Allotment in First-Price Auctions: An Experimental Investigation

Supplementary Material

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This manuscript collects additional analysis related to the paper *Allotment in First-Price Auctions: An Experimental Investigation*. In particular, Section A derives the equilibrium predictions under the benchmark hypothesis of risk neutrality, as well as those associated with the behavioral assumptions identified in the paper to explain our experimental evidence. Section B presents additional empirical analysis on: *(i)* relationship between bidding behavior and response time; *(ii)* the empirical relevance of risk aversion, joy of winning and loser's regret (as self-assessed by subjects in the post-experiment questionnaire) in explaining bidding behaviors. Section C contains the experimental instructions.

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A. Theoretical Predictions

In this Section, we first outline the Nash Equilibrium bids in the three auction formats (*1A1U*, *1A2U* and *2A1U*) under the standard assumption of risk neutral bidders with standard preferences (Subsection A.1). These equilibrium bids immediately imply the predictions *H1-H3* listed in Subsection 3.3 of the paper.

We then amend the risk neutral hypothesis to incorporate assumptions (A1) and (A2) introduced in Section 5 of the paper, that, for convenience, we report here:

(A1) bidders' actual preferences are such that the ratio between the marginal benefit of a higher bid and its marginal cost is, *ceteris paribus*, greater than under risk neutrality;

(A2) bidders' actual preferences are such that, in the allotment treatments, for equal bids, the marginal utility from winning is strictly decreasing in successive units.

There are several model specifications that incorporate assumptions (A1) and (A2). Here we mainly focus on three: risk aversion, joy of winning and loser's regret.

In Subsections A.2-A.4, we show formally that, *in equilibrium*, these models generate predictions that are consistent with most of our experimental evidence. We summarize the equilibrium predictions as follows:

h1. **Differences in efficiency and revenue.** *In* 1A1U*, the item is efficiently allocated to the bidder with the highest private value. In the allotment treatments, instead, allocative efficiency is not guaranteed. In all treatments, the auctioneer's revenue is higher than bidders' surplus. Moreover, under joy of winning and loser's regret, the expected auctioneer's revenue is higher in* 1A1U *than in the allotment treatments, provided that* $w > w_1 + w_2 - (4/81)(w_1 - w_2)^3$.

h2. **Overbidding.** *In all treatments, bidders bid above the RN equilibrium level. In the allotment treatments, both bids are associated with overbidding. Moreover, under joy of winning and loser's regret, if* $w > w_1 + w_2 - (w_1 - w_2)^2/9$*, bids in* 1A1U *are, on average, greater than the sum of the two bids in the allotment treatments.*

h3. **Bid spread.** *In the allotment treatments,* 1A2U *and* 2A1U*, bidders place two different bids. Bidding functions (and thus bid spread) are identical in* 1A2U *and* 2A1U*.*

In Subsection A.5, we also discuss two additional preference specifications – inequity aversion and reference dependence with loss aversion – that, under certain conditions, fall in the class of preferences identified by assumptions (A1) and (A2).

The theoretical predictions *h0-h3* replicate most of the experimental results *R0-R3* presented in Section 4 in the paper. There are two partial experimental results that are not aligned with the above predictions: first, even though the size of bid spread does not differ between *1A2U* and *2A1U*, the probability of bid spread is lower in the latter (result *R2*, point *(ii)*); second, efficiency is higher in *2A1U* than in *1A2U* (result *R1*, point *(i)*). Actually, there is also a third result which is not statistically appreciable but is worth considering anyway: while the revenue ranking between *1A1U* and *1A2U* is unambiguous (with the former larger than the latter), the revenue in *2A1U* seems to be in between the other two formats. These results undermine the equivalence *1A2U* and *2A1U* that should hold in equilibrium.

In an attempt to rationalize these differences, we explore the implications of an additional behavioral hypothesis, summarized by assumption (A3):

(A3) when an auction has a unique Nash Equilibrium, bidders play their equilibrium strategies; when, instead, an auction has multiple Nash Equilibria, two types of behaviors are possible: *(i)* either a bidder adopts an equilibrium strategy ("equilibrium" bidding), or *(ii)* a bidder believes that the opponent is going to play any of her equilibrium strategies with equal probability and best responds to this belief ("shaky" bidding).

In Subsection A.6, we first illustrate, by means of a simple 3×3 example that mimics our *2A1U* auction, the intuition behind shaky bidding behavior and how it differs from traditional equilibrium selection criteria. We then show that the presence of shaky bidders in *2A1U* is able to reasonably explain the differences observed between *1A2U* and *2A1U*. In fact, we obtain the following predictions:

h1'. **Differences in efficiency and revenue.** *Under joy of winning and loser's regret, in* 2A1U*, if a shaky bidder and an equilibrium bidder face in the auctions, efficiency and revenue will be larger than in* 1A2U*, provided that* $(w_1 - w_2)$ *is sufficiently small.*

h3'. **Bid spread.** *Shaky bidders in* 2A1U *would place the same bid in the two auctions.*

A.1 Risk neutrality

Suppose that bidders' valuations are *i.i.d.* uniform random variables drawn from the interval $[0, \bar{V}]$ in *1A1U* and from the interval $[0, \bar{v}]$ in *2A1U* in *1A2U*, with $\bar{V} = 2\bar{v}$. (For convenience, in the rest of this Section we will use the normalization $\bar{v} = 1$ (hence $\bar{V} = 2$) and will omit bidders' indexes, unless necessary.)

Under risk neutrality, the symmetric Bayes-Nash equilibrium of a single unit first-price auction with two bidders with values x *i.i.d.* according to a uniform distribution over $[z_1, z_2]$ is $(x + z_1)/2$. Hence, the symmetric equilibrium of *1A1U* is: $a^{RN}(V) = \frac{1}{2}V$. Since equilibrium bids are strictly increasing, the unit will be assigned to the bidder with the highest valuation; as a consequence, the expected overall surplus is simply the expected value of the highest valuation, which is equal to 4/3; expected revenue and expected bidders' surplus are both equal to 2/3.

It is trivial to show that, in *1A2U* and in *2A1U*, every bidder places two identical bids according to the same equilibrium bidding function used in a single-unit first-price auction, namely:[1]

$$b_1^{RN}(v) = b_2^{RN}(v) = c_1^{RN}(v) = c_2^{RN}(v) = \frac{1}{2}v.$$

As a consequence, the allocation and the expected overall surplus, revenue and bidders' surplus will be the same as in *1A1U*.

A.2 Joy of winning

Following Cooper and Fang (2008) and Roider and Schmitz (2012), we model joy of winning by adding a strictly positive bonus to the difference between value and bid whenever a bidder obtains any item from the auction(s). Therefore, in *1A1U*, bidder i's (perceived) payoff is $V_i - a_i + w$ if she wins the auction, and zero otherwise, where $w > 0$ measures the extra-benefit from winning. In *1A2U* and *2A1U*, a bidder can win one or two units. We denote by w_1 the

[1] We do not provide a formal proof of this result. However, it can be obtained from the model under the joy of winning hypothesis, by setting $w_1 = w_2 = 0$.

joy of winning one unit only and by $w_1 + w_2$ the joy of winning both units. In practice, w_2 is the joy of winning an additional second unit. To match assumption (A2), we assume, as it seems reasonable, that $w_1 > w_2 \geq 0$: the joy winning an additional unit decreases as the number of units already obtained increases. Formally, in *1A2U*, bidder i's (perceived) payoff is $2v_i - b_{i,1} - b_{i,2} + w_1 + w_2$ if she wins both units, $v_i - b_{i,1} + w_1$ if she wins (only) one unit, and zero otherwise. Similarly, in *2A1U* bidder i's (perceived) payoff is $2v_i - c_{i,1} - c_{i,2} + w_1 + w_2$ if she wins both auctions, $v_i - c_{i,j} + w_1$ if she wins only the j-th auction ($j = 1, 2$), and zero otherwise.

A.2.1 Equilibrium bids in *1A1U*

In the symmetric equilibrium of *1A1U*, each bidder bids according to:

$$a^{JoW}(V) = \frac{1}{2}V + w.$$

To see this, notice that this case can be interpret as a situation in which the true bidder's value is $x = V + w$, which is thus uniformly distributed over $[w, 2 + w]$. Hence, the symmetric equilibrium is $(x + w)/2$, or $V/2 + w$.

A.2.2 Equilibrium bids in *2A1U*

In *2A1U*, let $c_{i,j}$ denote bidder i's bid in auction j. There are two specular equilibria. The first is:

$$\begin{cases} c_{1,1}^{JoW}(v_1) = c_H(v_1),\ c_{1,2}^{JoW}(v_1) = c_L(v_1), \\ c_{2,1}^{JoW}(v_2) = c_L(v_2),\ c_{2,2}^{JoW}(v_2) = c_H(v_2); \end{cases}$$

the second is:

$$\begin{cases} c_{1,1}^{JoW}(v_1) = c_L(v_1),\ c_{1,2}^{JoW}(v_1) = c_H(v_1), \\ c_{2,1}^{JoW}(v_2) = c_H(v_2),\ c_{2,2}^{JoW}(v_2) = c_L(v_2), \end{cases}$$

where, letting $\Delta = w_1 - w_2$, and assuming $\Delta < 3/2$,

$$c_H(v) = \begin{cases} \frac{1}{2}v + \frac{2w_1+w_2}{3} & \text{if } \ 0 \leq v \leq 1 - \frac{2}{3}\Delta \\ \frac{1}{2} + \frac{w_1+2w_2}{3} & \text{if } \ 1 - \frac{2}{3}\Delta < v \leq 1 \end{cases}, \qquad c_L(v) = \frac{1}{2}v + \frac{w_1 + 2w_2}{3}. \tag{1}$$

Notice that $c_H(v) > c_L(v)$ for all $v \in (0, 1)$. Hence, in the first equilibrium, bidder 1 places a high bid c_H in the first auction and a low bid c_L in the second, while bidder 2 makes the opposite. In the second equilibrium, bidder 1 places a low bid in the first auction and a high bid in the second, while bidder 2 makes the opposite.

