Online Supplement

Auctions with endogenous participation and an uncertain number of bidders: Experimental evidence

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1 Additional tables and figures

Table 1: Frequency and number of observations of each realization of c within each session

Possible realizations of c	Frequency	Number of observations
1	0.021	1
2	0.021	1
3	0.021	1
4	0.083	4
5	0.083	4
6	0.021	1
7	0.104	5
8	0.021	1
9	0.063	3
10	0.042	2
11	0.063	3
12	0.021	1
13	0.021	1
14	0.083	4
15	0.063	3
16	0.042	2
17	0.021	1
18	0.083	4
19	0.063	3
20	0.063	3

Table 2: Probit estimates of determinants of entry without lagged number of bidders (reporting marginal effects)

	All 48 periods		Last 24 periods
	(1)	(2)	(3)
FP_{it}	0.009	0.018	0.009
	(0.012)	(0.029)	(0.019)
$Informed_i \cdot FP_{it}$	0.009	0.009	0.013
	(0.007)	(0.007)	(0.012)
$Informed_i \cdot EC_{it}$	0.007	0.007	0.016
	(0.006)	(0.006)	(0.010)
v_{it}	0.010***		0.012***
	(0.000)		(0.000)
$v_{it} \cdot FP_{it}$		0.010***	
		(0.000)	
$v_{it} \cdot EC_{it}$		0.010***	
		(0.000)	
c_{it}	-0.034***	-0.034***	-0.036***
	(0.001)	(0.001)	(0.002)
$ln(t+1)$	-0.024**	-0.024**	-0.067*
	(0.008)	(0.008)	(0.032)
$Male_i$	0.025	0.025	0.028
	(0.028)	(0.028)	(0.034)
$SafeChoices_i$	-0.021***	-0.021***	-0.025**
	(0.006)	(0.006)	(0.008)
$FirstFormat_i$	-0.022	-0.022	-0.026
	(0.011)	(0.011)	(0.018)
$RiskOrder_i$	-0.022	-0.022	-0.040*
	(0.012)	(0.012)	(0.019)
Observations	10944	10944	5472
Clusters	19	19	19
Log Likelihood	-5103.052	-5102.938	-2310.619
Pseudo R^2	0.299	0.299	0.372

Notes: Standard errors (in parentheses) clustered at the session level.
$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Table 3: Summary statistics for bidding conditional on observed entry behavior by number of bidders

Treatment	Observed bids of auction winner	Predicted winning bids	Observed bids of auction losers	Predicted losing bids
One bidder ($m = 1$)				
FPI	5.459	0.000	-	-
	(17.802)	(0.000)		
FPU	38.857	22.948	-	-
	(25.240)	(20.916)		
Two bidders ($m = 2$)				
FPI	57.435	53.165	31.305	28.355
	(19.706)	(22.922)	(18.494)	(28.272)
FPU	53.963	34.305	28.173	17.744
	(19.947)	(19.323)	(19.402)	(19.254)
ECI	-	74.260	45.963	50.805
		(19.622)	(22.274)	(24.075)
ECU	-	73.521	42.317	50.306
		(20.059)	(22.815)	(23.559)
Three bidders ($m = 3$)				
FPI	65.372	63.735	35.573	35.627
	(17.729)	(16.492)	(19.042)	(28.632)
FPU	59.083	41.852	30.384	19.928
	(16.890)	(16.438)	(18.821)	(19.147)
ECI	-	75.533	47.911	50.917
		(19.178)	(22.722)	(22.653)
ECU	-	76.838	44.183	51.411
		(18.320)	(22.960)	(23.112)

Notes: Table contains means with standard deviations in parentheses.
Since an English clock auction ends automatically when there is only one bidder, we exclude this case.
Predicted bids are calculated based on the realized c and v_i of the bidder.

Table 4: Random effects estimates of the responsiveness to bids English clock auctions to theoretical predictions conditional on observed entry

	ECI		ECU	
	All 48 periods	Last 24 periods	All 48 periods	Last 24 periods
	(1)	(2)	(3)	(4)
Equilibrium bid	0.785***	0.611***	0.894***	0.796***
	(0.044)	(0.040)	(0.037)	(0.081)
m_{it}	2.172*	0.461	1.758**	0.519
	(1.000)	(1.093)	(0.668)	(1.616)
$ln(t+1)$	3.686*	7.408***	17.402**	10.411
	(1.495)	(1.553)	(5.342)	(5.972)
$Male_i$	1.202	0.847	1.455	0.611
	(1.852)	(2.224)	(1.651)	(1.943)
$SafeChoices_i$	-0.499	1.243*	0.686	1.821**
	(0.654)	(0.542)	(0.538)	(0.682)
$FirstFormat_i$	3.429*	4.392*	4.069*	0.934
	(1.349)	(1.898)	(1.812)	(2.153)
$RiskOrder_i$	0.316	-0.958	-0.278	-2.193
	(0.845)	(1.843)	(0.723)	(2.584)
$Constant$	-12.869*	-20.348*	-73.667**	-39.513
	(6.124)	(8.110)	(23.890)	(22.363)
Observations	509	559	264	283
Clusters	9	10	9	10
R^2 of overall model	0.453	0.320	0.713	0.487

