Online Appendix for

Demographics and FDI: Lessons from China's One-Child Policy

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Online Appendix A – Proof of production aggregation

We omit time subscripts for simplicity. From equations (1), (2), and (3), we obtain,

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} K_i^{\alpha_i} L_i^{1-\alpha_i} \left[1 + \left(\frac{FDI_i}{K_i} \right)^{\alpha_i} \left(\frac{L_i^r}{L_i} \right)^{1-\alpha_i} \right] . \tag{A.1}$$

Assuming frictionless cross-country capital flows, condition (7) implies the equilibrium condition:

$$r^* + \delta = MPK_{i,t} = MPK_{i,t}^r . \tag{A.2}$$

Combining equations (A.2), (2), and (3), we obtain,

$$FDI_i \cdot L_i = K_i \cdot L_i^r . \tag{A.3}$$

Equation (A.1), combined with (A.3) and (4) becomes,

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} K_i^{\alpha_i} L_i^{-\alpha_i} \bar{L}_i . \tag{A.4}$$

Adding the term $K_i \cdot L_i$ to both sides of equation (A.3) leads to $(K_i + FDI_i) \cdot L_i = K_i \cdot (L_i + L_i^r)$, which implies,

$$\frac{\bar{K}_i}{\bar{L}_i} = \frac{K_i}{L_i} , \tag{A.5}$$

given (4), and given that $\bar{K}_i = K_i + FDI_i$. Combining (A.4) with (A.5) we obtain

$$\bar{Y}_i = \bar{A}_i^{1-\alpha_i} \left(\frac{\bar{K}_i}{\bar{L}_i} \right)^{\alpha_i} \bar{L}_i ,$$

which coincides with equation (6), proving the aggregation result. Q.E.D.

Online Appendix B – Proof of Proposition (3.3.1)

Proof of Proposition 3.3.1

We start by solving equation (14) as a difference equation, using the simplified form,

$$x_{j+1} = \theta x_j + \zeta_j \ , \quad j = 1, ..., T \ , \tag{B.1}$$

where,

$$x_j \equiv a_{b,j} \ , \tag{B.2}$$

$$\theta \equiv 1 + r^* \ , \tag{B.3}$$

$$\zeta_j \equiv w_{b+j-1} - c_{b,j} \ . \tag{B.4}$$

Combining (16) with (B.1), provides us with the initial condition for the differential equation given by (B.1), which is given by,

$$x_1 = 0 \ . \tag{B.5}$$

Using successive substitutions,

$$\left.\begin{array}{r} \left.\begin{array}{l} (B.1) \overset{j=1}{\Longrightarrow} x_2 = \theta x_1 + \zeta_1 \\ (B.1) \overset{j=2}{\Longrightarrow} x_3 = \theta x_2 + \zeta_2 \end{array}\right\} \Rightarrow x_3 = \theta\left(\theta x_1 + \zeta_1\right) + \zeta_2 \\ (B.1) \overset{j=3}{\Longrightarrow} x_4 = \theta x_3 + \zeta_3 \qquad\qquad\qquad \end{array}\right\} \Rightarrow x_4 = \theta\left[\theta\left(\theta x_1 + \zeta_1\right) + \zeta_2\right] + \zeta_3 \ ,$$

which can be rewritten as,

$$x_4 = \theta^3 x_1 + \sum_{\ell=1}^{3} \theta^{3-\ell} \zeta_\ell \ . \tag{B.6}$$

Generalizing (B.6), we obtain,

$$x_j = \theta^{j-1} x_1 + \sum_{\ell=1}^{j-1} \theta^{j-\ell-1} \zeta_\ell \ , \quad j = 2, ..., T \ . \tag{B.7}$$

Dividing both sides of (B.7) by θ^{j-1} gives a useful form of the solution, namely,

$$\frac{x_j}{\theta^{j-1}} = x_1 + \sum_{\ell=1}^{j-1} \frac{\zeta_\ell}{\theta^\ell} \ , \quad j = 2, ..., T \ . \tag{B.8}$$

Imposing the initial condition given by (B.5) on (B.8) leads to,

$$\frac{x_j}{\theta^{j-1}} = \sum_{\ell=1}^{j-1} \frac{\zeta_\ell}{\theta^\ell} , \qquad j = 2, ..., T . \tag{B.9}$$

Using (B.2), (B.3), and (B.4), (B.9) becomes,

$$\frac{a_{b,j}}{(1+r^*)^{j-1}} = \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^\ell} ,$$

or, more conveniently, after dividing both sides of the last equation by $1 + r^*$,

$$\frac{a_{b,j}}{(1+r^*)^{j-2}} = \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^{\ell-1}} , \qquad j = 2, ..., T . \tag{B.10}$$

With the solution of the wealth-accumulation path at hand given by equation (B.10), we proceed in order to impose the next necessary and sufficient condition for an optimum, which is the Euler equation

$$c_{b,j+1} = [\beta (1+r^*)]^\eta c_{b,j} , \qquad j = 1, ..., T-1 . \tag{B.11}$$

Solving (B.11) forward leads to equation (17). In order to solve for the optimum level of consumption $c_{b,1}$ which drives the whole optimal consumption path in (17), we can extend equation (B.10) to $j = T+1$, and impose the terminal condition given by (15). Specifically, a necessary condition for an optimum is that (15) binds, i.e.,

$$a_{b,T+1} = 0 . \tag{B.12}$$

Extending equation (B.10) to $j = T + 1$, and imposing (B.12) gives,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} . \tag{B.13}$$

In order to calculate the right-hand side of (B.13) we substitute equation (17) to get,

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \sum_{\ell=1}^{T} \frac{[\beta (1+r^*)]^{\eta(\ell-1)}}{(1+r^*)^{\ell-1}} ,$$

which simplifies to

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \sum_{\ell=1}^{T} \psi^{\ell-1} \ ,$$

and finally,

$$\sum_{\ell=1}^{T} \frac{c_{b,\ell}}{(1+r^*)^{\ell-1}} = c_{b,1} \frac{1-\psi^T}{1-\psi} \ . \tag{B.14}$$

To calculate the left-hand side of (B.13) we use the following change of indices:

$$t = b + \ell - 1 \ . \tag{B.15}$$

Equation (B.15) gives,

$$\ell = t - b + 1 \ , \tag{B.16}$$

which further implies,

$$\ell = 1 \overset{(B.16)}{\Longrightarrow} t = b \ , \tag{B.17}$$

and

$$\ell = T \overset{(B.16)}{\Longrightarrow} t = T - b + 1 \ . \tag{B.18}$$

Substituting (B.15), (B.16), (B.17), and (B.18) into the left-hand side of (B.13) we obtain,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \sum_{t=b}^{b+T-1} \frac{w_t}{(1+r^*)^{t-b}} \ . \tag{B.19}$$

Based on (5), equation (12) implies,

$$w_t = e^{g_{\bar{A}}(t-b)} w_b \ . \tag{B.20}$$

Before we substitute (B.20) into (B.19), notice that

$$w_t = 0 \ , \quad \text{for all } t \in \{T_R + 1, ..., T\} \ . \tag{B.21}$$

Substituting (B.20) and (B.21) into (B.19) gives,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = w_b \sum_{t=b}^{b+T_R-1} \left(\frac{e^{g_{\bar{A}}}}{1+r^*} \right)^{t-b} \ ,$$

or,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = w_b \sum_{j=0}^{T_R-1} \xi^j \ ,$$

which simplifies to,

$$\sum_{\ell=1}^{T} \frac{w_{b+\ell-1}}{(1+r^*)^{\ell-1}} = \frac{1-\xi^{T_R}}{1-\xi} w_b \ . \tag{B.22}$$

Substituting (B.22) and (B.14) into (B.13) gives,

$$c_{b,1} = \frac{1-\xi^{T_R}}{1-\xi} \frac{1-\psi}{1-\psi^T} w_b \ , \tag{B.23}$$

which proves equation (20).

