_{i1}^{SO}}{\partial \beta} = \frac{16384c^4 d\delta\theta r(d\delta r - 16c)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is negative for all admissible values of r.

$\frac{\partial z_{i1}^{SO}}{\partial \theta} = \frac{d\delta r(d\delta r - 16c)}{d^2\delta^2\theta r^2 - 16384\beta c^4} - \frac{d^3\delta^3\theta r^3(d\delta r - 16c)}{\left(d^2\delta^2\theta r^2 - 16384\beta c^4\right)^2}$, which is negative for all admissible values of r .

A10. Lemma 5

Let us compare the following difference in appropriations for each firm k where $k = i, j$,

$$w_{k1}^* - w_{k1}^{SO} = \frac{\theta(36c - d\delta r)}{432c^2} - \frac{64\beta c^2\theta(16c - d\delta r)}{16384\beta c^4 - d^2\delta^2\theta r^2}$$

$$= \frac{d^3\delta^3 r^3\theta^2 + 11264\beta c^4 d\delta r\theta - 36cd^2\delta^2 r^2\theta^2 + 147456\beta c^5\theta}{7077888\beta c^6 - 432c^2 d^2\delta^2 r^2\theta},$$

the above term is positive if and only if

$$r > \frac{4c^2 d\delta\left(27\theta - 704\beta c^2\right)}{\sqrt[3]{81c^3 d^6\delta^6\theta^2\left(9\theta - 736\beta c^2\right) + 48\gamma}} + \frac{4\sqrt[3]{27c^3 d^6\delta^6\theta^2\left(9\theta - 736\beta c^2\right) + 16\gamma}}{3^{2/3} d^3\delta^3\theta} + \frac{12c}{d\delta} \equiv \hat{r}.$$

where $\gamma = \sqrt{3}\sqrt{\beta c^8 d^{12}\delta^{12}\theta^3\left(1362944\beta^2 c^4 + 357372\beta c^2\theta - 6561\theta^2\right)}$.

But we also need that $r < \dfrac{16c}{\delta d}$. Hence, this term is positive if $r \in [\hat{r}, \dfrac{16c}{\delta d}]$.

A11. Lemma 6

Let us consider the following difference in appropriations for each firm k where $k = i, j$,

$$\tilde{w}_{k1} - w_{k1}^{SO} = 16\beta c^2\theta\left(\frac{4d\delta r - 64c}{16384\beta c^4 - d^2\delta^2\theta r^2} + \frac{324c - 9d\delta r}{62208\beta c^4 - d^2\delta^2\theta r^2}\right)$$

The above result is positive if

$$16\beta c^2\theta\left(\frac{4d\delta r - 64c}{16384\beta c^4 - d^2\delta^2\theta r^2} + \frac{324c - 9d\delta r}{62208\beta c^4 - d^2\delta^2\theta r^2}\right) > 0$$

$$\frac{4d\delta r - 64c}{16384\beta c^4 - d^2\delta^2\theta r^2} + \frac{324c - 9d\delta r}{62208\beta c^4 - d^2\delta^2\theta r^2} > 0$$

and solving for r we obtain

$$r > \frac{4\left(\frac{\sqrt[3]{5}\,\sqrt[3]{5c^3 d^6\delta^6\theta^2(10985\theta - 650592\beta c^2) + 432\Gamma}}{\theta} + \frac{5^{2/3} c^2 d^4\delta^4\left(845\theta - 19008\beta c^2\right)}{\sqrt[3]{5c^3 d^6\delta^6\theta^2(10985\theta - 650592\beta c^2) + 432\Gamma}} + 65cd^2\delta^2\right)}{15d^3\delta^3} \equiv \tilde{r},$$

where $\Gamma = \sqrt{15}\sqrt{\beta c^8 d^{12}\delta^{12}\theta^3\left(12266496\beta^2 c^4 + 2144140\beta c^2\theta - 54925\theta^2\right)}$.

A12. Lemma 7

Let us consider the following difference in abatement efforts for each firm k where $k = i, j$,

$$z_{k1}^{SO} - z_{k1}^{*} = d\delta\theta r\left(\frac{d\delta r - 36c}{62208ac^4 - d^2\delta^2\theta r^2} + \frac{d\delta r - 16c}{d^2\delta^2\theta r^2 - 16384ac^4}\right)$$

The above result is positive if

$$d\delta\theta r\left(\frac{d\delta r - 36c}{62208ac^4 - d^2\delta^2\theta r^2} + \frac{d\delta r - 16c}{d^2\delta^2\theta r^2 - 16384ac^4}\right) > 0$$

$$\frac{d\delta r-36c}{62208ac^4-d^2\delta^2\theta r^2}+\frac{d\delta r-16c}{d^2\delta^2\theta r^2-16384ac^4}>0$$

solving for θ we obtain $\theta > \dfrac{64\left(179ac^3 d\delta r-1584ac^4\right)}{5d^2\delta^2 r^2}$.

A13. Lemma 8

From Lemma 2 we have first-period equilibrium appropriations without abatement are higher than with abatement

$$w_{k1}^* > \tilde{w}_{k1}$$

if r^* holds.
From Lemma 5 we have first-period equilibrium appropriations with abatement are higher than socially optimum levels

$$w_{k1}^* > w_{k1}^{SO}$$

if $r \in [\hat{r}, \frac{16c}{\delta d}]$ holds.
From Lemma 6 we have first-period equilibrium appropriations with abatement are higher than socially optimum levels

$$\tilde{w}_{k1} > w_{k1}^{SO}$$

if $\tilde{r}$ holds.

Thus by combining Lemma 2 and 5 we obtain $r \in [\hat{r}, r^*]$, in this case $\hat{r} < r^*$. Similarly by combining Lemma 2 and 6 we obtain $r \in [\tilde{r}, r^*]$, in this case $\tilde{r} < r^*$.

This gives us the following 3 cases:
1. if $r \in [\hat{r}, r^*]$ or $r \in [\tilde{r}, r^*]$, we get

$$w_{k1}{}^* > \tilde{w}_{k1} > w_{k1}^{SO}.$$

2. if $r > r^*$ and $r > \hat{r}$ and $\hat{r} = \tilde{r}$, we get

$$\tilde{w}_{k1} > w_{k1}{}^* > w_{k1}^{SO}.$$

3. if $r > r^*$ and $r < \tilde{r}, \hat{r}$ and $\tilde{r} < \hat{r}$, we get

$$w_{k1}^{SO} > \tilde{w}_{k1} > w_{k1}{}^*.$$