To show that the two above are indeed equilibria, it suffices to show that, for a bidder, it is optimal to bid c_H in one auction and c_L in the other if the other bidder makes the opposite. Consider bidder 1, with value v_1; suppose that bidder 2 bids according to $c_L(v_2)$ in the first auction and to $c_H(v_2)$ in the second. The expected payoff of bidder 1 is

$$\Pi(c_{1,1}, c_{1,2}; v_1) = (v_1 - c_{1,1} + w_1)\Pr(c_{1,1} > c_L(v_2)) + (v_1 - c_{1,2} + w_2)\Pr(c_{1,2} \geq c_H(v_2))$$

$$+(w_1 - w_2)\Pr(c_{1,1} \leq c_L(v_2), c_{1,2} \geq c_H(v_2)).$$

Notice first that, for bidder 1, bidding above $c_L(1)$ in the first auction is strictly dominated by bidding exactly $c_L(1)$ (in both situations bidder 1 would win for sure, but in the second at a strictly lower price); likewise bidding above $c_H(1)$ in the second auction is strictly dominated by bidding exactly $c_H(1)$. Similarly, bidding below $c_L(0)$ in the first auction and below $c_H(0)$ in the second is strongly dominated by bidding slightly above $c_L(0)$ in the first (in the first situation, bidder 1 will win nothing for sure, while in the second, she will have a strictly positive probability of winning auction 1 at a profitable price).

Restricting our attention to undominated strategies, we can identify four relevant cases.

Case 1: $c_L(0) < c_{1,1} \leq c_L(1)$; $c_{1,2} \leq c_H(0)$. In this case, bidder 1's probability of winning the second auction is zero; her expected payoff is thus:

$$\Pi(c_{1,1}, c_{1,2}; v_1) = (v_1 - c_{1,1} + w_1)\Pr(c_{1,1} > c_L(v_2)) = \frac{2}{3}(v_1 - c_{1,1} + w_1)(3c_{1,1} - w_1 - 2w_2).$$

Notice that, for $v_1 > (2/3)\Delta$, this strategy is dominated (by bidding slightly above $c_H(0)$). Instead, for types $0 \leq v_1 \leq (2/3)\Delta$, the expression above is maximized at $c_{1,1} = \frac{1}{2}v_1 + \frac{2w_1+w_2}{3}$.

Case 2: $c_L(0) < c_{1,1} \leq c_L(1)$; $c_H(0) < c_{1,2} \leq c_H(1)$; $c_{1,2} \leq c_{1,1} + \Delta/3$. In this case, bidder 1's probability of winning the first auction or both is strictly positive, but her probability of winning the second auction but not the first is zero; her expected payoff is thus:

$$\Pi(c_{1,1}, c_{1,2}; v_1) = (v_1 - c_{1,1} + w_1)\Pr(c_{1,1} > c_L(v_2)) + (v_1 - c_{1,2} + w_2)\Pr(c_{1,2} \geq c_H(v_2))$$

$$= \frac{2}{3}(v_1 - c_{1,1} + w_1)(3c_{1,1} - w_1 - 2w_2) + \frac{2}{3}(v_1 - c_{1,2} + w_2)(3c_{1,2} - 2w_1 - w_2).$$

Notice that, for $v_1 \leq (2/3)\Delta$, this strategy is dominated (by bidding exactly $c_H(0)$ in the second auction). Instead, for types $v > (2/3)\Delta$, the expression above is maximized at:

- $c_{1,1} = \frac{1}{2}v_1 + \frac{2w_1+w_2}{3}$, $c_{1,2} = \frac{1}{2}v_1 + \frac{w_1+2w_2}{3}$, for $\frac{2}{3}\Delta < v_1 \leq 1 - \frac{2}{3}\Delta$;
- $c_{1,1} = \frac{1}{2} + \frac{w_1+2w_2}{3}$, $c_{1,2} = \frac{1}{2}v_1 + \frac{w_1+2w_2}{3}$, for $1 - \frac{2}{3}\Delta < v_1 \leq 1$.

Case 3: $c_L(0) < c_{1,1} \leq c_L(1)$; $c_H(0) < c_{1,2} \leq c_H(1)$; $c_{1,2} > c_{1,1} + \Delta/3$. In this case, bidder 1's probability of winning the second auction or both is strictly positive, while her probability of winning the first auction only is zero; her expected payoff is thus:

$$\Pi(c_{1,1}, c_{1,2}; v_1) = (v_1 - c_{1,1} + w_2)\Pr(c_{1,1} > c_L(v_2)) + (v_1 - c_{1,2} + w_1)\Pr(c_{1,2} \geq c_H(v_2))$$

$$= \frac{2}{3}(v_1 - c_{1,1} + w_2)(3c_{1,1} - w_1 - 2w_2) + \frac{2}{3}(v_1 - c_{1,2} + w_1)(3c_{1,2} - 2w_1 - w_2).$$

Notice that, for $v_1 \leq \Delta/3$, this strategy is dominated (by bidding exactly $c_L(0)$ in the first auction). Instead, for types $v_1 > \Delta/3$, is maximized at:

- $c_{1,1} = \frac{1}{2}v_1 + \frac{w_1+5w_2}{6}$, $c_{1,2} = \frac{1}{2}v_1 + \frac{5w_1+w_2}{6}$, for $\Delta/3 < v_1 \leq 1 - \Delta$;
- $c_{1,1} = \frac{1}{2}v_1 + \frac{w_1+5w_2}{6}$, $c_{1,2} = \frac{1}{2} + \frac{w_1+2w_2}{3}$, for $1 - \Delta < v_1 \leq 1$.

Case 4: $c_{1,1} \leq c_L(0)$; $c_H(0) < c_{1,2} \leq c_H(1)$. In this case, bidder 1's probability of winning the first auction is zero; her expected payoff is thus:

$$\Pi(c_{1,1}, c_{1,2}; v_1) = (v_1 - c_{1,2} + w_1)\Pr(c_{1,2} > c_H(v_2)) = \frac{2}{3}(v_1 - c_{1,2} + w_1)(3c_{1,2} - 2w_1 - w_2).$$

Notice that, for $v_1 > \Delta/3$, this strategy is dominated (by bidding slightly above $c_L(0)$). Instead, for types $v_1 \leq \Delta/3$, is maximized at $c_{1,2} = \frac{1}{2}v_1 + \frac{5w_1+w_2}{6}$.

It can easily be shown that bidder 1's payoff is higher in cases 1 and 2 than in cases 3 and 4. Hence, we conclude that the optimal bids of bidder 1, value v_1, are

- $c_{1,1} = \frac{1}{2}v_1 + \frac{2w_1+w_2}{3}$, $c_{1,2} = \frac{1}{2}v_1 + \frac{w_1+2w_2}{3}$, for $0 \leq v_1 \leq 1 - \frac{2}{3}\Delta$;
- $c_{1,1} = \frac{1}{2} + \frac{w_1+2w_2}{3}$, $c_{1,2} = \frac{1}{2}v_1 + \frac{w_1+2w_2}{3}$, for $1 - \frac{2}{3}\Delta < v_1 \leq 1$;

which correspond to $c_{1,1} = c_H$, $c_{1,2} = c_L$.

A.2.3 Equilibrium bids in *1A2U*

In the symmetric equilibrium of *1A2U*, each bidder bids according to:

$$b_1^{JoW}(v) = c_H(v), \quad b_2^{JoW}(v) = c_L(v),$$

where $c_H(v)$ and $c_L(v)$ are given by (1).

To see that this is indeed an equilibrium, notice that the optimization problem for a bidder in *1A2U* is essentially a constrained version of the optimization problem for a bidder in *2A1U*, where the constraint is $b_1 \geq b_2$. In fact, in *1A2U*, if bidder 2 bids according to $b_{2,1}(v_2) = c_H(v_2)$ and $b_{2,2}(v_2) = c_L(v_2)$, then the expected payoff of bidder 1, type v_1 is

$$\Pi(b_{1,1}, b_{1,2}; v_1) = (v_1 - b_{1,1} + w_1)\Pr(b_{1,1} > c_L(v_2)) + (v_1 - b_{1,2} + w_2)\Pr(b_{1,2} \geq c_H(v_2)),$$

which, under the constraint $b_{1,1} \geq b_{1,2}$, is identical to the expected payoff of bidder 1 in *2A1U*, when bidder 1 bids $b_{1,1}$ in the first auction and $b_{1,2}$ in the second, and bidder 2 bids according to $c_L(v_2)$ in the first auction and $c_H(v_2)$ in the second. Hence, since the optimal bids of bidder 1 in *2A1U* do indeed satisfy the constraint, they are admissible solution also in *1A2U*.

A.2.4 Bid spread in *1A2U* and *2A1U*

Since $w_1 > w_2$, it is immediate to show that $c_H(v) > c_L(v)$, for all $v \in (0, 1)$. Hence, we have bid spread both in *1A2U* and *2A1U*.

A.2.5 Overbidding

Since $w > 0$ and $w_1 > w_2 \geq 0$, it is immediate to show that $a^{JoW}(V) > a^{RN}(V)$ for all $V \in (0, 2]$ and that $c_L(v) > c_j^{RN}(v) = b_j^{RN}(v)$ for all $v \in (0, 1]$, which implies that both bids in *1A2U* and *2A1U* display overbidding.

A.2.6 Efficiency in *1A1U*, inefficiency in *1A2U* and *2A1U*

Allocative efficiency in *1A1U* is implied by the symmetric nature of the equilibrium and by the fact that equilibrium bidding functions are strictly increasing. As a consequence, the bidder with the highest private value places the highest bid and wins the auction. In both *1A2U* and *2A1U*, the bidder with the highest private value wins for sure one unit, as $c_H(v^h) > c_L(v^l)$, where v^h and v^l denotes the highest and the lowest between v_1 and v_2. However, there is a strictly positive probability that the bidder with the lowest private values wins one unit with her highest bid. This occurs whenever $c_H(v^l) > c_L(v^h)$, or when $v^h - v^l < (2/3)\Delta$, which, provided that Δ does not exceed 3/2, can happen with strictly positive probability.

