New evidence on US monetary policy activism and the Taylor rule: Appendix

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Appendix A: Further details regarding model

Identifying the time-varying parameters in the multiplier matrix

Restrictions on U_t^*

In terms of U_t^*, we impose a diagonality restriction on $(U_t^* U_t^{*\prime})$ such that the n^2 individual parameters in $u_t^* = vec\left(U_t^*\right)$ are comprised of $n^* = n\left(n+1\right)/2$ unrestricted parameters (located in the lower triangular component of U_t^*) and $n^2 - n^*$ restricted parameters.

To better explain this, first note that we can introduce a parameter expansion term κ_t that allows us to amend the specification of Θ_t as

$$\begin{aligned}\Theta_t &= U_t I(S_t) I(S_t) D_t \\ &= U_t \kappa_t I(S_t) I(S_t) \kappa_t^{-1} D_t \\ &= U_t^* I(S_t) I(S_t) D_t^*\end{aligned} \tag{A1}$$

where $\kappa_t = (U_t^{*\prime} U_t^*)^{\frac{1}{2}}$ is an $(n \times n)$ diagonal matrix and, by construction, $U_t^* = U_t \kappa_t$.

Given (A1), it follows that

$$\beta_t^* = U_t \kappa_t I(S_t) = U_t^* I(S_t). \tag{A2}$$

$$\alpha_t^* = I(S_t) \kappa_t^{-1} D_t = I(S_t) D_t^* \tag{A3}$$

such that $\beta_t = U_t I(S_t)$ and $\alpha_t = I(S_t) D_t$ as per equations (8) and (9).

Pursuant to this decomposition, conditional on S_t we can obtain α_t^* and β_t^* by estimating the individual parameters in $d_t^* = vec\left(D_t^*\right)$ and $u_t^* = vec(U_t^*)$ for $t = 2, ..., T$.

The diagonality of κ_t implies that U_t^* is orthogonal and satisfies $U_{it}^{*\prime} U_{jt}^* = 0$ for $i, j = 1, ..., n$ and $i \neq j$, where U_{it}^* is the ith column of U_t^*. It is noted that, given the diagonal κ_t^* and the idempotent nature of $I(S_t)$, we have

$$\kappa_t I(S_t) I(S_t) \kappa_t^{-1} = I(S_t) \tag{A4}$$

such that we are able to transition from the second row of (A1) to the first row and Θ_t does not depend on the expansion matrix κ_t.

To understand the ramifications of the orthogonality condition for estimating the pass-through parameters, assume that $S_t = r + 1 = 2$. In this case Θ_t has rank $r = 1$ and the relevant vector describing the relationship between interest rates, inflation and output is $\beta_{t,1} = U_{t,1}^* \kappa_{t,1}^{-1}$, where $U_{t,1}^*$ is the first column of U_t^* and $\kappa_{t,1} = \left(U_{t,1}^{*\prime} U_{t,1}^*\right)^{\frac{1}{2}}$. Now assume that $S_t = 3$ such that the multiplier matrix $\beta_{t,2}$ (consisting of two vectors) is given by $U^*_{t,2}\kappa^{-1}_{t,2}$, where $U^*_{t,2}$ is the first two columns of U^*_t and $\kappa_{t,2} = \left(U^{*\prime}_{t,2}U^*_{t,2}\right)^{\frac{1}{2}}$. Since κ_t is diagonal, the first column of $\beta_{t,2}$ is given by $U^*_{t,1}\kappa^{-1}_{t,1}$ and therefore is the same vector observed when $S_t = 2$. In other words, as the state of the system shifts from $S_t = 2$ to $S_t = 3$ we add a second orthogonal vector describing the relationship between the system of variables whilst continuing to retain the first vector.

It follows that if the rank of the multiplier matrix never exceeds unity, the first column in U_t uniquely determines the monetary policy weights associated with inflation and output; since $U^{*\prime}_{it}U^*_{jt} = 0$ for any t, there is *no other* set of monetary policy weights that is orthogonal to the set estimated.

Accordingly, if the probability of $S_t > 2$ is zero, the inflation and output targeting parameters can be deduced using $\beta_{t,1} = U^*_{t,1}\kappa^{-1}_{t,1}$, with the selection matrix $I(S_t)$ acting as a switch to turn the Taylor-type rule on or off in accordance with $\beta_t = U_t I(S_t)$ (viz. when $S_t = 1$ the Taylor-type rule is switched off and when $S_t = 2$ the Taylor-type rule is switched on). This is undertaken endogenously in our model and therefore allows for the estimation of the time-varying probability of interest rates following a Taylor-type rule at time t by reference to the posterior distribution of the selection matrix. This probability is trivially and exactly given by the proportion of times $\beta_{t,1}$ is 'selected', requiring no approximation or marginal likelihood derivation.

To ensure the satisfaction of the orthogonality conditions for U^*_t we specifically restrict the first $j-1$ elements of the jth column, for $j = 2, ..., n$, by solving for the $j-1$ elements $\left(u^*_{1j}, u^*_{2j}, ..., u^*_{(j-1)j}\right)_t$ that satisfy the orthogonality constraints

$$\begin{bmatrix} U^*_{1t} \perp U^*_{jt} \\ U^*_{2t} \perp U^*_{jt} \\ \vdots \\ U^*_{(j-1)t} \perp U^*_{jt} \end{bmatrix} \tag{A5}$$

where the solution to (A5) is conditional on the first $j-1$ columns of U^*_t (e.g. $U^*_{1t}, U^*_{2t}, ..., U^*_{(j-1)t}$) and the $n-j+1$ unrestricted elements of U^*_{jt}.

This approach allows us to recover the restricted elements recursively beginning with the second column of U^*_t and ending with the nth column. We can show that the solution for the $j-1$ restrictions pertaining to the jth column of U^*_t can be obtained as (omitting the t subscript for notational convenience)

$$\begin{bmatrix} u_{1j}^{*} \\ u_{2j}^{*} \\ \vdots \\ u_{(j-1)j}^{*} \end{bmatrix} = \begin{bmatrix} u_{11}^{*} & u_{12}^{*} & \cdots & u_{1(j-1)}^{*} \\ u_{21}^{*} & u_{22}^{*} & & u_{2(j-1)}^{*} \\ \vdots & & \ddots & \vdots \\ u_{(j-1)1}^{*} & u_{(j-1)2}^{*} & & u_{(j-1)(j-1)}^{*} \end{bmatrix}^{\prime -1} \begin{bmatrix} -\sum\limits_{i=j}^{n} u_{i1}^{*} u_{ij}^{*} \\ -\sum\limits_{i=j}^{n} u_{i2}^{*} u_{ij}^{*} \\ \vdots \\ -\sum\limits_{i=j}^{n} u_{i(j-1)}^{*} u_{ij}^{*} \end{bmatrix}. \tag{A6}$$

Pursuant to the above restrictions, it follows that $U_t^{*\prime} U_t^{*}$ is a diagonal matrix. The adoption of $\kappa_t = (U_t^{*\prime} U_t^{*})^{\frac{1}{2}}$ thereby produces a diagonal matrix that, given $U_t = U_t^{*} \kappa_t^{-1}$, also satisfies the orthonormality condition for U_t, $(U_t^{\prime} U_t) = I_n$.

