Appendix for "Does household debt affect the size of the fiscal multiplier?"

Juan Zurita*

*Department of Economics, University of Technology Sydney.

Appendix A: A Simple Model

Table 1: OLS Estimations

	Australia	Norway	Sweden	US	Japan	UK	France	Germany	Canada	Italy
	Log Real GDP									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
RealGDP(t-1)	0.90***	0.83***	0.90***	0.96***	0.88***	0.99***	1.06***	0.99***	0.98***	1.07***
	(0.026)	(0.044)	(0.046)	(0.018)	(0.068)	(0.040)	(0.049)	(0.033)	(0.030)	(0.031)
GovExp(t-1)	-0.0098	0.253**	0.217	0.206**	2.667*	0.0542	-0.00284	0.255*	0.0798	0.152
	(0.0179)	(0.084)	(0.157)	(0.0654)	(1.160)	(0.0567)	(0.0909)	(0.120)	(0.0450)	(0.139)
HDebt(t-1)	0.0721	0.665**	0.461	0.481**	18.30*	0.346	-0.640	2.310*	0.792*	0.390
	(0.0562)	(0.233)	(0.445)	(0.146)	(8.176)	(0.291)	(0.737)	(0.922)	(0.329)	(0.254)
GovExpxHDebt	-0.0141	-0.140**	-0.0892	-0.134**	-1.374*	-0.0307	0.0590	-0.208*	-0.0725*	-0.182
	(0.0130)	(0.0475)	(0.0846)	(0.0407)	(0.612)	(0.0269)	(0.0674)	(0.0837)	(0.030)	(0.123)
Constant	-3.02***	-4.38***	-1.694	-1.626**	-30.95*	-0.0716	2.322	-4.130*	-1.837*	0.888*
	(0.760)	(1.280)	(1.804)	(0.498)	(14.72)	(0.596)	(1.561)	(1.646)	(0.906)	(0.379)
N	207	163	107	221	105	214	161	198	205	157
Year	x	x	x	x	x	x	x	x	x	x
Quarter	x	x	x	x	x	x	x	x	x	x

Note: This table shows OLS estimations for the model describe in equation ??.

Appendix B: Data Sources

Table 2: Data Sources

Country	Source	From	To
United States	FRED / BIS	1965Q3	2019Q4
United Kingdom	FRED / BIS	1967Q4	2020Q3
Canada	FRED / BIS	1970Q1	2020Q2
Germany	FRED / BIS	1971Q4	2020Q2
Italy	FRED / BIS	1981Q1	2020Q2
France	FRED / BIS	1978Q4	2020Q2
Japan	FRED / BIS	1994Q3	2020Q2
Australia	FRED / BIS	1971Q1	2019Q4
Sweden	FRED / BIS	1993Q1	2019Q4
Norway	FRED / BIS	1993Q1	2019Q4

Note: This table summarises the data sources and time period for each country considered in the analysis.

Appendix C: Bayesian Econometric Method

Likelihood function

We use the collapse Gibbs sampling algorithm developed by Koop *et al.* (2006) to carry out efficient posterior simulations. Following Strachan & van Dijk (2006) and Gefang (2012) we derive the likelihood representation.

$$x_t = \omega_t \times \Phi + F(z_t)\left(\omega_t \times \Phi^z\right) + \varepsilon_t \tag{1}$$

where $\omega_t = (x_{t-1}, ..., x_{t-p})$, $\Phi = (\Gamma_1^{'}, ..., \Gamma_p^{'}, \xi')'$ and $\Phi^z = (\Gamma_1^{z'}, ..., \Gamma_p^{z'}, \xi^{z'})'$.

To simply our notation, let's define $X_0 = (x_1', x_2', ..., x_T')$ and $X = (X, F^z X)$, where $X = (\omega_1', \omega_2', ..., \omega_T')'$

and $F^z = diag(F(z_1), F(z_2), ..., F(z_T))$. Then, we set $B = (\Phi', \Phi^{z'})$ and $E = (\varepsilon_1', \varepsilon_2', ..., \varepsilon_T')'$. With this notation, we write model (1):

$$X_0 = X\Phi + F^z X\Phi^z + E = XB + E \tag{2}$$

With this notation, the likelihood function of equation (1) is:

$$L(x|\Sigma, B, \gamma, c, \mu) \propto |\Sigma|^{-(T/2)} exp\left\{-\frac{1}{2} tr\Sigma^{-1}E'E\right\} \tag{3}$$

Vectorising our model in equation (2), we obtain:

$$x_0 = xb + e, \tag{4}$$

where $x_0 = vec(X_0)$, $b = vec(B)$ and $e = vec(E)$. As a result, $E(ee') = V_e = \Sigma \otimes I_T$.

Reexpresing the term in braquets of equation (3), we obtain:

$$-\tfrac{1}{2} tr\Sigma^{-1}E'E = -\tfrac{1}{2}[e'(\Sigma^{-1} \otimes I_T)e]$$

$$= -\frac{1}{2}[s^2 + (b - \hat{b})'V^{-1}(b - \hat{b})], \tag{5}$$

where $s^2 = x_0' M_\nu x_0,\ M_\nu = \Sigma^{-1} \otimes [I_T - X(X'X)^{-1}X'], \hat{b} = [I_n \otimes X(X'X)^{-1}X']x_0)$and $V = \Sigma \otimes (X'X)^{-1}$. We can rewrite our likelihood as:

$$L(x|\Sigma, B, \gamma, c, \mu) \propto |\Sigma|^{-(T/2)} exp\left\{-\frac{1}{2}[s^2 + (b - \hat{b})'V^{-1}(b - \hat{b})]\right\} \tag{6}$$

It can be seen that the likelihood of b is normal condition on the rest of the parameters.

Priors

Following Strachan & van Dijk (2006) the full prior distribution for the parameters in a given model is then given by

$p(\Sigma, b, \gamma, c, \mu, \eta|M_\omega) = p(b|\Sigma, \gamma, c, \eta, \mu, M_\omega)p(\Sigma|M_\omega)p(\gamma|M_\omega)p(c|M_\omega)p(\eta|M_\omega)p(\mu|M_\omega)$

Each Parameter probability.

$p(b|\Sigma, \gamma, c, \eta, \mu, M_\omega) = N(0, \eta^{-1} I_k)$

$p(\Sigma|M_\omega) = |\Sigma|^{-(n+1)/2}$

$p(\gamma|M_\omega) = Gamma(1, 0.001)$

$p(c|M_\omega) = Uniform(0.1, 0.9)$

$p(\eta|M_\omega) = Gamma(3, 4)$

$p(\mu|M_\omega) = N(\mu_0, \Sigma_\mu)$

Posterior Probabilities

Bayesian inference methods have the advantage of using the support for alternative models by comparing posterior densities. Using Savage-Dickey density ratios (SDDR), we derive Bayes factors to compute the posterior probabilities and select the transition function (Verdinelli & Wasserman, 1995). The SDDR is a specific expression of the Bayes factor when we compare the marginal probability of the restricted model (null hypothesis) and the unrestricted model. In our case, the marginal probability of the restricted model is the point where all coefficients of b are zero, except for the ones corresponding to the intercepts. This model nests within all models since the conditional posterior of b is Normal. Taking the inverse $1/BF$ yields the support under the alternative hypothesis compared to the null hypothesis.

$$SDDR = \frac{p(b=0|\Sigma,\gamma,c,\mu,M_\omega)}{p(b|\Sigma,\gamma,c,\mu,M_\omega)}$$

Once we know the Bayes Factors for each of our models and their prior probabilities, we are able to compute the corresponding posterior densities via simple algebra.

Posterior computation

Combining the likelihood function with our prior $p(\Sigma)$, we can see that the posterior of Σ, conditional on γ, c, is Inverted Wishart with scale matrix $E'E$ and degrees of freedom T. The posterior distribution of b, conditional on $\Sigma, \gamma, c, \mu, \eta$, is normal with mean $\overline{b}$=$\overline{V}_b V^{-1}\hat{b}$ and covariance matric $\overline{V} = \Sigma \otimes (X'X + \eta I_k)^{-1}$.

