Price leadership and firm size asymmetry: an experimental analysis

Supplemental Online Appendix

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Appendix A

Proof of Proposition 1 To solve for the subgame perfect equilibrium of this game of timing, we use backward induction.[1] We first solve for the subgame perfect equilibrium of each price-setting subgame. There are four such subgames, corresponding to the two leader-follower games and the two simultaneous move games. It will be clear from our characterization that each of these subgames has a unique equilibrium.

Consider first the case in which both sellers moved in the same round, whether this round is $t = 1$ or $t = 2$, so that the price-setting game is one with simultaneous moves. In this case, Ghemawat (1986) has shown that the subgame has a unique equilibrium in non-degenerate mixed strategies in which sellers set prices according to the following cumulative distributions,[2]

$$F_l(p) = \begin{cases} 0 & \text{if } p < \underline{p}_1, \\ \frac{k_s}{k_l+k_s-d} - (\frac{k_s}{k_l})\frac{(d-k_s)\overline{p}}{(k_l+k_s-d)p} & \text{if } \underline{p}_l \le p < \overline{p}, \\ 1 & \text{if } p \ge \overline{p}, \end{cases} \tag{1}$$

[1]Formally, the game is defined as follows. Let $T_i = \{A_i, W_i\}$ be the set of possible times at which to announce a price for seller i, where $A_i = Announce$ denotes announcing a price in round 1 and $W_i = Wait$ denotes announcing a price in round 2. Seller i's (pure) strategy space is then $S_i^{OD} = T_i \times B_j^{OD}$ where B_j^{OD} is the set of functions that map $\{(A_j, W_i) \times P, (A_i, A_j), (W_j, A_i), (W_i, W_j)\}$ into P, $i, j \in \{1, 2\}$, $j \neq i$. In the analysis below, the firms may use behavior strategies.

[2]See also Deneckere et al. (1992) for a similar analysis in a slightly different context.

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$$F_s(p) = \begin{cases} 0 & \text{if } p < \underline{p}_l, \\ 1 + \frac{d-k_s}{k_l+k_s-d}(1 - \frac{\overline{p}}{p}) & \text{if } \underline{p}_l \leq p < \overline{p}, \\ 1 & \text{if } p \geq \overline{p}, \end{cases} \tag{2}$$

where

$$\underline{p}_l \equiv \frac{\overline{p}(d-k_s)}{k_l}.$$

Letting a 0 superscript denote simultaneous moves, if both sellers set their price in the same round t, seller l's equilibrium expected payoff is $\delta^{t-1}\pi_l^0 = \delta^{t-1}\overline{p}(d-k_s)$ and seller s's is equal to $\delta^{t-1}\pi_s^0 = \delta^{t-1}\pi_l^0\frac{k_s}{k_l}$. In the remainder of the proof, the superscript 1 refers to the leader's price and payoff, while the superscript 2 refers to the follower's.

Suppose now that seller l chose to set price in Round 1, while seller s chose to set price in Round 2. By setting its price equal to the reservation price $\overline{p}$, seller l can then guarantee itself a discounted payoff of $\overline{p}[m + \delta(d-m-k_s)]$. In this case, it sells a total quantity equal to $d - k_s$, of which a portion m is sold in the first round. Note that since profit made on the m impatient consumers is received immediately, it is not discounted. Alternatively, if seller l sets p and is the low-price seller, its profit is equal to $p[m + \delta(k_l - m)]$. It follows that the highest price $\underline{p}_l^1$ below which seller l would prefer to be the high-price seller at $\overline{p}$ rather than the low-price seller at $\underline{p}_l^1$ solves the following equation for p,

$$\overline{p}[m + \delta(d-m-k_s)] = p[m + \delta(k_l - m)].$$

Hence,

$$\underline{p}_l^1 = \frac{(1-\delta)m + \delta(d-k_s)}{(1-\delta)m + \delta k_l}.$$