We proceed with deriving the optimal path of wealth for the representative household in cohort b.

$$\left.\begin{array}{l} (B.10) \Rightarrow a_{b,j} = (1+r^*)^{j-2} \sum_{\ell=1}^{j-1} \frac{w_{b+\ell-1} - c_{b,\ell}}{(1+r^*)^{\ell-1}} \\ \qquad\qquad\qquad\qquad \ell = 1 \Rightarrow i = 0 \\ \underbrace{i = \ell - 1}_{\substack{\Downarrow \\ \ell = i+1}} \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \\ \qquad\qquad\qquad\qquad \ell = j-1 \Rightarrow i = j-2 \end{array}\right\} \Rightarrow a_{b,j} = (1+r^*)^{j-2} \sum_{i=0}^{j-2} \frac{w_{b+i} - c_{b,i+1}}{(1+r^*)^i} \ , \tag{B.24}$$

which holds for $j = 2, ..., T$. Based on (5), equation (12) implies,

$$w_{b+i} = e^{g_{\bar{A}} i} w_b \ , \quad i = 0, ..., T_R - 1 \ . \tag{B.25}$$

Equation (17) gives,

$$c_{b,i+1} = \left[\beta^\eta (1+r^*)^\eta\right]^i c_{b,1} \ , \quad i = 0, ..., T-1 \ . \tag{B.26}$$

After substituting (B.25) and (B.26) into (B.24) we obtain,

$$a_{b,j} = (1+r^*)^{j-2} \left(w_b \sum_{i=0}^{j-2} \xi^i - c_{b,1} \sum_{i=0}^{j-2} \psi^i \right) \ ,$$

which simplifies to,

$$a_{b,j} = (1+r^*)^{j-2}\left(\frac{1-\xi^{j-1}}{1-\xi}w_b - \frac{1-\psi^{j-1}}{1-\psi}c_{b,1}\right) \quad , \quad j = 1, ..., T_R + 1 \ . \tag{B.27}$$

Substituting (B.23) into equation (B.27) gives,

$$a_{b,j} = (1+r^*)^{j-2}\left(\frac{1-\xi^{j-1}}{1-\xi} - \frac{1-\xi^{T_R}}{1-\xi}\frac{1-\psi^{j-1}}{1-\psi^{T}}\right) \ w_b \ , \quad j = 1, ..., T_R + 1 \ ,$$

which can be written as,

$$a_{b,j} = (1+r^*)^{j-2}\,\frac{1-\xi^{T_R}}{1-\xi}\left(\frac{1-\xi^{j-1}}{1-\xi^{T_R}} - \frac{1-\psi^{j-1}}{1-\psi^{T}}\right) \ w_b \ , \quad j = 1, ..., T_R + 1 \ . \tag{B.28}$$

The reason why (B.28) holds for j only up to period T_R+1 is that, after period T_R the wage earnings are zero, i.e,

$$w_{b+j-1} = 0 \ , \quad j = T_R + 1, ..., T. \tag{B.29}$$

We can now solve for the optimal wealth path after period T_R+1, taking the wealth in period T_R+1, i.e., a_{b,T_R+1} as given. Specifically, after setting $j = T_R + 1$, (B.28) gives,

$$a_{b,T_R+1} = (1+r^*)^{T_R-1}\,\frac{1-\xi^{T_R}}{1-\xi}\left(1 - \frac{1-\psi^{T_R}}{1-\psi^{T}}\right) \ w_b \ . \tag{B.30}$$

Starting equation (14) from $j = T_R + 1$ and on, and after taking into account (B.29),

$$\left.\begin{array}{c} \left.\begin{array}{l} (14)\,,(B.29) \overset{j=T_R+1}{\Longrightarrow} a_{b,T_R+2} = \theta a_{b,T_R+1} - c_{b,T_R+1} \\ (14)\,,(B.29) \overset{j=T_R+2}{\Longrightarrow} a_{b,T_R+3} = \theta a_{b,T_R+2} - c_{b,T_R+2} \end{array}\right\} \Rightarrow a_{b,T_R+3} = \theta\left(\theta a_{b,T_R+1} - c_{b,T_R+1}\right) - c_{b,T_R+2} \\ (14)\,,(B.29) \overset{j=T_R+3}{\Longrightarrow} a_{b,T_R+4} = \theta a_{b,T_R+3} - c_{b,T_R+3} \end{array}\right\} \Rightarrow$$

$$\Rightarrow a_{b,T_R+4} = \theta\left[\theta\left(\theta a_{b,T_R+1} - c_{b,T_R+1}\right) - c_{b,T_R+2}\right] - c_{b,T_R+3}$$

which can be written in a condensed form as,

$$a_{b,T_R+4} = \theta^3 a_{b,T_R+1} - \sum_{\ell=1}^{3}\theta^{3-\ell}c_{b,T_R+\ell} \ ,$$

and can be generalized to,

$$a_{b,T_R+j} = \theta^{j-1} a_{b,T_R+1} - \sum_{\ell=1}^{j-1} \theta^{j-\ell-1} c_{b,T_R+\ell} \ ,$$

and rewritten as,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \sum_{\ell=1}^{j-1} \frac{c_{b,T_R+\ell}}{\theta^{\ell}} \right) \ , \quad j = 2, ..., T - T_R \ . \tag{B.31}$$

From (17) we obtain,

$$c_{b,T_R+\ell} = (\theta\psi)^{\ell-1} c_{b,T_R+1} \ , \quad j = 1, ..., T - T_R \ . \tag{B.32}$$

Combining (B.32) with (B.31) leads to,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - c_{b,T_R+1} \sum_{\ell=1}^{j-1} \frac{(\theta\psi)^{\ell-1}}{\theta^{\ell}} \right) \ ,$$

which is

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \theta^{-1} c_{b,T_R+1} \sum_{\ell=1}^{j-1} \psi^{\ell} \right) \ ,$$

and simplifies to,

$$a_{b,T_R+j} = \theta^{j-1} \left(a_{b,T_R+1} - \theta^{-1} \frac{1-\psi^{j-1}}{1-\psi} c_{b,T_R+1} \right) \ , \quad j = 2, ..., T - T_R \ . \tag{B.33}$$

Using again (17),

$$c_{b,T_R+1} = (\theta\psi)^{T_R} c_{b,1} \ . \tag{B.34}$$

Substituting (B.30), (B.23) and (B.34) into (B.33) we obtain,

$$a_{b,T_R+j} = \theta^{j-1} \left[\theta^{T_R-1} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{T_R}}{1-\psi^{T}} \right) - \theta^{T_R-1} \frac{1-\psi^{j-1}}{1-\psi} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-\psi}{1-\psi^{T}} \right] w_b \ ,$$

which simplifies, after some algebra, to,

$$a_{b,T_R+j} = \theta^{T_R+j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{T_R+j-1}}{1-\psi} \right) w_b \ , \quad j = 2, ..., T - T_R \ . \tag{B.35}$$

It remains to adjust the indices in (B.35), setting

$$\ell = T_R + j \ , \tag{B.36}$$

which implies,

$$j = \ell - T_R \ . \tag{B.37}$$

Substituting (B.36) and (B.37) into (B.35), after using (B.3) gives,

$$a_{b,\ell} = (1+r^*)^{j-2} \frac{1-\xi^{T_R}}{1-\xi} \left(1 - \frac{1-\psi^{\ell-1}}{1-\psi}\right) w_b \ , \quad \ell = T_R + 2, ..., T \ . \tag{B.38}$$

Cpmbining (B.38) with (B.28) proves (21), completing the proof of the Proposition. Q.E.D.

Online Appendix C – Proof of Proposition (3.4.1)

Proof of Proposition 3.4.1 We start from equation (25), dropping the country index i, namely

$$K_t = \sum_{j=0}^{T-1} a_{t-j,j+1} L_{t-j,j+1} \ , \tag{C.1}$$

where, according to equation (21),

$$a_{t-j,j+1} = (1+r^*)^{j-1} \frac{1-\xi^{T_R}}{1-\xi} \phi(j) \, w_{t-j} \ , \tag{C.2}$$

$$\phi(j) \equiv \begin{cases} \frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T} & , \quad j = 0, ..., T_R \\ 1 - \frac{1-\psi^j}{1-\psi^T} & , \quad j = T_R+1, ..., T-1 \end{cases} , \tag{C.3}$$

in which the indices change must be noticed. Combining (C.1) and (C.2) we obtain,

$$K_t = \frac{1-\xi^{T_R}}{1-\xi} \sum_{j=0}^{T-1} (1+r^*)^{j-1} \phi(j) \, w_{t-j} L_{t-j,j+1} \ . \tag{C.4}$$

Given (5), and given that the size of a cohort does not change during the cohort's lifetime (deterministic death time) notice that,

$$L_{t-j,j+1} = L_{t-j} = e^{-g_{\bar{L}} j} L_t \ , \tag{C.5}$$

and that, based on (5) again, equation (12) implies,

$$w_{t-j} = e^{-g_{\bar{A}} j} w_t \ . \tag{C.6}$$

We can relate L_{t-j} in equation (C.5) to $\bar{L}_t$ for all $j \in \{0, ..., T-1\}$, since,

$$\bar{L}_t = \sum_{j=0}^{T-1} L_{t-j} \ . \tag{C.7}$$

Specifically, combining (C.5) with (C.7), we obtain,

$$\bar{L}_t = L_t \sum_{j=0}^{T-1} e^{-g_{\bar{L}} j} \ ,$$

which simplifies to,

$$\bar{L}_t = \frac{1-e^{-g_{\bar{L}}T}}{1-e^{-g_{\bar{L}}}} L_t \ . \tag{C.8}$$

Combining (C.8) with (C.5) gives,

$$L_{t-j,j+1} = L_{t-j} = e^{-g_{\bar{L}}j} \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} \bar{L}_t \ . \tag{C.9}$$

With (C.9), (C.6) and (12) at hand, we return to (C.4), obtaining,

$$K_t = (1-\alpha)\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} \bar{A}_t \bar{L}_t \sum_{j=0}^{T-1} (1+r^*)^{j-1} \phi(j)\, e^{-(g_{\bar{A}}+g_{\bar{L}})j} \ ,$$

which can be re-written more concisely as,

$$K_t = \frac{1-\alpha}{1+r^*} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} \left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \bar{L}_t \sum_{j=0}^{T-1} \chi^j \phi(j) \ . \tag{C.10}$$