Following Koop *et al.* (2006), we run the following Gibbs sampler:

1. Initialize $(b, \Sigma, \gamma, c, \eta, \mu)$.

2. Draw $\Sigma | b, \gamma, c, \eta . \mu$ from $IW(E'E, T)$.

3. Draw $b | \Sigma, \gamma, c, \eta, \mu$ from $N(\bar{b}, \overline{V}_b)$.

4. Draw $\gamma | \Sigma, b, c, \eta, \mu$ using M-H algorithm.

5. Draw $c | \Sigma, b, \gamma, \eta, \mu$ using Griddy - Gibbs sampler.

6. Draw $\eta | \Sigma, b, \gamma, c, \mu$ from $G(\overline{v}_\eta, \overline{\mu}_\eta)$.

7. Draw $\mu | \Sigma, b, \gamma, c, \eta$ from $N(\mu_0, \Sigma_\mu)$.

8. Repeat steps 1-7.

We use Bayes factors to compare the models. The null hypothesis is the point where all coefficient of b are zero, except for the ones corresponding to the intercepts. Under this hypothesis we have the model with no explanatory variables except for the intercepts. This model nests within all models. Taking the inverse $1/BF$ yields the support under the alternative hypothesis compared to the null hypothesis.

We use algebra to compute the posterior probability. By definition:

$$BF_{1,2} = \frac{p(Data/M_1)}{p(Data/M_2)}$$

Using Bayes Rule, we obtain:

$$P(M_1/Data) = \frac{p(Data/M_1)p(M_1)}{p(Data/M_1)p(M_1) + p(Data/M_2)p(M_2)}$$

$$P(M_1/Data) = \frac{BF_{1,2}p(M_1)}{BF_{1,2}p(M_1) + p(M_2)}$$

$$P(M_1/Data) = \frac{BF_{1,2}p(M_1)}{BF_{1,2}p(M_1) + (1 - p(M_1))}$$

By Posterior Odds:

$$P(M_2/Data) = 1 - P(M_1/Data)$$

Another channel to compute the posterior probability.

$$\frac{p(M_1/Data)}{p(M_2/Data)} = PosteriorOdds = BayesFactor * PriorOdds$$

$$1 = P(M_1/Data) + P(M_2/Data)$$

$$P(M_2/Data) = P(M_1/Data)/PosteriorOdds$$

Generalised Impulse Response Functions

Following Pesaran & Shin (1998) we define the generalised impulse response function (GIRF) of x_t at horizon n by the following equation:

$$GI_x(n, \delta, \Omega_{t-1}) = E(x_{t+n}|\varepsilon = \delta, \Omega_{t-1}) - E(x_{t+n}|\Omega_{t-1}) \tag{7}$$

where $x_t = (x_{1t}, x_{2t}, ..., x_{mt})$, Ω_{t-1} is a non-decreasing information set which denotes the known history of the economy up to time *t-1*, and δ identifies the composition of shocks. An advantage of the generalised impulse response functions is that they are shock and history dependent for non linear models, in comparison to linear systems whose impulse responses are history invariant.

Rather than shocking all elements of ε_t, an alternative approach could involve shocking only one element, for example the j^{th} element, and then integrating out the effects of other shocks using either an assumed distribution or the historically observed distribution of errors. In this case, the GIRF is written as:

$$GI_x(n, \delta_j, \Omega_{t-1}) = E(x_{t+n}|\varepsilon_{jt} = \delta_j, \Omega_{t-1}) - E(x_{t+n}|\Omega_{t-1}) \tag{8}$$

Under the assumption the ε_t has a normal distribution, we can reexpress the conditional expecting value of ε_t as

$$E(\varepsilon_t|\varepsilon_{jt} = \delta_j) = (\sigma_{1j}, \sigma_{2j}, ..., \sigma_{mj})' \times \sigma_{jj}^{-1} \times \delta_j = \sum \varepsilon_t \times \sigma_{jj}^{-1} \times \delta_j \tag{9}$$

Finally, by setting $\delta_j = \sqrt{\sigma_{jj}}$ we can write the generalised impulse response function as

$$\Psi_j(n) = \delta_j^{-1/2} \boldsymbol{A_n} \sum \varepsilon_j, \qquad n = 1, 2, ... \tag{10}$$

where $\boldsymbol{A}$ represents the $m \times m$ coefficient matrix. Equation 10 quantifies the impact of one-standard error shock to the j^{th} equation at time t on the expected values of x at time $t+n$. For more details, refer to Pesaran & Shin (1998), Potter (2000) and Koop *et al.* (1996).

Orthogonalized impulse response functions

Suggested by Sims (1980), the orthogonal approach consist of resolving the problem surrounding the selection of δ by using a Cholesky decomposition of the error covariance matrix $E(\varepsilon_t \varepsilon_t') = \boldsymbol{\Sigma}$:

$$\boldsymbol{PP'} = \boldsymbol{\Sigma} \tag{11}$$

where $\boldsymbol{P}$ is an $m \times m$ lower triangular matrix. Undet this approach the orthogonalized impulse function of a unit shock to the j^{th} equation on x_{t+n} is given by

$$\psi_j^o(n) = \boldsymbol{A_n P e}_j, \qquad n = 0, 1, 2, ..., \tag{12}$$

where $\boldsymbol{e}_j$ is an $m \times 1$ vector with ones at its j_{th} elements and zeros elsewhere.

Appendix D: Empirical Results

Posterior Probabilities

Table 3: Posterior probabilities for models with different transition functions.

STVECM Specification	Australia	Sweden	Norway	US	UK	Canada	Germany	Italy	France	Japan
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**
					Lag p = 6					
$f = HousingPrices, z_t = \triangle^{y/y} p_{t-1}$	**0.980**	**0.640**	0.195	**0.742**	0.001	0.002	0.000	**0.772**	0.062	**0.503**
$f = HouseholdDebt, z_t = \triangle^{y/y} h_{t-1}$	0.013	0.242	0.144	0.176	**0.998**	**0.894**	**0.992**	0.173	0.105	0.173
$f = HousingPrices, z_t = \triangle^{q/q} p_{t-1}$	0.007	0.111	**0.455**	0.024	0.000	0.054	0.000	0.046	**0.832**	0.053
$f = HouseholdDebt, z_t = \triangle^{q/q} h_{t-1}$	0.000	0.007	0.206	0.058	0.001	0.051	0.007	0.009	0.001	0.271
					Lag p = 5					
$f = HousingPrices, z_t = \triangle^{y/y} p_{t-1}$	**0.831**	**0.889**	**0.616**	**0.970**	0.000	0.000	0.000	**0.986**	0.012	**0.644**
$f = HouseholdDebt, z_t = \triangle^{y/y} h_{t-1}$	0.162	0.079	0.084	0.025	**1.000**	**0.973**	**0.970**	0.013	0.047	0.083
$f = HousingPrices, z_t = \triangle^{q/q} p_{t-1}$	0.007	0.032	0.290	0.001	0.000	0.017	0.000	0.001	**0.941**	0.010
$f = HouseholdDebt, z_t = \triangle^{q/q} h_{t-1}$	0.000	0.000	0.009	0.004	0.000	0.010	0.030	0.000	0.000	0.263
					Lag p = 4					
$f = HousingPrices, z_t = \triangle^{y/y} p_{t-1}$	**0.907**	**0.968**	0.044	**1.000**	0.000	0.000	0.000	**0.984**	0.007	**0.670**
$f = HouseholdDebt, z_t = \triangle^{y/y} h_{t-1}$	0.092	0.023	0.200	0.000	**1.000**	**0.999**	**0.898**	0.016	0.412	0.098
$f = HousingPrices, z_t = \triangle^{q/q} p_{t-1}$	0.001	0.009	**0.751**	0.000	0.000	0.001	0.000	0.000	**0.581**	0.003
$f = HouseholdDebt, z_t = \triangle^{q/q} h_{t-1}$	0.000	0.000	0.006	0.000	0.000	0.000	0.102	0.000	0.000	0.228

Note: We use the Savage Dickey Ratio to compute posterior probabilities under the assumption of uniform equal prior distribution. f represents housing prices or household debt to GDP in the transition function. In the first panel, all models were run with lag $p = 6$, while in the second and third panels with use lags $p = 5$ and $p = 4$. Estimation sample for each country can be found in Table 2.

Models Comparison

We use posterior probabilities, calculated from the Bayes factors, to examine which transition variable plays a more important role in triggering regime changes. Table 3 presents our results, which were led by the assumption that each of the variables chosen as a transition function was the most likely to account for the higher percentage of the posterior mass. In our benchmark model, we compared Bayesian posterior probabilities for a model with lag $p=6$. Table 3 also displays posterior probabilities for models with lags $p=4$ and $p=5$.