Restrictions on D_t^{*}

Similar to the restrictions for U_t^{*}, there are $n(n-1)/2$ restrictions to ensure that the rows of D_t^{*} (hence the columns of $D_t^{*\prime}$) are orthogonal. The restrictions are placed on the first $j-1$ elements of the jth row of D_t^{*} for $j = 2, ..., n$. As is the case for the restricted elements of U_t^{*}, we can show that the restricted $j-1$ elements in the jth row of D_t^{*} can be obtained as (again, omitting the t subscript)

$$\begin{bmatrix} d_{j1}^{*} \\ d_{j2}^{*} \\ \vdots \\ d_{j(j-1)}^{*} \end{bmatrix} = \begin{bmatrix} d_{11}^{*} & d_{12}^{*} & \cdots & d_{1(j-1)}^{*} \\ d_{21}^{*} & d_{22}^{*} & & d_{2(j-1)}^{*} \\ \vdots & & \ddots & \vdots \\ d_{(j-1)1}^{*} & d_{(j-1)2}^{*} & & d_{(j-1)(j-1)}^{*} \end{bmatrix}^{-1} \begin{bmatrix} -\sum_{i=j}^{n} d_{1i}^{*} d_{ji}^{*} \\ -\sum_{i=j}^{n} d_{2i}^{*} d_{ji}^{*} \\ \vdots \\ -\sum_{i=j}^{n} d_{(j-1)i}^{*} d_{ji}^{*} \end{bmatrix}. \tag{A7}$$

Pursuant to the above conditions, the n^2 parameters in $d_t^{*} = vec\,(D_t^{*})$ are comprised of $n\,(n+1)\,/2$ unrestricted parameters (located in the upper triangular component of D_t^{*}) and $n\,(n-1)\,/2$ restricted parameters (with an analogous interpretation for the parameters in $u_t^{*} = vec\,(U_t^{*})$, except that the $n\,(n+1)\,/2$ unrestricted parameters in u_t^{*} are located in the lower triangular component of U_t^{*}).

Appendix B: Priors and estimation

The following proper and independent prior densities are adopted for the model parameters.

$$vec\left(\left[\begin{array}{cc} c' & B' \end{array}\right]'\right) \sim N(\underline{g}=0, \Sigma_g = 100),$$

$$\Sigma \sim IW(\underline{v}=2, \underline{D}),$$

$$\underline{D} = 0.036\left[\begin{array}{ccc} var(\triangle\pi_t) & 0 & 0 \\ 0 & var(\triangle i_t) & 0 \\ 0 & 0 & var(\triangle y_t) \end{array}\right],$$

$$P_{i,\cdot} \sim D\left(\tau_{i1}, \tau_{i2}, ..., \tau_{i,4}\right), \quad i = 1, 2, ..., 4.$$

$$\sigma_i^2 \sim IG(\frac{\vartheta_i}{2}, \frac{\vartheta_i f_i}{2}), \qquad i = 1, ..., n^* = 4,$$

$$\rho \sim U(\underline{\rho} = 0.999, \overline{\rho} = 1),$$

where $P_{i,\cdot}$ is the ith row of P. IW, D, IG and U refer to the inverse-Wishart, Dirichlet, inverse-Gamma and Uniform densities respectively. The parameters c, B and Σ are regime-dependent such that $c = c_j$, $j = 1, 2, ..., 4$, (with an analogous representation for B, Σ) whereby c_j is the intercept when $S_t = j$. Since S_t is identified by the rank of the long-run multiplier matrix, we do not need to impose any 'labelling' restrictions on c, B or Σ.

The priors for the transition matrix governing S_t are set to $\tau_{1j} = \tau_{3j} = \tau_{4j} = (2, 5, 2, 2)$ and $\tau_{2j} = (10, 100, 10, 10)$. This ensures that our priors are consistent with existing research which a priori imposes the existence of a single long-run relationship between interest rates, output and inflation (for example, Clarida, Gali and Gertler (2000) and Lubik and Shorfheide (2004)).

We set $\vartheta_i = 0.0196$ and $f_i = 1$. The prior for ρ is based on the discussion in Koop, Leon-Gonzalez and Strachan (2011). Finally, we assume $\sigma_d = 1$ such that the distribution of the initial state is $d_0^* \sim N\left(0, I_{n^*}\right)$.

The Metropolis-in-Gibbs sampler used in the paper consists of the eight steps detailed in this Appendix. The following notation holds for each step.

$$
\begin{aligned}
\widetilde{x}_T &= (x_1, x_2, ..., x_T) \\
\widetilde{S}_T &= (S_1, S_2, ..., S_T)\,, \quad p = (p_{11}, ..., p_{1(n+1)}, ..., p_{(n+1)1}, ..., p_{(n+1)(n+1)}), \\
\sigma &= (\sigma_1, ..., \sigma_{n^*})\,, \;\; n^* = n\,(n+1)\,/2, \\
g &= vec\left(\left[\begin{array}{cc} c' & B' \end{array}\right]'\right), \\
\widetilde{u}^*_T &= (u^*_1, ..., \; u^*_T)\,, \quad \widetilde{d}^*_T = (d^*_1, ..., \; d^*_T)\,.
\end{aligned}
$$

Step 1: Draw $\widetilde{S}_T$ given $\widetilde{x}_T, \widetilde{d}^*_T, \widetilde{u}^*_T, g, p$ and Σ.

Draws of $\widetilde{S}_T$ are based on the multi-move Gibbs Sampling algorithm of Carter and Kohn (1994) (see, also, Chib, 1996). The conditional posterior probability for $S_t = j$, $j \in \{1, 2, ..., n+1\}$, is

$$
\Pr(S_t = j|\widetilde{x}_t, \widetilde{d}^*_t, \widetilde{u}^*_t, g, \Sigma, S_{t+1}) = \frac{p(S_t = j|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, S_{t+1})}{\sum_{i=1}^{n+1} p(S_t = i|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, S_{t+1})}
$$

$$
p(S_t|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, S_{t+1}) \propto p(S_t|\widetilde{x}_t, d^*_t, u^*_t, g, \Sigma)p(S_{t+1}|S_t)
$$

where $p(S_{t+1}|S_t)$ is the transition probability and $p(S_t|\widetilde{y}_t, d^*_t, u^*_t, g, \Sigma)$ is obtained using Hamilton's (1989) basic filter. s_{jt} is set to 0 or 1 according to:

(i.) $s_{1t} = 1$ if a draw u from the $U(0,1)$ distribution is less than or equal to $\Pr(S_t = 1|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, \widetilde{S}_{\neq t})$.

(ii.) $s_{jt} = 1$ if u is between $\sum_{i=1}^{j-1} \Pr(S_t = i|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, \widetilde{S}_{\neq t})$ and $\sum_{i=1}^{j} \Pr(S_t = i|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, \widetilde{S}_{\neq t})$ for $j = 2, ..., n$.

(iii.) $s_{(n+1)t} = 1$ if u is between $\sum_{i=1}^{n} \Pr(S_t = i|\widetilde{x}_T, d^*_t, u^*_t, g, \Sigma, \widetilde{S}_{\neq t})$ and 1.

Step 2: Draw p given $\widetilde{S}_T$

Draws of p are based on the approach proposed in Kim and Nelson (1998). Conditional on $\widetilde{S}_T$, the transition parameters are independent of the remaining parameters and are drawn from the Dirichlet distribution as follows

$$
P_{i,\cdot}|\widetilde{S}_T \sim D\left(\vartheta_{i1} + \tau_{i1}, \vartheta_{i2} + \tau_{i2}, ..., \vartheta_{i,n+1} + \tau_{i,n+1}\right), \quad i = 1, 2, ..., n+1
$$

where D is the Dirichlet density, $P_{i,\cdot}$ is the ith row of P and ϑ_{ij} is the total number of transitions from $S_{t-1} = i$ to $S_t = j$, $(i, j = 1, 2, ..., n+1$ and $t = 2, 3, ..., T)$. The τ_{ij}

$(i, j = 1, 2, ..., n+1)$ are prior hyper-parameters.