Recalling that $\xi = e^{g_{\bar{A}}}/(1+r^*)$ and $\chi \equiv (1+r^*)/e^{g_{\bar{A}}+g_{\bar{L}}}$, notice that,

$$\chi\xi = e^{-g_{\bar{L}}} \ . \tag{C.11}$$

Using (C.11) we can simplify (C.10) into,

$$K_t = \nu \cdot \left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t \bar{L}_t \ . \tag{C.12}$$

where

$$\nu \equiv \frac{1-\alpha}{1+r^*} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-\chi\xi}{1-(\chi\xi)^T} \sum_{j=0}^{T-1} \chi^j \phi(j) \ . \tag{C.13}$$

The constant ν in (C.13) corresponds to the constant ν in (35). To prove that (C.13) and (35) are equivalent, observe that, based on (C.3),

$$\sum_{j=0}^{T-1} \chi^j \phi(j) = \sum_{j=0}^{T_R} \chi^j \left(\frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T}\right) + \sum_{j=T_R+1}^{T-1} \chi^j \left(1 - \frac{1-\psi^j}{1-\psi^T}\right) \ . \tag{C.14}$$

For calculating the first summation of the right-hand side of (C.14),

$$\sum_{j=0}^{T_R} \chi^j \left(\frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T}\right) = \frac{1}{1-\xi^{T_R}} \left[\sum_{j=0}^{T_R} \chi^j - \sum_{j=0}^{T_R} (\chi\xi)^j\right] - \frac{1}{1-\psi^T} \left[\sum_{j=0}^{T_R} \chi^j - \sum_{j=0}^{T_R} (\chi\psi)^j\right] \ ,$$

which simplifies to,

$$\sum_{j=0}^{T_R}\chi^j\left(\frac{1-\xi^j}{1-\xi^{T_R}}-\frac{1-\psi^j}{1-\psi^T}\right)=\frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi}\right]-$$
$$-\frac{1}{1-\psi^T}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\psi)^{T_R+1}}{1-\chi\psi}\right]. \tag{C.15}$$

Regarding the second summation of the right-hand side of (C.14), observe that

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}+\chi^{T_R+2}+...+\chi^{T-1},$$

which simplifies to,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}\left(1+\chi+\chi^2+...+\chi^{T-T_R-2}\right),$$

or,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\chi^{T_R+1}\frac{1-\chi^{T-T_R-1}}{1-\chi},$$

i.e.,

$$\sum_{j=T_R+1}^{T-1}\chi^j=\frac{\chi^{T_R+1}-\chi^T}{1-\chi}. \tag{C.16}$$

Based on (C.16), the second summation of the right-hand side of (C.14) simplifies to,

$$\sum_{j=T_R+1}^{T-1}\chi^j\left(1-\frac{1-\psi^j}{1-\psi^T}\right)=\frac{\chi^{T_R+1}-\chi^T}{1-\chi}-\frac{1}{1-\psi^T}\left[\frac{\chi^{T_R+1}-\chi^T}{1-\chi}-\frac{(\chi\psi)^{T_R+1}-(\chi\psi)^T}{1-\chi\psi}\right]. \tag{C.17}$$

Substituting (C.15) and (C.17) into (C.14) we obtain, after some algebra,

$$\sum_{j=0}^{T-1}\chi^j\phi(j)=\frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi}-\frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi}\right]+$$
$$+\frac{1}{1-\psi^T}\left[\frac{1-(\chi\psi)^T}{1-\chi\psi}-\frac{1-\chi^T}{1-\chi}\right]+\frac{\chi^{T_R+1}-\chi^T}{1-\chi}. \tag{C.18}$$

Finally, combining (C.18) with (C.13), we obtain the expression in (35) for ν.

It remains to obtain the expressions for $FDI_{i,t}/\bar{Y}_{i,t}$ and $K_{i,t}/\bar{Y}_{i,t}$ given by (34) and (37). Equation (32) can be rewritten as,

$$S_t = K_{t+1} - (1-\delta) K_t \ . \tag{C.19}$$

Observe that equation (C.12) holds for all t, namely,

$$\frac{K_{t+1}}{\bar{A}_{t+1}\bar{L}_{t+1}} = \frac{K_t}{\bar{A}_t\bar{L}_t} = \nu \cdot \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \ ,$$

which means that,

$$\frac{K_{t+1}}{K_t} = \frac{\bar{A}_{t+1}\bar{L}_{t+1}}{\bar{A}_t\bar{L}_t} \ ,$$

and based on (5), it is,

$$K_{t+1} = e^{g_{\bar{A}} + g_{\bar{L}}} K_t \ . \tag{C.20}$$

Combining (C.20) with (C.19) we arrive at,

$$S_t = \left(e^{g_{\bar{A}} + g_{\bar{L}}} - 1 + \delta\right) K_t \ . \tag{C.21}$$

Recall from equation (6) that,

$$r^* + \delta = \alpha \frac{\bar{Y}_t}{\bar{K}_t} \ ,$$

which can be rewritten as,

$$\frac{\bar{K}_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ ,$$

or,

$$\frac{K_t}{\bar{Y}_t} + \frac{FDI_t}{\bar{Y}_t} = \frac{\alpha}{r^* + \delta} \ . \tag{C.22}$$

From equation (6) we obtain,

$$\bar{Y}_t = \left(\frac{\bar{K}_t}{\bar{A}_t\bar{L}_t}\right)^{\alpha} \bar{A}_t\bar{L}_t \ ,$$

and based on (11) it is,

$$\bar{Y}_t = \left(\frac{\alpha}{r^* + \delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_t\bar{L}_t \ . \tag{C.23}$$

Combining (C.23) with (C.12), we obtain,

$$\frac{K_t}{\bar{Y}_t} = \nu \ . \tag{C.24}$$

From (C.24) and (C.22) we prove equation (34). After dividing both sides of (C.21) by $\bar{Y}_t$ and substituting (C.24), we arrive at equation (37), proving the Proposition. Q.E.D.

Online Appendix D – Proof of Propositions (3.4.2) and (3.4.3)

Proof of Proposition 3.4.2 Our main goal is to characterize what happens to the FDI/GDP ratio as $g_{\bar{L}}$ decreases. From equation (34) we can see that

$$\frac{\partial\left(\frac{FDI_t}{\bar{Y}_t}\right)}{\partial g_{\bar{L}}} > 0 \Leftrightarrow \frac{\partial \nu}{\partial g_{\bar{L}}} < 0 \; . \tag{D.1}$$

According to equation (21),

$$a_{b,j+1} = (1+r^*)^{j-1} \frac{1-\xi^{T_R}}{1-\xi} \phi(j)\, w_b \; , \tag{D.2}$$

where,

$$\phi(j) \equiv \begin{cases} \frac{1-\xi^j}{1-\xi^{T_R}} - \frac{1-\psi^j}{1-\psi^T} & , \quad j = 0, ..., T_R \\ 1 - \frac{1-\psi^j}{1-\psi^T} & , \quad j = T_R+1, ..., T-1 \end{cases} . \tag{D.3}$$

It is straightforward to show that,

$$\frac{1-\xi^{T_R}}{1-\xi} > 0 \text{ , for all } \xi > 0,\, \xi \neq 1 \; . \tag{D.4}$$

To see that (D.4) is true, notice that the signs of the numerator and the denominator of $\left(1-\xi^{T_R}\right)/(1-\xi)$ will be the same, no matter if $0<\xi<1$ or $\xi>1$. Therefore, according to (D.2) and (D.4), the only way to guarantee that accumulated wealth, $a_{b,j+1}$, along the lifecycle of a cohort (leaving out $a_{b,1} = a_{b,T+1} = 0$) are positive, is to pick calibrating parameters r^*, $g_{\bar{A}}$, $g_{\bar{L}}$, β and η, so that ξ and ψ in (D.3) guarantee that,

$$\phi(j) > 0 \text{ , for all } j \in \{1, ..., T-1\} \; . \tag{D.5}$$

Returning now to (D.1), combining (C.11) and (C.13), ν can be re-written as,

$$\nu \equiv \frac{1-\alpha}{1+r^*} \frac{1-\xi^{T_R}}{1-\xi} \frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} \sum_{j=0}^{T-1} \chi^j \phi(j) \; . \tag{D.6}$$

Using a similar argument to this for proving (D.4), we can see that

$$\frac{1-e^{-g_{\bar{L}}}}{1-e^{-g_{\bar{L}}T}} > 0 \ , \quad \text{for all } g_{\bar{L}} \neq 0 \ . \tag{D.7}$$

