Appendix: Measurement Error

As mentioned in section 2.2.7, some of the individual characteristics variables probably involve measurement error. Such measurement error would mean that the coefficient estimates for the regressions from Tables 3, 4, and 5 are biased.[1] In this appendix, we explore the direction of this bias. In particular, we first state assumptions that allow us to pin down the direction of the bias. These assumptions will ensure that the coefficient estimates exhibit so-called attenuation bias, i.e., that asymptotically they have the correct sign, but are biased toward zero. Then, we discuss to what extent these assumptions are appropriate in our experiment.

Consider the regression equation:

$$y = \beta_0 + \beta_1 x_1^* + \ldots + \beta_n x_n^* + v,$$

where $E(v) = 0$ and $E(x_i^* v) = 0$ for $i = 1, \ldots, n$. We would like to estimate this regression. However, $x_1^*, \ldots, x_n^*$ are not directly observed. Instead of each x_i^*, we observe a noisy measure of it, $x_i = x_i^* + e_i$, where e_i is the measurement error.[2]

To get to an estimable equation, replace each x_i^* in the regression above with $x_i - e_i$:

$$y = \beta_0 + \beta_1 x_1 + \ldots + \beta_n x_n + (v - \beta_1 e_1 - \ldots - \beta_n e_n), \tag{1}$$

where the term in parentheses is the new error term.

Fix i. What is the direction of the bias in the estimate of β_i if we run OLS based on equation 1? In general, it is not possible to give a definitive answer. To pin down the direction of the bias, we make the following assumptions.

A1: e_i is uncorrelated with $e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n$.

A2: x_i^* is uncorrelated with $x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_n^*$.

A3: e_i is uncorrelated with $x_1^*, \ldots, x_{i-1}^*, x_{i+1}^*, \ldots, x_n^*$.

A4: e_i is uncorrelated with v.

[1] To be precise, we are talking about asymptotic bias, i.e., about inconsistency.

[2] We do not need $E(e_i) = 0$, i.e., the measure to be unbiased. If $E(e_i) \neq 0$, this does not affect the OLS estimates of $\beta_1, \ldots, \beta_n$ based on equation 1: $E(e_i)$ will simply be absorbed into the constant.

A5: Either:

a) x_i^* and e_i are uncorrelated or

b) x_i^* and x_i are dummy variables, $Prob(x_i = 1|x_i^* = 1) > Prob(x_i = 1|x_i^* = 0)$, and $Prob(x_i = 0|x_i^* = 0) > Prob(x_i = 0|x_i^* = 1)$.[3]

Claim:

Fix i and consider $\widehat{\beta}_i$, the estimate of β_i based on an OLS regression of y on $x_1, \ldots, x_n$.
(A) If $Prob(e_i = 0) = 1$ (i.e., there is no measurement error) and A2 holds, then $\widehat{\beta}_i$ is a consistent estimate of β_i.
(B) If $Prob(e_i = 0) < 1$ (i.e., there is measurement error) and A1-A5 hold, then $\widehat{\beta}_i$ suffers from attenuation bias.

Before we prove the claim, let us discuss A1-A5 in the context of our experiment. In the regressions reported in Tables 3, 4, and 5, the independent variables are A_{risk}, N_{risky}, $N_{patient}$, $Trust$, $Trustworthy$, Alt, Dom, Str, $FOSD$, PA, and $Male$.[4] How do A1-A5 apply to each of these variables?[5,6] A1, A3, and A4 are probably innocuous. They merely say that the measurement error in a given variable is uncorrelated with the measurement errors in the other independent variables, with the true values of the other variables, and with the error term in the regression that uses the true values of the independent variables.

In the case of each of our dummies $Trust$, $Trustworthy$, Dom, Str, $FOSD$, and PA, A5 is also innocuous because it just says that (i) the observed variable equaling 1 is more likely if the underlying variable equals 1 than if the underlying variable equals 0 (e.g., that observing $Trust = 1$ is more likely if the subject really is trusting than if she is not) and, similarly, (ii) the observed variable equaling 0 is more likely if

[3]The two inequalities can be replaced by the weaker requirement that $\frac{Prob(x_i=1|x_i^*=0)}{Prob(x_i=1|x_i^*=1)} < \frac{Prob(x_i=0|x_i^*=0)}{Prob(x_i=0|x_i^*=1)}$.

[4]Or, in the case of Table 4, their sums for two subjects paired for a repeated game.

[5]We do not think of A_{risk} as measuring an underlying true characteristic. It is merely a gimmick allowing us to keep track of the two subjects who seem to misunderstand the risk attitude elicitation task. At any rate, we are not interested in the coefficient estimate for A_{risk}, so it does not matter whether A1-A5 hold for this variable.