Step 3: Draw $\widetilde{u}_T^*$ given $\widetilde{x}_T, \widetilde{S}_T, \widetilde{d}_T^*, g, \Sigma$ and ρ

We draw u_t^* recursively, starting from the first column of u_t^* to the nth column. Given the other parameters and latent variables, the elements in the jth column of u_T^*, for $j = 1, ..., n$, can be estimated pursuant to the following linear, Gaussian model

$$\triangle \widehat{x}_t^{*(j)} = \left(\widetilde{x}_t^{j:n} + \widetilde{x}_t^{1:j-1} \widehat{U}_t^{(j)} \right) u_t^{*(j)} + \varepsilon_t^{(j)\prime},$$

$$u_t^{*(j)} = \rho u_{t-1}^{*(j)} + \eta_t^{(j)},$$

where

$$\widehat{U}_t^{(j)} = \begin{bmatrix} u_{11,t}^* & u_{12,t}^* & \cdots & u_{1(j-1),t}^* \\ u_{21,t}^* & u_{22,t}^* & & u_{2(j-1),t}^* \\ \vdots & & \ddots & \vdots \\ u_{(j-1)1,t}^* & u_{(j-1)2,t}^* & & u_{(j-1)(j-1),t}^* \end{bmatrix}^{\prime -1} \begin{bmatrix} -u_{j1,t}^* & \cdots & -u_{n1,t}^* \\ -u_{j2,t}^* & & -u_{n2,t}^* \\ \vdots & & \vdots \\ -u_{j(j-1),t}^* & \cdots & -u_{n(j-1),t}^* \end{bmatrix},$$

$\triangle \widehat{x}_t^{*(j)} = (\triangle x_t - \triangle x_{t-1} B_t - c_t - x_{t-1} (U_t^* I(S_t) D_t^*)_{\neq j})'$, $(U_t^* I(S_t) D_t^*)_{\neq j}$ is the resulting matrix after removing the jth column of U_t^*, the jth row of D_t^* and both the jth row and column of $I(S_t)$., and $u_t^{*(j)}$ is a $(n-j+1)$ vector containing the $(n-j+1)$ free parameters that are in the jth column of u_t^*. Further, $\widetilde{x}_t^{a:b}$ is the matrix comprised of the ath to bth columns of $\left((I(S_t) D_t^*)_j' \otimes x_{t-1} \right)$, and $(I(S_t) D_t^*)_j$ is the jth row of $I(S_t) D_t^*$. Finally, $\eta_t^{(j)} \sim N(0, I_{n-j+1})$, $u_0^{*(j)} \sim N\left(0, I_{n-j+1} \frac{1}{1-\rho^2}\right)$ with I_{n-j+1} being an identity matrix of size $(n-j+1)$.

Accordingly, the Kalman filter and smoother can be used to obtain the conditional density of $\widetilde{u}_T^*$. Draws are obtained pursuant to the simulation smoother method in Durbin and Koopman (2002).

Step 4: Draw the latent variable autoregressive parameter ρ given $\widetilde{u}_T^*$

Given $\widetilde{u}_T^*$ we draw ρ. However, drawing ρ is not straightforward as this parameter enters into the distribution of u_0^*. As such, we draw ρ using a Metropolis-Hastings step as per Koop et. al. (2011).

Step 5: Draw the latent variable $\widetilde{d}_T^*$ given $\widetilde{x}_T, \widetilde{S}_T, \widetilde{u}_T^*, g, \Sigma$ and σ

The draws for d_t^* are obtained using the same approach as that adopted for u_t^* in Step 3. In particular, we draw d_t^* recursively, starting from first row of d_t^* to the nth row. Given the rest of the parameters and the latent variables, the elements in the jth row of d_T^*, for $j = 1, ..., n$, can be estimated pursuant to the following linear, Gaussian model

$$\triangle\widehat{x}_t^{*(j)} = \left(\widetilde{z}_t^{j:n} + \widetilde{z}_t^{1:j-1}\widehat{D}_t^{(j)}\right) d_t^{*(j)} + \varepsilon_t^{(j)\prime},$$

$$d_t^{*(j)} = d_{t-1}^{*(j)} + \zeta_t^{(j)},$$

where

$$\widehat{D}_t^{(j)} = \begin{bmatrix} d_{11,t}^* & d_{12,t}^* & \cdots & d_{1(j-1),t}^* \\ d_{21}^*, t & d_{22,t}^* & & d_{2(j-1),t}^* \\ \vdots & & \ddots & \vdots \\ d_{(j-1)1,t}^* & d_{(j-1)2,t}^* & & d_{(j-1)(j-1),t}^* \end{bmatrix}^{-1} \begin{bmatrix} -d_{1j,t}^* & \cdots & -d_{1n,t}^* \\ -d_{2j,t}^* & & -d_{2n,t}^* \\ \vdots & & \vdots \\ -d_{(j-1)j,t}^* & \cdots & -d_{(j-1)n,t}^* \end{bmatrix},$$

$\triangle\widehat{x}_t^{*(j)} = (\triangle x_t - \triangle x_{t-1}B_t - c_t - x_{t-1}\left(U_t^* I(S_t) D_t^*\right)_{\neq j})'$, $(U_t^* I(S_t) D_t^*)_{\neq j}$ is the resulting matrix after removing the jth column of U_t^*, the jth row of D_t^* and both the jth row and column of $I(S_t)$, and $d_t^{*(j)}$ is an $(n-j+1)$ vector containing the $(n-j+1)$ free parameters in the jth row of d_t^*. Further, $\widetilde{z}_t^{a:b}$ is the matrix comprised of the ath to bth columns of $\left(I_n \otimes x_{t-1}\left(U_t^* I(S_t)\right)_j\right)$, and $\left(U_t^* I(S_t)\right)_j$ is the jth column of $U_t^* I(S_t)$. Finally, $\zeta_t^{(j)} \sim N\left(0, Q^j\right)$ with Q^j being a $((n-j+1)\times(n-j+1))$ diagonal matrix containing the following elements $\left(\sigma^2_{1+(j-1)(n+1-j)+\sum_{k=0}^{j-1}k}, ..., \sigma^2_{j(n+1-j)+\sum_{k=0}^{j-1}k}\right)$.

Step 6: Draw the latent variable variances σ given $\widetilde{d}_T^*$

Conditional on $\widetilde{d}_T$, it can be shown that the conditional posterior density of the regime-dependent σ_i is inverse gamma

$$\sigma_i|\widetilde{d}_{iT}^* \sim IG\left(0.5\left(\vartheta_i + T\right),\ 0.5\left(\vartheta_i f_i + \sum_2^T (d_{it}^* - d_{it-1}^*)^2\right)\right), \quad i = 1, 2, .., n^*.$$

Step 7: Draw the autoregressive parameters and intercepts g given $\widetilde{y}_T, \widetilde{S}_T, \widetilde{d}_T^*, \widetilde{u}_T^*$ and Σ

Conditional on the remaining parameters, the mean and variance of g, which contains regime dependent parameters, can be obtained from the following linear, Gaussian SUR equation

$$\triangle x_t^* = (I_n \otimes z_t)\, g + \varepsilon_t$$

where $\triangle x_t^* = \triangle y_t' - \alpha_t^{*\prime} \beta_t^{*\prime} y_{t-1}'$ and $z_t = \begin{bmatrix} 1 & \triangle x_{t-1} \end{bmatrix}$. Given the adoption of a multivariate normal prior for g, the conditional posterior density of g is also multivariate normal

$$g | \widetilde{x}_T, \widetilde{d}_T^*, \widetilde{u}_T^*, \widetilde{S}_T, \Sigma \sim MVN(\overline{g}, \overline{\Sigma}_g)$$

where $\overline{\Sigma}_g = \left(X_g'(\Sigma^{-1} \otimes I_T) X_g + \Sigma_g \right)^{-1}$, $\overline{g} = \overline{\Sigma}_g \left(\Sigma_g^{-1} \underline{g} + X_g'(\Sigma^{-1} \otimes I_T) \triangle X^* \right)$, $\triangle X^* = \begin{bmatrix} \triangle x_1^* & \ldots & \triangle x_n^* \end{bmatrix}'$, $\triangle x_i^* = \begin{bmatrix} \triangle x_{i1}^* & \ldots & \triangle x_{iT}^* \end{bmatrix}'$ and $X_g = I_n \otimes \begin{bmatrix} z_1' & \ldots & z_T' \end{bmatrix}'$.