In order to find the sign of $\partial\nu/\partial g_{\bar{L}}$ from (D.6), notice that, among constants ξ, ψ, and χ, only $\chi = (1+r^*)/e^{g_{\bar{A}}+g_{\bar{L}}}$ depends on $g_{\bar{L}}$. Therefore, let's express χ as,

$$\chi = e^{-g}\kappa \ ,$$

where $\kappa \equiv (1+r^*)/e^{g_{\bar{A}}}$, and we express $g_{\bar{L}}$ as "g", for notational simplicity. Therefore,

$$\nu = \zeta \underbrace{\frac{1-e^{-g}}{1-e^{-gT}}}_{f(g)} \cdot \underbrace{\sum_{j=0}^{T-1} \left(e^{-g}\kappa\right)^j \phi(j)}_{h(g)} \ , \tag{D.8}$$

where

$$\zeta = \frac{1-\alpha}{1+r^*}\frac{1-\xi^{T_R}}{1-\xi} > 0 \ .$$

Based on (D.8),

$$\frac{\partial\nu}{\partial g} = \zeta\left[f'(g)\,h(g) + f(g)\,h'(g)\right] \ . \tag{D.9}$$

Notice that,

$$f(g)\,h(g) > 0 \ , \text{ and } h'(g) < 0 \ . \tag{D.10}$$

Since $\zeta > 0$, equations (D.9) and (D.10) imply that,

$$\frac{\partial\nu}{\partial g} < 0 \Longleftrightarrow \frac{f'(g)}{f(g)}\frac{h(g)}{-h'(g)} < 1 \ . \tag{D.11}$$

From the definition of $h(g)$ in (D.8) we see that

$$\frac{h(g)}{-h'(g)} = \frac{\sum_{j=0}^{T-1}\left(e^{-g}\kappa\right)^j\phi(j)}{\sum_{j=0}^{T-1} j\left(e^{-g}\kappa\right)^j\phi(j)} \ , \tag{D.12}$$

i.e.,

$$\frac{h(g)}{-h'(g)} = \frac{\overbrace{\phi(0)}^{\overset{0}{\parallel}} + e^{-g}\kappa\ \phi(1) + ... + (e^{-g}\kappa\ )^{T-1}\phi(T-1)}{0\cdot\phi(0) + e^{-g}\kappa\ \phi(1) + ... + (T-1)(e^{-g}\kappa\ )^{T-1}\phi(T-1)} \ . \tag{D.13}$$

From (D.12) and (D.13) we can see that

$$\frac{h(g)}{-h'(g)} = 1 \text{ , if } T = 2 \ , \tag{D.14}$$

and

$$\frac{h(g)}{-h'(g)} < 1 \text{ , if } T > 2 \ . \tag{D.15}$$

Coming now to $f(g)$,

$$f'(g) = \frac{e^{-g}\left(1-e^{-gT}\right) - Te^{-gT}\left(1-e^{-g}\right)}{\left(1-e^{-gT}\right)^2} \ . \tag{D.16}$$

After some algebra, from the definition of $f(g)$ in (D.8) we see that,

$$\frac{f'(g)}{f(g)} = \frac{1}{e^g-1} - \frac{T}{e^{gT}-1} \ . \tag{D.17}$$

Based on (D.17), we can see that,

$$\frac{f'(g)}{f(g)} < 1 \Longleftrightarrow \frac{T}{e^{gT}-1} - \frac{1}{e^g-1} + 1 > 0 \ , \tag{D.18}$$

which implies,

$$\frac{f'(g)|_{T=2}}{f(g)|_{T=2}} < 1 \Longleftrightarrow \frac{2}{e^{2g}-1} - \frac{1}{e^g-1} + 1 > 0 \Longleftrightarrow \frac{e^g\left(e^g-1\right)}{e^{2g}-1} > 0 \ , \tag{D.19}$$

which is a true statement for all $g \neq 0$ (i.e., for all $g_{\bar{L}} \neq 0$). Combining (D.19) with (D.14) proves the part of the proposition that refers to $T = 2$.

For $T > 2$, inequality (D.18) is not guaranteed to be true, therefore, combining (D.17), (D.12), (D.11) and (D.1), proves inequality (43) of the proposition, completing the proof. Q.E.D.

Proof of Proposition 3.4.3

Based on equation (34), the inequality given by (44) holds if,

$$\frac{\partial \nu}{\partial r^*} > 0 \ . \tag{D.20}$$

Therefore, we focus on providing sufficient conditions for (D.20). Based on (35),

$$\nu = f(r^*)\, g(r^*)\, h(r^*) \underbrace{\left[j(r^*) + k(r^*) + \ell(r^*)\right]}_{\substack{\shortparallel \\ m(r^*)}} \ , \tag{D.21}$$

where,

$$f(r^*) = \frac{1-\alpha}{1+r^*} \ , \tag{D.22}$$

$$g(r^*) = \frac{1-\chi\xi}{1-(\chi\xi)^T} \ , \tag{D.23}$$

$$h(r^*) = \frac{1-\xi^{T_R}}{1-\xi^T} \ , \tag{D.24}$$

$$j(r^*) = \frac{1}{1-\xi^{T_R}}\left[\frac{1-\chi^{T_R+1}}{1-\chi} - \frac{1-(\chi\xi)^{T_R+1}}{1-\chi\xi}\right] \ , \tag{D.25}$$

$$k(r^*) = \frac{1}{1-\psi^T}\left[\frac{1-(\chi\psi)^T}{1-\chi\psi} - \frac{1-\chi^T}{1-\chi}\right] \ , \tag{D.26}$$

$$\ell(r^*) = \frac{\chi^{T_R+1}-\chi^T}{1-\chi} \ . \tag{D.27}$$

Therefore, based on the notation given by (D.21),

$$\begin{aligned} \frac{\partial \nu}{\partial r^*} &= f'(r^*)\, g(r^*)\, h(r^*)\, m(r^*) \\ &\quad + f(r^*)\, g'(r^*)\, h(r^*)\, m(r^*) \\ &\quad + f(r^*)\, g(r^*)\, h'(r^*)\, m(r^*) \\ &\quad + f(r^*)\, g(r^*)\, h(r^*) \underbrace{m'(r^*)}_{\substack{\shortparallel \\ j'(r^*)+k'(r^*)+\ell'(r^*)}} \end{aligned} \tag{D.28}$$

Therefore, we need to investigate the signs of $f(r^*)$, $g(r^*)$, $h(r^*)$, $jk\ell(r^*)$ as well as the signs of $f'(r^*)$, $g'(r^*)$, $h'(r^*)$, $jk\ell'(r^*)$.

From (D.22), we can immediately see that,

$$f(r^*) > 0 \text{ and } f'(r^*) < 0 \text{ , as long as } r^* > -1 \text{ .} \tag{D.29}$$

Similarly, since $\chi = (1 + r^*)/e^{g_{\bar{A}}+g_{\bar{L}}}$ and $\xi = e^{g_{\bar{A}}}/(1 + r^*)$, (D.23) can be rewritten as,

$$g(r^*) = \frac{1 - e^{-g_{\bar{L}}}}{1 - e^{-g_{\bar{L}}T}} \text{ ,}$$

which implies,

$$g(r^*) > 0 \text{ and } g'(r^*) = 0 \text{ , for all } r^* \in \mathbb{R} \text{ .} \tag{D.30}$$

Regarding the signs of $h(r^*)$ and $h'(r^*)$, let's re-define $h(r^*)$ as

$$h(r^*) \equiv n(\xi(r^*)) \text{ ,} \quad \text{where } n(\xi) \equiv \frac{1 - \xi^{T_R}}{1 - \xi^T} \text{ , and } \xi(r^*) \equiv \frac{e^{g_{\bar{A}}}}{1 + r^*} \text{ .} \tag{D.31}$$

Notice from (D.31) that

$$h(r^*) > 0 \quad \text{for all } \xi \neq 1 \text{ ,} \tag{D.32}$$

and,

$$h'(r^*) \equiv n'(\xi(r^*))\,\xi'(r^*) \text{ .} \tag{D.33}$$

Equation (D.31) implies,

$$\xi'(r^*) < 0 \text{ , for all } r^* > -1 \text{ ,} \tag{D.34}$$

and, after some algebra,

$$n'(\xi) = \xi^{T+T_R-1} \frac{T\left(\frac{1}{\xi^{T_R}} - 1\right) - T_R\left(\frac{1}{\xi^T} - 1\right)}{\left(1 - \xi^T\right)^2} \text{ .} \tag{D.35}$$

Therefore, (D.33), (D.34), and (D.35) imply,

$$h'(r^*) < 0 \Leftrightarrow T\left(\frac{1}{\xi^{T_R}} - 1\right) > T_R\left(\frac{1}{\xi^T} - 1\right) \text{ .} \tag{D.36}$$

At this point, given (D.30), equation (D.28) implies,

$$\frac{\partial \nu}{\partial r^*} = f' \cdot g \cdot h \cdot m + f \cdot g \cdot h' \cdot m + f \cdot g \cdot h \cdot m' \ ,$$

i.e.,

$$\frac{\partial \nu}{\partial r^*} = f \cdot g \cdot h \cdot m \cdot \left(\frac{f'}{f} + \frac{h'}{h} + \frac{m'}{m} \right) \ ,$$

and based on (D.21),

$$\frac{\partial \nu}{\partial r^*} = \nu \cdot \left(\frac{f'}{f} + \frac{h'}{h} + \frac{m'}{m} \right) \ . \tag{D.37}$$

Given that $f'/f = d\left[\ln\left(f\right)\right]/dr^*$, equation (D.22) implies,

$$\frac{f'\left(r^*\right)}{f\left(r^*\right)} = \frac{d\left[\ln\left(\frac{1-\alpha}{1+r^*}\right)\right]}{dr^*} = \frac{-1}{1+r^*} \ . \tag{D.38}$$

Similarly, equation (D.24) combined with (D.31) implies,

$$\frac{h'\left(r^*\right)}{h\left(r^*\right)} = \frac{d\left[\ln\left(\frac{1-\xi^{T_R}}{1-\xi^{T}}\right)\right]}{dr^*} = \frac{\xi}{1+r^*} \cdot \left(\frac{T_R \xi^{T_R - 1}}{1-\xi^{T_R}} - \frac{T\xi^{T-1}}{1-\xi^{T}} \right) \ . \tag{D.39}$$

Combining (D.37) with (D.38) and (D.39) proves (44). Q.E.D.

