

# Online Appendix

## A Additional Materials

The tables in this appendix show the summary statistics of the variables used followed by cointegration, stationarity and tests of exogenous conditioning.

The figures are related to robustness exercises shown in the main IRF exercises.

Table A.1: Descriptive Statistics

	Mean	Median	Max	Min	Std. dev.	Mean	Median	Max	Min	Std. dev.
	<i>y</i>					<i>e</i>				
Canada	4.54	4.58	4.98	4.03	0.29	-4.33	-4.25	-3.64	-4.87	0.33
France	4.54	4.58	4.83	4.18	0.20	-4.74	-4.80	-3.96	-5.21	0.31
Germany	4.54	4.59	4.86	4.18	0.20	-4.72	-4.80	-3.90	-5.19	0.34
Italy	4.52	4.59	4.69	4.22	0.14	-4.76	-4.80	-4.00	-5.24	0.34
Japan	4.56	4.60	4.78	4.13	0.18	0.28	0.17	1.19	-0.23	0.37
UK	4.54	4.59	4.90	4.09	0.25	-4.99	-5.04	-4.11	-5.48	0.35
US	4.51	4.59	4.97	3.95	0.30					
	<i>q</i>					<i>cpi</i>				
Canada	1.87	2.01	2.64	0.68	0.59	0.007	0.006	0.032	-0.010	0.007
France	2.33	2.54	3.25	0.76	0.65	0.007	0.005	0.038	-0.004	0.008
Germany	1.48	1.60	2.25	0.30	0.52	0.005	0.004	0.021	-0.004	0.005
Italy	2.61	2.52	3.47	1.48	0.42	0.011	0.007	0.062	-0.005	0.012
Japan	1.32	1.37	2.11	0.47	0.36	0.002	0.001	0.024	-0.010	0.005
UK	3.18	3.33	3.75	2.09	0.43	0.009	0.007	0.052	-0.003	0.008
US	1.84	2.06	2.94	0.48	0.67	0.008	0.007	0.038	-0.022	0.007
	<i>r</i>					<i>green</i>				
Canada	0.013	0.010	0.045	0.000	0.010	0.98	0.98	1.11	0.82	0.08
France	0.012	0.009	0.041	-0.002	0.011	0.94	0.89	1.18	0.75	0.14
Germany	0.009	0.008	0.028	-0.002	0.008	1.02	0.98	1.20	0.88	0.10
Italy	0.016	0.011	0.049	-0.001	0.014	0.85	0.82	1.09	0.60	0.13
Japan	0.006	0.001	0.029	0.000	0.007	0.96	0.95	1.14	0.79	0.10
UK	0.014	0.013	0.037	0.001	0.010	0.89	0.82	1.13	0.71	0.13
US	0.010	0.011	0.035	0.000	0.008	0.91	0.86	1.11	0.76	0.11
	<i>l</i>									
Canada	0.016	0.014	0.040	0.003	0.009					
France	0.015	0.013	0.039	-0.001	0.010					
Germany	0.012	0.012	0.026	-0.001	0.007					
Italy	0.019	0.013	0.048	0.003	0.012					
Japan	0.008	0.004	0.022	0.000	0.006					
UK	0.016	0.013	0.036	0.002	0.009					
US	0.015	0.014	0.035	0.004	0.008					

Notes: Data presented in natural logs.

Table A.2: Unit root test (Weighted Symmetric) for domestic variables at 5% significance