Similarly, define $\underline{p}_s^2$ to be the highest price seller s would not want to match as a second-mover. It is clear that

$$\underline{p}_s^2 = \overline{p}\left(\frac{d-k_l}{k_s}\right).$$

We now show that the following inequality holds, $\underline{p}_s^2 < \underline{p}_l^1$. Indeed, comparing $\underline{p}_s^2$ and $\underline{p}_l^1$, a straightforward calculation yields that $\underline{p}_s^2 < \underline{p}_l^1$ if and only if

$$(1-\delta)m + \delta(k_l - k_s) > 0,$$

which clearly holds since $k_l > k_s$. Hence, seller s will match any price greater than or equal to $\underline{p}_l^1$ by seller l. It then follows that as a price leader, seller l maximizes its payoff by setting its price equal to $\overline{p}$. In equilibrium, seller s follows by optimally matching $\overline{p}$ and sells its capacity at that price. Thus, equilibrium payoffs in the unique subgame perfect Nash equilibrium of this price-setting game are given by

$$\pi_l^1 = \overline{p}[m + \delta(d-m-k_s)],$$

and

$$\pi_s^2 = \delta \overline{p} k_s.$$

Now consider the case in which seller s chose to set price in round 1, while seller l decided to wait. We show that if a condition on the value of m is satisfied, in the unique equilibrium of this subgame, the small seller is the low-price seller, while the large seller follows by setting its price equal to $\overline{p}$. To this effect, note that by setting its price equal to $\overline{p}$, seller l can guarantee itself a discounted profit equal to $\delta \overline{p}(d - k_s)$. It follows that the highest price it will not find profitable to match as a follower is given by

$$\underline{p}_l^2 = \overline{p}\left(\frac{d - k_s}{k_l}\right).$$

Thus, any price less than $\underline{p}_l^2$ set by seller s in the first round will not be matched by seller l in the second round. Hence, by setting its price equal to $\underline{p}_l^2$, seller s can guarantee itself a payoff of

$$\underline{p}_l^2[m + \delta(k_s - m)].$$

Alternatively, seller s may set its price equal to $\overline{p}$ and be sure to earn

$$\overline{p}[m + \delta(d - m - k_l)],$$

by the serving residual demand. Thus, seller s will prefer to set $\underline{p}_l^2$ to setting $\overline{p}$ if and only if

$$\underline{p}_l^2[m + \delta(k_s - m)] > \overline{p}[m + \delta(d - m - k_l)],$$

which, after re-arranging terms, yields

$$m < \left(\frac{\delta}{1-\delta}\right)(k_l - k_s). \tag{3}$$

The right-hand side of (7) is clearly strictly greater than zero since $k_l > k_s$. Suppose (7) holds, then it follows that in the unique subgame perfect equilibrium of this subgame, seller s sets its price equal to $\underline{p}_l$ and seller l follows with a price equal to $\overline{p}$. Equilibrium payoffs are equal to

$$\pi_s^1 = \underline{p}_l^2[m + \delta(k_s - m)],$$

and

$$\pi_l^2 = \delta \overline{p}(d - k_s).$$

Noting that $\pi_l^2 = \delta \pi_l^0 = \delta \overline{p}(d - k_s)$, we obtain the following payoff matrix for the reduced form of the full game.

		Seller l	
		A	W
Seller s	A	π_s^0, π_l^0	$\pi_s^1, \delta\pi_l^0$
	W	$\delta\overline{p}k_s, \pi_l^1$	$\delta\pi_s^0, \delta\pi_l^0$