After adjusting the model lag to $p=5$, our evidence shows that the variables chosen as a transition function in the benchmark model remain the same. This means that using housing prices in the transition function for Australia (83%), Sweden (89%), Norway (61%), United States (97%), Italy (98%), France (94%) and Japan (64%), helps us to account for the highest percentage of the posterior mass. On the other hand, household debt to GDP accounts for the higher percentage of the posterior mass for United Kingdom (100%), Canada (99%) and Germany (89%).

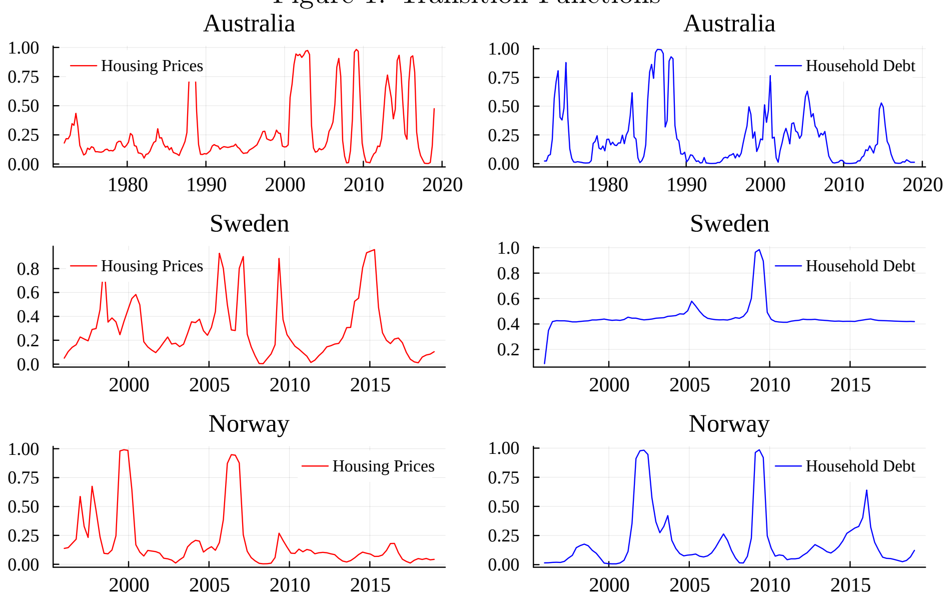
When we adjust the model lag to $p=4$, our results show that the selection of variable for the transition function does not change, and the probability to account for a higher percentage of the posterior mass increases for all countries, except for France. Our results led us to conclude that changing the model lag does not affect the transition variable that accounts for the highest percentage of the posterior mass.

Transition Variables

Figures 1 and 2 show the values of the smooth transition functions for Australia, Sweden, Norway, and the world's seven largest economies when we use housing prices (left column) and household debt to GDP (right column), expressed as year-to-year variation. We take advantage of the regime changes to identify periods of low and high household debt.

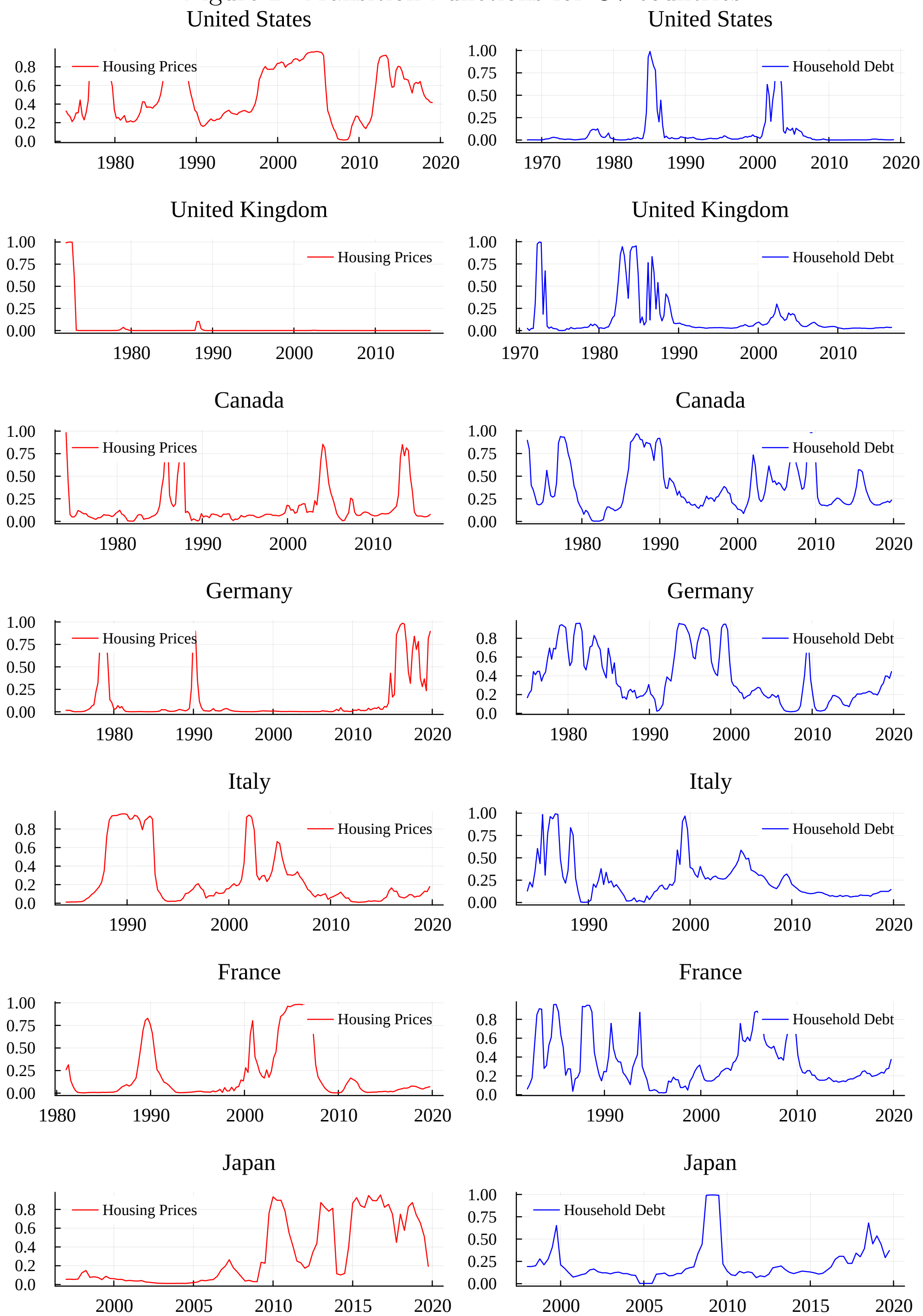
Figures 3 and 4 display smooth transition functions when we use quarter-to-quarter variations to identify regime changes.

Figure 1: Transition Functions



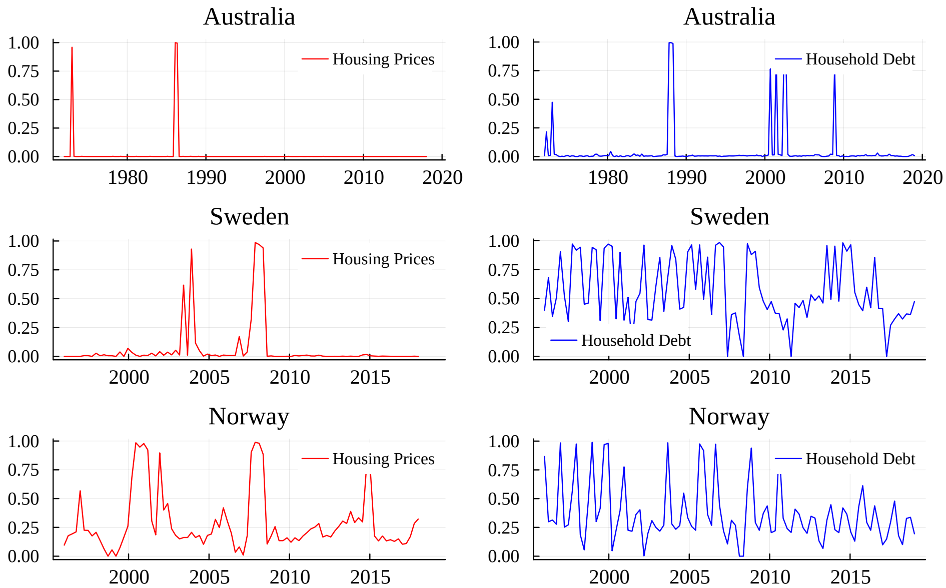
Note: This figure presents the transition functions when we use housing prices (left column) and household debt to GDP (right column) in Australia, Sweden and Norway. Time series are included as year-to-year variation. Estimation sample for each country can be found in Table 2.

