Supplementary Material for
"Equilibrium Selection in Similar Repeated Games: Experimental Evidence on the Role of Precedents"

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Appendix A

In the paper we assert that for the parameterizations of the prisoner's dilemma (PD) stage game that we study in the experiment and given our induced discount factor, it is possible to support play of the efficient XX outcome as a sequential equilibrium of the indefinitely repeated PD. In this appendix we explain how we verified this claim following the logic of the contagious sequential equilibrium construction of Kandori (1992).

Consider the 2×2 game given in Table 1 of the paper, where action X corresponds to cooperation and action Y corresponds to defection. Denote the period discount factor (equivalently the continuation probability) by δ and the population or matching group size by M. In our experiment as in one of the environments studied by Kandori, these M players are randomly and anonymously matched in each period of the infinitely repeated game and can only draw upon their own personal histories in the previous $t = 0,1,2,...$ matches of the game; player i has no knowledge of the identities of any player nor of what transpires in any matches other than his own and no communication with other players is allowed. Following Kandori (1992), assume that there are just two types of players. All players start off as c-types, playing the cooperative strategy. A player remains a c-type if he has never experienced a defection, but permanently becomes a d-type if he initiates or experiences and defection. The sequential equilibrium associated with this "contagious strategy" is characterized by the type-dependent strategy wherein c-types play cooperate (action X) and d-types play defect (action Y). Thus, trust is attached to the society as a whole. A single defection triggers a contagious wave of defection until the economy arrives at the outcome of mutual defection by all M players. The threat of such a contagion may deter defection from occurring and thus enable play of the efficient, cooperative strategy by all M players in every period.

Let D_t denote the total number of d-type players as of period t and let A denote an $M \times M$ transition matrix with elements $a_{ij} = \Pr[D_{t+1} = j | D_t = i]$. Let B denote another

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$M \times M$ matrix with elements $b_{ij} = \Pr[D_{t+1} = j | D_t = i$ and one d-type player deviates to cooperation (plays X) at time t.]. Denote the conditional probability that a d-type player randomly meets a c-type player when there are already i d-type players as the ith element of the column vector

$$\rho = \frac{1}{M-1}[M-1, M-2, ..., 1, 0]'.$$

Finally, let e_i denote a $1 \times M$ row vector with ith element equal to 1 and all other elements equal to 0. Kandori's lemma, that the contagious strategy constitutes a sequential equilibrium, requires that two conditions be satisfied. The first condition, adapted to the game of Table 1 is that

$$\frac{10}{1-\delta} \geq \sum_{t=0}^{\infty} \delta^t e_1 A^t \rho (T-10) = e_1 (I - \delta A)^{-1} (T-10).$$

Here, all payoffs have been normalized relative to the mutual defection payoff of 10. This condition states that the present value of the normalized payoff from playing the cooperative strategy, (left hand side), equals or exceeds the present value of the normalized payoff from defecting in every period (right hand side), where $e_1 A^t \rho$ denotes the probability of meeting a c-type player given that the player initiated a contagious wave of defection at time 0. This condition can be rewritten as:

$$\frac{10}{T-10} \geq (1-\delta) e_1 (I - \delta A)^{-1}. \tag{1}$$

A second condition is that once a contagious wave of defection has been initiated, no d-type player has an incentive to revert to play of the cooperative strategy in an effort to slow down the contagious process. This second condition, adapted to the game of Table 1, is that a d-type player finds a one-shot deviation to cooperation unprofitable given that $D_t = k$, for all $k = 2, 3, ...M$:

$$\sum_{t=0}^{\infty} \delta^t \left[e_k A^t \rho (T-10) \right] \geq \left(\frac{M-k}{M-1} - \frac{k-1}{M-1} \right) (10) + \delta \sum_{t=0}^{\infty} \delta^t \left[e_k B A^t \rho (T-10) \right]. \tag{2}$$

This condition states that the present normalized value from defecting forever when there are k d-type players (left hand side) is greater than the present normalized payoff from a one time deviation to the cooperative strategy by the d-type player who then reverts to playing the defection strategy forever afterward. Note that $\frac{M-k}{M-1}$ is the probability the deviating d-type player meets a c-type player, $\frac{k-1}{M-1}$ is the probability the deviating d-type player meets a d-type player and $e_k B$ represents the distribution of d-type players in the next period given that in the current period there are k d-type players and the one d-type, the candidate player, deviates to the cooperative strategy in the current period.

Using our parameterization of the indefinitely repeated PD, where $T \in \{25, 30\}$, $M = 10$ and $\delta = 5/6$, we have verified numerically that conditions (1) and (2) are always satisfied.[1]

[1] The Mathematica program we used to verify these conditions is available upon request.

It follows that a social norm of mutual cooperation in all periods comprises a sequential equilibrium of both versions of the infinitely repeated PD that we study in our experiment. Of course, many other equilibria are also possible, including mutual defection in all periods.