Step 8: Draw the covariance Σ given $\widetilde{x}_T, \widetilde{S}_T, \widetilde{d}_T^*, \widetilde{u}_T^*$, and g

Given the adoption of an inverse Wishart prior for Σ, the conditional posterior of Σ is also inverse Wishart with

$$\Sigma | \widetilde{x}_T, \widetilde{d}_T^*, \widetilde{u}_T^*, \widetilde{S}_T, g \sim IW(\underline{v} + T, \underline{D} + \overline{A})$$

where $\overline{A} = \sum_{t=1}^T \varepsilon_t' \varepsilon_t$, $\varepsilon_t = \triangle x_t' - \alpha_t^{*\prime} \beta_t^{*\prime} x_{t-1}' - (I_n \otimes z_t)\, g$, and $z_t = \begin{bmatrix} 1 & \triangle x_{t-1} \end{bmatrix}$.

Appendix C: Parameter estimates

Table C1: Estimated posterior medians and standard deviations of the parameters

	Median	Std Dev		Median	Std Dev		Median	Std Dev
p_{00}	0.8617	0.0451	$c_{3,1}$	0.1611	0.058300148	$B_{33,1}$	0.0124	0.1013
p_{01}	0.0493	0.0172	$c_{3,2}$	n/a		$B_{31,2}$	n/a	
p_{02}	0.2361	0.1087	$c_{3,3}$	n/a		$B_{32,2}$	n/a	
p_{03}	0.2369	0.1084	$B_{11,0}$	0.1859	0.0991	$B_{33,2}$	n/a	
p_{10}	0.1160	0.0405	$B_{12,0}$	-0.1437	0.0507	$B_{31,3}$	n/a	
p_{11}	0.9354	0.0192	$B_{13,0}$	0.3963	0.1199	$B_{32,3}$	n/a	
p_{12}	0.1762	0.0862	$B_{11,1}$	0.5246	0.0741	$B_{33,3}$	n/a	
p_{13}	0.1761	0.0858	$B_{12,1}$	-0.0187	0.0174	$\Sigma_{11,0}$	1.2895	0.2078
p_{20}	0.0097	0.0067	$B_{13,1}$	0.0417	0.0400	$\Sigma_{12,0}$	0.5571	0.2651
p_{21}	0.0067	0.0043	$B_{11,2}$	n/a		$\Sigma_{13,0}$	0.3278	0.1184
p_{22}	0.5006	0.1508	$B_{12,2}$	n/a		$\Sigma_{22,0}$	5.2092	0.8071
p_{23}	0.0548	0.0375	$B_{13,2}$	n/a		$\Sigma_{23,0}$	0.3383	0.2350
p_{30}	0.0097	0.0067	$B_{11,3}$	n/a		$\Sigma_{33,0}$	1.0057	0.1511
p_{31}	0.0067	0.0043	$B_{12,3}$	n/a		$\Sigma_{11,1}$	0.0480	0.0095
p_{32}	0.0543	0.0378	$B_{13,3}$	n/a		$\Sigma_{12,1}$	-0.0001	0.0251
p_{33}	0.5008	0.1507	$B_{21,0}$	0.6831	0.2001	$\Sigma_{13,1}$	0.0168	0.0103
ρ	0.9994	0	$B_{22,0}$	-0.4814	0.1009	$\Sigma_{22,1}$	1.1135	0.2050
σ_1	0.0012	0.0001	$B_{23,0}$	0.2969	0.2403	$\Sigma_{23,1}$	-0.0339	0.0491
σ_2	0.0012	0.0001	$B_{21,1}$	0.4694	0.3668	$\Sigma_{33,1}$	0.2053	0.0367
σ_3	0.0012	0.0001	$B_{22,1}$	0.0325	0.0795	$\Sigma_{11,2}$	n/a	
σ_4	0.0012	0.0001	$B_{23,1}$	-0.0634	0.2015	$\Sigma_{12,2}$	n/a	
σ_5	0.0012	0.0001	$B_{21,2}$	n/a		$\Sigma_{13,2}$	n/a	
σ_6	0.0012	0.0001	$B_{22,2}$	n/a		$\Sigma_{22,2}$	n/a	
$c_{1,0}$	-0.0475	0.1152	$B_{23,2}$	n/a		$\Sigma_{23,2}$	n/a	
$c_{1,1}$	0.0932	0.0340	$B_{21,3}$	n/a		$\Sigma_{33,2}$	n/a	
$c_{1,2}$	n/a		$B_{22,3}$	n/a		$\Sigma_{11,3}$	n/a	
$c_{1,3}$	n/a		$B_{23,3}$	n/a		$\Sigma_{12,3}$	n/a	
$c_{2,0}$	-0.1655	0.2392	$B_{31,0}$	0.0179	0.0879	$\Sigma_{13,3}$	n/a	
$c_{2,1}$	-0.3034	0.2572	$B_{32,0}$	-0.0053	0.0433	$\Sigma_{22,3}$	n/a	
$c_{2,2}$	n/a		$B_{33,0}$	0.2687	0.1067	$\Sigma_{23,3}$	n/a	
$c_{2,3}$	n/a		$B_{31,1}$	-0.1791	0.1352	$\Sigma_{33,3}$	n/a	
$c_{3,0}$	-0.1657	0.1108	$B_{32,1}$	-0.0393	0.0300			

Notes: Coefficients labelled 'n/a' are omitted since the rank of the multiplier matrix Θ_t is almost always 0 or 1. p_{ab} is the probability of a transition from rank a to rank b. ρ is the AR(1) coefficient for the latent variable u_t^*. σ_i is the ith element of $vec(Q)$, being the variance of d_t^*. $c_{i,r}$ is the intercept for variable i ($i \in$ {interest rates, inflation, output}) in rank r. $B_{ij,r}$ is the lagged response of the change in variable j for the change in variable i in rank r. $\Sigma_{ij,r}$ is the i,jth element of the variance of ε_t in rank r.

Appendix D: Comparison of predicted versus actual variables.

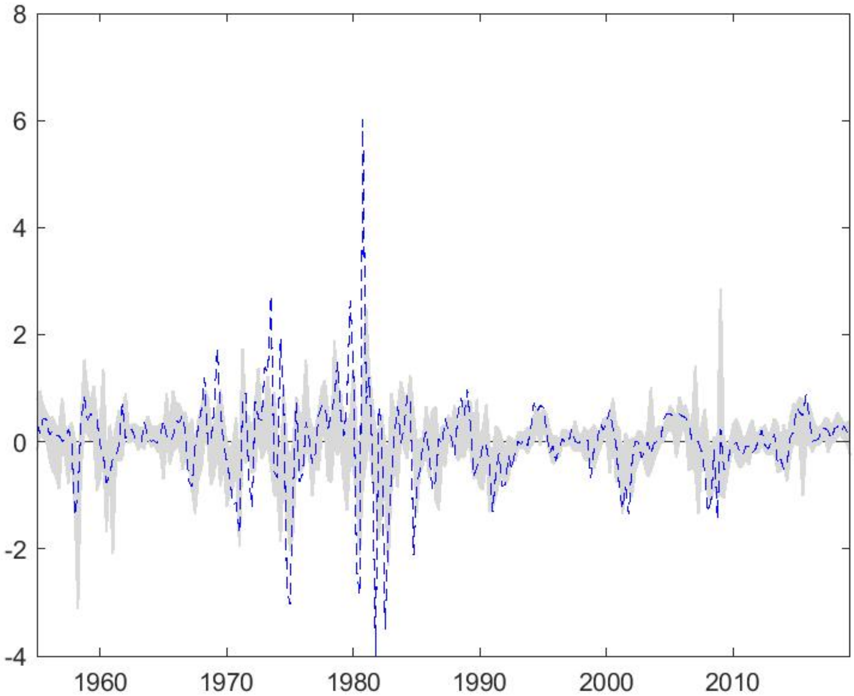

Figure D1. Dashed line is the change in the actual nominal interest rate. Shaded area is the 99 per cent credible interval of predicted values.