Figure 2: Transition Functions for G7 countries



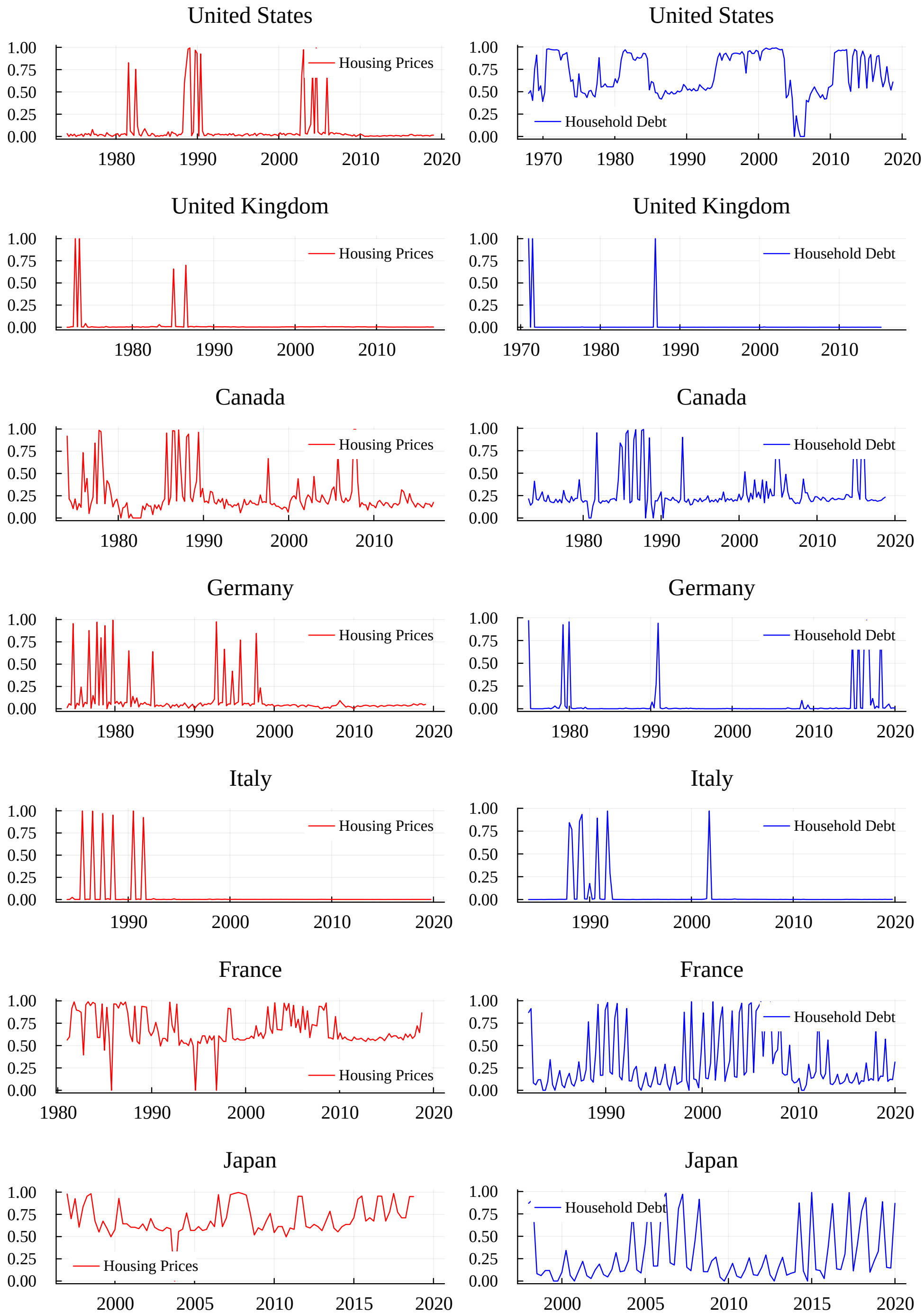
Note: This figure presents the transition functions when we use housing prices (left column) and household debt to GDP (right column) in G7 countries. Time series are included as year-to-year variation. Estimation sample for each country can be found in Table 2.

Figure 3: Transition Functions with quarter-to-quarter time series



Note: This figure presents the transition functions when we use housing prices (left column) and household debt to GDP (right column) in Australia, Sweden and Norway. Time series are included as quarter-to-quarter variation. Estimation sample for each country can be found in Table 2.

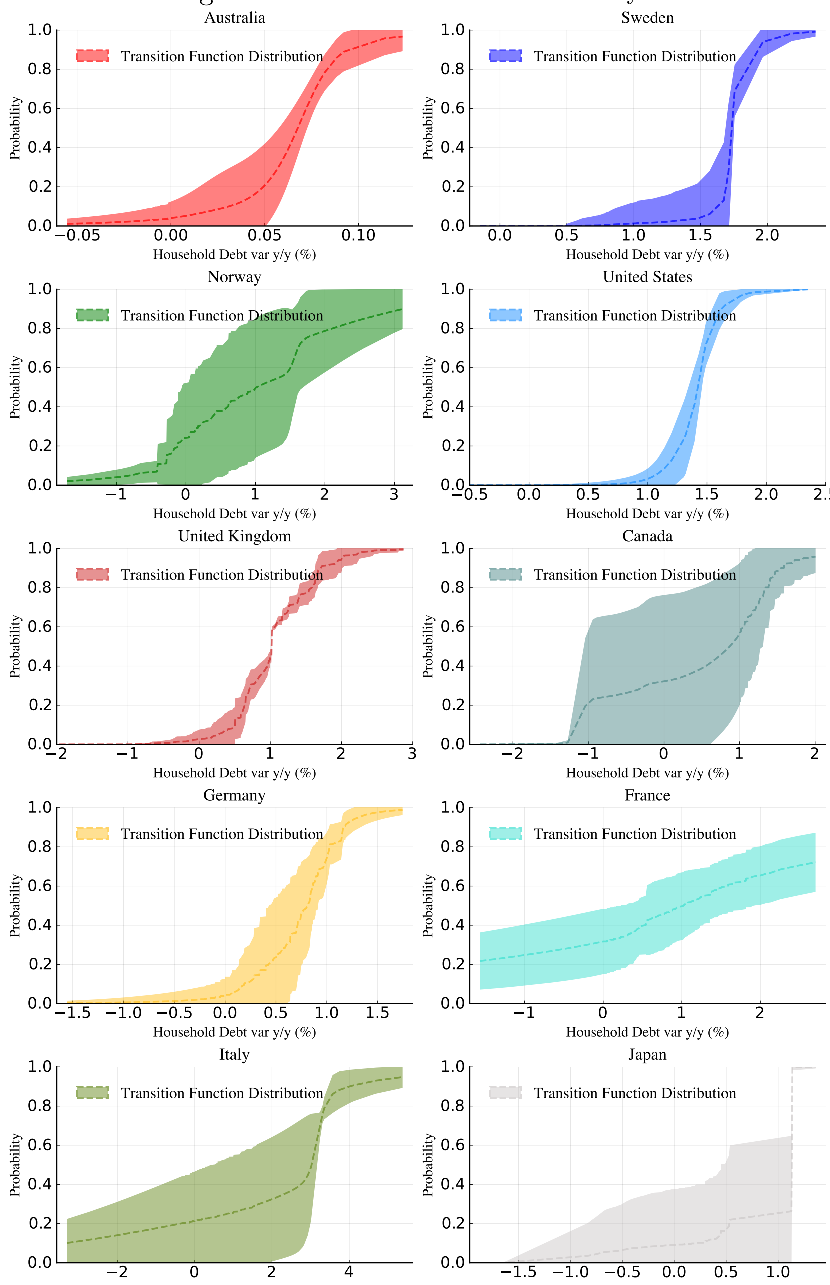
Figure 4: Transition Functions with quarter-to-quarter time series



Note: This figure presents the transition functions when we use housing prices (left column) and household debt to GDP (right column) in G7 countries. Time series are included as quarter-to-quarter variation. Estimation sample for each country can be found in Table 2.