Since $m > 0$ and $\delta < 1$, a simple comparison of the possible payoffs for seller l shows that $\pi_l^1 > \delta\pi_l^0$, so that, independently of seller s's action, A is optimal for seller l. Moreover, if $\delta > \frac{d-k_s}{k_l}$, seller s's best-reply to A by seller l is to play W. Recall that for the above payoff matrix to be generated by equilibrium behavior in the pricing subgames, (7) must hold as well. However, a straightforward calculation shows that if $\delta > \frac{d-k_l}{d-k_s}$, then the right-hand side of (7) is strictly greater than $d - k_l$. Since $m < d - k_l$ by assumption, it follows that if $\delta > \hat{\delta}$, the game has a unique equilibrium and the path of play described in the proposition is the unique equilibrium path of play. □

Appendix B: Sample instructions

B.1 General

This is an experiment in the economics of market decision-making. Various research agencies have provided funds for the conduct of this research. The instructions are simple and if you follow them carefully and make good decisions, you may earn a considerable amount of money that will be paid to you in cash at the end of the experiment. It is in your best interest to fully understand the instructions, so please feel free to ask any questions at any time. It is important that you do not talk and discuss your information with other participants in the room until the session is over.

All transactions in today's experiment will be in experimental francs. These experimental francs will be converted to real US dollars at the end of the experiment at the rate of ___________ experimental francs = \$1. Notice that the more experimental dollars you earn, the more US dollars you earn. What you earn depends partly on your decisions and partly on the decisions of others.

In this experiment we are going to conduct markets in which you will be a participant in a sequence of trading periods. In every period, you will be a **seller** of a fictitious good X. The 16 participants in today's experiment will be randomly rematched every period into 8 markets with 2 sellers in each market. Therefore, the specific person who is the other seller in your market will change randomly after each period. The experiment consists of 3 sections where each section will comprise of 20 trading periods.

In each period, each seller is endowed with a fixed amount of units of good X to sell i.e. good X costs you nothing to produce. This endowment of good X can be large or small. In each market, one seller will receive a large endowment and the other seller will receive a small endowment. If you receive a large endowment you will continue to receive a large endowment throughout the experiment. Similarly, if you receive a small endowment you will continue to receive a small endowment throughout the experiment. The information about the size of your and the other seller's endowment will be displayed on all decision screens. The size of the small endowment will change in periods 21 and 41.

As a seller you can sell multiple units of good X every period but each buyer will purchase exactly one unit of the good each period. The buyers in today's experiment

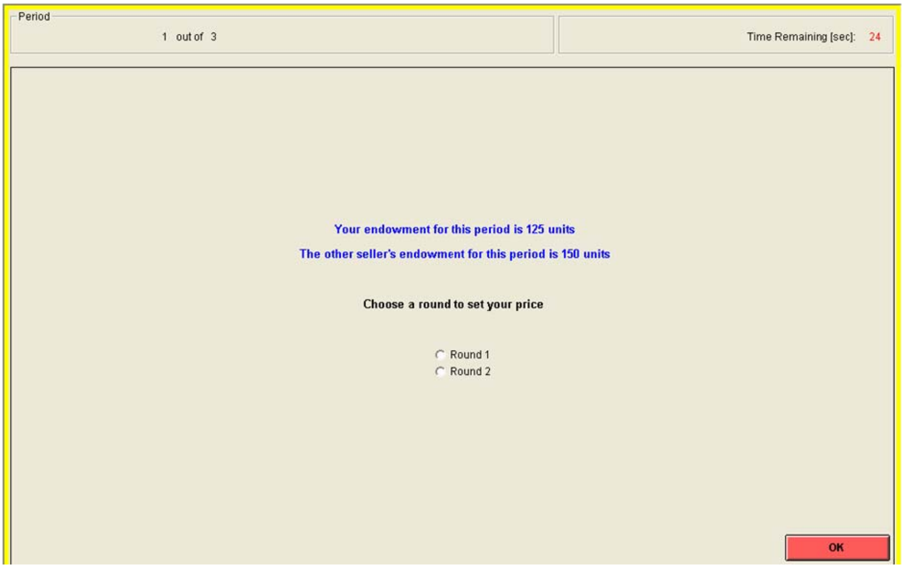

Fig. 1 Round decision screen

are simulated by computerized "robots". There are 200 buyers in each of the 8 markets. The buyers will not pay a price greater than 50 experimental francs for the single unit of good X. This maximum price is the same for all buyers and sellers throughout the experiment, and will be displayed on everyone's decision screen as shown in Fig. 1. Sellers are not allowed to post a price above this maximum.