Appendix B

Procedures

The subjects were recruited from the undergraduate population at the University of Pittsburgh for three-hour sessions.[2] Subjects had no prior experience participating in any treatment of our experiment. Upon arrival in the laboratory they were randomly assigned to separate computer terminals and received written instructions which were also read aloud in an effort to make the instructions public knowledge.[3] After the instructions were read, subjects were required to complete a quiz that checked their comprehension of the instructions. A sample copy of the instructions and the quiz is given in Appendix B. After review of the quiz questions and correction of any incorrect quiz answers, subjects began playing an indefinitely repeated version of the stage game shown in Table 1 of the paper with a known, treatment specific value of T, entering their choices, X or Y, on a computer screen when prompted.

Prior to the first round of each new supergame we elicited subjects' beliefs as to the number of the other 9 players in their matching group of size 10 who would choose X in the first round of that supergame. Subjects were aware of the payoff matrix that would be in effect, i.e., the value of T, each time that we elicited these beliefs. We only elicited beliefs prior to the first round of each new supergame; beyond the first round we did *not* elicit beliefs so as to avoid being too intrusive regarding subjects' decision processes and delaying completion of the experiment in a timely manner. Moreover, we did not incentivize this belief elicitation to avoid further complicating the game; nevertheless, as we shall discuss below, the beliefs we elicited are very informative about subsequent play. Beliefs were elicited in all sessions except the first two sessions we ran (sessions 1 and 2 in Table 2 of the paper).

Following the input of their belief in the first round of each new supergame, subjects played the round by choosing X or Y. At the end of the first round and each subsequent round of the supergame, subjects were reminded of the game matrix and of their own choice for the round and were also informed about their matched player's choice, and their own and their matched player's payoffs for the round. Then a six-sided die was rolled to determine whether the supergame would continue with another round. Subjects took turns rolling the die. Subjects were provided with a complete history of their own past play from all rounds of all prior supergames. The game payoffs in Table 1 of the paper represent the monetary payoffs in terms of cents (US$). Subjects were paid their payoffs from all rounds of all supergames played and in addition they were given a show-up payment of $5. Total earnings for subjects averaged about $17 (including the $5 show-up fee), and sessions typically lasted about 90 minutes.

[2]We typically finished a session well before this time limit, in about 1.5 hours or less, so as to avoid possible end game effects associated with the finite 3-hour time horizon of a session.

[3]The experiment was conducted using z-Tree (Fischbacher, 2007).

Appendix C

Instructions used in *PD30-SH10* and *SH10-PD30*. (Instructions for *PD25-SH15* and *SH15-PD25* are identical except for different values for T).

Instructions

You are about to participate in an economic decision-making experiment. Please read these instructions carefully, as the amount of money you earn may depend on how well you understand these instructions. If you have a question at any time, please feel free to ask the experimenter. We ask that you not talk with the other participants once the experiment has begun.

Overview

There are 20 participants in today's session. The computer program will begin by randomly dividing you up into two groups of size 10. Your assignment to a group of size 10 will not change for the duration of the experiment. You will only interact with members of your own group of size 10 for the entire experiment.

This experimental session is made up of a number of "sequences." Each sequence consists of an indefinite number of "rounds". In each round you will be randomly and anonymously matched with one of the other 9 participants in you group of size 10 and asked to make a decision. All matchings with the other 9 participants are equally likely, that is, in each round you have a 1/9 chance of being matched with any single participant in your group of size 10.

At the beginning of the first sequence, you will be assigned an ID number which is used only for record identification purposes. You will never be told the ID number or the identity of the person you are matched with in any round, nor will he/she ever be told your identity—even after the end of the session.

In each round, both you and the other participant matched to you must choose between two possible decisions labeled "X" and "Y". You make your decision without knowing the decision of the other participant. You simply click on the radio button next to your decision and then click the OK button. You can change your mind anytime prior to clicking the OK button. After all participants have made their decisions, you will be informed of the decision made by the participant matched to you, and likewise, that other participant will learn of your decision.

Your decision, together with the decision of the participant matched to you results in one of four possible outcomes: X,X, X,Y, Y,X and Y,Y, where the first letter refers to your decision and the second letter refers to the decision of the participant matched to you. The outcome determines your payoff for the round as well as the payoff of the participant matched to you for that round. There are two possible ways in which the four outcomes translate into payoffs, which we refer to as payoff table 1 and payoff table 2. These are shown below.

Table 1	Other's decision: X	Other's decision: Y
Your decision X	Your payoff: 20 Other's payoff: 20	Your payoff:0 Other's payoff: 30
Your decision Y	Your payoff: 30 Other's payoff: 0	Your payoff:10 Other's payoff: 10

Table 2	Other's decision: X	Other's decision: Y
Your decision X	Your payoff: 20 Other's payoff: 20	Your payoff:0 Other's payoff: 10
Your decision Y	Your payoff: 10 Other's payoff: 0	Your payoff:10 Other's payoff: 10

Notice that the only difference between payoff table 1 and payoff table 2 is the payoff that you receive in the outcome where you choose Y and the other participant chooses X, or symmetrically, the payoff the other participant receives when you choose X and the other participant chooses Y. In all other cases the payoffs are unchanged across the two payoff tables.