Domestic Variables	CV	Canada	France	Germany	Italy	Japan	UK	US
$y$ (with trend)	-3.24	-2.16	-1.60	-2.24	-1.04	-0.31	-2.09	-1.81
$y$ (no trend)	-2.55	1.25	0.81	0.68	0.32	1.88	0.28	0.58
$\Delta y$	-2.55	-5.72	-5.56	-5.61	-5.54	-5.92	-4.76	-4.97
$q$ (with trend)	-3.24	-2.88	-2.41	-3.21	-2.16	-1.90	-1.38	-2.17
$q$ (no trend)	-2.55	-0.82	-0.78	-0.93	-2.00	-1.70	-0.08	0.53
$\Delta q$	-2.55	-8.34	-8.01	-6.10	-5.43	-8.05	-8.79	-7.52
$r$ (with trend)	-3.24	-3.33	-3.57	-4.26	-3.28	-2.34	-3.20	-2.31
$r$ (no trend)	-2.55	-0.79	-0.56	-1.46	-0.56	-0.94	-0.39	-0.14
$\Delta r$	-2.55	-6.90	-9.07	-4.25	-7.74	-6.01	-7.87	-4.70
$l$ (with trend)	-3.24	-4.53	-3.54	-3.50	-2.98	-1.59	-4.01	-4.11
$l$ (no trend)	-2.55	-0.21	-0.13	-0.35	-0.98	0.85	0.55	-0.20
$\Delta l$	-2.55	-7.93	-6.30	-6.61	-4.94	-5.95	-7.26	-7.85
$e$ (with trend)	-3.24	-1.70	-2.36	-2.26	-2.26	-1.54	-2.09	
$e$ (no trend)	-2.55	0.42	-1.19	-1.09	-0.87	-0.17	-0.60	
$\Delta e$	-2.55	-8.07	-8.16	-6.29	-8.56	-5.63	-9.32	
$cpi$ (with trend)	-3.24	-2.84	-0.90	-3.48	0.60	-2.80	-1.29	-1.77
$cpi$ (no trend)	-2.55	-1.08	0.42	-2.84	2.49	-1.88	0.32	-0.34
$\Delta cpi$	-2.55	-8.96	-8.98	-9.59	-13.30	-10.65	-8.27	-10.91
$green$ (with trend)	-3.24	-2.76	-0.37	-2.03	-2.08	-0.37	-0.59	-1.75
$green$ (no trend)	-2.55	-1.31	-0.90	-1.32	-1.10	-1.36	-1.08	-1.88
$\Delta green$	-2.55	-3.72	-2.88	-2.33	-4.30	-2.57	-2.46	-2.01

**Notes:** Stationarity test for level with and without trend, and first difference. CV denotes critical value.

Table A.3: Unit root test (Weighted Symmetric) for foreign variables at 5% significance

Foreign Variables	CV	Canada	France	Germany	Italy	Japan	UK	US
$y^*$ (with trend)	-3.24	-1.72	-1.58	-1.45	-1.69	-1.70	-1.41	-1.68
$y^*$ (no trend)	-2.55	0.55	0.85	0.53	0.84	0.49	1.37	0.94
$\Delta y^*$	-2.55	-4.94	-6.05	-4.59	-5.91	-4.97	-5.86	-5.53
$q^*$ (with trend)	-3.24	-2.17	-2.31	-2.10	-2.45	-2.29	-2.49	-3.09
$q^*$ (no trend)	-2.55	0.45	-0.55	-0.39	-0.41	0.18	-0.36	-0.74
$\Delta q^*$	-2.55	-7.62	-8.28	-8.05	-8.29	-7.83	-8.15	-8.42
$r^*$ (with trend)	-3.24	-2.34	-4.37	-3.02	-4.06	-2.42	-3.09	-3.26
$r^*$ (no trend)	-2.55	0.00	-0.38	0.06	-0.70	0.24	-0.04	-0.51
$\Delta r^*$	-2.55	-4.79	-6.13	-6.99	-6.22	-6.99	-6.53	-6.00
$l^*$ (with trend)	-3.24	-4.11	-3.56	-2.85	-3.75	-4.36	-3.59	-4.02
$l^*$ (no trend)	-2.55	-0.07	0.07	0.24	0.26	0.06	0.12	0.17
$\Delta l^*$	-2.55	-7.88	-6.87	-6.82	-7.13	-7.94	-7.03	-7.52
$e^*$ (with trend)	-3.24	-1.81	-2.20	-2.19	-2.23	-2.16	-2.15	-1.54
$e^*$ (no trend)	-2.55	-0.23	-0.82	-0.57	-0.93	-0.67	-0.85	0.26
$\Delta e^*$	-2.55	-9.01	-8.61	-8.71	-8.43	-8.50	-8.35	-8.39
$cpi^*$ (with trend)	-3.24	-1.49	-0.66	0.42	-0.89	-1.06	-0.85	-1.07
$cpi^*$ (no trend)	-2.55	-0.02	0.94	2.06	0.54	0.45	0.63	0.32
$\Delta cpi^*$	-2.55	-10.73	-8.96	-8.41	-9.18	-10.49	-9.45	-9.58
$green^*$ (with trend)	-3.24	-1.81	-1.60	-1.01	-1.44	-1.96	-1.67	-1.33
$green^*$ (no trend)	-2.55	-1.89	-1.36	-1.32	-1.54	-1.86	-1.52	-1.02
$\Delta green^*$	-2.55	-2.00	-2.70	-2.22	-2.01	-2.10	-2.49	-3.23

Notes: See notes to Table A.2.