Trading Instructions for periods 1–21

In this section, the size of the large endowment is _______ units while the amount of small endowment is _______ units.

Each trading period is subdivided into 2 rounds. At the beginning of each period, you first decide the **round** in which to set your price. You can choose to set your price either in Round 1 or Round 2. You will then be asked to set a **price** for good X. An example of the decision screen in shown in Fig. 1.

After both you and the other seller have made their choice, the rounds chosen by both sellers are displayed on your screen.

There are 2 different cases, depending on the choice of round made by you and the other seller.

Case 1: If you chose to set your price in Round 1 or if both you and the other seller chose to price in the same round, you will be asked to choose the price you wish to charge per unit of good X. An example of the decision screen is shown in Fig. 2 for the case where both sellers chose Round 2 to make their pricing decision.

Case 2: If you chose to set your price in Round 2 and the other seller chose to set her price in Round 1, then you will learn the price of the other seller prior to making your pricing decision. An example of your decision screen is shown in Fig. 3.

Fig. 2 Price decision screen

The buyers, your quantity sold and your profit

Again, there are 200 computer simulated buyers in today's experiment. The buyers will not pay a price greater than 50 experimental francs for the single unit of good X.

If both sellers choose to set their prices in the same round (either Round 1 or Round 2) then the buyers will first purchase from the seller offering the lower price, and the remaining buyers will purchase from the higher priced seller**.** For example: Suppose Seller 1 (with, for example, an endowment of 150 units) sets a price of 26.36 and Seller 2 sets a price of 48.12 then Seller 1 sells her entire endowment of 150 units while Seller 2 sells the remaining $200 - 150 = 50$ units.

However, if one seller chose to set her price in Round 1 and the other seller chose to set her price in Round 2, then 40 out of the 200 buyers will buy from the first seller to set a price, whether or not this seller posted the lowest price. The remaining 160 buyers will wait until both sellers have set their price and will purchase from the seller offering the lower price. Hence, if one of the two sellers sets her price in Round 2, 160 buyers will wait until the end of Round 2 before purchasing.

Typically, your profit is equal to your price times your quantity sold. **However, if either you or your paired seller or both set her price in Round 2, the quantity sold in Round 2 will be applied a factor of 0.8 when calculating your profit.**

There are 4 cases, depending on the rounds in which you and the other seller set their price. In all four cases, your profit for the period depends on the price set by you and the other seller in that period. The attached sheet labeled "Factored Quantity

Fig. 3 Price decision screen

Sheet for Periods 1–20" provides the numerical value for the specific factor is multiplied by "your price" in the profit calculation in the different cases outlined below.

Case 1: You and the other seller set their price in Round 1. Note that in this case, your entire quantity sold comes from purchases made by buyers in Round 1. Your profit is computed as follows:

$$\text{Your profit} = \text{your price} \times \text{your endowment} \quad \text{if your price} < \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times (200 - \text{other seller's endowment})$$
$$\text{if your price} > \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times \left(\frac{\text{your endowment}}{\text{your endowment} + \text{other's endowment}} \right) \times 200$$
$$\text{if your price} = \text{other seller's price}$$

Case 2: You and the other seller set their price in Round 2. Note that in this case, your entire quantity sold comes from purchases made by buyers in Round 2. Thus, a factor of 0.8 is applied in the profit calculations. Your profit is computed as follows:

$$\text{Your profit} = \text{your price} \times 0.8 \times \text{your endowment}$$

$$\text{if your price} < \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times 0.8 \times (200 - \text{other seller's endowment})$$
$$\text{if your price} > \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times 0.8 \times \left(\frac{\text{your endowment}}{\text{your endowment} + \text{other's endowment}}\right) \times 200$$
$$\text{if your price} = \text{other seller's price}$$