Online Appendix E – Proof of Proposition (3.6.1)

Proof of Proposition 3.6.1 The proof relies on combining equations (48) and (49). Based on (48),

$$\frac{FDI_{\hat{t}+\ell}}{\bar{Y}_{\hat{t}+\ell}} = \frac{\alpha}{r^* + \delta} - \left(\frac{\alpha}{r^* + \delta}\right)^{-\frac{\alpha}{1-\alpha}} \frac{K_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} \ . \tag{E.1}$$

We focus on relating the dynamics of $\bar{L}_{\hat{t}+\ell}$ with the dynamics of $\bar{K}_{\hat{t}+\ell}$ in (E.1). During the transition, each cohort grows at rate $g_L(t)$, given by,

$$g_L(t) = \begin{cases} g_{L,1} & , \quad t \leq \hat{t} \\ g_{L,2} & , \quad t > \hat{t} \end{cases} \ . \tag{E.2}$$

Moreover,

$$\bar{L}_{\hat{t}+\ell} = \sum_{j=0}^{T-1} L_{\hat{t}+\ell-j} \ , \ \ \ell = 1, ..., T \ . \tag{E.3}$$

From (E.2), we can see that,

$$L_{\hat{t}+\ell-j} = \begin{cases} e^{-g_{L,2}\ell - g_{L,1}(j-\ell)} L_{\hat{t}+\ell} & , \quad j \geq \ell \\ e^{-g_{L,2}j} L_{\hat{t}+\ell} & , \quad j < \ell \end{cases} \ , \ \ \ell = 1, ..., T \ , \ \ j = 0, ..., T-1 \ ,$$

which can be summarized as,

$$L_{\hat{t}+\ell-j} = \Lambda(j,\ell)\, L_{\hat{t}+\ell} \ , \tag{E.4}$$

where $\Lambda(j,\ell)$ is given by (51). Combining (E.3) and (E.4) we obtain,

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell} \sum_{j=0}^{T-1} \Lambda(j,\ell) \ \ ,$$

and based on the definition of $\Lambda(j,\ell)$ from (51),

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell} \left[\sum_{j=0}^{\ell-1} e^{-g_{L,2}j} + \sum_{j=\ell}^{T-1} e^{-\left(g_{L,2} - g_{L,1}\right)\ell - g_{L,1}j} \right] \ , \tag{E.5}$$

where the convention

$$\sum_{i=a}^{b} x_i = 0 \ , \text{ if } \ a > b \ , \tag{E.6}$$

applies. After some algebra, (E.5) implies,

$$\bar{L}_{\hat{t}+\ell} = L_{\hat{t}+\ell}\left[\frac{1-e^{-g_{L,2}\ell}}{1-e^{-g_{L,2}}} + e^{-g_{L,2}\ell}\frac{1-e^{-g_{L,1}(T-\ell)}}{1-e^{-g_{L,1}}}\right] . \tag{E.7}$$

Combining (E.4) and (E.7) gives,

$$L_{\hat{t}+\ell-j} = \Lambda(j,\ell)\,\lambda(\ell)\,\bar{L}_{\hat{t}+\ell} , \tag{E.8}$$

where $\lambda(\ell)$ is given by (52).

With (E.8) at hand, we can now relate the dynamics of $\bar{L}_{\hat{t}+\ell}$ with the dynamics of $\bar{K}_{\hat{t}+\ell}$. The definition of $\bar{K}_{\hat{t}+\ell}$ is,

$$\bar{K}_{\hat{t}+\ell} = \sum_{j=0}^{T-1} a_{\hat{t}+\ell-j,j+1} L_{\hat{t}+\ell-j} . \tag{E.9}$$

From (49) we obtain,

$$a_{\hat{t}+\ell-j,j+1} = (1+r^*)^{j-1}\frac{1-\xi^{T_R}}{1-\xi}\phi(j)\,w_{\hat{t}+\ell-j} , \tag{E.10}$$

where $\phi(j)$ is given by equation (42). From (12),

$$w_{\hat{t}+\ell} = (1-\alpha)\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}_{\hat{t}+\ell} , \tag{E.11}$$

therefore,

$$w_{\hat{t}+\ell-j} = w_{\hat{t}+\ell}e^{-g_{\bar{A}}j} . \tag{E.12}$$

Combining (E.9) with (E.8), (E.11), and (E.12), gives

$$\frac{\bar{K}_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} = (1-\alpha)\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}\frac{1-\xi^{T_R}}{1-\xi}\lambda(\ell)\sum_{j=0}^{T-1}(1+r^*)^{j-1}\Lambda(j,\ell)\,\phi(j)\,e^{-g_{\bar{A}}j} . \tag{E.13}$$

Keeping in mind that $\xi \equiv e^{-g_{\bar{A}}}/(1+r^*)$, (E.13) becomes,

$$\frac{\bar{K}_{\hat{t}+\ell}}{\bar{A}_{\hat{t}+\ell}\bar{L}_{\hat{t}+\ell}} = \frac{1-\alpha}{1+r^*}\left(\frac{\alpha}{r^*+\delta}\right)^{\frac{\alpha}{1-\alpha}}\frac{1-\xi^{T_R}}{1-\xi}\lambda(\ell)\sum_{j=0}^{T-1}\xi^{-j}\Lambda(j,\ell)\,\phi(j) . \tag{E.14}$$

Combining (E.14) with (E.1), leads to (50), proving the proposition. Q.E.D.

Online Appendix F – Sensitivity Analysis

Case 1: shortening the economic lifespan from its benchmark value $T = 50$ to $T = 45$, and the retirement time from its benchmark value $T_R = 45$, to $T_R = 40$

Here, the interest rate is $r^* = 10.67\%$ and all calibration parameters appear in Table F.1.

Table F.1 Calibration parameters (annual values, % rates).

	$g_{\bar{A}}$	α	β	δ	η
China	3.08	46.76	99.15	17.17	57.81
India	2.52	20.07	98.01	16.94	35.78

The goodness of fit to key calibration targets (as in Table 3 in the main body of the paper), is given by Table F.2.

Table F.2 Initial calibration targets (%).

	Savings rate 1985		FDI/GDP ratio 1995	
	China	India	China	India
Data	33.75	14.88	2.90	0.19
Model	28.69	13.23	2.90	0.19

The goodness of fit of the transition dynamics in this case appear in Figure F.1.

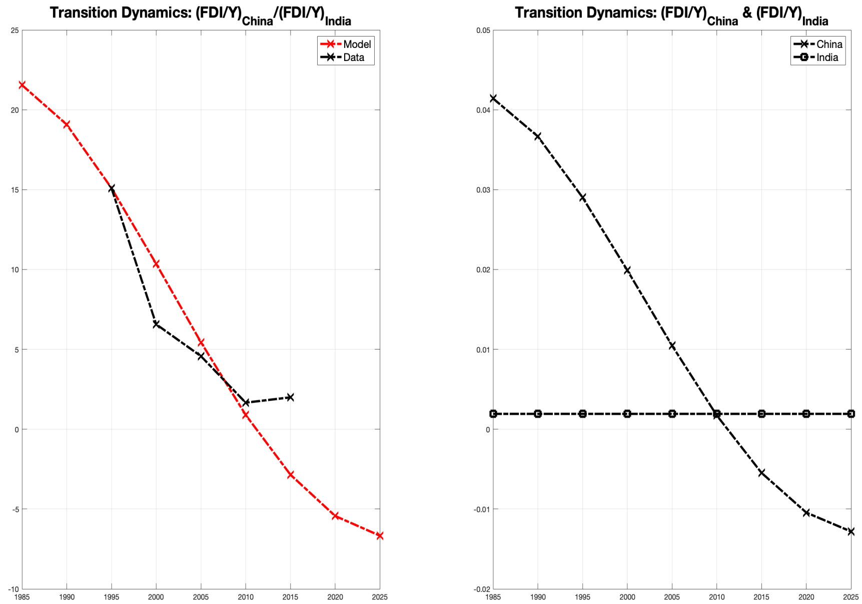

Figure F.1

In brief, the goodness of fit in this case of shorter life expectancy and shorter retirement age (that may fit to China's agrarian regime some decades ago), reveals that the model-implied effects of an exogenous demographic intervention such as the one-child policy are robust to shortening life expectancy and to having an earlier retirement age.