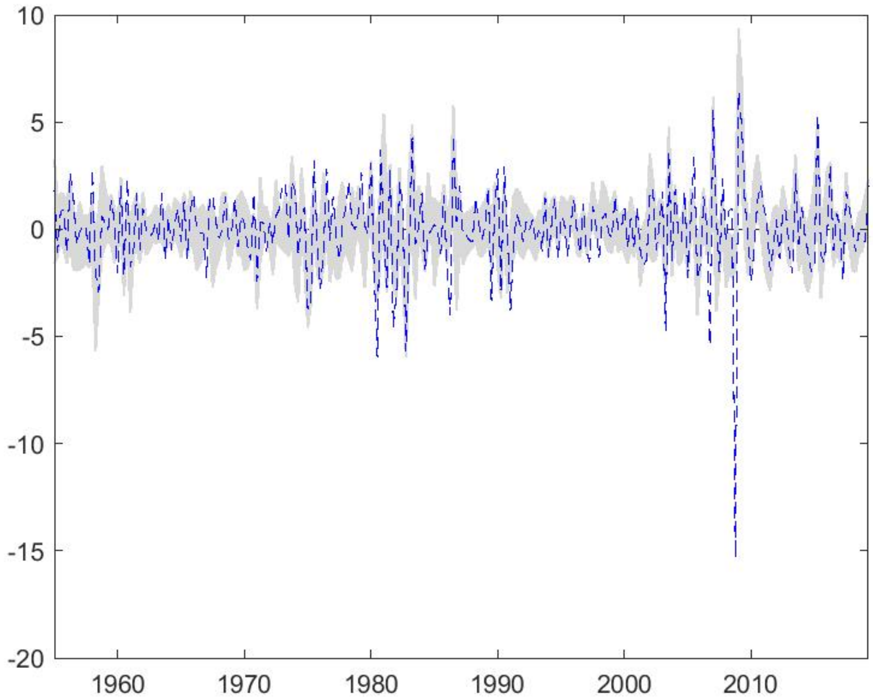

Figure D2. Dashed line is the change in actual inflation. Shaded area is the 99 per cent credible interval of predicted values.

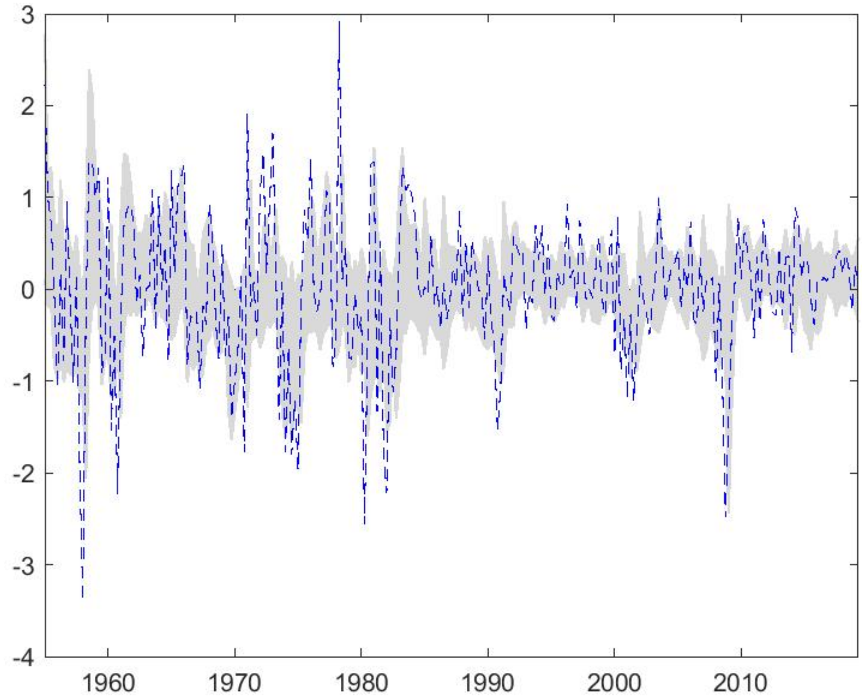

Figure D3. Dashed line is the change in the actual output gap. Shaded area is the 99 per cent credible interval of predicted values.

Appendix E: Robustness to alternative output gap measures

To examine the sensitivity of the results to the measure of the output gap, we re-estimate the model by substituting linearly detrended real GDP per capita in place of our preferred CBO-based measure of the output gap.

The time-varying probabilities of monetary policy activism in Figure E1 are extremely similar across the two measures of the output gap, as are the estimates of the probability of $S_t = 2$ (whereby the rank of the system is equal to unity) in Figure E2.

The time-varying values of $\beta_{\pi,t}$ and the associated probabilities regarding $\beta_{\pi,t} > 1$ (in Figures E3 and E4 respectively) are also similar, although the probabilities obtained when estimating the model using the linearly-detrended real GDP per capita measure appear to be within an overly tight range. Nevertheless, the overall patterns for the probability that $\beta_{\pi,t} > 1$ are similar across the two sets of estimates, resulting in monetary policy activism probabilities (Figure E1) that are largely insensitive to the choice of output gap.

The primary difference between the two models estimated using alternative output gap measures lies, as expected, in the estimates for $\beta_{y,t}$. A key issue with the estimates of $\beta_{y,t}$ when using the linearly detrended output gap measure is that it produces overly pronounced shifts in the monetary policy weights during 1996 and 2011. These disproportionate shifts clearly stem from the detrending choice, with the CBO-based estimates avoiding such issues.

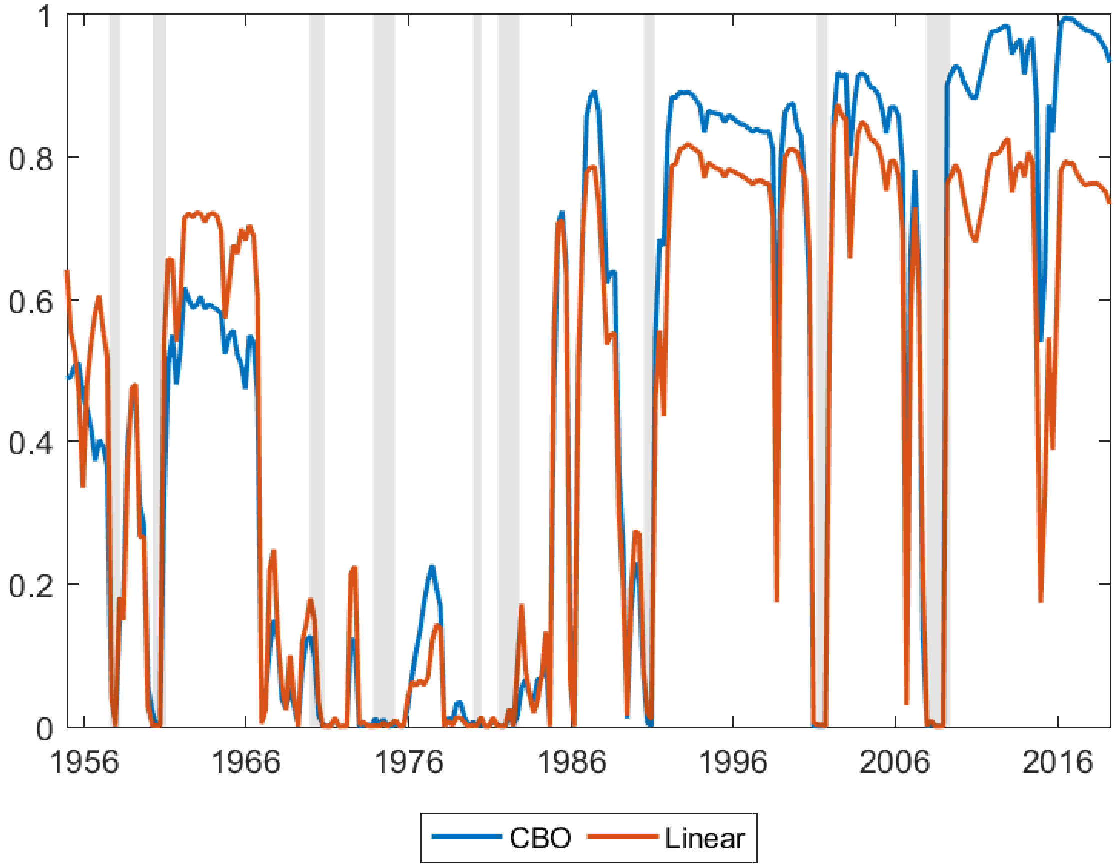

Figure E1. Time-varying probability of active monetary policy, $P(activism_t)$. CBO is based on the estimates when using the CBO-based measure of the output gap. Linear is based on the estimates when using linearly detrended real GDP per capita as the output gap measure. Shaded lines are NBER-dated recessions.