Table A.4: Lags and Cointegration

	<i>VARX</i>		Cointegrating
	Domestic Lag	Foreign Lag	Relationships
Canada	2	1	1
France	2	1	1
Germany	2	1	2
Italy	2	1	2
Japan	2	1	2
UK	2	1	1
US	2	1	2

Table A.5: Weak exogeneity test at 5% significance

Country	CV	$q^*$	$r^*$	$e^*$
Canada	3.91	0.01	0.03	
France	3.91	4.99	1.13	
Germany	3.06	0.40	1.51	
Italy	3.06	2.39	0.87	
Japan	3.06	0.71	1.04	
UK	3.91	0.73	1.79	
US	3.06			1.28

Figure A.1: Green Patent Shock: Impact on the US Economy

Structural GIRF

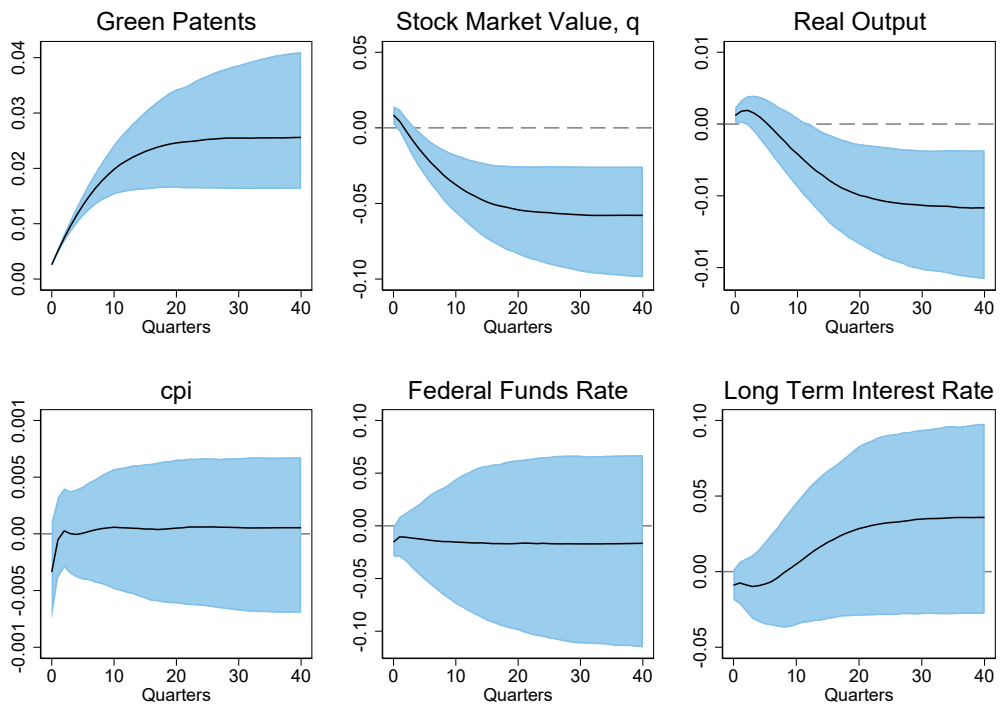


Figure A.2: Green Patent Shock: Impact on G7 Stock Market Values

Structural GIRF

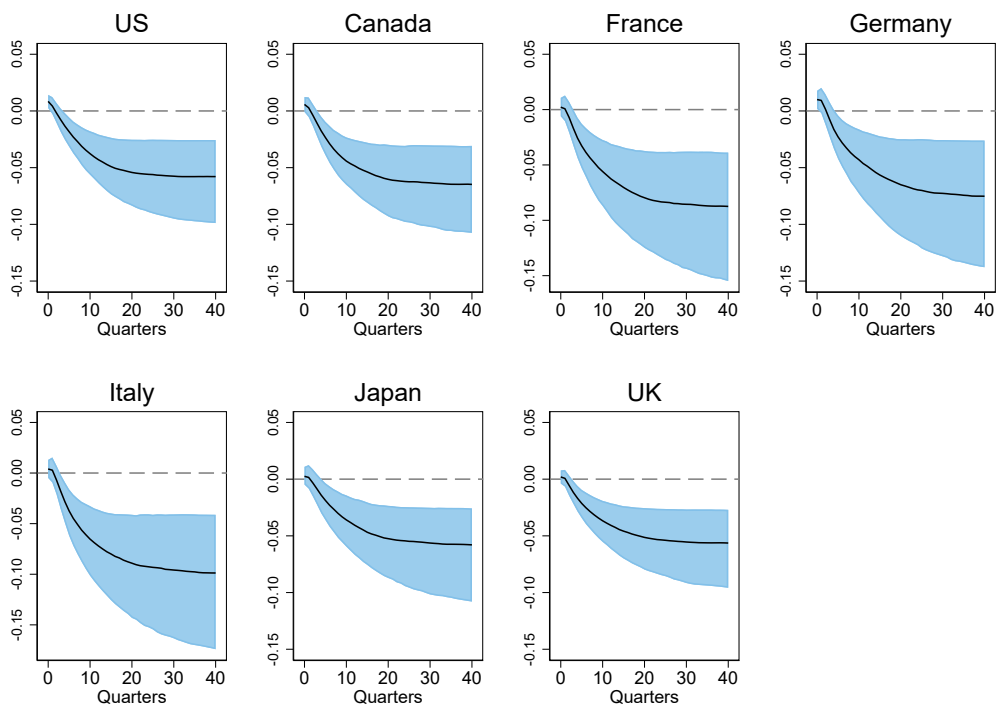
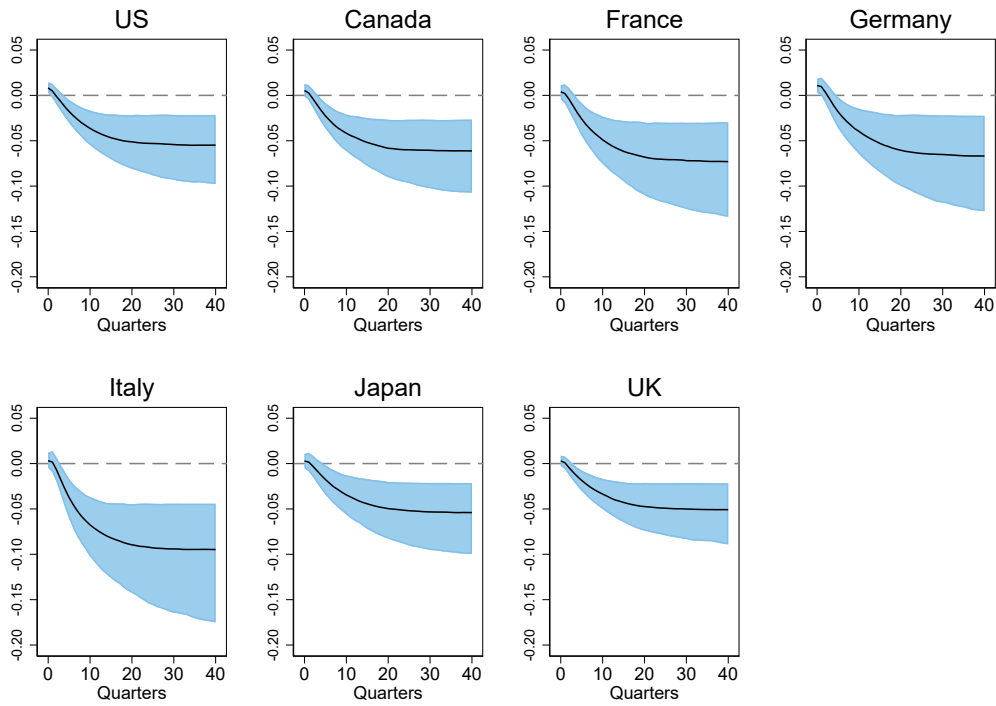


Figure A.3: US green patent shock and responses of stock markets (bilateral financial flow)



A5Picture1



Figure A.4: US green patent shock and responses of stock markets (varying bilateral trade, 1980-2022)