Case 2: expanding the economic lifespan from its benchmark value $T = 50$ **to** $T = 55$**, while keeping the retirement time to its benchmark value** $T_R = 45$

Here, the analysis refers to the recent improvements in healthcare in China that have led to more longevity (75 years). Because a higher life expectancy motivates more savings, here the interest rate is set to a slighlty lower value, with $r^* = 8.95\%$. Table F.3 gives the calibration parameters in this case

Table F.3 Calibration parameters (annual values, % rates).

	$g_{\bar{A}}$	α	β	δ	η
China	3.11	46.74	99.14	17.14	57.77
India	2.52	20.05	98.01	16.91	35.77

Table F.2 shows the goodness of fit to key calibration targets.

Table F.4 Initial calibration targets (%).

	Savings rate 1985		FDI/GDP ratio 1995	
	China	India	China	India
Data	33.75	14.88	2.90	0.19
Model	32.30	14.68	2.90	0.19

In this case we can see from that the increase in life expectancy helps the model to better match the savings calibration targets. This is intuitive, because households that live longer must save more in order to finance a longer post-retirement period. The goodness of fit of the transition dynamics in this case of more longevity appear in Figure F.2.

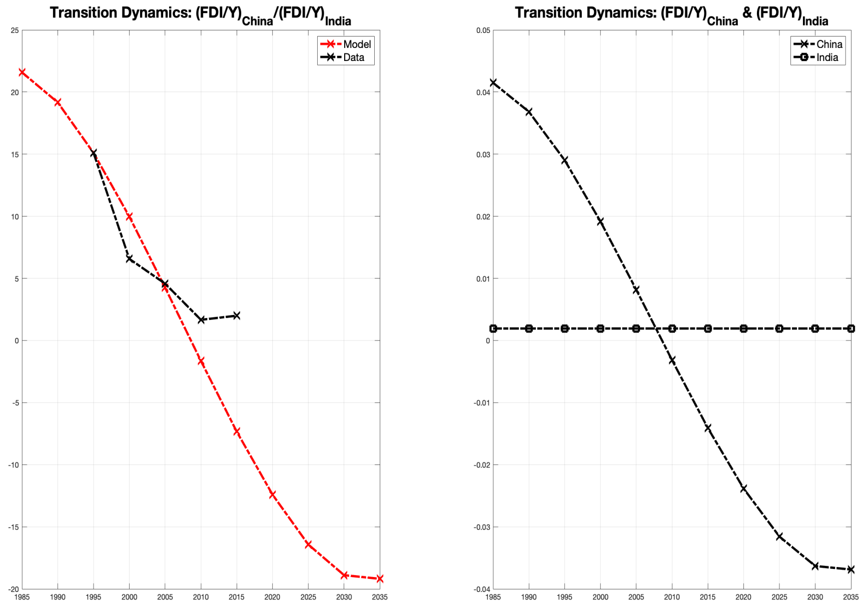

Figure F.2

In this case we can see again that the model's implied effects of China's one-child policy on the FDI/GDP ratio are robust to life-expectancy changes.

Online Appendix G – Literature review

Literature related to China's high savings rate

Curtis et al. (2015) and Choukhmane (2020) hypothesize that reduced fertility implies fewer children to support parents in their old age, thereby inducing parents to increase their own savings. Wei and Zhang (2011) explain the increased savings rate as a competitive response to the policy-induced sex ratio imbalance: families save more to increase the wealth of their sons in order to enhance their position in the competition for increasingly scarce spouses. Imrohoroglu and Zhao (2018) emphasize the long-term care insurance traditionally provided by families, and how the one-child policy has decreased the ability of families to provide it. Parents are thus forced to self-insure and do so by saving more. Other related work includes Chamon and Prasad (2010) and Yang et al. (2013). Finally, Zhang (2017) provides a comprehensive overview of the socio-economic effects of the one-child policy in China.

Another likely reason behind the documented increase in China's the savings rate is the remarkable improvement in life expectancy in China (to compare the progression of life expectancy indices between China and India, see https://data.worldbank.org/). Accordingly, the associated health care and medical costs have increased tremendously, all of which encourage Chinese households to save more. Moreover, in the past decades, the geographical mobility of young Chinese cohorts is much higher than the previous generation due to the drastic relaxation of the residential registration system (Hukou system). Hence, the monetary cost for supporting elder parents has also increased due to mobility-induced spatial separations, which also compels elderly parents to save more for retirement.

Regarding the extent of the change in the Chinese savings rate since 1980, there is some disagreement in the literature. Using the gross domestic savings to GDP ratio as a measure according to the World Bank, the Chinese savings rate increased from 33.4% in 1982 to 47.5% in 2014, a 14.1 percentage points increase. Choukhmane et al. (2020) used the Chinese Urban Household Survey (CEIC data) and showed an increase of 20 percentage points from 10% in 1980 to approximately 30% in 2015. Imrohoroglu and Zhao (2018) document the savings rate in China as increasing from 20% to 40%, an extreme view in the literature that we adopt for illustrative purposes.

Literature related to China's high capital returns

Bai et al. (2006) were the first to document the high capital returns in China (exceeding 20% post 1993) carefully. They conclude that China's high investment rate is consistent with the observed high returns. Nevertheless, mapping the documented high returns reported by Bai et al. (2006) to the aggregative concept of MPK under perfect foresight that we employ in this paper is not a straightforward task. Cochrane, in the discussion of Bai et al. (2006, p. 99), notes that the comparatively high return in China should be adjusted for differences in risk. Nordhaus and Cooper's discussion of Bai et al. (2006) emphasizes that a sudden conversion of land from agricultural to residential use is a process that can increase capital returns (capital gains) in ways that are not captured in standard equilibrium capital theory analysis. The discussion appears on pages 93-98, following Bai et al. (2006). Bai et al. (2006, Table 1 and Figure 2, pp. 72-75) also report a nearly 60% decline in capital returns in China from 1993-2001. This dramatic decline cannot be fully attributed to a TFP-growth decline, possibly validating the comments by Nordhaus and Cooper in Bai et al. (2006, pp. 93-98). Part of this decline can be explained, however, by the anticipated rapid decline in

China's population growth rate, as reported in Figures 1 and 2 of the present paper. Song et al. (2011) explore the seeming contradiction implicit in China's simultaneous high capital returns and high capital outflows. Their model rests on the internal reallocation of capital out of low growth firms that are large, externally financed, and whose capital needs are low. In contrast, high growth, high productivity firms are small and subject to capital constraints. They thus finance their rapidly increasing investments out of internally generated funds alone. As a result, the surplus capital from low growth firms migrates abroad, while the relative growth in the high productivity firms allows the high overall capital returns to be observed. A more recent study also reporting high capital returns in China and focusing on the link between these returns and the housing boom in China, is Chen and Wen (2017). Nothing in the present model depends on the precise level of capital returns.

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Appendix H - Data Descriptions and Sources

year	China FDI/GDPratio (%)	India FDI/GDPratio (%)	Ratio	log(ratio)
1981	0.155	0.027	5.822	0.765
1982	0.239	0.022	10.731	1.031
1983	0.430	0.002	255.079	2.407
1984	0.661	0.006	111.110	2.046
1985	0.824	0.032	25.571	1.408
1986	0.710	0.028	25.256	1.402
1987	0.613	0.043	14.218	1.153
1988	0.707	0.016	44.807	1.651
1989	0.751	0.043	17.615	1.246
1990	0.709	0.048	14.783	1.170
1991	0.800	0.022	35.780	1.554
1992	1.750	0.070	24.988	1.398
1993	3.930	0.146	26.934	1.430
1994	4.198	0.246	17.046	1.232
1995	3.832	0.476	8.044	0.905
1996	4.074	0.544	7.489	0.874
1997	4.402	0.801	5.495	0.740
1998	4.251	0.555	7.655	0.884
1999	3.450	0.431	7.997	0.903
2000	3.321	0.637	5.216	0.717
2001	3.744	0.974	3.845	0.585
2002	3.781	0.934	4.048	0.607
2003	3.033	0.561	5.407	0.733
2004	3.092	0.660	4.687	0.671
2005	4.240	0.783	5.418	0.734
2006	4.290	1.978	2.168	0.336
2007	4.404	2.070	2.127	0.328
2008	4.322	3.518	1.228	0.089
2009	3.234	2.952	1.095	0.040
2010	4.914	2.209	2.224	0.347
2011	4.811	2.597	1.853	0.268
2012	4.178	1.816	2.300	0.362
2013	4.290	1.989	2.157	0.334
2014	5.132	2.829	1.814	0.259

Table H.1 Data on FDI/GDP ratios

Foreign Direct Investment[1]

We use four different data sources to cross-verify the FDI inflows and outflows of China and India.