Transition Functions

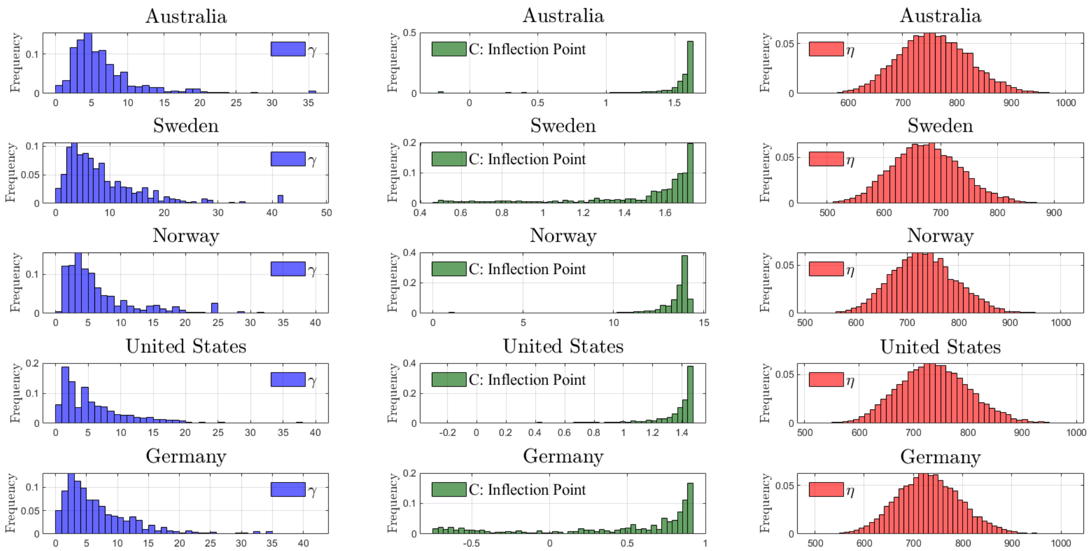
Figure 5: Transition Function Probability



Note: This figure presents the transition probability functions for all countries in our sample when we consider household debt-to-GDP as a transition variable.

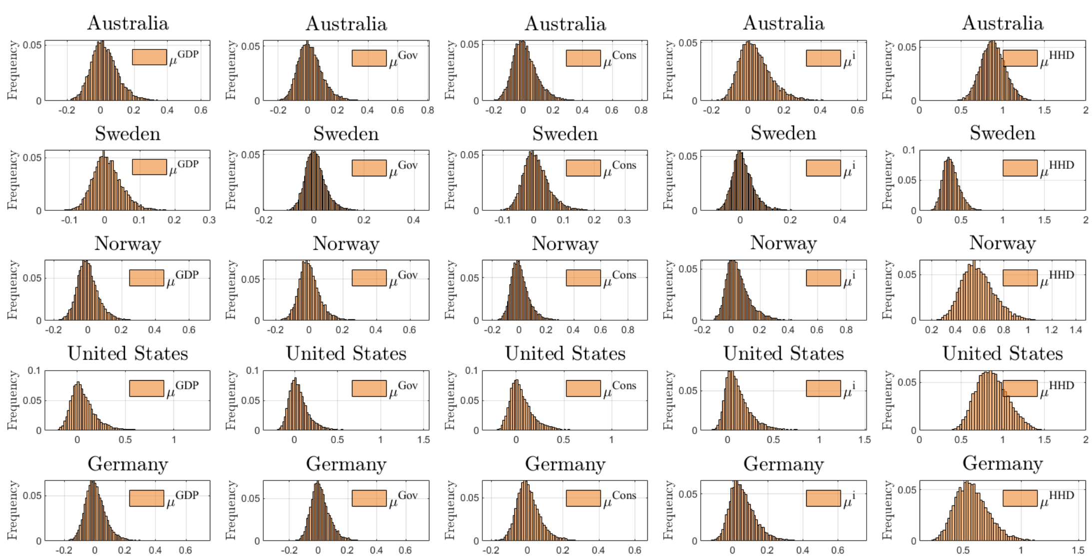
Key Parameters Posterior Probabilities Distributions

Figure 6: Posterior Probabilities: γ, η and c



Note: This graph shows the posterior probabilities for γ (speed of transition between regimes), c (inflection point in the transition variables) and η (shrinkage parameter for lags estimates).

Figure 7: Posterior Probabilities: μ



Note: This figure shows posterior probabilities for μ, which identifies the linear deterministic trends (steady states) of the variables included in the model. GDP, Gov, Cons, i and HHD stand for Gross Domestic Product, government expenditure, private consumption, interest rate and household debt to GDP ratio.

GIRFs: Cumulative Government Spending Multipliers

Tables 4 and 5 display the generalised impulse response functions for government spending when we select lag $p = 4$ and $p = 5$.

Table 4: GIRFs: Government Spending Multipliers - Lag 4

	Australia		Norway		United States		Germany		Sweden	
Horizon	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	0.622	0.062	0.663	0.359	-0.938	0.941	0.360	1.719	2.309	2.266
	(0.056)	(0.044)	(0.091)	(0.083)	(0.122)	(0.022)	(0.029)	(0.022)	(0.113)	(0.128)
2	-0.528	-2.565	0.311	0.915	-0.312	0.157	0.844	1.077	2.545	4.268
	(0.100)	(0.086)	(0.168)	(0.229)	(0.172)	(0.051)	(0.060)	(0.075)	(0.265)	(0.503)
4	-4.591	-3.085	1.466	0.143	-0.824	0.756	2.193	1.956	1.549	7.699
	(0.108)	(0.229)	(0.291)	(0.239)	(0.504)	(0.067)	(0.133)	(0.084)	(0.378)	(0.583)
8	-5.573	-0.749	0.896	1.250	-1.004	2.002	2.341	3.589	0.525	17.30
	(0.229)	(0.874)	(0.394)	(0.219)	(1.526)	(0.136)	(0.258)	(0.286)	(0.411)	(2.254)
12	-4.628	1.429	0.445	1.217	-3.73	2.331	-0.754	2.773	-4.852	20.356
	(0.209)	(0.818)	(0.441)	(0.263)	(2.153)	(0.219)	(0.207)	(0.331)	(1.030)	(4.107)
16	-3.323	1.006	1.503	2.627	-0.779	4.157	0.668	3.789	-3.154	19.477
	(0.167)	(0.726)	(1.398)	(0.286)	(3.241)	(0.277)	(0.224)	(0.150)	(1.035)	(5.912)
20	-0.376	1.963	5.198	2.534	-5.008	4.234	1.133	2.951	-4.126	12.75
	(0.202)	(0.836)	(0.972)	(0.397)	(2.722)	(0.549)	(0.258)	(0.175)	(1.119)	(2.546)

Notes: Fiscal multipliers represent the percent change of GDP after increasing government expenditures by 1 percent. Standard deviation in brackets. Lag $p = 4$. Estimation sample for each country can be found in Table 2.

