Online Appendix for "A Model of Endogenous Education Quality: the Role of Teachers"

Mauro Rodrigues and Danilo Souza[1]

A Additional Tables and Figures

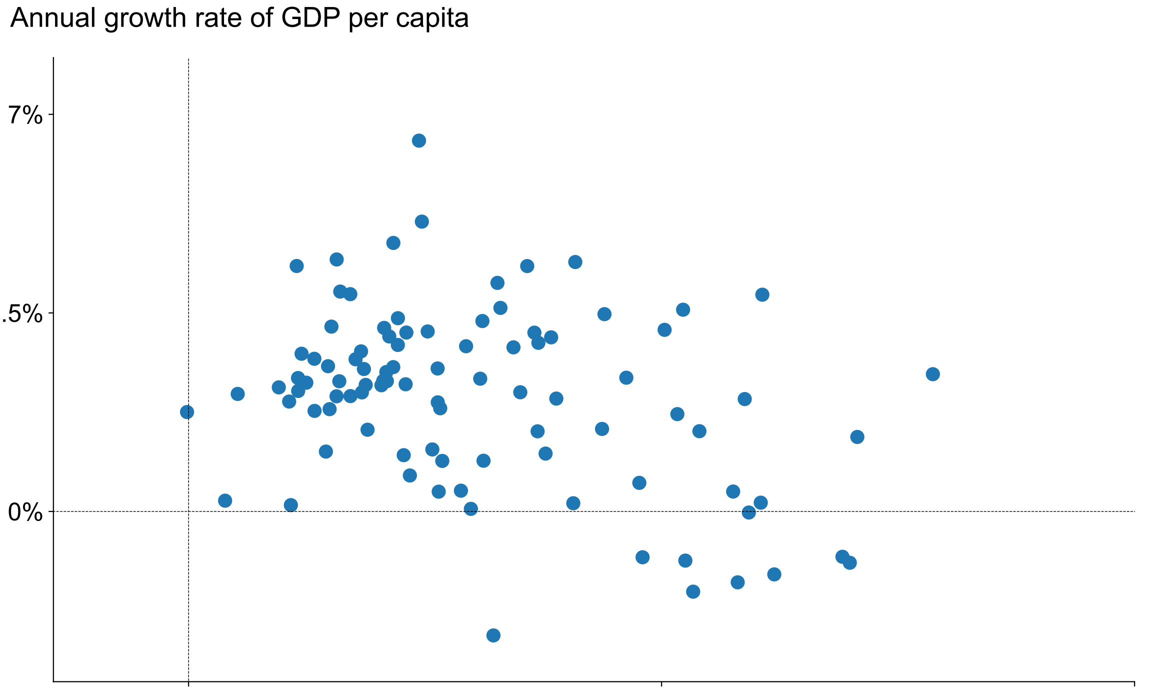

Fig. A.1: Average growth in years of schooling and growth in GDP *per capita*, 1970-2010. *Source: Barro-Lee Database (average years of schooling) and The Maddison Project (GDP per capita).*

[1]Rodrigues: Department of Economics, University of Sao Paulo (mrodrigues@usp.br); Souza: Department of Economics, University of Sao Paulo (danilosouza@usp.br).