In terms of magnitude, notice that the expected total surplus in *1A1U* is simply the expectation of the highest between V_1 and V_2, namely $\mathrm{E}[V^h] = 4/3$. The expected total surplus in *1A2U* and *2A1U* are clearly identical. Consider *2A1U*: from the symmetry between the two auctions, the expected surplus in *2A1U* is simply twice the expected surplus in any of the two auctions, say auction 1. The expected surplus in auction 1 can be written as

$$\mathrm{E}[S_1] = \int_{1-\frac{2}{3}\Delta}^{1} v_1 dv_1 + \int_0^{\frac{2}{3}\Delta}\int_0^{1-\frac{2}{3}\Delta} v_1 dv_1 dv_2 + \int_{\frac{2}{3}\Delta}^{1}\int_{v_2-\frac{2}{3}\Delta}^{1-\frac{2}{3}\Delta} v_1 dv_1 dv_2 + \int_{\frac{2}{3}\Delta}^{1}\int_0^{v_2-\frac{2}{3}\Delta} v_2 dv_1 dv_2.$$

The first three integrals correspond to situations in which bidder 1 (the one who bids according to $c_H(\cdot)$) wins the auction; the last integral corresponds to situations in which bidder 2 (the one who bids according to $c_L(\cdot)$) wins the auction. By simple algebra, one obtains that $\mathrm{E}[S_1] = 2/3 - 2\Delta^2(9-4\Delta)/81$. Hence, the expected total surplus in *2A1U* and *1A2U* is equal to $\mathrm{E}[S] = 4/3 - 4\Delta^2(9-4\Delta)/81$, which, under the assumption $\Delta < 3/2$, is always lower than the expected total surplus in *1A1U*.

A.2.7 Bidders bid more in *1A1U* than in *1A2U* and *2A1U*

We show that the expected value of a bidder's bid in *1A1U* is greater than the expected value of the sum of the two bids in *1A2U* and *2A1U*, provided that $w > w_1 + w_2 - \Delta^2/9$. Now, the expected value of a^{JoW} is simply $1/2 + w$. The expected value of the sum of the two bids in *1A2U* and *2A1U* is simply the sum of the expected values of $c_H(v)$ and $c_L(v)$. The expected value of $c_H(v)$ is $\frac{1}{4} + \frac{2w_1+w_2}{3} - \frac{\Delta^2}{9}$; the expected value of $c_L(v)$ is $\frac{1}{4} + \frac{w_1+2w_2}{3}$. The results follows immediately.

A.2.8 Auctioneer's revenue is higher in *1A1U* than in *1A2U* and *2A1U*

The expected revenue in *1A1U* is simply $\mathrm{E}[V^h]/2 + w = 2/3 + w$.

The expected revenue in *1A2U* and *2A1U* is identical. Take *2A1U*. From the symmetry between the two auctions, the expected revenue in *2A1U* is simply twice the expected revenue from one of the two auctions, say auction 1. The expected revenue from auction 1 can be written as

$$\mathrm{E}[R_1] = \int_{1-\frac{2}{3}\Delta}^{1}\left(\frac{1}{2} + \frac{w_1+2w_2}{3}\right) dv_1 + \int_0^{\frac{2}{3}\Delta}\int_0^{1-\frac{2}{3}\Delta}\left(\frac{v_1}{2} + \frac{2w_1+w_2}{3}\right) dv_1 dv_2$$

$$+ \int_{\frac{2}{3}\Delta}^{1}\int_{v_2-\frac{2}{3}\Delta}^{1-\frac{2}{3}\Delta}\left(\frac{v_1}{2} + \frac{2w_1+w_2}{3}\right) dv_1 dv_2 + \int_{\frac{2}{3}\Delta}^{1}\int_0^{v_2-\frac{2}{3}\Delta}\left(\frac{v_2}{2} + \frac{w_1+2w_2}{3}\right) dv_1 dv_2.$$

The first integral is the revenue when bidder 1's value is greater than $1-(2/3)\Delta$, in which bidder 1 wins for sure; the second integral corresponds to the case in which bidder 2's value is lower than $(2/3)\Delta$, in which case bidder 1 wins even if her value is below $1-(2/3)\Delta$; the third integral corresponds to the case in which bidder 2's value is greater than $(2/3)\Delta$, but still bidder 1 wins; the fourth integral is the revenue when bidder 2 wins. By simple algebra, one obtains that $\mathrm{E}[R_1] = (2+3w_1+3w_2-(4/27)\Delta^3)/6$. Hence, the expected revenue in *1A1U* is greater than the expected revenue in *1A2U* and *2A1U* if and only if $w > w_1 + w_2 - (4/81)\Delta^3$.

A.3 Loser's regret

Suppose that, at the end the auction, each bidder is informed of the winning bid(s) and suffers a loss in utility (regret) whenever she realizes that she missed an opportunity to win in the sense that an item (or both) has been awarded to her opponent at a price that is below her own valuation for that item. In particular, in *1A1U*, bidder i's (perceived) payoff is $V_i - a_i$ if she wins the auction; it is $-r$ if she does not win the auction and $V_i > a_j$, where $r > 0$ measures the utility loss associated with loser's regret; it is zero otherwise. In *1A2U* and *2A1U*, bidder i may experience loser's regret in three circumstances: *(i)* she does not obtain any unit and discovers that both winning bids were lower than her valuation; *(ii)* she does not obtain any unit and discovers that one of the winning bids (but not the other) was lower than her valuation; *(iii)* she wins one unit and discovers that her opponent's winning bid was lower than her valuation. Denote by r_1, r_2 and r_3 the (fixed) level of regret associated with these three circumstances. Reasonably, r_1 should be no smaller than r_2 and r_3. Formally, in *1A2U*, bidder i's (perceived) payoff is $2v_i - b_{i,1} - b_{i,2}$ if she wins both units; it is $v_i - b_{i,1} - r_3$ if she wins one unit and $b_{j,1} < v_i$; it is $v_i - b_{i,1}$ if she wins one unit and $b_{j,1} \geq v_i$; it is $-r_2$ if she wins zero units and $b_{j,1} \geq v_i > b_{j,2}$; it is $-r_1$ if she wins zero units and $v_i > b_{j,1} (\geq b_{j,2})$; it is zero otherwise. In *2A1U*, bidder i's (perceived) payoff is $2v_i - c_{i,1} - c_{i,2}$ if she wins both auctions; it is $v_i - c_{i,1} - r_3$ $(v_i - c_{i,2} - r_3)$ if she wins only the first (second) auction and $c_{j,2} < v_i$ $(c_{j,1} < v_i)$; it is $v_i - c_{i,1}$ $(v_i - c_{i,2})$ if she wins only the first (second) auction and $c_{j,2} \geq v_i$ $(c_{j,1} \geq v_i)$; it is $-r_2$ if she does not win any auction and either $c_{j,1} < v_i \leq c_{j,2}$ or $c_{j,2} < v_i \leq c_{j,1}$; it is $-r_1$ if she does not win any auction and $c_{j,1} < v_i$ and $c_{j,2} < v_i$.

Our next result concerns the relation between joy of winning and loser's regret. The intuition suggests that these two psychological features should yield very similar behavioral responses; after all, both generate an extra-benefit from winning relative to losing: the joy of winning in the former case, the avoidance of (possible) regret in the latter. Lemma 1 show that this intuition is correct: under certain (reasonable) conditions, joy of winning and loser's regret are equivalent, thus generate the same predictions.

LEMMA 1: *In* 1A1U*, equilibrium bids under joy of winning with parameter $w > 0$ are identical to those under loser's regret with parameter $r = w$. In* 1A2U *and* 2A1U*, equilibrium bids under joy of winning with parameters $w_1 > w_2 \geq 0$ are identical to those under loser's regret with parameters $r_2 = w_1$, $r_3 = w_2$ and $r_1 = w_1 + w_2$.*

Proof.

- *1A1U.* The expected payoff of bidder 1, type v_1, when she bids a_1 and bidder 2 bids according to $a_2(\cdot)$ is

$$\begin{aligned}\Pi(a_1; v_1) =& (v_1 - a_1) \Pr(a_1 \geq a_2(v_2)) - r \Pr(v_1 \geq a_2(v_2) > a_1) \\ =& (v_1 - a_1 + r) \Pr(a_1 \geq a_2(v_2)) - r \Pr(v_1 > a_2(v_2)).\end{aligned}$$

  The first term of the above expression is the same as the expected payoff of bidder 1, type v_1, under joy of winning with parameter r. The second term, instead is independent of a_1. As a consequence, best responses and thus equilibria will be the same.

- *1A2U.* The expected payoff of bidder 1, type v_1, when she bids $(b_{1,1}, b_{1,2})$ and bidder 2 bids according to $(b_{2,1}(\cdot), b_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(b_{1,1}, b_{1,2}) =& (v_1 - b_{1,1}) \Pr(b_{1,1} > b_{2,2}(v_2)) + (v_1 - b_{1,2}) \Pr(b_{1,2} > b_{2,1}(v_2)) \\ & - r_1 \Pr(b_{1,1} < b_{2,2}(v_2), b_{2,1} < v_1) - r_2 \Pr(b_{1,1} < b_{2,2}(v_2), b_{2,2} < v_1 < b_{2,1}) \\ & - r_3 \Pr(b_{1,1} > b_{2,2}(v_2), b_{1,2} < b_{2,1}(v_2), b_{2,1} < v_1).\end{aligned}$$

Under the assumption $r_1 = r_2 + r_3$, the above expression reduces to

$$\begin{aligned}\Pi(b_{1,1}, b_{1,2}) =& (v_1 - b_{1,1}) \Pr(b_{1,1} > b_{2,2}(v_2)) + (v_1 - b_{1,2}) \Pr(b_{1,2} > b_{2,1}(v_2)) \\ & - r_2 \Pr(b_{1,1} < b_{2,2}(v_2) < v_1) - r_3 \Pr(b_{1,2} < b_{2,1}(v_2) < v_1) \\ =& (v_1 - b_{1,1} + r_2) \Pr(b_{1,1} > b_{2,2}(v_2)) + (v_1 - b_{1,2} + r_3) \Pr(b_{1,2} > b_{2,1}(v_2)) \\ & - r_2 \Pr(b_{2,2}(v_2) < v_1) - r_3 \Pr(b_{2,1}(v_2) < v_1).\end{aligned}$$

The first two terms of the above expression are equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters r_2 and r_3. The second term, instead is independent of $b_{1,1}$ and $b_{1,2}$. As a consequence, best responses and thus equilibrium will be the same.

