Belief Formation: An Experiment With Outside Observers
Online Supplement

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Abstract

In this note, we briefly highlight one technical issue concerning the estimation of models of stochastic best response over a continuous action space.

1 Stochastic Best Response Over A Continuous Action Space

Using the notation of Costa-Gomes and Weizsäcker (2008), let y_g denote a generic belief statement and b_g denote a subject's true belief. In this case, the expected payoff from reporting y_g when the true belief is b_g is:

$$\begin{aligned} \bar{v}(y_g, b_g) &= A - c\left[b_{g,1}[(y_{g,1}-1)^2 + y_{g,2}^2 + y_{g,3}^2]\right] \\ &- c\left[b_{g,2}[y_{g,1}^2 + (y_{g,2}-1)^2 + y_{g,3}^2]\right] \\ &- c\left[b_{g,3}[y_{g,1}^2 + y_{g,2}^2 + (y_{g,3}-1)^2]\right]. \end{aligned} \tag{1}$$

It is assumed that the true belief is unobserved, but that players state a probabilistic payoff maximising response, which follows a logistic distribution with parameter $\lambda^b \geq 0$. The density, therefore, of stating belief y_g is then:

$$r_g(y_g, b_g, \lambda^b) = \frac{\exp[\lambda^b \bar{v}(y_g, b_g)]}{\int_{s\in\Delta^2} \exp[\lambda^b \bar{v}(s, b_g)]}. \tag{2}$$

Although one would typically assume that b_g, $y_g \in \{x \in \mathbb{R}^3_+ : x_1 + x_2 + x_3 = 1\}$, doing so can lead to positive values of the log-likelihood function, which might be a little unusual to most readers. To see this, suppose that suppose that $b_g = (0.8, 0.15, 0.05)$, $y_g = (1, 0, 0)$ and that $\lambda^b = 2$. Then in this case, $r_g(y_g, b_g, \lambda^b) \approx 3.795576 > 1$, which would imply that $\log(r_g) > 0$.

Fortunately, one can simply rescale the beliefs suitably so that $r_g < 1$. In particular, if instead, we treat stated beliefs (y_g) as ranging from 0 to 100, so that:

$$\begin{aligned} \bar{v}'(y_g, b_g) &= \big[A - c\left[b_{g,1}[(y_{g,1} - 100)^2 + y_{g,2}^2 + y_{g,3}^2]\right] \\ &- c\left[b_{g,2}[y_{g,1}^2 + (y_{g,2} - 100)^2 + y_{g,3}^2]\right] \\ &- c\left[b_{g,3}[y_{g,1}^2 + y_{g,2}^2 + (y_{g,3} - 100)^2]\right]\big] /10000 \end{aligned}$$

and we plug $\bar{v}'(y_g, b_g)$ into (2), then we get, for the specific example, that $r'_g = 3.794058 \times 10^{-4} < 1$. As can be seen, these two numbers differ by, essentially, a factor of 10^4.

In Tables 1 and 2, we provide estimation results for the 10 best and 10 worst subjects using both estimation methods. That is, if we do not scale the stated beliefs and if we do. As can be seen, the estimates are all nearly identical. The only difference is that the log likelihoods differ by approximately $N \ln(10000) \approx 1842.1$.

As a final point, the observant reader may notice that we actually restricted subjects' belief statements to being between 0 and 100, with one number after the decimal, rather than the continuous space that we use to estimate the belief formation models. We estimated the models using a continuous response space largely for computational ease. If we assume a discrete space where each belief could be at a precision of 3 decimals, then for each of the 20 periods (because b_g changes each period), we need to calculate 501,501 values in the denominator of Equation 2. Obviously this can add up to substantial computational time. As a test, we estimated the DSG-NC game for our EWA (Level 2 Prior) model assuming a discrete space and the continuous space reported in the paper. For the continuous space estimation, it took 455.297 seconds to obtain our estimated coefficient, while for the discrete space, it took **3898.769** seconds and led to essentially the same estimated coefficients (See Table 3 in this file). Therefore, we feel justified in using this approach in order to save vast amounts of computational time.

Table 1: Estimation results: 10 most accurate and 10 least accurate observers (unscaled stated beliefs)

(a) 10 most accurate observers

	DSG-C	nDSG-C	DSG-NC	nDSG-NC
λ^b_{EWA}	9.475	4.140	2.274	3.645
δ	0.000	0.526	0.729	0.895
ϕ	0.741	0.389	0.666	0.099
λ^a_{EWA}	0.083	0.124	0.400	0.170
LL	412.0	273.2	217.1	224.0

(b) 10 least accurate observers

	DSG-C	nDSG-C	DSG-NC	nDSG-NC
λ^b_{EWA}	1.758	1.534	0.578	1.572
δ	0.516	0.000	0.981	0.392
ϕ	0.940	0.852	0.986	0.741
λ^a_{EWA}	0.162	0.145	10.000	5.547
LL	177.4	178.7	146.3	183.6

Table 2: Estimation results: 10 most accurate and 10 least accurate observers (scaled stated beliefs)

(a) 10 most accurate observers

	DSG-C	nDSG-C	DSG-NC	nDSG-NC
λ^b_{EWA}	9.450	4.133	2.272	3.638
δ	0.000	0.526	0.729	0.895
ϕ	0.741	0.389	0.666	0.099
λ^a_{EWA}	0.083	0.125	0.401	0.170
LL	-1430.4	-1569.0	-1625.0	-1618.2

(b) 10 least accurate observers

	DSG-C	nDSG-C	DSG-NC	nDSG-NC
λ^b_{EWA}	1.755	1.533	0.577	1.571
δ	0.516	0.000	0.981	0.392
ϕ	0.940	0.852	0.986	0.741
λ^a_{EWA}	0.162	0.145	10.000	5.548
LL	-1664.7	-1663.5	-1695.8	-1658.6

Table 3: Comparing the Discrete vs. Continuous Estimation Procedures

	DSG-NC	
	Discrete	Continuous
λ^b_{EWA}	1.192	1.192
δ	0.879	0.879
ϕ	0.99240	0.99236
λ^a_{EWA}	3.1016	3.1261

References

Costa-Gomes, M. A., and G. Weizsäcker (2008): "Stated Beliefs and Play in Normal Form Games," *Review of Economic Studies*, 75, 729–762.