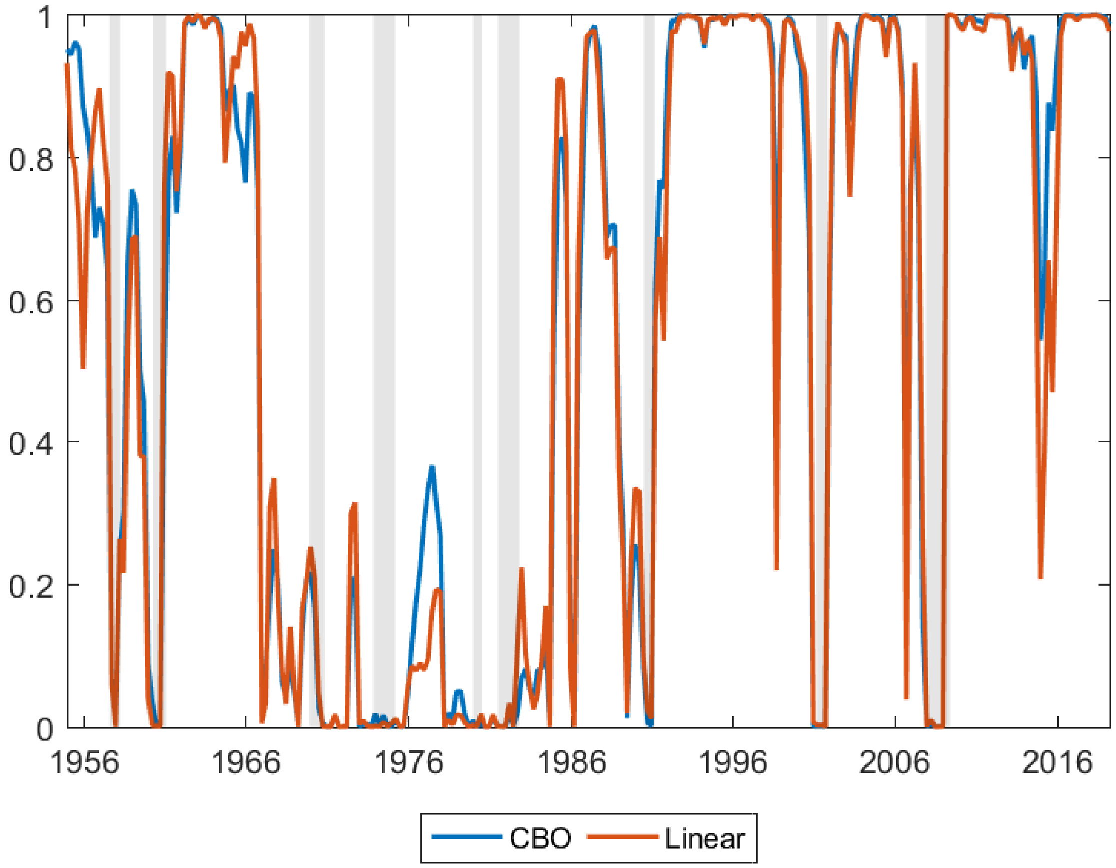

Figure E2. Time-varying probability of $S_t = 2$ (i.e. rank=1). CBO is based on the estimates when using the CBO-based measure of the output gap. Linear is based on the estimates when using linearly detrended real GDP per capita as the output gap measure. Shaded lines are NBER-dated recessions.

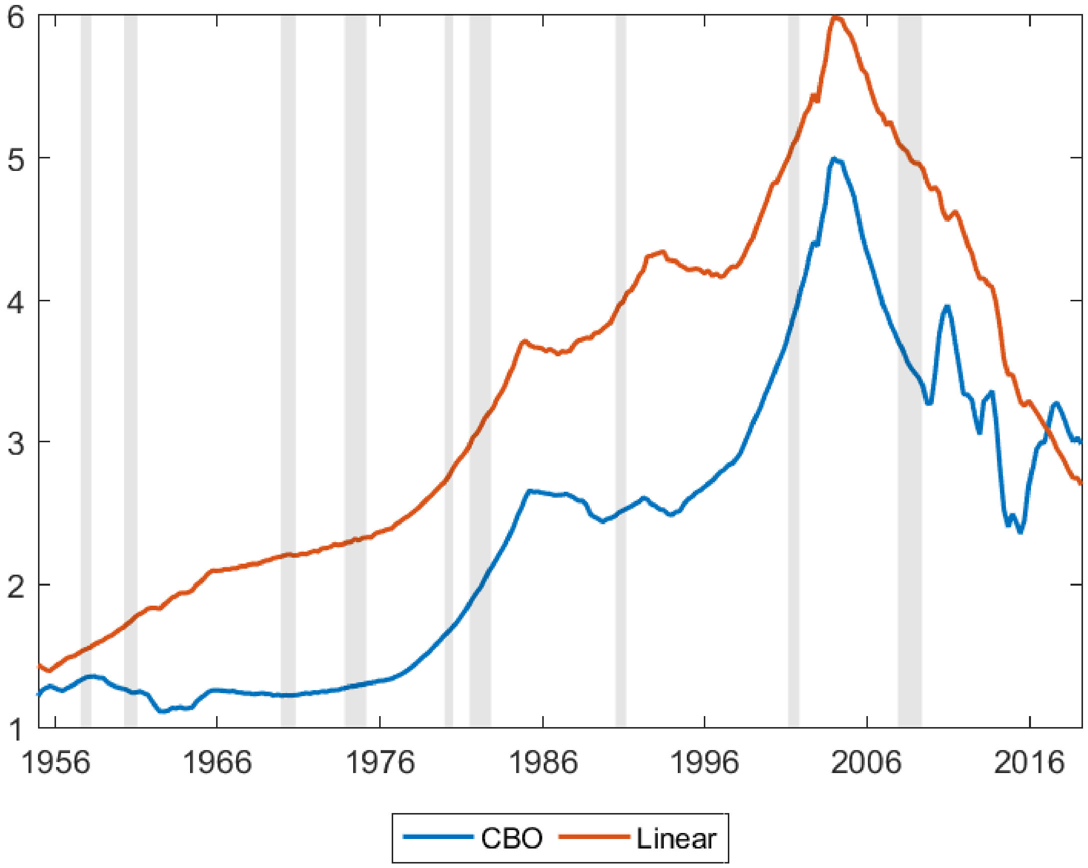

Figure E3. Time-varying estimates of the inflation targeting coefficient $\beta_{\pi,t}$. CBO is based on the estimates when using the CBO-based measure of the output gap. Linear is based on the estimates when using linearly detrended real GDP per capita as the output gap measure. Shaded lines are NBER-dated recessions.