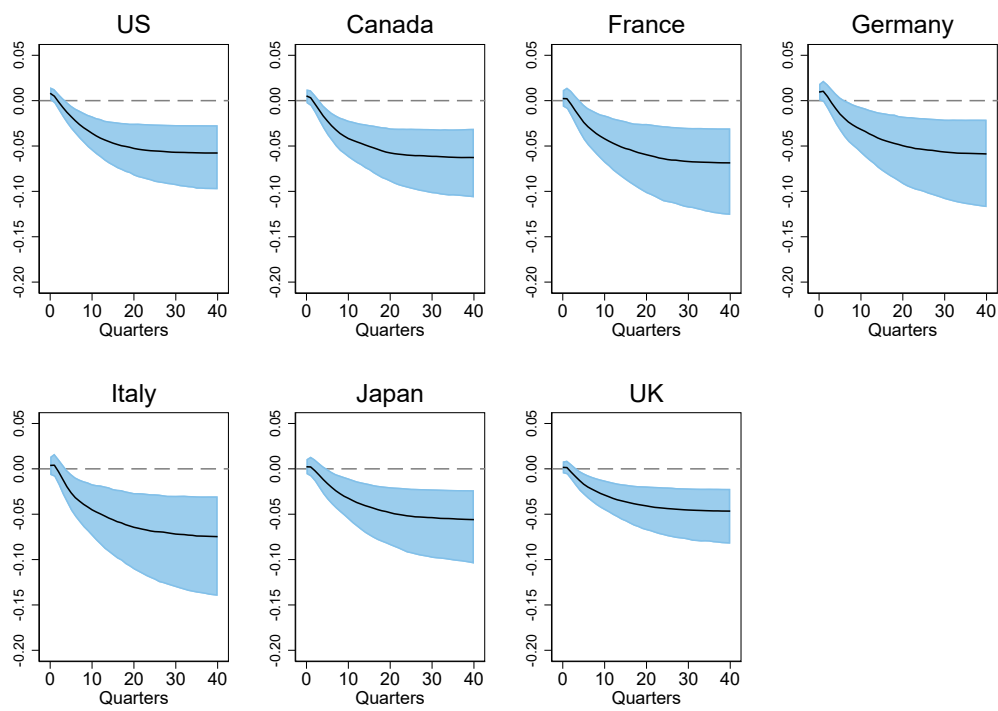
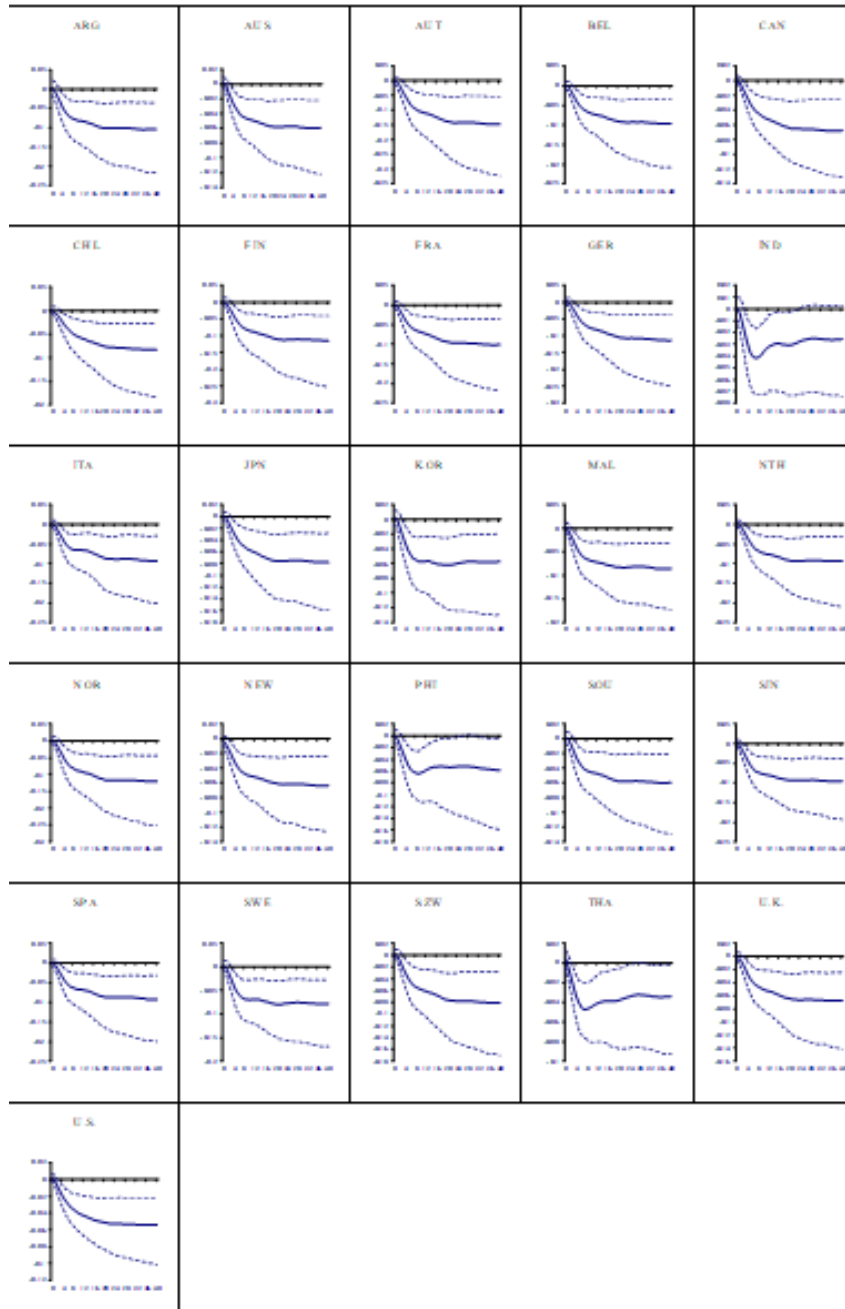