- *2A1U*. The expected payoff of bidder 1, type v_1, when she bids $(c_{1,1}, c_{1,2})$ and bidder 2 bids according to $(c_{2,1}(\cdot), c_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(c_{1,1}, c_{1,2}) =& (v_1 - c_{1,1}) \Pr(c_{1,1} > c_{2,1}) + (v_1 - c_{1,2}) \Pr(c_{1,2} > c_{2,2}) \\ & - r_1 \Pr(c_{1,1} < c_{2,1} \leq v_1, c_{1,2} < c_{2,2} \leq v_1) \\ & - r_2 [\Pr(c_{1,1} < c_{2,1} \leq v_1, c_{1,2} < c_{2,2}, v_1 < c_{2,2}) \\ & + \Pr(c_{1,1} < c_{2,1}, v_1 < c_{2,1}, c_{1,2} < c_{2,2} \leq v_1)] \\ & - r_3 [\Pr(c_{1,1} \geq c_{2,1}, c_{1,2} < c_{2,2} \leq v_1) + \Pr(c_{1,1} < c_{2,1} \leq v_1, c_{1,2} \geq c_{2,2})].\end{aligned}$$

Under the assumption $r_1 = r_2 + r_3$, the above expression reduces to

$$\begin{aligned}\Pi(c_{1,1}, c_{1,2}) =& (v_1 - c_{1,1} + r_2) \Pr(c_{1,1} \geq c_{2,1}) + (v_1 - c_{1,2} + r_2) \Pr(c_{1,2} \geq c_{2,2}) \\ & - (r_2 - r_3) \Pr(c_{1,1} \geq c_{2,1}, c_{1,2} \geq c_{2,2}) \\ & - (r_2 - r_3) [\Pr(c_{1,1} < c_{2,1}, v_1 < c_{2,1}, c_{1,2} \geq c_{2,2}) \\ & + \Pr(c_{1,1} < c_{2,1}, v_1 < c_{2,1}, c_{1,2} < c_{2,2} \leq v_1)] \\ & - r_2 \Pr(c_{2,1} \geq v_1) - r_3 \Pr(c_{2,2} \geq v_1),\end{aligned}$$

or, equivalently, to

$$\begin{aligned}\Pi(c_{1,1}, c_{1,2}) =& (v_1 - c_{1,1} + r_2) \Pr(c_{1,1} \geq c_{2,1}) + (v_1 - c_{1,2} + r_2) \Pr(c_{1,2} \geq c_{2,2}) \\ & - (r_2 - r_3) \Pr(c_{1,1} \geq c_{2,1}, c_{1,2} \geq c_{2,2}) \\ & - (r_2 - r_3) [\Pr(c_{1,1} \geq c_{2,1}, c_{1,2} < c_{2,2}, v_1 < c_{2,2}) \\ & + \Pr(c_{1,1} < c_{2,1} \leq v_1, c_{1,2} < c_{2,2}, v_1 < c_{2,2},)] \\ & - r_2 \Pr(c_{2,2} \geq v_1) - r_3 \Pr(c_{2,1} \geq v_1).\end{aligned}$$

In both expressions, the first two lines are equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters r_2 and r_3. The expression in the third and fourth lines is nonpositive, but is equal to zero if bidders play the strategies of the equilibrium under joy of winning. Finally, the last line is independent of $c_{1,1}$ and $c_{1,2}$, thus it does not affect best responses and equilibria. As a consequence, the equilibrium under joy of winning with parameters r_2 and r_3 is also an equilibrium of *2A1U* under loser's regret.

A.4 Risk Aversion

When bidders are risk averse, their utility function $u(\cdot)$ is strictly concave.[2] In *1A1U*, bidder i's payoff is $u(V_i - a_i)$ if she wins the auction and zero otherwise. In *1A2U*, bidder i's payoff is $u(2v_i - b_{i,1} - b_{i,2})$ if she wins both units, $u(v_i - b_{i,1})$ if she wins one unit only and zero otherwise. Finally, in *2A1U*, bidder i's payoff is $u(2v_i - c_{i,1} - c_{i,2})$ if she wins both auctions, $u(v_i - c_{i,1})$ if she wins only the first auction, $u(v_i - c_{i,2})$ if she wins only the second auction and zero otherwise.

A.4.1 Equilibrium bids in *1A2U*

We characterize a symmetric equilibrium in differentiable, strictly increasing strategies. In *1A2U*, the expected payoff of a bidder with value v is:

$$\Pi(b_1, b_2; v) = u(2v - b_1 - b_2)F(b_1^{-1}(b_2)) + u(v - b_1)[F(b_2^{-1}(b_1)) - F(b_1^{-1}(b_2))],$$

where $u(\cdot)$ is a strictly increasing and strictly concave function, with $u(0) = 0$.

If the optimal bids (b_1^*, b_2^*) are such that $b_1^* > b_2^*$ (we will verify this condition in the sequel), then they must satisfy the following first order necessary conditions:

$$\begin{cases} -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*)) - u'(v - b_1^*)\left[F(b_2^{-1}(b_1^*)) - F(b_1^{-1}(b_2^*))\right] \\ +u(v - b_1^*)f(b_2^{-1}(b_1^*))(b_2^{-1}(b_1^*))' = 0, \\ -u'(2v - b_1^* - b_2^*)F(b_1^{-1}(b_2^*))+ \\ + \left[u(2v - b_1^* - b_2^*) - u(v - b_1^*)\right] f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))' = 0. \end{cases} \tag{2}$$

In a symmetric equilibrium, it must be that $b_1^* = b_1(v)$ and $b_2^* = b_2(v)$. Thus, the first order conditions for a symmetric equilibrium are:

$$\begin{cases} -u'(2v - b_1(v) - b_2(v))F(b_1^{-1}(b_2(v))) - u'(v - b_1(v))[F(b_2^{-1}(b_1(v))) \\ -F(b_1^{-1}(b_2(v)))] + u(v - b_1(v))f(b_2^{-1}(b_1(v)))(b_2^{-1}(b_1(v)))' = 0, \\ -u'(2v - b_1(v) - b_2(v))F\left(b_1^{-1}(b_2(v))\right) \\ + \left[u(2v - b_1(v) - b_2(v)) - u(v - b_1(v))\right] f\left(b_1^{-1}(b_2(v))\right)\left(b_1^{-1}(b_2(v))\right)' = 0. \end{cases}$$

This system of differential equations, together with the boundary conditions $b_1(0) = b_2(0) = 0$ and $b_1(\bar{v}) = b_2(\bar{v}) = \bar{b}$ defines a symmetric equilibrium $(b_1^{RA}(v), b_2^{RA}(v))$.[3]

A.4.2 Equilibrium bids in *2A1U*

The expected payoff of a bidder with value v in *2A1U* is:

$$\begin{aligned} \Pi(c_1, c_2; v) =& u(v - c_1)F(c_1^{-1}(c_1)) + u(v - c_2)F(c_2^{-1}(c_2)) \\ & +[u(2v - c_1 - c_2) - u(v - c_1) - u(v - c_2)]F(\min(c_1^{-1}(c_1); c_2^{-1}(c_2)). \end{aligned}$$

Suppose that, as in a symmetric equilibrium, the optimal bids c_1^*, c_2^* are such that $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$. Notice that, at this point, the expected payoff is not differentiable. However, it

[2] Apart from differentiability, we do not make any assumption on the particular form of the utility function. Hence, our results apply to any specific form of risk aversion. Moreover, they are also independent from the probability distribution of values.

[3] Provided that, $b_1^{RA}(v) > b_2^{RA}(v)$, for all $v \in (0, \bar{v})$.

admits right and left partial derivatives. In particular, the right partial derivatives must be non-positive and the left partial derivatives must be nonnegative (otherwise, any increase or decrease in bids would be profitable):

$$\frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^+} \leq 0, \quad \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_1^-} \geq 0;$$

$$\frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^+} \leq 0, \quad \frac{\partial \Pi(c_1^*, c_2^*)}{\partial c_2^-} \geq 0.$$

Let us focus on the first two conditions (by symmetry, the same holds for the other two conditions), which are:

$$-u^{'}(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))^{'} \leq 0, \tag{3}$$

$$-u^{'}(2v - c_1^* - c_2^*)F(c_1^{-1}(c_1^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_2^*)]f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))^{'} \geq 0. \tag{4}$$

Since $c_1^{-1}(c_1^*) = c_2^{-1}(c_2^*)$, the first term of (3) coincides with that of (4). Let us concentrate on the second term. Since $u(\cdot)$ is strictly concave, the incremental ratio $[u(z+h) - u(z)]/h$ is strictly decreasing in z. Therefore, we have

$$u(2v - c_1^* - c_2^*) - u(v - c_2^*) < u(v - c_1^*).$$

But then, (3) and (4) cannot simultaneously hold. This means that the optimal bids (c_1^*, c_2^*) are such that $c_1^{-1}(c_1^*) \neq c_2^{-1}(c_2^*)$. In particular, they imply that there are no symmetric equilibria.

Now, suppose that the optimal bids (c_1^*, c_2^*) are such that $c_1^{-1}(c_1^*) > c_2^{-1}(c_2^*)$. The expected payoff is now differentiable at (c_1^*, c_2^*) and the first order conditions are:

$$\begin{cases} -u^{'}(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) - u^{'}(v - c_1^*)\left[F(c_1^{-1}(c_1^*)) - F(c_2^{-1}(c_2^*))\right] \\ +u(v - c_1^*)f(c_1^{-1}(c_1^*))(c_1^{-1}(c_1^*))^{'} = 0 \\ -u^{'}(2v - c_1^* - c_2^*)F(c_2^{-1}(c_2^*)) + [u(2v - c_1^* - c_2^*) - u(v - c_1^*)] \times \\ \times f(c_2^{-1}(c_2^*))(c_2^{-1}(c_2^*))^{'} = 0 \end{cases}$$

Notice the perfect analogy with the first order conditions for optimal bids in *1A2U* - see system (2). Therefore, in *2A1U* there is an equilibrium in which:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_1^{RA}(v_1),\ c_{1,2}^{RA}(v_1) = b_2^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_2^{RA}(v_2),\ c_{2,2}^{RA}(v_2) = b_1^{RA}(v_2). \end{cases}$$

In this equilibrium, bidder 1 respectively places the highest bid in the first auction and the lowest bid in the second, while bidder 2 makes the opposite. As under joy of winning, there is also a specular equilibrium in which bidder 1 respectively places the lowest bid in the first auction and the highest bid in the second, while bidder 2 makes the opposite:

$$\begin{cases} c_{1,1}^{RA}(v_1) = b_2^{RA}(v_1),\ c_{1,2}^{RA}(v_1) = b_1^{RA}(v_1), \\ c_{2,1}^{RA}(v_2) = b_1^{RA}(v_2),\ c_{2,2}^{RA}(v_2) = b_2^{RA}(v_2). \end{cases}$$

A.4.3 Bid spread in *1A2U* and *2A1U*

Given the analogy between *1A2U* and *2A1U* in terms of equilibrium bids, we focus on *1A2U* and show that, if $(b_1^{RA}(v), b_2^{RA}(v))$ constitutes a symmetric equilibrium in differentiable strategies, then it must be that $b_1^{RA}(v) > b_2^{RA}(v)$, for all $v \in (0, \bar{v})$.