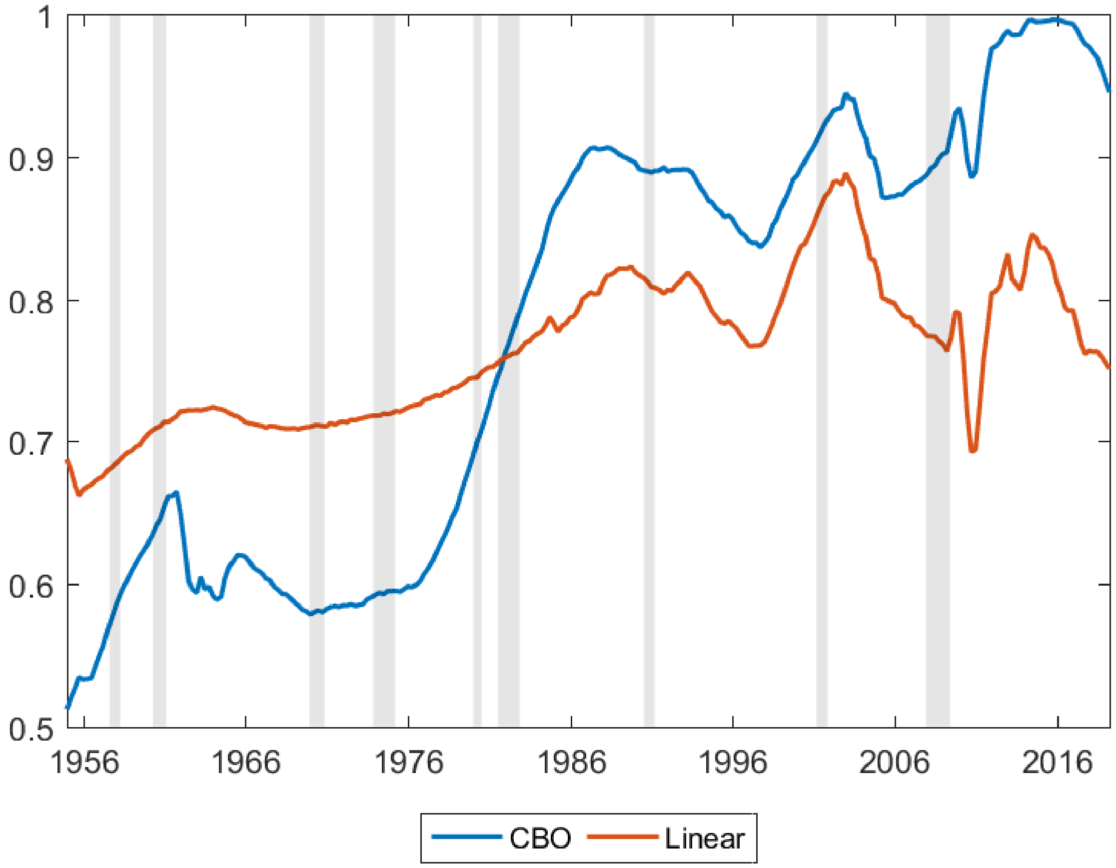

Figure E4. Time-varying probability of the inflation targeting coefficient exceeding unity at time t, $P(\beta_{\pi,t} > 1)$. CBO is based on the estimates when using the CBO-based measure of the output gap. Linear is based on the estimates when using linearly detrended real GDP per capita as the output gap measure. Shaded lines are NBER-dated recessions.

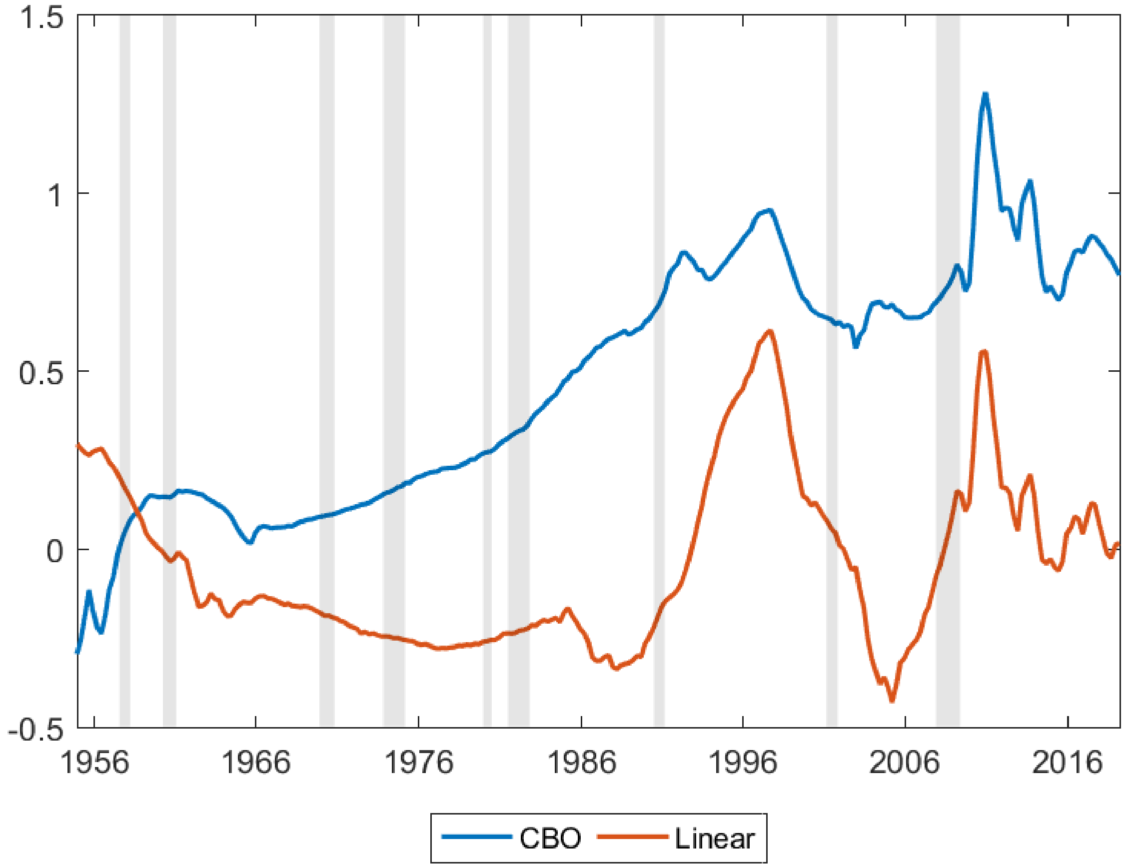

Figure E5. Time-varying estimates of the output targeting coefficient $\beta_{y,t}$. CBO is based on the estimates when using the CBO-based measure of the output gap. Linear is based on the estimates when using linearly detrended real GDP per capita as the output gap measure. Shaded lines are NBER-dated recessions.

Appendix F: Gap between monetary policy weights when rank is allowed to vary and rank is set to unity