Figure A.5: Generalized Impulse Responses of a US green patent shock and responses of stock markets (model with 33 countries)



## B Impulse Responses

### B.1 Generalized Impulse Response Functions

We present the GIRF from equation (25), which showed the solution of the GVAR. Thus, GIRFs are defined as,

$$GIRF(x_t; \varepsilon_{ilt}, n) = \mathbb{E}(x_{t+n} | \varepsilon_{ilt} = \sqrt{\sigma_{ii, \ell \ell}}, \mathcal{I}_{t-1}) - \mathbb{E}(x_{t+n} | \mathcal{I}_{t-1}) \quad (\text{B.1})$$

where  $\mathcal{I}_{t-1}$  is the information set at time  $t - 1$ ;  $\sigma_{ii, \ell \ell}$  is the diagonal element of the variance covariance matrix  $\Sigma_\varepsilon$  corresponding to the  $\ell^{\text{th}}$  equation in the  $i^{\text{th}}$  country,  $n$  is the horizon, and  $x_t$  is the vector with all domestic variables.

Assuming that the error terms follow a multivariate normal distribution, we can rewrite the GIRF in its final form:

$$GIRF(x_t; u_{lt}, n) = \frac{e'_j A_n G_0^{-1} \sum_\varepsilon e_l}{\sqrt{e'_l \sum_\varepsilon e_l}}, \quad n = 0, 1, 2, \dots; l, j = 1, 2, \dots, k \quad (\text{B.2})$$

where  $G$  and  $A$  are the matrices taken from equation (25),  $e' = (0, 0, \dots, 0; 1, 0, \dots, 0)'$  is a selection vector with unity as the  $\ell^{\text{th}}$  element in the case of a country-specific shock. GIRFs are invariant to the ordering of the variables but are not strictly speaking structural shocks since they allow for correlation of the error terms (the error terms are not orthogonal).

### B.2 Structural Generalized Impulse Response Functions

Generalized IRFs do not identify shocks in any strictly structural or causal sense. We overcome this limitation by using the Structural Generalized IRF (SGIRF). Consequently, we can order the shock following an economic rationale. For the above model let the structural shocks be given by  $v_t = P\varepsilon_t$ . The identification conditions using the recursive identification approach of Sims (1980) and thus matrix  $P$  is lower triangle. See Smith and Galesi (2017) for a more detailed description.

## References

**Sims, C.** (1980). Macroeconomics and Reality. *Econometrica*, 48, 1-48.

**Smith, V. L. and Galesi, A.** (2017) *GVAR Toolbox 2.0. User Guide*, mimeo.