Consider a bidder with value $v \in (0, \bar{v})$ who finds optimal to bid $b_1^* = b_2^* = b^*$. Then, it must be the case that deviations are non-profitable, namely:

$$\frac{\partial \Pi(b^*, b^*)}{\partial b_1} \leq 0, \quad \frac{\partial \Pi(b^*, b^*)}{\partial b_2} \geq 0.$$

In a symmetric equilibrium, $b_1^* = b_1(v)$, $b_2^* = b_2(v)$, and since $b_1^* = b_2^* = b^*$, then $b_1(v) = b_2(v) = b^*$. The conditions above reduce to

$$b_2^{'}(x) \geq \frac{u(v-b^*)}{u^{'}(2v-2b^*)} \frac{f(v)}{F(v)}$$

and

$$b_1^{'}(x) \leq \frac{u(2v-2b^*) - u(v-b^*)}{u^{'}(2v-2b^*)} \frac{f(v)}{F(v)}.$$

Since $u(\cdot)$ is strictly concave and $u(0) = 0$, we have that, for $y > 0$, $2u(y) > u(2y)$. Therefore,

$$\frac{u(v-b^*)}{u^{'}(2v-2b^*)} \frac{f(v)}{F(v)} > \frac{u(2v-2b^*) - u(v-b^*)}{u^{'}(2v-2b^*)} \frac{f(v)}{F(v)},$$

which implies that $b_2^{'}(v) > b_1^{'}(v)$. Thus, whenever the two bids are equal, the bidding function corresponding to the lowest bid is steeper than that of the highest bid. However, this is not possible under the assumption that $b_1(\cdot)$ and $b_2(\cdot)$ are differentiable.

A.4.4 Overbidding

It is well known that $a^{RA}(V) > a^{RN}(V)$. Let us prove overbidding in *1A2U* and *2A1U*. Starting from *1A2U*, suppose that the opponent bids according to generic (strictly increasing) bidding strategies, $b_1(\cdot)$ and $b_2(\cdot)$. Under risk neutrality, the first order conditions that define the optimal lowest bid b_2^* of a bidder with value v is

$$(v - b_2^*) f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))^{'} = F(b_1^{-1}(b_2^*)). \tag{5}$$

Now, suppose that under risk aversion, the bidder bids according to b_2^* defined by (5). The partial derivative of her expected payoff with respect to b_2 evaluated at b_2^* is

$$-u^{'}(2v - b_1 - b_2^*)F(b_1^{-1}(b_2^*)) + [u(2v - b_1 - b_2^*) - u(v - b_1)] f(b_1^{-1}(b_2^*))(b_1^{-1}(b_2^*))^{'}.$$

By using (5), the last expression can be written as

$$\frac{u(2v - b_1 - b_2^*) - u(v - b_1)}{v - b_2^*} - u^{'}(2v - b_1 - b_2^*),$$

which is strictly positive. Indeed, notice that the first addend is the incremental ratio of $u(\cdot)$ from $(v - b_1)$ to $(2v - b_1 - b_2^*)$. The second term is the derivative of $u(\cdot)$ evaluated at $(2v - b_1 - b_2^*)$. Since $u(\cdot)$ is strictly concave, the first term is strictly greater than the second. This shows that, under risk aversion, for any possible bidding strategy adopted by the opponent, a bidder has an incentive to increase her lowest bid with respect to the corresponding equilibrium level under risk neutrality.

A.4.5 Efficiency in *1A1U*, inefficiency in *1A2U* and in *2A1U*

The argument is identical to the one used under joy of winning (see Subsection A.2.6). Inefficiency in *1A2U* and in *2A1U* follows directly from bid spread.

A.5 Inequity aversion and loss aversion

Inequity aversion. Consider the following simple model of inequity averse bidders: suppose that bidder i suffers a utility loss ε_1 whenever the number of units won by her is lower than the number of units won by the other bidder, and a utility loss ε_2 in the opposite situation (with $\varepsilon_1 > \varepsilon_2 \geq 0$). Then, this model would imply exactly the same predictions as those obtained from a model of joy of winning with parameters $w = \varepsilon_1 - \varepsilon_2$, $w_1 = \varepsilon_1$, $w_2 = -\varepsilon_2$.

To see this, notice that, under inequity aversion, in *1A1U* the expected payoff of bidder 1, type v_1, when she bids a_1 and bidder 2 bids according to $a_2(\cdot)$ is

$$\begin{aligned}\Pi(a_1; v_1) =& (v_1 - a_1) \Pr(a_1 \geq a_2(v_2)) - \varepsilon_1 \Pr(a_1 < a_2(v_2)) - \varepsilon_2 \Pr(a_1 \geq a_2(v_2)) \\ =& (v_1 - a_1 + \varepsilon_1 - \varepsilon_2) \Pr(a_1 \geq a_2(v_2)) - \varepsilon_1.\end{aligned}$$

Notice that, up to the constant $-\varepsilon_1$, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameter $w = \varepsilon_1 - \varepsilon_2$.

In *1A2U*, the expected payoff of bidder 1, type v_1, when she bids $(b_{1,1}, b_{1,2})$ and bidder 2 bids according to $(b_{2,1}(\cdot), b_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(b_{1,1}, b_{1,2}) =& (v_1 - b_{1,1}) \Pr(b_{1,1} \geq b_{2,2}(v_2)) + (v_1 - b_{1,2}) \Pr(b_{1,2} \geq b_{2,1}(v_2)) \\ & - \varepsilon_1 \Pr(b_{1,1} < b_{2,2}(v_2)) - \varepsilon_2 \Pr(b_{1,2} \geq b_{2,1}(v_2)) \\ =& (v_1 - b_{1,1} + \varepsilon_1) \Pr(b_{1,1} \geq b_{2,2}(v_2)) + (v_1 - b_{1,2} - \varepsilon_2) \Pr(b_{1,2} \geq b_{2,1}(v_2)) - \varepsilon_1.\end{aligned}$$

Notice that, up to the constant $-\varepsilon_1$, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters $w_1 = \varepsilon_1$ and $w_2 = -\varepsilon_2$.

In *2A1U*, the expected payoff of bidder 1, type v_1, when she bids $(c_{1,1}, c_{1,2})$ and bidder 2 bids according to $(c_{2,1}(\cdot), c_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(c_{1,1}, c_{1,2}) =& (v_1 - c_{1,1}) \Pr(c_{1,1} \geq c_{2,1}(v_2)) + (v_1 - c_{1,2}) \Pr(c_{1,2} \geq c_{2,2}(v_2)) \\ & - \varepsilon_1 \Pr(c_{1,1} < c_{2,1}(v_2), c_{1,2} < c_{2,2}(v_2)) - \varepsilon_2 \Pr(c_{1,1} \geq c_{2,1}(v_2), c_{1,2} \geq c_{2,2}(v_2)) \\ =& (v_1 - c_{1,1} + \varepsilon_1) \Pr(c_{1,1} \geq c_{2,1}(v_2)) + (v_1 - c_{1,2} + \varepsilon_1) \Pr(c_{1,2} \geq c_{2,2}(v_2)) \\ & - (\varepsilon_1 + \varepsilon_2) \Pr(c_{1,1} \geq c_{2,1}(v_2), c_{1,2} \geq c_{2,2}(v_2)) - \varepsilon_1.\end{aligned}$$

Notice that, up to the constant $-\varepsilon_1$, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters $w_1 = \varepsilon_1$ and $w_2 = -\varepsilon_2$.

Loss aversion. Suppose that bidders have reference dependent preferences with loss aversion. In the spirit of Köszegi and Rabin (2006), we assume that, beyond the standard utility, bidders have a gain/loss utility which depends on the deviations from a reference point, where the reference point captures the expectation of the bidders on the likely outcome of the auction(s). In particular, suppose that, in the allotment treatments, a bidder enjoys a gain if she wins both units, suffers a loss if she wins none, and experiences no gain/loss if she wins one units. In *1A1U*, instead, a bidder enjoys a gain if she wins (the only unit sold), and suffers a loss if she loses. These simple assumptions captures the idea that a bidder, prior to participating in the auction(s), expects to win an "average" number of units during the auction. Let $\lambda_G \geq 0$ and $\lambda_L > \lambda_G$ denote the gain and loss parameters, respectively. Then, this model would imply exactly the same predictions as those obtained from a model of joy of winning with parameters $w = \lambda_G + \lambda_L$, $w_1 = \lambda_L$, $w_2 = \lambda_G$.

To see this, notice that, under loss aversion, in *1A1U* the expected payoff of bidder 1, type v_1, when she bids a_1 and bidder 2 bids according to $a_2(\cdot)$ is

$$\begin{aligned}\Pi(a_1; v_1) =&(v_1 - a_1)\Pr(a_1 \geq a_2(v_2)) - \lambda_L \Pr(a_1 < a_2(v_2)) + \lambda_G \Pr(a_1 \geq a_2(v_2))\\ =&(v_1 - a_1 + \lambda_G + \lambda_L)\Pr(a_1 \geq a_2(v_2)) - \lambda_L.\end{aligned}$$

Notice that, up to the constant λ_L, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameter $w = \lambda_G + \lambda_L$.