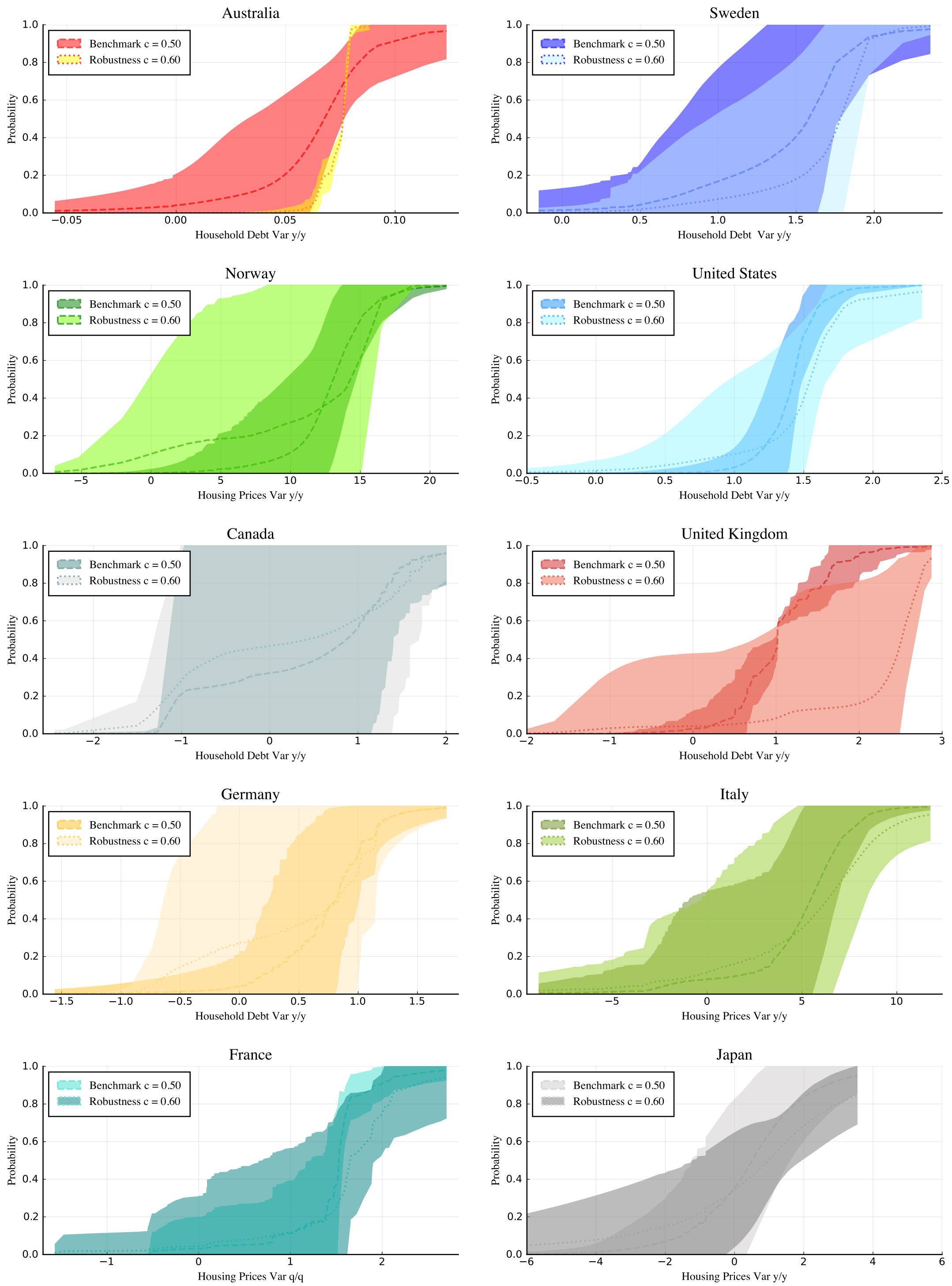
Table 5: GIRFs: Government Spending Multipliers - Lag 5

	Australia		Norway		United States		Germany		Sweden	
Horizon	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*	GIRF *Low*	GIRF *High*
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	-0.281	0.498	1.021	0.708	-0.262	1.228	0.538	0.342	-0.773	0.183
	(0.049)	(0.041)	(0.136)	(0.077)	(0.049)	(0.159)	(0.035)	(0.039)	(0.089)	(0.090)
2	-1.689	0.300	1.908	0.684	-3.279	3.263	0.981	0.909	-0.187	-4.441
	(0.155)	(0.128)	(0.487)	(0.081)	(0.287)	(0.395)	(0.158)	(0.210)	(0.200)	(0.369)
4	-3.084	2.176	1.056	-0.426	-3.219	3.300	2.735	1.938	0.773	-6.661
	(0.421)	(0.187)	(0.453)	(0.106)	(0.237)	(0.434)	(0.327)	(0.247)	(0.620)	(1.187)
8	-13.714	5.003	2.411	-1.319	-3.325	-9.144	2.544	3.947	6.146	-5.117
	(0.898)	(0.291)	(0.988)	(0.162)	(0.299)	(3.063)	(0.203)	(0.329)	(1.978)	(1.891)
12	-9.559	9.872	-1.101	-2.769	-7.343	0.450	0.414	4.31	10.059	7.016
	(0.716)	(0.426)	(0.773)	(0.191)	(0.708)	(1.215)	(0.344)	(0.345)	(3.117)	(2.279)
16	-4.031	8.090	0.555	-1.299	-11.184	2.019	0.291	9.375	8.790	12.182
	(0.745)	(0.481)	(0.596)	(0.295)	(0.774)	(2.178)	(0.380)	(0.377)	(1.636)	(3.074)
20	-2.999	6.222	0.638	-0.626	-8.466	1.235	-0.151	6.803	9.085	6.357
	(0.587)	(0.565)	(0.714)	(0.257)	(0.316)	(1.334)	(0.689)	(0.285)	(0.769)	(1.069)

Notes: Fiscal multipliers represent the percent change of GDP after increasing government expenditures by 1 percent. Standard deviation in brackets. Lag $p = 5$. Estimation sample for each country can be found in Table 2.

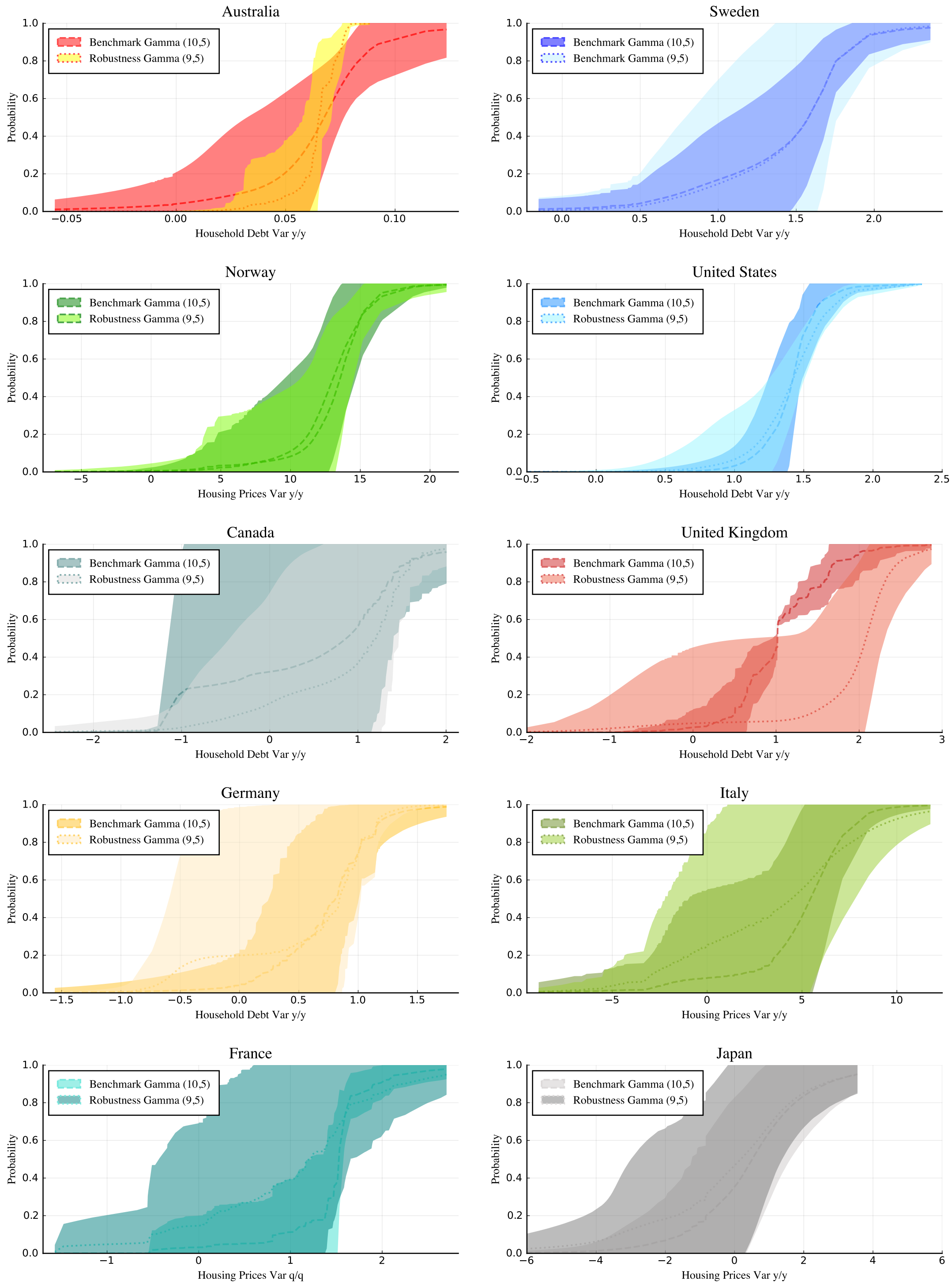
The Role of Transition Functions

Figure 8: Transition Function Probability: The point of inflection parameter



Note: This figure presents the transition probability functions for all countries in our sample after changing the point of inflection parameter in the transtition function. The shaded areas represent standard deviations.