1. OECD: 1990-2013. Historic time series from OECD FDI statistics to end-2013 (http://www.oecd.org/daf/inv/investment-policy/fdi-statistics-according-tobmd3.htm).
2. National Accounts: 1982 – 2014. National Bureau of Statistics China (NBS-China) provides FDI outflow and inflow information (http://datE.stats.gov.cn/english/index.htm).
3. UNCTAD (United Nations Conference on Trade and Development): 1981-2013.The UNCTAD work program on FDI Statistics documents and analyzes global and regional trends in FDI.
4. DataStream: 1981-2016 (Quarterly). Thomson Reuters DataStream provides quarterly data on FDI inflows and outflows for China and India.[2]

Population Estimates and Forecasts: 1950-2100. United Nations: probabilistic population projections based on the world population prospects (the 2015 revision).[3]

GDP Series: 1990-2014, 2015-2018 (estimates). Work Bank, PPP adjusted at constant 2011 international USD.

Capital Stock -GDP ratio (K/Y ratio): PWT 9.0 (The Penn World Table).

FDI data come from four sources: (a) National Accounts, (b) OECD, (c) Datastream, and (d) UNCTAD. These sources cover different years, so we specify which we use in each context and document the correlation among these data sources. National account data for India is downloaded from the RBI website (https://rbi.org.in/Scripts/SDDSView.aspx) and it is identical to the data provided by OECD. So, we only report the OECD source.

[1] All FDI statistics from different sources use 2010 USD as the base dollar value.
[2] The quarterly data sources are composed by Oxford Economics (http://www.oxfordeconomics.com/).
[3] United Nations (2015). Probabilistic Population Projections based on the World Population Prospects: The 2015 Revision. Population Division, DESA. http://esE.un.org/unpd/ppp/.

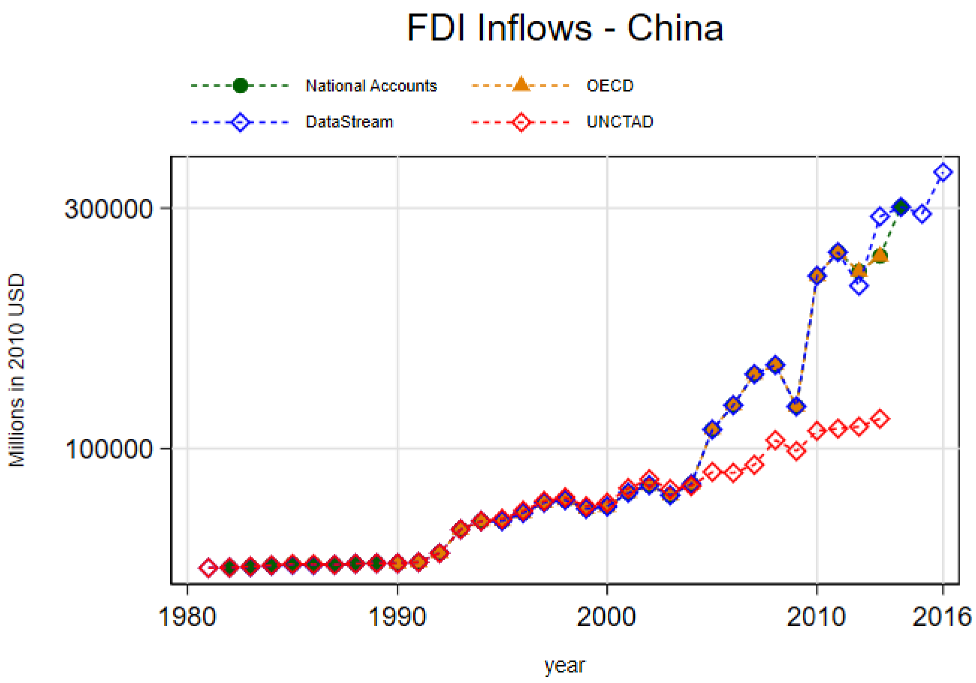

Figure H.1

The sources used in the paper are National-accounts data for the period 1982-2014 and Datastream data for years 2015-2016. National-accounts data and Datastream data overlap over the period 1982-2014 with a correlation coefficient of 99.79%.

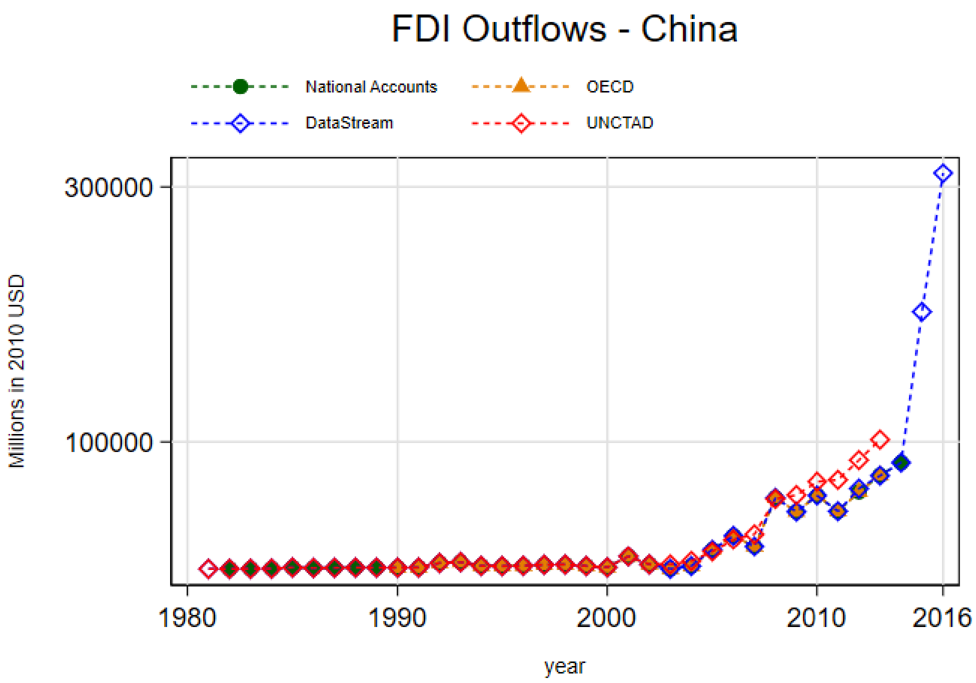

Figure H.2

The sources used in the paper are National-accounts data for the period 1982-2014 and Datastream data for years 2015-2016. National-accounts data and Datastream data overlap over the period 1982-2014 with a correlation coefficient 99.99%.

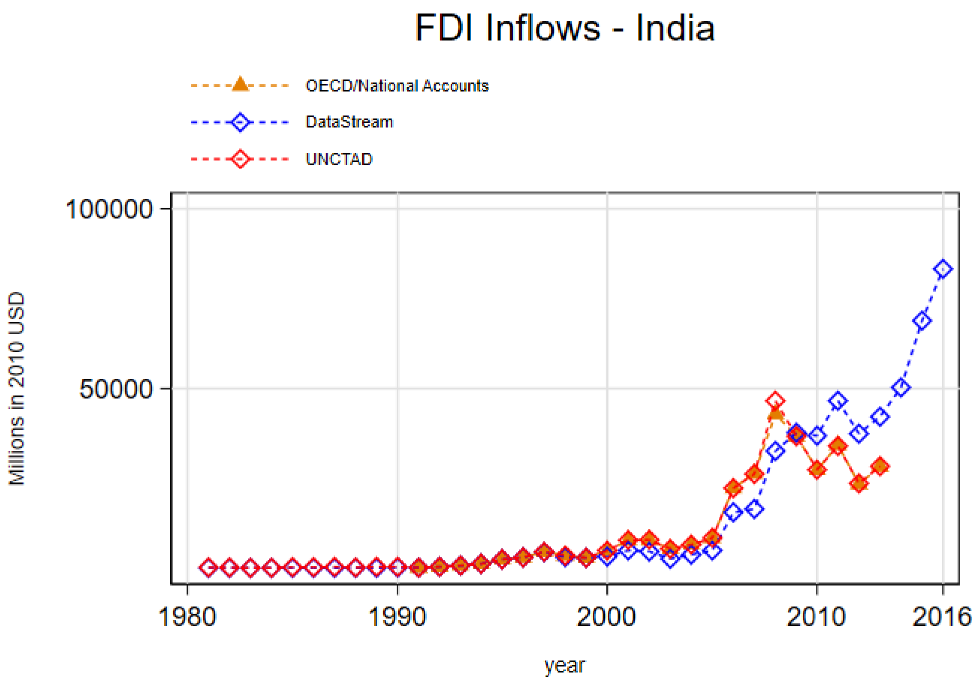

Figure H.3

The sources used in the paper are UNCTAD data for the period 1981-2013 and Datastream data for years 2014-2016. UNCTAD data and Datastream data overlap over the period 1981-2013 with a correlation coefficient of 92.56%. The reason we have chosen UNCTAD data for the period 1981-2013 is because, (a) for the period between 1981 and 1989 Datastream reports zero values (but not missing values), and (b) the two data sources overlap over the period 1991-2013 with a correlation coefficient of 99.87%.