In *1A2U*, the expected payoff of bidder 1, type v_1, when she bids $(b_{1,1}, b_{1,2})$ and bidder 2 bids according to $(b_{2,1}(\cdot), b_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(b_{1,1}, b_{1,2}) =&(v_1 - b_{1,1})\Pr(b_{1,1} \geq b_{2,2}(v_2)) + (v_1 - b_{1,2})\Pr(b_{1,2} \geq b_{2,1}(v_2))\\ &-\lambda_L \Pr(b_{1,1} < b_{2,2}(v_2)) + \lambda_G \Pr(b_{1,2} \geq b_{2,1}(v_2))\\ =&(v_1 - b_{1,1} + \lambda_L)\Pr(b_{1,1} \geq b_{2,2}(v_2)) + (v_1 - b_{1,2} + \lambda_G)\Pr(b_{1,2} \geq b_{2,1}(v_2)) - \lambda_L.\end{aligned}$$

Notice that, up to the constant $-\lambda_L$, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters $w_1 = \lambda_L$ and $w_2 = \lambda_G$.

In *2A1U*, the expected payoff of bidder 1, type v_1, when she bids $(c_{1,1}, c_{1,2})$ and bidder 2 bids according to $(c_{2,1}(\cdot), c_{2,2}(\cdot))$ is

$$\begin{aligned}\Pi(c_{1,1}, c_{1,2}) =&(v_1 - c_{1,1})\Pr(c_{1,1} \geq c_{2,1}(v_2)) + (v_1 - c_{1,2})\Pr(c_{1,2} \geq c_{2,2}(v_2))\\ &-\lambda_L \Pr(c_{1,1} < c_{2,1}(v_2), c_{1,2} < c_{2,2}(v_2)) + \lambda_G \Pr(c_{1,1} \geq c_{2,1}(v_2), c_{1,2} \geq c_{2,2}(v_2))\\ =&(v_1 - c_{1,1} + \lambda_L)\Pr(c_{1,1} \geq c_{2,1}(v_2)) + (v_1 - c_{1,2} + \lambda_L)\Pr(c_{1,2} \geq c_{2,2}(v_2))\\ &-(\lambda_L - \lambda_G)\Pr(c_{1,1} \geq c_{2,1}(v_2), c_{1,2} \geq c_{2,2}(v_2)) - \lambda_L.\end{aligned}$$

Notice that, up to the constant $-\lambda_L$, the above expression is equal to the expected payoff of bidder 1, type v_1, under joy of winning with parameters $w_1 = \lambda_L$ and $w_2 = \lambda_G$.

A.6 Shaky bidding behavior in *2A1U*

To illustrate the intuition behind shaky bidding behavior, consider the following 3×3 symmetric game.

	A	*B*	*C*
A	1,1	5,4	6,6
B	4,5	3,3	4,5
C	6,6	5,4	1,1

This game can be thought of as an extremely stylized version of our *2A1U*: strategy A corresponds to the strategy "high bid in the first auction, low bid in the second", strategy C corresponds to the strategy "low bid in the first auction, high bid in the second", strategy B corresponds to the strategy "an intermediate bid in both auctions".

There are two Nash Equilibria in pure strategies: (A, C) and (C, A). Notice that both Nash Equilibria have payoffs $(6, 6)$, so neither of them is payoff dominant. Also, risk dominance does not allow to select one equilibrium. To see this, notice that in a symmetric game with two Nash Equilibria, one equilibrium is risk dominant if the payoff each player obtains by playing her equilibrium strategy is larger than the payoff obtained by playing the other equilibrium strategy, when the other player plays her two equilibrium strategies with equal probability. Formally, if (s_1^*, s_2^*) and (s_1^{**}, s_2^{**}) are the two Nash Equilibria of the game, (s_1^*, s_2^*) is risk dominant if, for each player i,

$$s_i^* = \arg \max_{s_i \in \{s_i^*, s_i^{**}\}} \frac{1}{2} u_i(s_i, s_{-i}^*) + \frac{1}{2} u_i(s_i, s_{-i}^{**}).$$

It is easy to verify that none of the equilibria in the game above satisfies this condition. In fact,

$$\frac{1}{2} u_1(A, C) + \frac{1}{2} u_1(A, A) = \frac{1}{2} u_1(C, C) + \frac{1}{2} u_1(C, A) = \frac{7}{2},$$

and the same is true for player 2. Thus, in this game neither Nash Equilibrium stands out. Hence, solving the coordination problem is a difficult task and it seems sensible to think that a player may consider avoiding the risk of mis-coordination and look for a safer strategy. Shaky bidding reflects exactly such a behavior. Now, what does shaky bidding behavior imply in a game like this? According to assumption (A3), a shaky bidder will pick the strategy that maximizes her payoff when the other player randomizes (with equal probability) between her two equilibrium strategies. Formally, a shaky bidder i will play the strategy $\hat{s}_i$ such that

$$\hat{s}_i = \arg \max_{s_i \in S_i} \frac{1}{2} u_i(s_i, s_{-i}^*) + \frac{1}{2} u_i(s_i, s_{-i}^{**}).$$

This definition is similar to the definition of risk dominance. There is however a crucial difference: a shaky bidder chooses among all her available strategies, whereas, in the definition of payoff dominance, bidders choose within the set of equilibrium strategies (and, moreover, choose consistently). As a consequence, when shaky bidders are involved, the outcome will not necessarily be an equilibrium outcome. If we apply the definition of shaky bidding behavior to the game above, we obtain that a shaky bidder would select strategy B, which indeed is not part of any Nash Equilibrium. In fact,

$$\frac{1}{2} u_1(B, C) + \frac{1}{2} u_1(B, A) = 4 > \frac{7}{2},$$

and the same holds for player 2.

A.6.1 Shaky bidding under joy of winning

Consider the joy of winning model, and suppose that, in *2A1U*, bidder i believes that her opponent is going to play the following mixed strategy: with probability 1/2, she will play $c_H(v)$ in the first auction and $c_L(v)$ in the second; with probability 1/2, she will play $c_L(v)$ in the first auction and $c_H(v)$ in the second, where $c_H(v)$ and $c_L(v)$ are defined by (1). Bidder i's best response would be to make the same bid in both auction, where this common bid is given by:[4]

$$c(v) = \begin{cases} \frac{2w_1 + w_2}{3} & \text{if} \quad 0 \le v \le \frac{\Delta}{3} \\ \frac{1}{2} v + \frac{w_1 + w_2}{2} & \text{if} \quad \frac{\Delta}{3} < v \le \hat{v} = 1 - \frac{\sqrt{3\Delta(1-\Delta)}}{3} \\ \frac{1}{2} + \frac{w_1 + 2w_2}{3} & \text{if} \quad \hat{v} = 1 - \frac{\sqrt{3\Delta(1-\Delta)}}{3} < v \le 1 \end{cases}$$

[4]This result and those that follow are obtained under the (reasonable) assumption that $\Delta < 3/4$.

The proof of this result follows the same lines as the one in Subsection A.2.2.

Now suppose that an equilibrium bidder (i.e. one who bids $c_H(v)$ in one auction and $c_L(v)$ in the other) and a shaky bidder (i.e. one who bids $c(v)$ in both auctions) play one against the other. The structure of bids immediately implies that efficiency would be larger than in *1A2U* where two equilibrium bidders face. This can also be shown formally: if bidder 1 is a shaky bidder (i.e. one who bids according to $c(v)$ in both auctions) and bidder 2 is an equilibrium bidder (i.e. one who bids according to $c_H(v)$ in the first auction and $c_L(v)$ in the second), the expected surplus in the first auction would be

$$\mathrm{E}[S_1] = \int_{\hat{v}}^{1} v_1 dv_1 + \int_{0}^{\hat{v}-\frac{\Delta}{3}} \int_{v_2+\frac{\Delta}{3}}^{\hat{v}} v_1 dv_1 dv_2$$

$$+ \int_{0}^{\hat{v}-\frac{\Delta}{3}} \int_{0}^{v_2+\frac{\Delta}{3}} v_2 dv_1 dv_2 + \int_{\hat{v}-\frac{\Delta}{3}}^{1-\frac{2}{3}\Delta} \int_{0}^{\hat{v}} v_2 dv_1 dv_2 + \int_{1-\frac{2}{3}\Delta}^{1} \int_{0}^{\hat{v}} v_2 dv_1 dv_2.$$

The first two integrals correspond to situations in which bidder 1 (the shaky bidder) wins the auction; the last three integrals correspond to situations in which bidder 2 (the equilibrium bidder) wins the auction.

In the same vain, the expected surplus in the second auction would be

$$\mathrm{E}[S_2] = \int_{\hat{v}}^{1} v_1 dv_1 + \int_{0}^{\frac{2}{3}\Delta} \int_{0}^{\frac{\Delta}{3}} v_1 dv_1 dv_2 + \int_{\frac{\Delta}{3}}^{\hat{v}} \int_{0}^{v_1+\frac{\Delta}{3}} v_1 dv_2 dv_1$$

$$+ \int_{\hat{v}+\frac{\Delta}{3}}^{1} \int_{0}^{\hat{v}} v_2 dv_1 dv_2 + \int_{\frac{2}{3}\Delta}^{\hat{v}+\frac{\Delta}{3}} \int_{0}^{v_2-\frac{\Delta}{3}} v_2 dv_1 dv_2.$$

The first three integrals correspond to situations in which bidder 1 (the shaky bidder) wins the auction; the last two integrals correspond to situations in which bidder 2 (the equilibrium bidder) wins the auction.

After simple algebra, we obtain that, if a shaky bidder and an equilibrium bidder face in *2A1U*, the expected total surplus would be equal to $[54\hat{v}^3 - 2\Delta^3 + 162(1 + \hat{v} + \Delta\hat{v} - \hat{v}^2) + 9\Delta(3\Delta + 3\hat{v}^2 - 9 - 5\Delta\hat{v}]/162$, which is always larger than the expected total surplus in *1A2U*.