Figure 9: Transition Function Probability: The gamma (smooth transition) parameter



Note: This figure presents the transition probability functions for all countries in our sample after changing the point of inflection parameter in the transtition function. The shaded areas represent standard deviations.

The role of lags

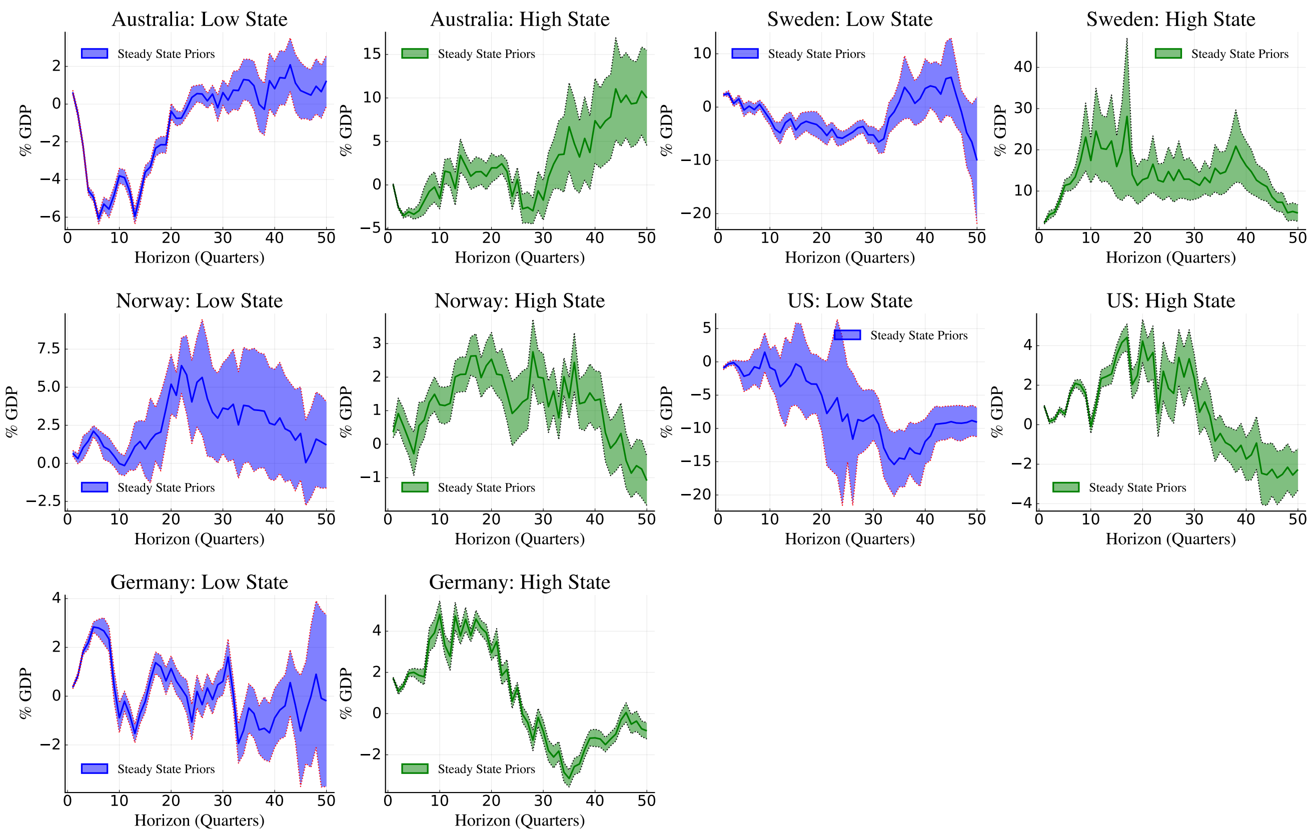
In Table 6 we extend the analysis of the lags when we consider the fiscal multiplier in periods of low and high household debt after four and eight quarters. Columns (7) to (18) show the fiscal multipliers comparison, after four and eight quarters, when we estimate the model with different lags.

Table 6: Fiscal Multipliers: The role of lags

	On Impact ($h = 1$)						After 4 quarters						After 8 quarters					
	Model p=6		Model p=5		Model p=4		Model p=6		Model p=5		Model p=4		Model p=6		Model p=5		Model p=4	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High	Low	High
Australia	0.831	-0.461	-0.281	0.498	0.622	0.062	5.080	-7.766	-3.084	2.176	-4.591	-3.085	3.302	-8.833	-13.71	5.003	-5.573	-0.749
	(0.059)	(0.151)	(0.049)	(0.041)	(0.056)	(0.044)	(0.343)	(1.563)	(0.421)	(0.187)	(0.108)	(0.229)	(0.753)	(1.766)	(0.898)	(0.291)	(0.229)	(0.874)
Sweden	0.192	1.115	-0.773	0.186	2.309	2.266	1.005	2.499	0.773	-6.661	1.549	7.699	2.132	2.164	6.146	-5.117	0.525	17.29
	(0.139)	(0.155)	(0.089)	(0.090)	(0.113)	(0.128)	(0.253)	(0.612)	(0.620)	(1.187)	(0.378)	(0.583)	(0.675)	(1.307)	(1.978)	(1.891)	(0.411)	(2.254)
Norway	0.271	0.178	1.021	0.708	0.663	0.359	0.771	1.168	1.056	-0.426	1.466	0.143	1.699	0.420	2.412	-1.319	0.896	1.250
	(0.042)	(0.100)	(0.136)	(0.077)	(0.091)	(0.083)	(0.130)	(0.154)	(0.453)	(0.106)	(0.291)	(0.239)	(0.069)	(0.642)	(0.988)	(0.162)	(0.394)	(0.219)
US	1.019	0.603	-0.262	1.228	-0.938	0.941	0.513	6.850	-3.219	3.300	-0.824	0.756	3.188	6.808	-3.325	-9.144	-1.004	2.003
	(0.118)	(0.032)	(0.049)	(0.159)	(0.122)	(0.022)	(0.443)	(0.859)	(0.237)	(0.434)	(0.504)	(0.067)	(0.579)	(1.288)	(0.299)	(3.063)	(1.526)	(0.136)
Germany	1.215	1.560	0.538	0.342	0.360	1.719	2.449	1.134	2.735	1.938	2.193	1.956	2.638	0.511	2.544	3.947	2.341	3.589
	(0.131)	(0.071)	(0.035)	(0.039)	(0.029)	(0.022)	(0.138)	(0.061)	(0.327)	(0.247)	(0.133)	(0.084)	(0.656)	(0.064)	(0.203)	(0.329)	(0.258)	(0.286)