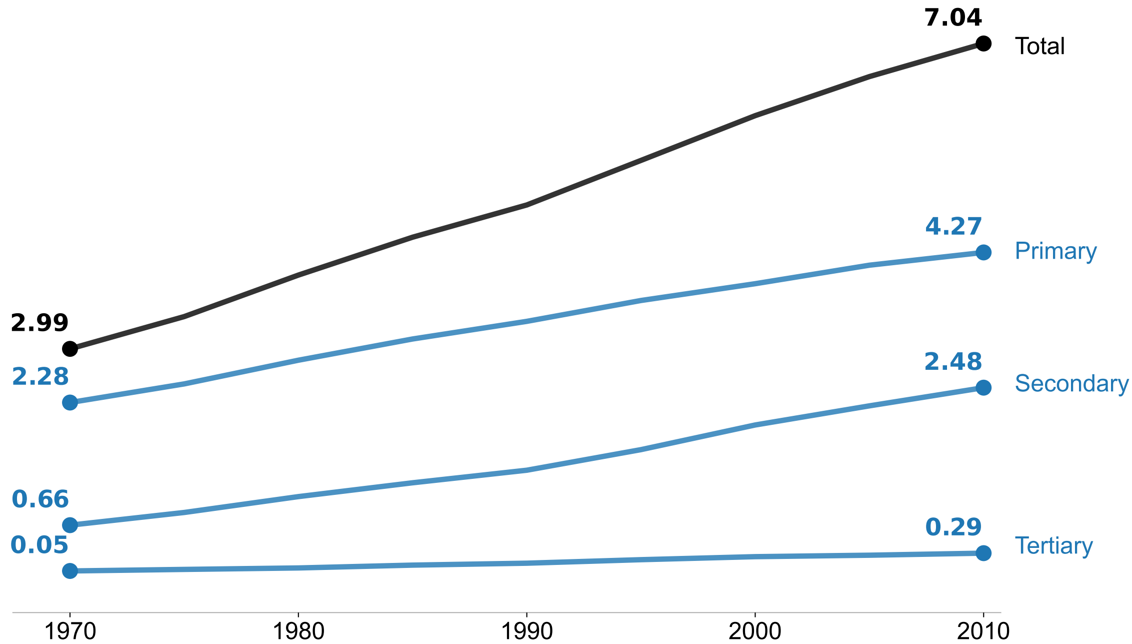

Fig. A.2: Weighted average years of schooling - Middle and Low Income countries. *Source: Barro-Lee Database.*

Table A.1: Calibrated Parameters

	$\upsilon = 0\%$	$\upsilon = 5\%$	$\upsilon = 10\%$	$\upsilon = 15\%$
Values for ψ				
$\eta = 13.4\%$	0.239	0.214	0.177	0.140
$\eta = 10\%$	0.241	0.212	0.172	0.133
$\eta = 6.8\%$	0.239	0.215	0.175	0.138
Values for ϕ				
$\eta = 13.4\%$	0.488	0.510	0.538	0.548
$\eta = 10\%$	0.488	0.509	0.534	0.545
$\eta = 6.8\%$	0.488	0.507	0.538	0.545
Values for Z				
$\eta = 13.4\%$	3.255	3.219	3.296	3.367
$\eta = 10\%$	3.047	3.056	3.178	3.264
$\eta = 6.8\%$	2.916	2.873	2.984	3.046

Table A.2: Country-specific Parameters for $\eta = 13.4\%$

	ψ			ϕ			Z		
	$v = 0\%$	$v = 5\%$	$v = 10\%$	$v = 0\%$	$v = 5\%$	$v = 10\%$	$v = 0\%$	$v = 5\%$	$v = 10\%$
LATAM	0.239	0.214	0.177	0.488	0.510	0.538	3.255	3.219	3.296
Brazil	0.412	0.377	0.352	0.555	0.498	0.489	3.000	3.070	3.094
Chile	0.276	0.251	0.223	0.425	0.481	0.495	2.649	2.637	2.662
Ecuador	0.464	0.432	0.407	0.425	0.495	0.486	3.149	3.154	3.136
Honduras	0.248	0.241	0.211	0.555	0.494	0.503	2.649	2.494	2.480
Mexico	0.204	0.187	0.154	0.425	0.466	0.515	3.554	3.410	3.398
Nicaragua	0.176	0.152	0.123	0.425	0.477	0.484	1.399	1.430	1.497
Panama	0.152	0.130	0.102	0.555	0.489	0.499	6.554	6.430	6.426
Uruguay	0.404	0.383	0.358	0.425	0.478	0.496	2.649	2.645	2.636