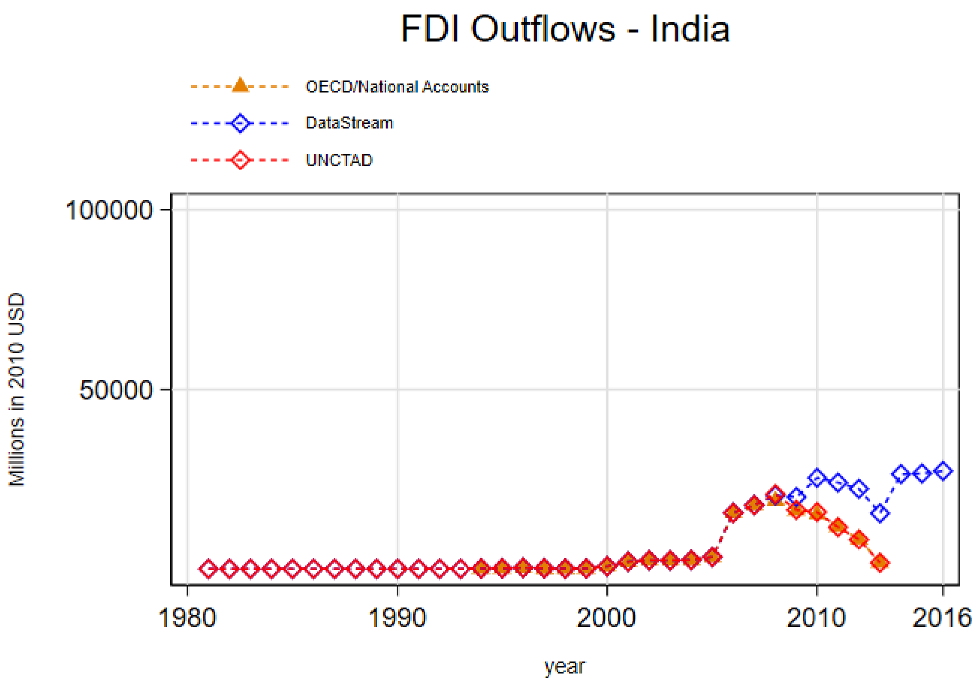

Figure H.4

The sources used in the paper are UNCTAD data for the period 1981-2013 and Datastream data for years 2014-2016. UNCTAD data and Datastream data overlap over the period 1981-2013 with a correlation coefficient of 89.32%. The reason we have chosen UNCTAD data for the period 1981-2013 is because, (a) for the period between 1981 and 1993 Datastream reports zero values (but not missing values), and (b) the two data sources overlap over the period 1994-2013 with a correlation coefficient of 99.86%.

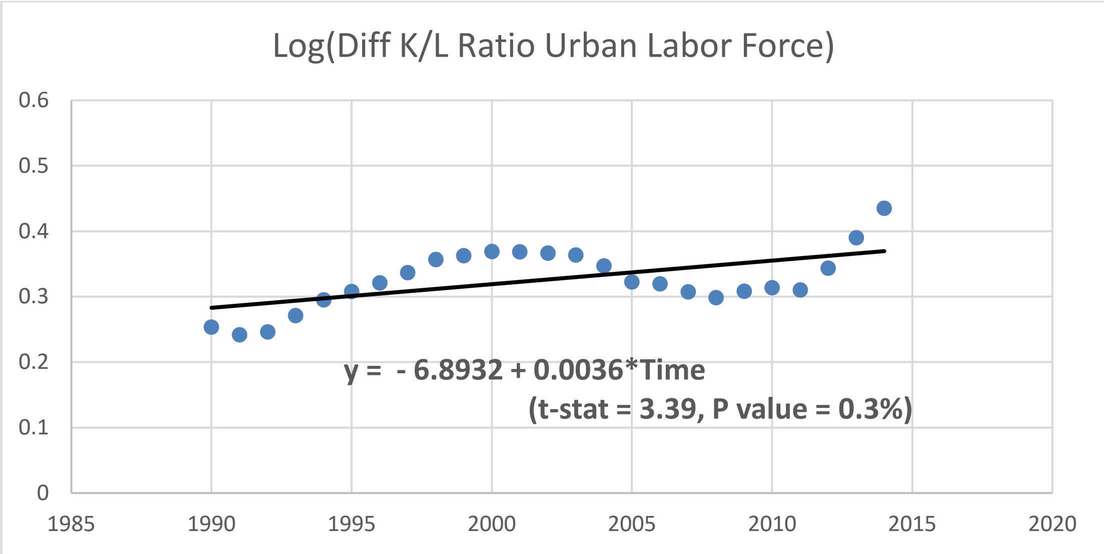

Figure H.5

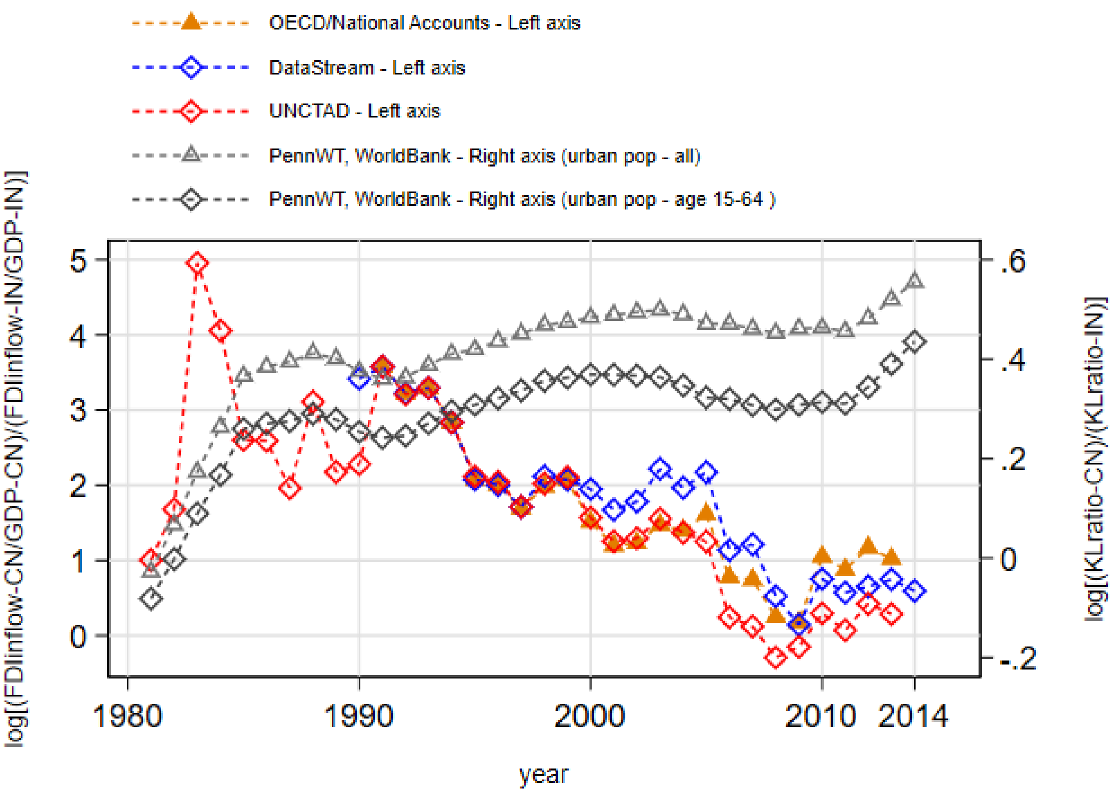

Figure H.6

To address the concern that large-scale internal migration in China would decrease the capital-labor ratio instead of increasing it, we use the urban population, restricted to ages 15-64 and perform a robustness check. Figure H.5 shows that the linear time trend coefficient (of the log K/L ratio of China over the K/L ratio of India) is positive and statistically significant (not equal to 0 with p-value at 0.3%). In Figure H.6 where we plot a similar data series as Figure 5 (in the paper) using this restricted sample, all the quantitative results remain.

The first two columns of Table H.2 provide the data appearing in Figure H.6 (without the logarithmic conversion of ratios). The last two columns of Table H.2 are the two new urban (working) population series appearing in Figure H.6.

year	Ratio_FDIY	Ratio_FullPop	Ratio_PopUrban	Ratio_PopUrbanWorking
1990	30.45	0.96	1.46	1.29
1991	35.73	0.99	1.43	1.27
1992	25.08	1.02	1.44	1.28
1993	26.94	1.07	1.47	1.31
1994	17.04	1.13	1.51	1.34
1995	7.96	1.17	1.52	1.36
1996	7.42	1.22	1.55	1.38
1997	5.54	1.27	1.57	1.40
1998	8.32	1.32	1.60	1.43
1999	7.95	1.36	1.61	1.44
2000	7.02	1.41	1.62	1.45
2001	5.33	1.50	1.63	1.45
2002	5.94	1.57	1.64	1.44
2003	9.19	1.66	1.65	1.44
2004	7.14	1.73	1.63	1.41
2005	8.79	1.72	1.60	1.38
2006	3.11	1.72	1.60	1.38
2007	3.36	1.71	1.59	1.36
2008	1.69	1.68	1.57	1.35
2009	1.16	1.70	1.59	1.36
2010	2.12	1.72	1.59	1.37
2011	1.77	1.72	1.58	1.36
2012	1.92	1.77	1.62	1.41
2013	2.11	1.87	1.68	1.48
2014	1.81	1.97	1.74	1.55

Table H.2