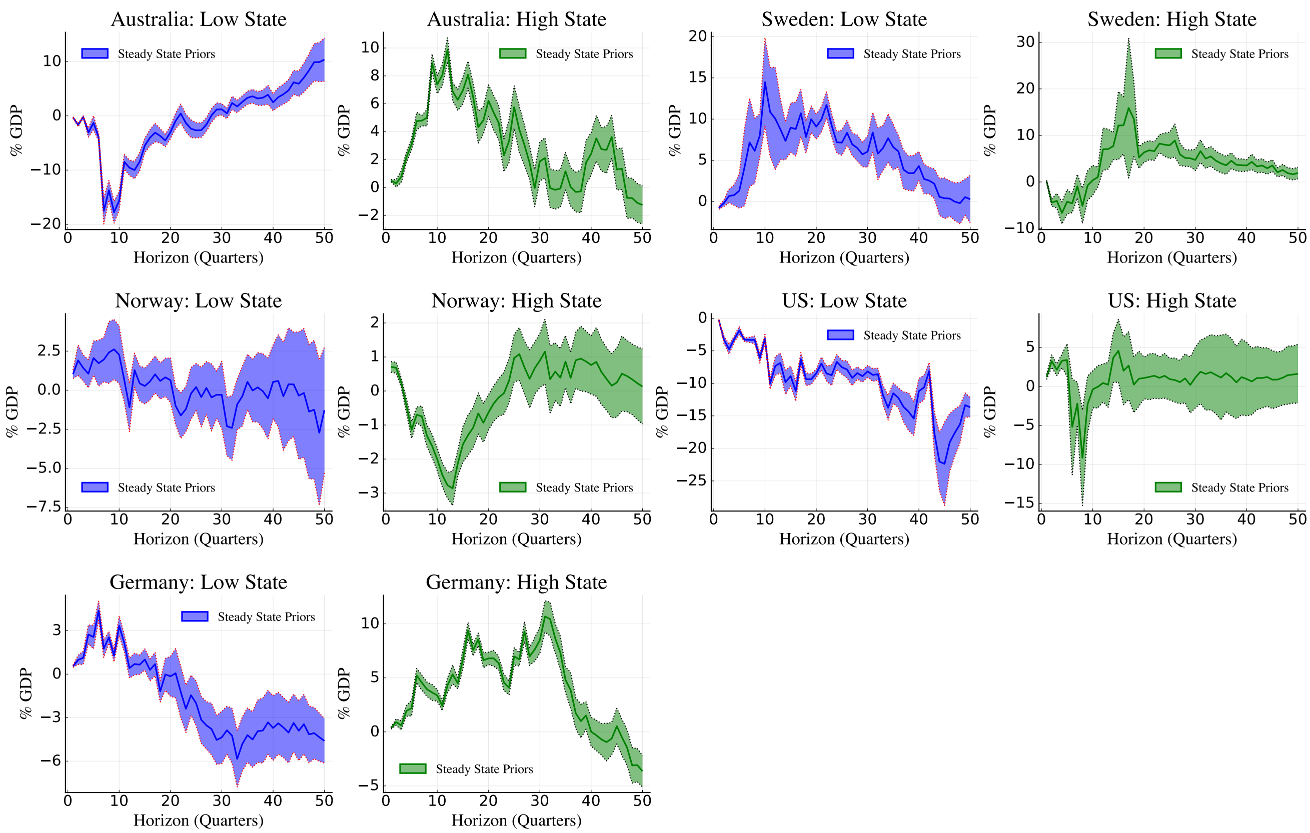
Notes: This table shows fiscal multipliers for a model with lag $p = 6$, $p = 5$ and $p = 4$. Fiscal multipliers represent the percent change of GDP after increasing government expenditures by 1 percent. Standard deviation in brackets. Estimation sample for each country can be found in table 2.

Figure 10: Government Spending Multiplier - Lag 4



Note: This figure presents government spending multiplier for Australia, Norway, United States and Germany. Mean responses (solid) and 95 % credibility bands (shaded areas). Estimation sample for each country can be found in Table 2.

Figure 11: Government Spending Multiplier - Lag 5



Note: This figure presents government spending multiplier for Australia, Norway, United States, United Kingdom, Germany, Italy, France, and Japan. Mean responses (solid) and 95 % credibility bands (shaded areas). Estimation sample for each country can be found in Table 2.

Table 7: State Dependent Local Projection Impulse Response Functions

	Norway	
Horizon	GIRF *Low*	GIRF *High*
	(1)	(2)
1	0.364	0.158
	(0.153)	(0.319)
2	-0.091	0.197
	(0.103)	(0.301)
4	-0.088	0.895
	(0.118)	(0.216)
8	-0.163	0.411
	(0.173)	(0.539)
12	0.050	-0.974
	(0.139)	(0.329)
16	-0.203	1.363
	(0.152)	(0.329)
20	0.011	-1.210
	(0.209)	(0.502)

Notes: This table shows the local projection estimation of fiscal multipliers when we use household debt-to-GDP ratio to identify low and high regimes. Fiscal multipliers represent the percent change of GDP after increasing government expenditures by 1 percent. Standard deviation in brackets. Estimation sample for can be found in Table 2.

Table 8: Fiscal Multiplier (4 quarters): STVAR vs State-Dependent Local Projections

	STVAR			STVAR for Robustness			State - Dependent Local Projections		
Country	Low State	High State	Difference	Low State	High State	Difference	Low State	High State	Difference
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Australia	5.080	-7.766	12.85	3.069	-3.911	6.98	0.135	0.592	-0.46
	(0.344)	(1.563)		(0.273)	(0.359)		(0.173)	(0.311)	
Sweden	1.005	2.499	-1.49	-0.203	-4.467	-4.26	0.405	-0.684	1.09
	(0.253)	(0.652)		(0.449)	(0.897)		(0.297)	(1.275)	
Norway	0.771	1.168	-0.39	1.484	2.346	-0.86	-0.029	N/A	
	(0.130)	(0.154)		(0.148)	(0.276)		(0.115)		
United States	0.513	6.850	-6.34	1.333	3.336	-2.01	0.176	0.276	-0.10
	(0.443)	(0.859)		(0.477)	(0.356)		(0.394)	(0.563)	
Germany	2.449	1.134	1.32	0.570	0.153	0.42	-0.042	-0.232	-0.19
	(0.138)	(0.060)		(0.099)	(0.153)		(0.110)	(0.331)	

Notes: This table shows fiscal multipliers, after four quarters, estimated with STVAR and SD-LP models. STVAR Robustness refers to the model that identifies a low regime with $F(z_t) <= 0.4$ and a high regime with $F(z_t) >= 0.6$. Lag $p = 6$.

Table 9: Fiscal Multiplier (12 quarters): STVAR vs State-Dependent Local Projections

	STVAR			STVAR for Robustness			State - Dependent Local Projections		
Country	Low State	High State	Difference	Low State	High State	Difference	Low State	High State	Difference
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Australia	5.618	0.436	5.18	6.419	3.393	3.03	0.020	0.853	-0.83
	(1.079)	(1.546)		(0.266)	(0.734)		(0.199)	(0.259)	
Sweden	1.374	3.713	-2.34	2.545	-2.309	4.85	-0.045	5.759	-5.80
	(0.480)	(2.804)		(0.639)	(0.652)		(0.447)	(2.134)	
Norway	0.492	0.042	0.45	-1.283	3.285	4.57	0.029	N/A	
	(0.084)	(0.604)		(0.459)	(0.696)		(0.101)		
United States	2.618	4.352	-1.73	6.558	4.194	2.36	1.078	-1.639	2.72
	(0.506)	(1.284)		(1.442)	(1.161)		(0.484)	(0.834)	
Germany	2.876	1.139	1.737	-2.329	4.934	7.263	-0.352	-0.042	-0.31
	(0.494)	(0.077)		(0.374)	(0.322)		(0.167)	(0.414)	

Notes: This table shows fiscal multiplers, after four quarters, estimated with STVAR and SD-LP models. STVAR Robustness refers to the model that identifies a low regime with $F(z_t) <= 0.4$ and a high regime with $F(z_t) >= 0.6$. Lag $p = 6$.

Appendix E: Ricardian vs Keynesian Perspective

Existing literature agrees that the macroeconomic effectiveness of fiscal policy is conditioned by the way that households respond to fiscal policy changes. The theory of Ricardian Equivalence posits that given a path of government expenditure, it does not matter whether additional fiscal expansions are financed with higher taxes or by issuing debt, because it does not alter households' consumption behavior in the long run. This suggests that fiscal expenditure expansions of fiscal expenditures do not impact aggregate demand.

However, the Ricardian Equivalence hypothesis relies on several assumptions about households' behavior and economic context that may not hold. In particular, it rests on the premise that households have infinite time horizons. As stated by Blanchard (1985); Blanchard & Fischer (1989); Cardia (1997); Blanchard & Perotti (2002) a probability of dying imposes finite horizons that causes

Ricardian Equivalence to fail. Other assumptions relied on by Ricardian Equivalence include the presence of no childless households, no liquidity constraints, no uncertainty, no differential borrowing rates, no distribution effects, no distortionary taxation, no interest rate or growth rate differential and rational households (Poterba & Summers, 1987; Seater, 1993).

In contrast to the Ricardian Equivalence hypothesis, Keynesian theories claim that increasing debt to finance fiscal expansions, while keeping the tax rate constant, could take advantage of households' positive marginal propensity to consume and stimulate the economy through an expansion of aggregate demand.

Whether Ricardian Equivalence holds or not has important implications for the way policymakers conduct fiscal policy. Where the above assumptions do not hold, increasing public expenditure may in fact be an effective tool to smooth business cycles. In our research we consider whether households constrained by high levels of household debt respond differently after receiving the benefits of a fiscal transfer. Shapiro & Slemrod (2003) has suggested that indebted households are more inclined to increase their savings or pay off debt rather than increase their consumption in response to a positive fiscal policy change (e.g. tax reduction or fiscal transfer). In this paper we hypothesize that high levels of household debt commitments condition households' behavior following the receipt of a fiscal transfer, weakening the effect of fiscal expansions.

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