Table A.3: Targeted Moments

	Model			
	$v = 5\%$	$v = 10\%$	$v = 15\%$	Data
Tax rate as a percentage of GDP				
Average				
$\eta = 13.4\%$	0.194	0.190	0.186	0.198
$\eta = 10\%$	0.193	0.186	0.181	0.198
$\eta = 6.8\%$	0.195	0.189	0.184	0.198
Standard Deviation				
$\eta = 13.4\%$	0.001	0.002	0.003	0.063
$\eta = 10\%$	0.001	0.002	0.003	0.063
$\eta = 6.8\%$	0.001	0.002	0.003	0.063
Ratio between teachers and market workers wage				
Average				
$\eta = 13.4\%$	1.088	1.095	1.099	1.078
$\eta = 10\%$	1.087	1.096	1.098	1.078
$\eta = 6.8\%$	1.089	1.102	1.104	1.078
Standard Deviation				
$\eta = 13.4\%$	0.452	0.478	0.505	0.281
$\eta = 10\%$	0.459	0.486	0.512	0.281
$\eta = 6.8\%$	0.466	0.495	0.522	0.281

B Model: Proofs of Propositions

Proof of Proposition 1. Equations (12) and (13) are the result of integrating the individual human capital production function for each group (market employees and teachers), given the innate ability threshold and the government's budget constraint. Thus,

$$\begin{aligned}
H_t &= \int_0^{a^\star-\bar{B}\theta} h_t^i(a^i)dF(a^i) + \int_{a^\star}^{\bar{B}} h_t^i(a^i)dF(a^i) \\
&= \int_0^{a^\star-\bar{B}\theta} Za^i s^\eta (h_{t-1}^T)^v f(a^i)da^i + \int_{a^\star}^{\bar{B}} Za^i s^\eta (h_{t-1}^T)^v f(a^i)da^i \\
&= Z(h_{t-1}^T)^v s^\eta \left(\int_0^{a^\star-\bar{B}\theta} a^i \frac{1}{\bar{B}} da^i + \int_{a^\star}^{\bar{B}} a^i \frac{1}{\bar{B}} da^i \right) \\
&= \frac{Zs^\eta (h_{t-1}^T)^v}{\bar{B}} \left(\left[\frac{(a^i)^2}{2} \right]_0^{a^\star-\bar{B}\theta} + \left[\frac{(a^i)^2}{2} \right]_{a^\star}^{\bar{B}} \right) \\
&= \frac{Zs^\eta (h_{t-1}^T)^v}{2\bar{B}} \left((a^\star)^2 - 2a^\star \bar{B}\theta + \bar{B}^2\theta^2 + \bar{B}^2 - (a^\star)^2 \right) \\
&= \frac{Z\bar{B}(1+\theta^2)s^\eta}{2}(h_{t-1}^T)^v - Zs^\eta (h_{t-1}^T)^v \theta a^\star \\
&= \frac{Z\bar{B}(1+\theta^2)s^\eta}{2}(h_{t-1}^T)^v - Zs^\eta (h_{t-1}^T)^v \theta \frac{w_t^T}{w_t^M} \frac{1}{Zs^\eta (h_{t-1}^T)^v} \\
&= \frac{Z\bar{B}(1+\theta^2)s^\eta}{2}(h_{t-1}^T)^v - \theta \frac{1}{w_t^M} \frac{1}{\theta} \frac{p\tau}{1-p\tau} w_t^M H_t \\
&= \frac{Z\bar{B}(1+\theta^2)s^\eta}{2}(h_{t-1}^T)^v - \frac{p\tau}{(1-p\tau)} H_t
\end{aligned}$$

and finally

$$H_t = \frac{Z\bar{B}}{2} \left[(1+\theta^2)(1-p\tau) \right] s^\eta (h_{t-1}^T)^v$$