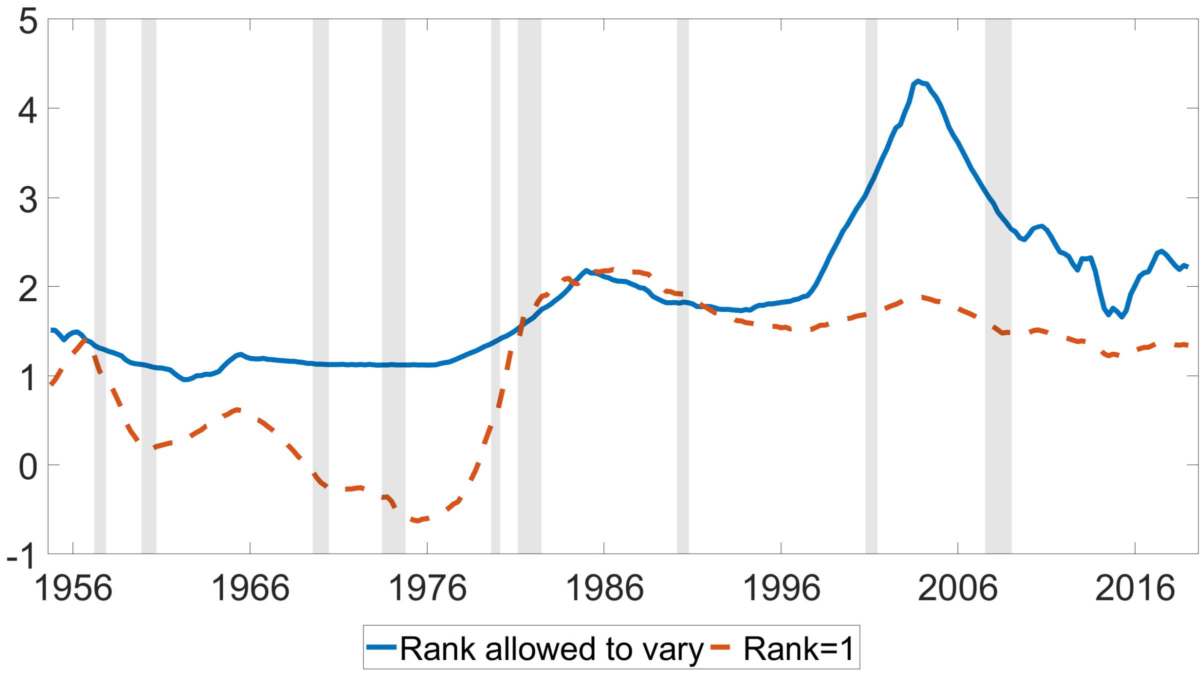

Figure F1: The gap between the estimated inflation targeting and output gap targeting weights when: (i) the rank is allowed to vary over time; and (ii) the rank=1 restriction is permanently imposed.

Appendix G: Time-varying smoothness in the model

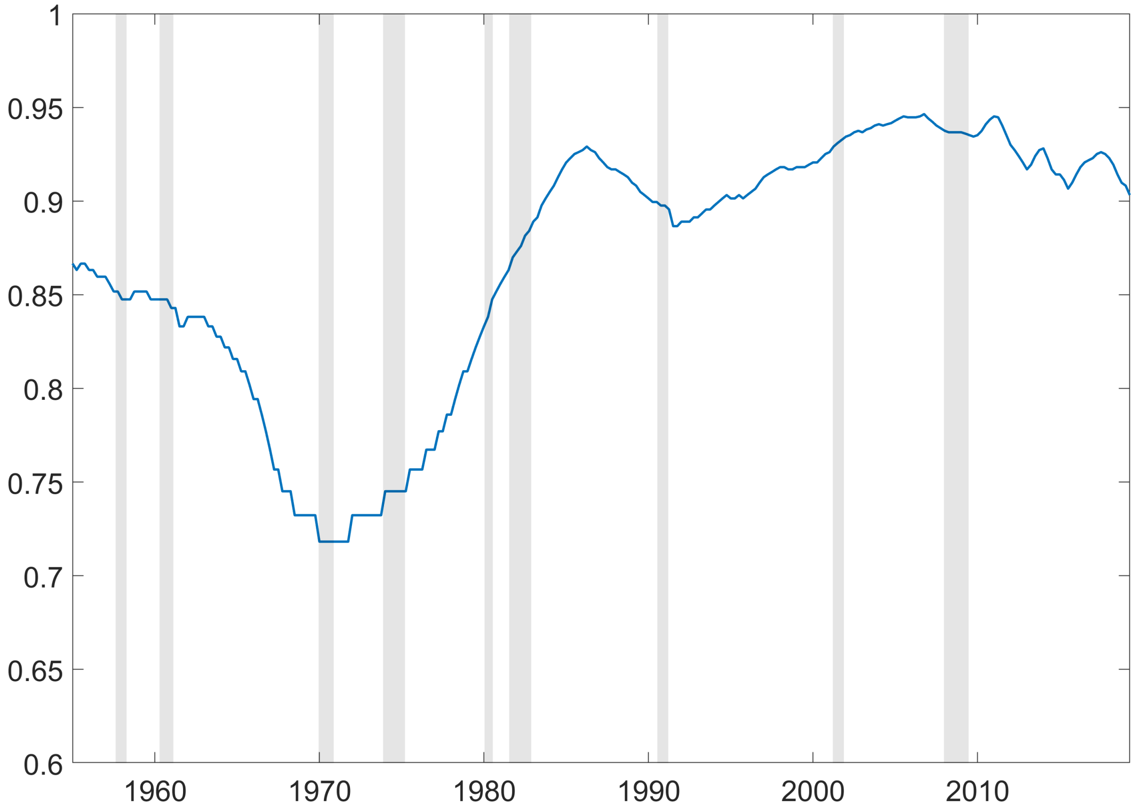

Figure G1. Implied time-varying smoothness (ρ). Shaded lines are NBER-dated recessions.

Appendix H: Comparison of activism and determinacy probabilities

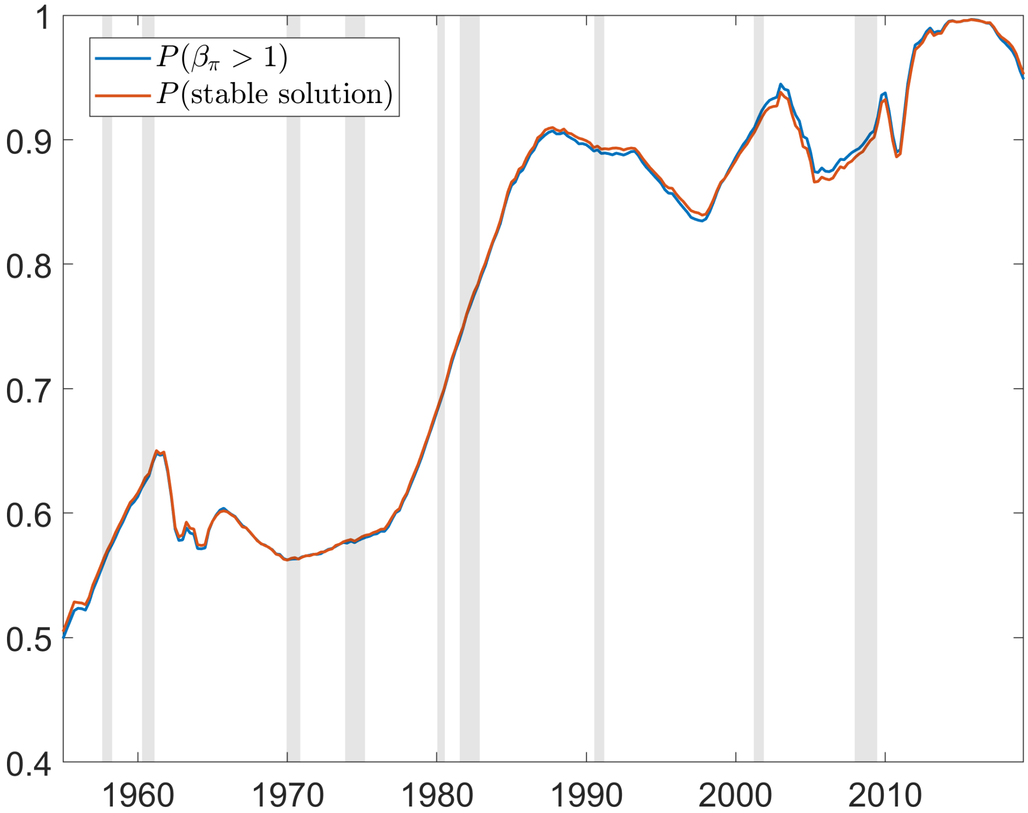

Figure H1: A comparison of the time-varying probability of monetary policy activism pursuant to $P(\beta_{\pi,t} > 1)$ and determinate monetary policy pursuant to the satisfaction of the Blanchard-Kahn conditions (denoted $P(\text{stable solution})$). Shaded lines are NBER-dated recessions.

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