As far as revenue is concerned, if bidder 1 is a shaky bidder (i.e. one who bids according to $c(v)$ in both auctions) and bidder 2 is an equilibrium bidder (i.e. one who bids according to $c_H(v)$ in the first auction and $c_L(v)$ in the second), the expected revenue in the first auction would be

$$\mathrm{E}[R_1] = \int_{\hat{v}}^{1} \left(\frac{1}{2} + \frac{w_1 + 2w_2}{3}\right) dv_1 + \int_{0}^{\hat{v}-\frac{\Delta}{3}} \int_{v_2+\frac{\Delta}{3}}^{\hat{v}} \left(\frac{1}{2}v_1 + \frac{w_1 + w_2}{2}\right) dv_1 dv_2$$

$$+ \int_{0}^{\hat{v}-\frac{\Delta}{3}} \int_{0}^{v_2+\frac{\Delta}{3}} \left(\frac{1}{2}v_2 + \frac{2w_1 + w_2}{3}\right) dv_1 dv_2 + \int_{\hat{v}-\frac{\Delta}{3}}^{1-\frac{2}{3}\Delta} \int_{0}^{\hat{v}} \left(\frac{1}{2}v_2 + \frac{2w_1 + w_2}{3}\right) dv_1 dv_2$$

$$+ \int_{1-\frac{2}{3}\Delta}^{1} \int_{0}^{\hat{v}} \left(\frac{1}{2} + \frac{w_1 + 2w_2}{3}\right) dv_1 dv_2.$$

The first two integrals correspond to situations in which bidder 1 (the shaky bidder) wins the auction; the last three integrals correspond to situations in which bidder 2 (the equilibrium bidder) wins the auction.

In the same vain, the expected revenue in the second auction would be

$$\mathrm{E}[R_2] = \int_{\hat{v}}^{1} \left(\frac{1}{2} + \frac{w_1 + 2w_2}{3}\right) dv_1 + \int_0^{\frac{2}{3}\Delta} \int_0^{\frac{\Delta}{3}} \left(\frac{2w_1 + w_2}{3}\right) dv_1 dv_2$$

$$+ \int_{\frac{\Delta}{3}}^{\hat{v}} \int_0^{v_1 + \frac{\Delta}{3}} \left(\frac{1}{2}v_1 + \frac{w_1 + w_2}{2}\right) dv_2 dv_1 + \int_{\hat{v}+\frac{\Delta}{3}}^{1} \int_0^{\hat{v}} \left(\frac{1}{2}v_2 + \frac{w_1 + 2w_2}{3}\right) dv_1 dv_2$$

$$+ \int_{\frac{2}{3}\Delta}^{\hat{v}+\frac{\Delta}{3}} \int_0^{v_2 - \frac{\Delta}{3}} \left(\frac{1}{2}v_2 + \frac{w_1 + 2w_2}{3}\right) dv_1 dv_2.$$

The first three integrals correspond to situations in which bidder 1 (the shaky bidder) wins the auction; the last two integrals correspond to situations in which bidder 2 (the equilibrium bidder) wins the auction.

After simple algebra, we obtain that, if a shaky bidder and an equilibrium bidder face in *2A1U*, the expected revenue would be equal to $[27\hat{v}^3 + 2\Delta^3 + 162(1 + w_1 + w_2) + 9\Delta(6\Delta\hat{v} - 6\Delta - \Delta^2\hat{v} - 9\hat{v})]/162$, which is larger than the expected revenue in *1A2U* provided that $\Delta < \hat{\Delta} \approx 0.35$.

It can easily be verified that the results above also apply to a model with loser's regret.

A.6.2 Shaky bidding under risk aversion

Consider a model with risk averse bidders, and suppose that, in *2A1U*, bidder i believes that her opponent is going to play the following mixed strategy: with probability 1/2, she will play $c_H(v)$ in the first auction and $c_L(v)$ in the second; with probability 1/2, she will play $c_L(v)$ in the first auction and $c_H(v)$ in the second. Then bidder i's expected payoff is:

$$\begin{aligned}\Pi(c_1, c_2; v) = &\frac{1}{2}\{u(v - c_1)[F(c_H^{-1}(c_1)) + F(c_L^{-1}(c_1))] \\ &+ u(v - c_2)[F(c_L^{-1}(c_2)) + F(c_H^{-1}(c_2))] \\ &+ [u(2v - c_1 - c_2) - u(v - c_1) - u(v - c_2)] \\ &\times [F(\min(c_H^{-1}(c_1); c_L^{-1}(c_2))) + F(\min(c_L^{-1}(c_1); c_H^{-1}(c_2)))]\}\end{aligned}$$

If the optimal bids (c_1^*, c_2^*) are such that $c_H^{-1}(c_1^*) < c_L^{-1}(c_2^*)$, they must satisfy the following first order conditions:

$$\begin{cases} u(v - c_1^*)(c_L^{-1}(c_1^*))' - u'(v - c_1^*)[c_L^{-1}(c_1^*) - c_H^{-1}(c_2^*)] - u(v - c_2^*)(c_H^{-1}(c_1^*))' \\ +u(2v - c_1^* - c_2^*)(c_H^{-1}(c_1^*))' - u'(2v - c_1^* - c_2^*)[c_H^{-1}(c_1^*) + c_H^{-1}(c_2^*)] = 0, \\ u(v - c_2^*)(c_L^{-1}(c_2^*))' - u'(v - c_2^*)[c_L^{-1}(c_2^*) - c_H^{-1}(c_1^*)] - u(v - c_1^*)(c_H^{-1}(c_2^*))' \\ +u(2v - c_1^* - c_2^*)(c_H^{-1}(c_2^*))' - u'(2v - c_1^* - c_2^*)[c_H^{-1}(c_1^*) + c_H^{-1}(c_2^*)] = 0. \end{cases}$$

The two conditions above are clearly satisfied when $c_1^* = c_2^*$.

B Additional results

B.1 Bidding behavior and response times

In this Subsection, we report parametric results on the relationship between response times and bidding behavior in the three treatments. We investigate into this issue in two steps.

First, we parametrically assess the existence of differences in response times across treatments. In this respect, we run parametric panel regressions (based on the same econometric specification of *Table 2* in the paper), with time responses as dependent variable and the treatment dummies and the linear time trend as independent variables. Results are reported in the first two columns of *Table I.* Time responses are significantly lower in *1A1U* than in any of the other two treatments. Moreover, by comparing the coefficients of the two treatment dummies, we find that time responsens are higher in *1A2U* than in *2A1U* ($\chi^2(1) = 5.28$, $p < 0.05$).

Second, we replicate the parametric analysis in *Tables 1-3* in the paper, on pooled data by including the time responses as additional determinant of the (sum of the) bids in the three treatments as well as the (size and probability of) bid spread in *1A2U* and *2A1U.* Results are reported in the remaining six columns of *Table I.* We find that both the size and the probability of bid spread are significantly and positively associated with the response times. However, the inclusion of the response times as additional determinant of the bidding behavior leaves virtually unchanged the coefficients of the treatment dummies.

Table I. Response times and bidding behavior in the three treatments

	Response Times		*Overbidding*				*Bid Spread*	
			Sum of Bids		Prob.		Size	Prob.
Value			0.659***	0.659***	0.001***	0.001***	0.066***	0.002***
			(0.006)	(0.006)	$(2.1 * 10^{-4})$	$(2.1 * 10^{-4})$	(0.003)	$(2.3 * 10^{-4})$
Period	−0.698***	−0.698***	−0.674***	−0.672***	−0.003*	−0.003	0.183***	0.011***
	(0.033)	(0.033)	(0.077)	(0.077)	(0.002)	(0.002)	(0.038)	(0.003)
1A2U&2A1U	7.502***		−4.087*		−0.065**			
	(2.313)		(2.155)		(0.032)			
1A2U		10.448***		−4.767*		−0.094**		
		(2.564)		(2.535)		(0.037)		
2A1U		4.557*		−3.442		−0.056	0.599	−0.056*
		(2.564)		(2.501)		(0.047)	(0.783)	(0.033)
Response Time			−0.037	−0.034	−0.002*	−0.001	0.122***	0.005**
			(0.044)	(0.044)	(0.001)	(0.001)	(0.020)	(0.002)
Constant	17.250***	17.250***	11.660***	11.625***			−3.066***	
	(1.902)	(1.828)	(1.989)	(2.012)			(0.825)	
lrl (lpl)	−8360.002	−8355.667	−10209.877	−10207.905	−1013.8334	−1011.941	−5311.769	−704.746
$Wald-\chi^2$	458.26	465.20	14723.25	14722.45	50.48	53.92	711.04	88.04
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Obs.	2430	2430	2430	2430	2430	2430	1620	1620

Notes. The first two columns report coefficient estimates (standard errors in parentheses) from two-way linear random effects models accounting for both potential individual dependency over repetitions and dependency within rematching group. The dependent variable is the response time spent by subjects to choose their bid(s) in the period. The remaining six columns replicate the analysis reported in *Tables 1-3* in the paper, by including the response times as additional determinant of the (sum of the) bids in the three treatments as well as the (size and probability of) bid spread in *1A2U* and *2A1U*.

B.2 Further insight on the empirical relevance of risk aversion, joy of winning and loser's regret

In this Subsection, we investigate the empirical relevance of the hypothesis of risk aversion, joy of winning and loser's regret by combining subjects' bids with the information from the post-experiment questionnaire.[5]

In particular, we draw from two questions: the first question asked subjects to self-report their risk attitude; the second question concerned the importance of winning to them and thus was meant to capture both joy of winning and loser's regret.[6]

Table II presents the proportions of subjects either reporting to be risk averse or giving importance to winning, for different levels of overbidding in the different treatments. In particular, for each subject, we first run a regression with the (sum of the) bid(s) as the dependent variable and the (sum of the) private value(s) and the constant term as controls. Given the coefficient of *Value*, each subject is classified according to the extent of overbidding. Then, for each of the four categories, *Table II* shows the proportions of subjects who reported to be risk averse or gave importance to winning.

[5]Measures of risk aversion based on incentivized elicitation mechanisms can be considered more robust than those inferred from post-experimental survey. However, Dohmen et al. (2011) provide robust evidence supporting a strong correlation between self-reported information on risk attitude and measures from standard incentivized experimental designs. The authors also show that the question about risk taking generates the best all-around predictor of risky behavior.