Equation (13) comes from equation (12) and the fact that $H_t + \theta . h_t^T = \int_0^{\bar{B}} h_t^i(a^i).dF(a^i) = \frac{Z.\bar{B}}{2} s^\eta . (h_{t-1}^T)^v$. Thus,

$$
\begin{aligned}
h_t^T &= \frac{1}{\theta}\left[\frac{Z.\bar{B}}{2}s^{\eta}.(h_{t-1}^T)^v - H_t\right] \\
&= \frac{1}{\theta}\left[\frac{Z.\bar{B}}{2}s^{\eta}.(h_{t-1}^T)^v - \frac{Z\bar{B}}{2}\left[(1+\theta^2)(1-p\tau)\right]s^{\eta}(h_{t-1}^T)^v\right]
\end{aligned}
$$

and finally

$$
h_t^T = \frac{Z\bar{B}}{2\theta}[1-(1+\theta^2)(1-p\tau)]s^{\eta}(h_{t-1}^T)^v
$$

□

Proof of Proposition 2. To reach equation (14) we first use equation (3) along with equations (11) and (9), the fact that $w_t^M = A$ and $s_t = s$ for a constant θ. Thus,

$$
\begin{aligned}
a_t^\star &= \frac{w_t^T}{w_t^M}\cdot\frac{1}{Z.s^{\eta}.(h_{t-1}^T)^v} \\
a_t^\star &= \frac{1}{\theta}\cdot\frac{p\tau}{1-p\tau}\cdot AH_t\cdot\frac{1}{A}\cdot\frac{1}{Z.s^{\eta}.(h_{t-1}^T)^v}
\end{aligned}
$$

Now we use the function that defines the stock of human capital of market workers, as in equation (12), to reach

$$
\begin{aligned}
a_t^\star &= \frac{1}{\theta}\cdot\frac{p\tau}{1-p\tau}\cdot\frac{Z\bar{B}}{2}\left[(1+\theta^2)(1-p\tau)\right]s^{\eta}(h_{t-1}^T)^v\cdot\frac{1}{Z.s^{\eta}.(h_{t-1}^T)^v} \\
a_t^\star = a^\star &= \frac{(1+\theta^2)}{\theta}\cdot\frac{\bar{B}}{2}\cdot p\cdot\tau
\end{aligned}
$$

Moreover, the fact that $a^\star$ is decreasing in θ follows directly from the partial derivative of $a^\star$ with respect to θ. Since $\theta \in (0,1)$, and $\bar{B}, p, \tau > 0$,

$$
\frac{\partial a^\star}{\partial\theta} = -\frac{(1-\theta^2)}{\theta^2}\cdot\frac{\bar{B}}{2}\cdot p\cdot\tau < 0
$$

$\square$

Proof of Proposition 3. We begin by taking the partial derivative of equation (13) with respect to θ to show the short-run effect:

$$\begin{aligned}
\frac{\partial h_t^T}{\partial \theta} &= \left[\frac{Z\bar{B}}{2} \cdot s^{\eta} \cdot \left(h_{t-1}^T\right)^{v}\right] \cdot \left[\frac{-2\theta^2(1-p\tau) - 1 + (1+\theta^2)(1-p\tau)}{\theta^2}\right] \\
\frac{\partial h_t^T}{\partial \theta} &= \left[\frac{Z\bar{B}}{2} \cdot s^{\eta} \cdot \left(h_{t-1}^T\right)^{v}\right] \cdot \left[\frac{-p\tau - \theta^2(1-p\tau)}{\theta^2}\right] < 0
\end{aligned}$$

To see the long-run effect of the same increase in θ we first take the log in both sides of the steady-state counterpart of equation (13)

$$\begin{aligned}
h^T &= \left[\frac{Z\bar{B}}{2\theta}[1 - (1+\theta^2)(1-p\tau)]s^{\eta}\right]^{\frac{1}{1-v}} \\
\ln(h^T) &= \frac{1}{1-v} \cdot \left[\ln\left(\frac{ZB}{2}\right) + \eta\ln(s) - \ln(\theta) + \ln(1 - (1+\theta^2)(1-p\tau))\right]
\end{aligned}$$

and, then, take the partial derivative of $\ln(h^T)$ with respect to $\ln(\theta)$, which is the same as calculating the elasticity of h^T with respect to θ.