Case 3: You set your price in Round 1 and the other seller set his/her price in Round 2. Note that in this case, you will sell 40 units in Round 1 and the rest in Round 2. *The factor of 0.8 is applied to the quantity sold in Round 2 only*. Your profit is computed as follows:

$$\text{Your profit} = \text{your price} \times [40 + 0.8 \times (\text{your endowment} - 40)]$$
$$\text{if your price} < \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times [40 + 0.8 \times (200 - 40 - \text{other seller's endowment})]$$
$$\text{if your price} > \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times \left[40 + 0.8 \times \left(\frac{\text{your endowment} - 40}{\text{your endowment} - 40 + \text{other's endowment}}\right) \times (200 - 40)\right]$$
$$\text{if your price} = \text{other seller's price}$$

Case 4: You set your price in Round 2 and the other seller set his/her price in Round 1. Note your entire quantity sold comes from purchases made by buyers in Round 2, and a factor of 0.8 is applied in the profit calculations. Your profit is computed as follows:

$$\text{Your profit} = \text{your price} \times 0.8 \times \text{your endowment}$$
$$\text{if your price} < \text{other seller's price}$$

$$\text{Your profit} = \text{your price} \times 0.8 \times (200 - \text{other seller's endowment})$$
$$\text{if your price} > \text{other seller's price}$$

Fig. 4 Outcome screen

$$\text{Your profit} = \text{your price} \times 0.8 \times \left(\frac{\text{your endowment}}{\text{your endowment} + \text{other's endowment} - 40}\right) \times (200 - 40)$$

$$\text{if your price} = \text{other seller's price}$$

Note **that if your price is the lower price, you are guaranteed to sell your whole endowment. If your price is the higher price, you will sell a quantity equal to 200 minus the other seller's endowment.** Finally, if sellers set the same price, the remaining buyers are split in proportion to the sellers' unsold endowments accordingly. At the end of each period, your profit is computed and displayed on the outcome screen as shown in Fig. 4.

Once the outcome screen is displayed you should record all of the trading information: your choice of round, your price, your quantity sold, your factored quantity sold and the other seller's choice of round and the other seller's price in your Personal Record sheet. Also, record your profit from this period and the total profit from all previous periods. Then click on the button on the lower right of your screen to begin the next trading period. Recall that you will be randomly re-matched with a different seller every period.

[ON SEPARATE PAGE IN ORIGINAL INSTRUCTIONS]

Trading Instructions for Periods 21–40

This section is similar to the previous section.

The only difference in this section is that the size of the small endowment will now be equal to _____ units while the size of the large endowment remains at ______ units. All trading instructions and profit calculations are identical. In this section again, you will be making both a round and a pricing decision.

The attached sheet labeled "Factored Quantity Sheet for Periods 21–40" provides the numerical value for the factor multiplied by "your price" in the profit calculation in the 4 different cases outlined in the general instructions. Recall that you will be randomly re-matched with a different seller every period.

Trading Instructions for Periods 41–60

This section is similar to the previous section.

The only difference in this section is that the size of the small endowment will now be equal to _____ units again, while the size of the large endowment remains at ______ units. All trading instructions and profit calculations are identical. In this section again, you will be making both a round and a pricing decision.

The attached sheet labeled "Factored Quantity Sheet for Periods 1–20" provides the numerical value for the factor multiplied by "your price" in the profit calculation in the 4 different cases outlined in the general instructions. Recall that you will be randomly re-matched with a different seller every period.

References

Deneckere, R., Kovenock, D., & Lee, R. (1992). A model of price leadership based on consumer loyalty. *Journal of Industrial Economics*, *40*, 147–156.

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