Which table?

Only one payoff table, either Table 1 or Table 2 will be in effect for all members of your group of size 10 and for all rounds of a given sequence. This table will be shown on every participant's computer screen prior to their making a decision. Specifically, you and the other 9 participants with whom you can be randomly matched in each round will see the exact same table, which will be clearly labeled as either Table 1 or Table 2 in the upper left hand corner. For further ease of reference, Table 1 will be displayed to all participants in a black color while Table 2 will be displayed to all participants in a red color. The table shown in each round will be the one used to determine your payoffs for that round.

At the start of each new sequence, the payoff table for all 10 members of your group may or may not change from the one used in the previous sequence. Therefore, at the start of each new sequence you will want to carefully examine the payoff table that is being used to determine payoffs in each round of the new sequence. If a change in payoff tables does occur, the change will only occur at the start of a new sequence of rounds; the payoff table shown in the first round of each new sequence will remain the same one in effect for all participants for all subsequent rounds of that sequence. The payoff table may or may not change again at the start of a new sequence.

When does a sequence of rounds end?

Each sequence consists of at least one round. Following the first round, and every subsequent round, a six-sided die will be rolled. (Participants will take turns rolling the die). If the die roll results in a 1, 2, 3, 4, or 5 being rolled, then the current sequence continues on with another round. If the die roll results in a 6 being rolled, then the current sequence is over. Depending on the amount of time remaining in the session, a new sequence of rounds may then begin. For the new sequence, the table used to determine payoffs may or may not change from the table used in the prior sequence of rounds. Thus, there is a 5/6 chance that a given sequence will continue from one round to the next and a 1/6 chance that the sequence will not continue beyond the current round. Each new sequence will consist of an indefinite number of rounds. In each round of the new sequence you will be randomly matched with one of the other 9 participants as before.

Expectations at the start of each new sequence

In the first round of each new sequence the computer program will ask you "How many of the other 9 participants (excluding you) in your group will choose X in this round?" Please enter your expectation, a number between 0 and 9 inclusive, in the blue input box on your screen. When you have entered this number and made an action choice of X or Y for the first round, click the OK button. You will only be asked for this expectation of the number of others choosing X in the first round of each new sequence.

Results, Record Sheets and your History of Play

At the end of each round you see a results screen reporting on your decision, the decision of the other participant matched to you, your payoff and the other participant's payoff. Please record this information on a new row of your record sheet under the appropriate headings. Then press the OK button to continue. For your convenience, a history of your past results will appear at the bottom of the results screen for ready reference.

Payments:

Your payoff for each round is a number, either 0, 10, 20 or 30. This number represents your payoff for the round in cents, that is, 1 point=1 cent, so your possible earnings each round are $0.00, $0.10, $0.20 or $0.30. At the end of the session you will paid the sum total of your payoff earnings from all rounds of all sequences, plus a payment of $5 for completing the session. Payments are made privately and in cash.

Questions?

Now is the time for questions. If you have a question, please raise your hand and the experimenter will answer your question in private.

Quiz

Before we begin, we ask you to complete the following quiz. The purpose of the quiz is to test your understanding of the rules of the experiment. After all participants have completed the quiz, we will analyze the quiz questions. If there are any mistakes, we will go over the relevant part of the instructions again.

Instructions: Circle or write in the blank space the correct answer to questions 1-11 below:

1. A change in the payoff table, if it occurs, will only occur at the start of a new sequence.
   Circle one: True False

2. I will be matched with the same other participant in all rounds of a sequence.
   Circle one: True False

3. Suppose it is round 3. The chance that the sequence continues on to round 4 is:
   Would your answer change if we replaced round 3 with round 13 and round 4 with round 14?
   Circle one: Yes No

   For the next four questions, assume payoff Table 1 is in effect.

4. If you choose X and the other participant matched with you chooses Y, then your payoff is and the other participant's payoff is .

5. If you choose X and the other participant matched with you chooses X, then your payoff is and the other participant's payoff is .

6. If you choose Y and the other participant matched with you chooses Y, then your payoff is and the other participant's payoff is .

7. If you choose Y and the other participant matched with you chooses X, then your payoff is and the other participant's payoff is .

   For the next four questions, assume payoff Table 2 is in effect.

8. If you choose X and the other participant matched with you chooses Y, then your payoff is and the other participant's payoff is .

9. If you choose X and the other participant matched with you chooses X, then your payoff is and the other participant's payoff is .

10. If you choose Y and the other participant matched with you chooses Y, then your payoff is and the other participant's payoff is .

11. If you choose Y and the other participant matched with you chooses X, then your payoff is and the other participant's payoff is .