[6]For any variable that is measured without error (this is almost certainly the case for $Male$), only A2 is relevant.

the underlying variable equals 0 than if the underlying variable equals 1. For dummy variables, A5 would fail only for the most perverse measures.

The meat is in A2 and, in the case of the non-dummy variables N_{risky}, $N_{patient}$, and *Alt*, in A5 a). Taking A1 as given, A2 is equivalent to the statement that x_i is uncorrelated with $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$. Inspection of Table 10 shows that A2 is probably not exactly correct for most variables. However, the absolute size of the correlations in the table are quite low, with the highest correlation in absolute size being 0.38. Thus, A2 probably provides a reasonable approximation.

A5 a) says that the measurement error is uncorrelated with the true value of the underlying variable. This is the key assumption in classical errors-in-variables. It is a very reasonable assumption in the case of N_{risky}, $N_{patient}$, and *Alt* with the following caveat: when the underlying variable is close to the boundary of admissible values, this must affect the measurement error. E.g., when a subject's true preference is to give nothing to the charity the measurement error in *Alt* cannot be negative and when a subject's true preference is to give everything to the charity the measurement error in *Alt* cannot be positive.

Proof of claim:
Rewrite equation (1) by putting all terms $\beta_j x_j, j \neq i$, into the error term:

$$y = \beta_0 + \beta_i x_i + (w - \beta_i e_i), \tag{2}$$

where $w = v + \beta_1 x_1^* + \ldots + \beta_{i-1} x_{i-1}^* + \beta_{i+1} x_{i+1}^* + \ldots + \beta_n x_n^*$ and $w - \beta_i e_i$ is the new error term.[7]

A1 (which holds automatically if $Prob(e_i = 0) = 1$) and A2 imply that x_i is uncorrelated with $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$. Thus, the coefficient on x_i is the same regardless of whether we take an orthogonal projection of y onto the span of $1, x_1, \ldots, x_n$ or onto the span of $1, x_i$. In other words, the estimate of β_i will be asymptotically the same regardless of whether we run OLS based on equation (1) or based on equation (2). Thus, instead of analyzing the bias in the estimate of β_i from OLS based on equation (1), we can analyze this bias from OLS based on equation (2).

If $Prob(e_i = 0) = 1$ and A2 holds, x_i is uncorrelated with the error term in

[7] $E(w - \beta_i e_i)$ may not be 0. This does not affect the OLS estimates of $\beta_1, \ldots, \beta_n$ based on equation 2: $E(w - \beta_i e_i)$ will simply be absorbed into the constant.

equation (2), so that the OLS esimate of β_i based on equation (2) is consistent. This proves (A). From here on assume $Prob(e_i = 0) < 1$.

In equation (2), A2-A4 guarantee that x_i is uncorrelated with w[8]; A3 and A4 guarantee that e_i is uncorrelated with w. In conjunction with A5 a), this ensures that we have classical errors-in-variables. In this case, it is well known that the estimate of β_i exhibits attenuation bias.[9]

Now suppose that A5 b) holds instead of A5 a). Given that the single independent variable is a dummy, the OLS estimate of β_i based on equation (2) asymptotically equals $E(y|x_i = 1) - E(y|x_i = 0)$. We can rewrite this difference as follows:

$$\begin{aligned} E(y|x_i = 1) &- E(y|x_i = 0) \\ &= \beta_0 + \beta_i E(x_i^*|x_i = 1) + E(w|x_i = 1) - \beta_0 - \beta_i E(x_i^*|x_i = 0) - E(w|x_i = 0) \\ &= \beta_i[E(x_i^*|x_i = 1) - E(x_i^*|x_i = 0)] \\ &= \beta_i[Prob(x_i^* = 1|x_i = 1) - Prob(x_i^* = 1|x_i = 0)] \end{aligned}$$

The first equality follows from taking conditional expectations of both sides of equation (2). The second equality follows because, by A2-A4, x_i is uncorrelated with w.[10] The third equality follows from the fact that x_i^* is a dummy and its expectation can be expressed as the probability that it equals 1. The term in brackets on the last line is clearly strictly less than 1 given that $Prob(e_i = 0) < 1$. Moreover, straightforward application of Bayes' rule shows that, by the inequalitites in A5 b), this term is strictly greater than 0. Thus, again, the estimate of β_i exhibits attenuation bias. Q.E.D.

[8] x_i is still correlated with the error term, $w - \beta_i e_i$, because it is correlated with e_i. This is what disrupts the usual consistency of the OLS estimate of β_i.

[9] See Wooldridge (2002), section 4.4.2.

[10] Given that x_i is a dummy, x_i being uncorrelated with w implies $E(w|x_i = 1) = E(w|x_i = 0)$.