[6]The first question focuses on risk aversion. Subjects were asked to report on a 7-point scale (with 1 indicating risk aversion and 7 risk seeking) whether, in general terms, they are willing to take or to avoid risks. The second question refers to joy of winning/loser's regret. Subjects were asked whether they agree (on the basis of a 7-point scale, with 1 indicating strong disagreement and 7 strong agreement) with the statement that in a generic period of the experiment, winning (at least one of) the unit(s) was very important to them, regardless of the corresponding monetary payoff.

Table II. Risk aversion and joy of winning/loser's regret in explaining overbidding

	1A1U			*1A2U*			*2A1U*			*Pooled*		
	Obs.	*RA*	*JoW*	*Obs.*	*RA*	*JoW*	*Obs.*	*RA*	*JoW*	*Obs.*	*RA*	*JoW*
$bid/value \leq 0.5$	0	0	0	11	0.182	0.364	7	0.286	0	18	0.222	0.222
$0.5 < bid/value \leq 0.625$	16	0.375	0.313	13	0.385	0.154	13	0.385	0.077	42	0.381	0.191
$0.625 < bid/value \leq 0.75$	29	0.379	0.276	18	0.444	0.278	19	0.526	0.211	66	0.439	0.258
$0.75 < bid/value$	9	0.777	0.222	12	0.5	0.333	15	0.333	0.267	36	0.5	0.278

Notes. This table shows the proportions of subjects that either reported to be risk averse or assigned much importance to winning, for different levels of overbidding. *RA* is a dummy that takes value 1 if the subject reports to be risk averse (1, 2, 3 in the first question). *JoW* takes value 1 if the subject agrees with the joy of winning/loser's regret statement (5, 6, 7 in the second question).

In *Table II*, when we look at the pooled data, the proportions of subjects that either reported to be risk averse or gave importance to winning increase with overbidding. Next, in *Table III*, we re-run the panel regressions in *Table 2* of the paper, by adding the two dummies from the questionnaire, *RA* (for risk aversion) and *JoW* (for joy of winning/loser's regret).

By looking at the regressions with pooled data (columns (4) and (5)), *RA* and *JoW* are both significant and have the expected sign. Robust evidence (although with different levels of significance) is found when we analyze each treatment separately (columns (1), (2) and (3)). Hence, subjects who either reported to be risk averse or gave importance to winning tend to place, on average, higher bids. Interestingly, relative to the results in *Table 2* in the paper, adding *RA* and *JoW* weakens the effect of allotment, as indicated by the lower significance of *1A2U&2A1U*.

Table III. Bids, risk aversion and joy of winning/loser's regret

	1A1U	*1A2U*	*2A1U*	*Pooled*	
	(1)	(2)	(3)	(4)	(5)
Value	0.681***	0.633***	0.660***	0.658***	0.658***
	(0.009)	(0.010)	(0.009)	(0.005)	(0.005)
RA	4.835**	6.217*	1.055	4.695**	4.664**
	(2.353)	(3.758)	(3.264)	(1.840)	(1.841)
JoW	2.467	4.530	10.847**	4.825**	5.015**
	(2.589)	(4.090)	(4.184)	(2.115)	(2.126)
Period	−0.493***	−0.640***	−0.800***	−0.649***	−0.649***
	(0.117)	(0.135)	(0.111)	(0.070)	(0.070)
1A2U&2A1U				−3.883*	
				(2.050)	
1A2U					−4.870**
					(2.382)
2A1U					−2.878
					(2.394)
Constant	4.870*	4.857	6.214**	7.747***	7.703***
	(2.503)	(2.975)	(2.677)	(2.987)	(2.109)
lrl	−3355.838	−3482.530	−3325.905	−10199.444	−10197.308
$Wald-\chi^2$	6020.51	3788.57	5442.95	14739.69	14739.89
$p > \chi^2$	0.000	0.000	0.000	0.000	0.000
Obs.	810	810	810	2430	2430

Notes. This table reports coefficient estimates (standard errors in parentheses) from two-way linear random effects models over all periods accounting for both potential individual dependency over repetitions and dependency within rematching group.

C Experimental instructions

In this Section, we present the instructions given to participants in the three treatments. Instructions were originally written in German. We first present the part of *Instructions* that is common to all treatments (*[All treatments]*) and, then, the part of *Detailed Instructions* which is specific of each to the three treatments (*[1A1U]*, *[1A2U]*, *[2A1U]*).

Instructions - ***[All treatments]***

Welcome. Thanks for participating in this experiment. If you follow the instructions carefully you can earn an amount of money that will be paid in cash at the end of the experiment. During the experiment you are not allowed to talk or communicate in any way with the other participants. If you have questions raise your hand and one of the assistants will come and answer it. The rules that you are reading are the same for all participants.

General rules. For showing up on time, you will receive 4 euro. During the experiment you will receive points. At the end of the experiment, the total number of points you have accumulated will be converted into Euro at the rate of 100 points = 3 euro. Your final payment will be composed of the show-up fee of 4 euro plus the amounts of money that you will earn during the experiment. Your final payment will be paid to you in cash immediately at the end of the experiment.

Detailed Instructions - ***[1A1U]***

In this experiment you will participate in one auction that involves one unit of a hypothetical good. If you acquire the unit, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn.

In total, two persons will participate in the auction. Thus, in addition to you, there is another participant who will also want to acquire the unit involved in the auction. Exactly like you, the other participant will be given a resale value before the beginning of the auction. This value will be, again, randomly drawn from an interval between 0 and 200 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore drawn independently of each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale value of the other participant, nor will the other participant be informed of your resale value.

Rules of the auction. The unit of the good involved in the auction will be auctioned off according to the following rules. You, as well as the other participant, will place one bid for the unit. The bid will be equivalent to the number of points that you are willing to pay to acquire the unit. Given the choices of the two participants, the highest bid wins the unit involved in the auction. This means that you will acquire the unit if you place the highest bid in the auction. In the case of identical bids, the winning bid will be randomly determined. If you acquire the unit, your earnings in points will be given by the difference between the resale value and your winning bid. If you do not acquire the unit, you will earn nothing. Note that you may also generate losses if you acquire the unit by bidding more than the resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following table reports the two hypothetical bids placed by the two participants in the auction. The two bids are ranked in order from the highest to the lowest. The participant placing the highest bid wins the unit and pays 87 points to acquire that unit.

Auction	
Rank	*Bid*
1	87
2	53

Repetitions of the experimental task. The experiment consists of 15 periods. In each period you will participate in an auction involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bid, the winning bid and how many points you have obtained in that auction.

Detailed Instructions - ***[1A2U]***

In this experiment you will participate in one auction that involves two units of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the auction have the same resale value.

In total, two persons will participate in the auction. Thus, in addition to you, there is another additional participant who will also want to acquire the two units involved in the auction. Exactly like you, the other participant will be given a resale value for each of the two units before the beginning of the auction. Also for the other participant, the two units are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the auction are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the auction, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

Rules of the auction. The two units of the good involved in the auction will be auctioned off according to the following rules. You and the other participant will each place two bids, one for each unit. Each bid will be equivalent to the number of points that you are willing to pay to acquire the corresponding unit. Given the choices of the two participants, the two highest bids win the two units involved in the auction. This means that you will acquire one unit if you place one of the two highest bids in the auction. Similarly, you will acquire both units if you place the two highest bids in the auction. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following table reports the four hypothetical bids placed by the two participants in the auction. The four bids are ranked in order from highest to lowest. The participant placing the highest bid wins the first unit and pays 87 points to acquire that unit.

The participant placing the second highest bid wins the second unit and pays 77 points to acquire that unit.

Auction	
Rank	*Bid*
1	87
2	77
3	66
4	53

Repetitions of the experimental task. The experiment consists of 15 periods. In each period, you will participate in an auction involving two units. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period, you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the two winning bids in the auction and how many points you have obtained in the auction.

Detailed Instructions - ***[2A1U]*** In this experiment you will participate in two simultaneous auctions, each involving one unit of a hypothetical good. For each unit you acquire, the experimenter will purchase that unit at its resale value. Your resale value will be communicated to you before the beginning of the auction. This value will be randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. Finally, the two units in the two auctions have the same resale value.

In total, two persons will participate in the two auctions. Thus, in addition to you, there is another participant who will also want to acquire the two units involved in the two auctions. Exactly like you, the other participant will be given the resale value of each of the two units before the beginning of the two auctions. Also for the other participant, the units in the two auctions are assigned the same resale value. The resale value will be, again, randomly drawn from an interval between 0 and 100 points, with every number in this interval having the same probability of being drawn. The resale values of the two participants in the two auctions are therefore independently drawn from each other. Thus, it is likely that you and the other participant will be given different resale values. During the two auctions, you will not be informed about the resale values of the other participant, nor will the other participant be informed of your resale values.

Rules of the auctions. The two units of the good involved in the two auctions will be auctioned off according to the following rules. You and the other participant will each place two bids, one in each of the two auctions. Each bid will be equivalent to the number of points that you are willing to pay to acquire the unit in the corresponding auction. Given the choices of the two participants, in each auction, the highest bid wins the corresponding unit. This means that you will acquire one unit if you place the highest bid in one of the two auctions. Similarly, you will acquire two units if you place the highest bids in both auctions. In case of identical bids, the winning bids will be randomly determined. For each unit you acquire, your earnings in points will be given by the difference between its resale value and your winning bid. If you do not acquire any unit, you will earn nothing. Note that you may also generate losses if you acquire a unit by bidding more than its resale value. Eventual losses will be subtracted from your total earnings in points.

Example. The following two tables report the bids placed by the two participants in the two auctions. In each auction, the two bids are ranked in order from highest to lowest. The participant placing the highest bid in the first auction wins the first unit and pays 87 points to acquire that unit. The participant placing the highest bid in the second auction wins the second unit and pays 77 points to acquire that unit.

Auction A

Rank	*Bid*
1	87
2	53

Auction B

Rank	*Bid*
1	77
2	66

Repetitions of the experimental task. The experiment consists of 15 periods. In each period you will participate in two simultaneous auctions, each involving one unit. In each period, you and the other participant will be given new resale values drawn according to the previous rules. In each period you will be randomly re-matched with another participant in such a way that you will never interact with the same opponent in two consecutive periods. The experiment is anonymous, meaning that you will not be told who the other group member is. At the end of each period, the computer will show your bids, the winning bids in the two auctions and how many points you have obtained in each of the two auctions.

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