$$\begin{aligned}
\frac{\partial \ln(h^T)}{\partial \ln(\theta)} &= \frac{1}{1-v} \cdot \left[\eta\frac{\partial \ln(s)}{\partial \ln(\theta)} - 1 + \frac{\partial \ln(1 - (1+\theta^2)(1-p\tau))}{\partial \ln(\theta)}\right] \\
\xi_{h^T,\theta} &= \frac{1}{1-v} \cdot \left[\eta\xi_{s,\theta} - 1 - \frac{2\theta^2(1-p\tau)}{1-(1+\theta^2)(1-p\tau)}\right]
\end{aligned}$$

which can be finally rewritten as the equation below by incorporating the function that relates s and θ as in equation (15).

$$\xi_{h^T,\theta} = \frac{1}{1-v} \cdot \left[\eta - 1 - \frac{2\theta^2(1-p\tau)}{1-(1+\theta^2)(1-p\tau)}\right]$$

Thus, since $\eta \in (0,1)$, it is clear that $\xi_{h^T,\theta} < 0$, which means that a policy to increase average years of schooling by hiring more teachers is followed by a decrease in the average human capital of teachers in both the short and long-run.

$\square$

Proof of Lemma 1. We begin by writing the average teachers quality of steady-state

$$h^T = \left[\frac{Z\bar{B}}{2\theta}[1-(1+\theta^2)(1-p\tau)]s^\eta\right]^{\frac{1}{1-v}}$$

Note that the stock of human capital of market workers of steady-state H is a function of h^T and given by

$$H = \frac{Z\bar{B}}{2}[(1+\theta^2)(1-p\tau)]s^\eta(h^T)^v$$

We then take the log in both sides of this equation to reach:

$$\ln(H) = \ln\left(\frac{Z\bar{B}}{2}[(1+\theta^2)(1-p\tau)]s^\eta\right) + v\ln(h^T)$$
$$\ln(H) = \ln\left(\frac{Z\bar{B}}{2}\right) + \ln(1+\theta^2) + \ln(1-p\tau) + \eta\ln(s) + v\ln(h^T)$$

and, then, take the partial derivative of $\ln(H)$ with respect to $\ln(\theta)$, which is the same as calculating the elasticity of H with respect to θ.

$$
\begin{aligned}
\frac{\partial \ln(H)}{\partial \ln(\theta)} &= \frac{\partial \ln(1+\theta^2)}{\partial \ln(\theta)} + \eta \frac{\partial \ln(s)}{\partial \ln(\theta)} + v \frac{\partial \ln(h^T)}{\partial \ln(\theta)} \\
\frac{\partial \ln(H)}{\partial \ln(\theta)} &= \frac{2\theta^2}{1+\theta^2} + \eta + v\xi_{h^T,\theta}
\end{aligned}
$$

which can be restated, by using equation (15) and $\xi_{h^T,\theta}$, as

$$
\xi_{H,\theta} = \frac{\eta - v}{1-v} + \frac{2\theta^2}{1-v}\left[\frac{1-v}{1+\theta^2} - \frac{v(1-p\tau)}{1-(1+\theta^2)(1-p\tau)}\right]
$$

□

Proof of Proposition 4. We begin by taking the partial derivative of equations (12) and (11) with respect to θ to show the short-run effect:

$$
\frac{\partial H_t}{\partial \theta} = \left[\frac{Z\bar{B}}{2} \cdot s^{\eta} \cdot \left(h_{t-1}^{T}\right)^{v}\right] \cdot 2\theta(1-p\tau) > 0
$$

$$
\frac{\partial GDP_t}{\partial \theta} = \frac{A}{1-p\tau} \cdot \frac{\partial H_t}{\partial \theta} > 0
$$

The long-run effect of the same increase in θ, however, follows directly from equation (16). The elasticity of the steady-state value of H_t – and, therefore, GDP_t – with respect to θ can be both positive and negative depending mainly on the values we assume for the education quantity and education quality rates of return (η and v respectively). □

Proof of Proposition 5. The long-run effect of an increase in τ or p on GDP follows directly from the partial derivative of $ln(H)$ with respect to $ln(\tau)$ or $ln(p)$, and the fact that GDP is proportional to H. First, notice that

$$\xi_{h^T,\theta} = \xi_{h^T,p} = \frac{p\tau(1+\theta^2)}{(1-v)[1-(1+\theta^2)(1-p\tau)]} > 0$$

which leads to

$$\begin{aligned}
\frac{\partial \ln(H)}{\partial \ln(\tau)} &= -\frac{p}{1-p\tau} + v\xi_{h^T,\theta} \lesseqgtr 0 \\
\frac{\partial \ln(H)}{\partial \ln(p)} &= -\frac{\tau}{1-p\tau} + v\xi_{h^T,p} \lesseqgtr 0
\end{aligned}$$

Then, after some algebra, we can write the elasticity of steady-state GDP with respect to τ and p as a function of $\frac{\partial \ln(H)}{\partial \ln(\tau)}$ and $\frac{\partial \ln(H)}{\partial \ln(p)}$, respectively.

$$\begin{aligned}
\frac{\partial \ln(GDP)}{\partial \ln(\tau)} = \xi_{GDP,\tau} &= \frac{p}{1-p\tau} + \frac{\partial \ln(H)}{\partial \ln(\tau)} \\
\frac{\partial \ln(GDP)}{\partial \ln(p)} = \xi_{GDP,p} &= \frac{\tau}{1-p\tau} + \frac{\partial \ln(H)}{\partial \ln(p)}
\end{aligned}$$

$$\xi_{GDP,\tau} = \xi_{GDP,p} = \frac{vp\tau(1+\theta^2)}{(1-v)[1-(1+\theta^2)(1-p\tau)]} > 0$$

□

Proof of Proposition 6. Given that adult's utility is logarithmic in consumption (private goods and public goods) and children's human capital, the first-order condition with respect to τ is necessary and sufficient to reach the $\tau^\star$ that maximizes u_t^i. We can rewrite the optimization problem by substituting all constraints in the utility function such as

$$\underset{\tau}{\text{maximize}} \quad \log\left[(1-\tau)w_t^M h_t\right] + \psi \log\left[\frac{(1-p)\tau}{1-p\tau} A H_t\right] + \phi \log\left[Z a^i s^\eta (h_t^T)^v\right]$$

which is also equivalent to

$$\underset{\tau}{\text{maximize}} \quad \log(1-\tau) + \psi \log\left[(1-p)\tau\right] + \phi v \log\left[1-(1+\theta^2)(1-p\tau)\right]$$

given that the private market worker take w_t^M, h_t, Z, a^i and s^η as given. Thus, $\tau^\star$ is the one that solves the first-order condition below for given values of ψ, ϕ, v, p, and θ.

$$\frac{-1}{1-\tau^\star} + \frac{\psi}{\tau^\star} + \frac{\phi v p(1+\theta^2)}{1-(1+\theta^2)(1-p\tau^\star)} = 0$$

The fact that $\tau^\star$ is increasing in θ follows directly from the Implicit Function Theorem. First, we define the first order condition as a function $F(\tau^\star, \theta) = 0$. Then notice that $\partial F/\partial\theta > 0$ and $\partial F/\partial\tau^\star < 0$. Finally, the derivative of $\tau^\star$ with respect to θ is given by

$$\frac{d\tau^\star}{d\theta} = -\frac{\partial F/\partial\theta}{\partial F/\partial\tau^\star} > 0$$

□