Notes: Standard errors (in parentheses) clustered at the session level.
$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Table 5: Random effects estimates of the responsiveness to bids in first-price auctions to theoretical predictions conditional on observed entry

	FPI		FPU	
	All 48 periods	Last 24 periods	All 48 periods	Last 24 periods
	(1)	(2)	(3)	(4)
Equilibrium bid	0.928***	0.917***	0.667***	0.584***
	(0.021)	(0.019)	(0.014)	(0.040)
m_{it}	-3.231***	-0.316	-1.293*	-0.588
	(0.787)	(0.644)	(0.548)	(0.934)
$ln(t+1)$	-1.110	-6.069	-4.004***	-19.947***
	(0.711)	(5.110)	(0.851)	(5.767)
$Male_i$	-2.902	-2.122	-3.747	-4.852*
	(1.556)	(1.871)	(2.227)	(1.893)
$SafeChoices_i$	0.586	0.604	2.239***	2.094**
	(0.738)	(0.852)	(0.461)	(0.765)
$FirstFormat_i$	0.151	1.856	-0.271	3.779
	(0.789)	(1.829)	(2.191)	(3.111)
$RiskOrder_i$	0.673	-0.738	-1.520	-1.516
	(1.330)	(1.171)	(2.093)	(2.373)
$Constant$	9.828*	18.425	33.915***	87.854***
	(4.722)	(14.136)	(5.614)	(20.251)
Observations	1161	586	1296	645
Clusters	9	9	10	10
R^2 of overall model	0.622	0.747	0.350	0.280

Notes: Standard errors (in parentheses) clustered at the session level.
$^{*}p < 0.05, ^{**} p < 0.01, ^{***} p < 0.001$

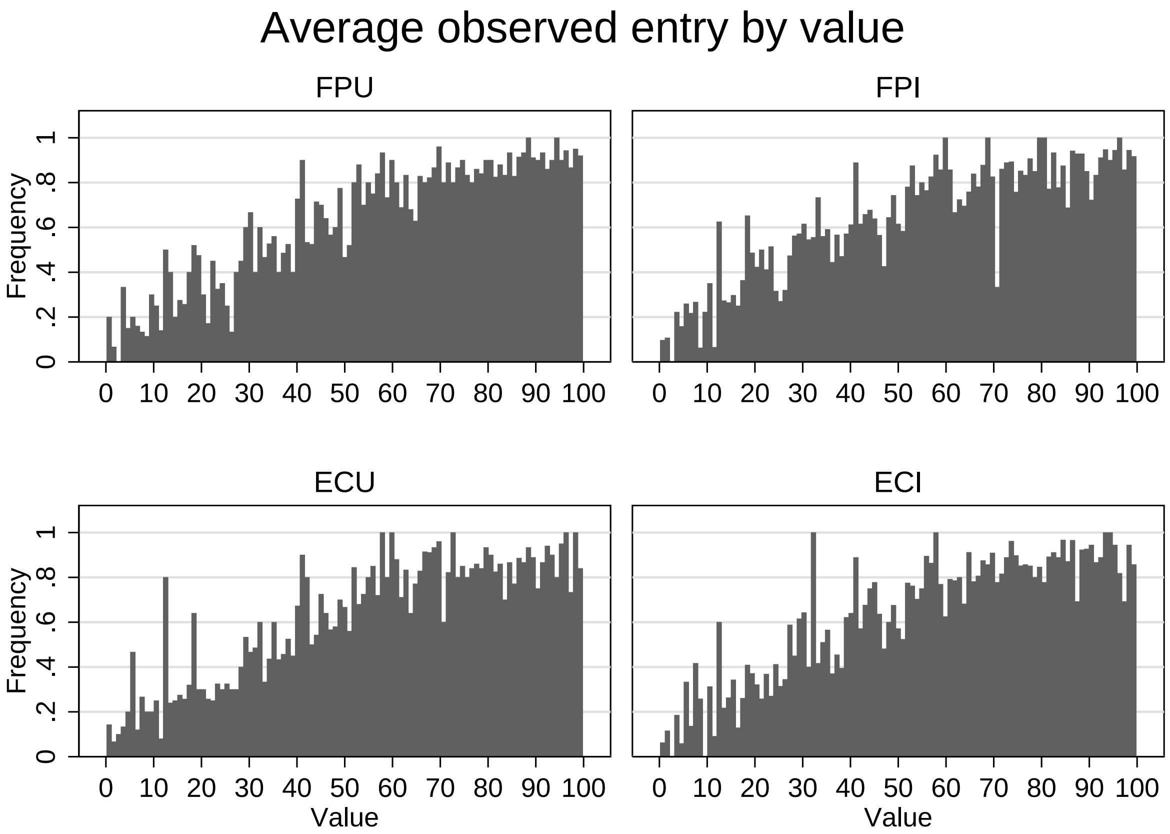

Figure 1: Observed entry by value

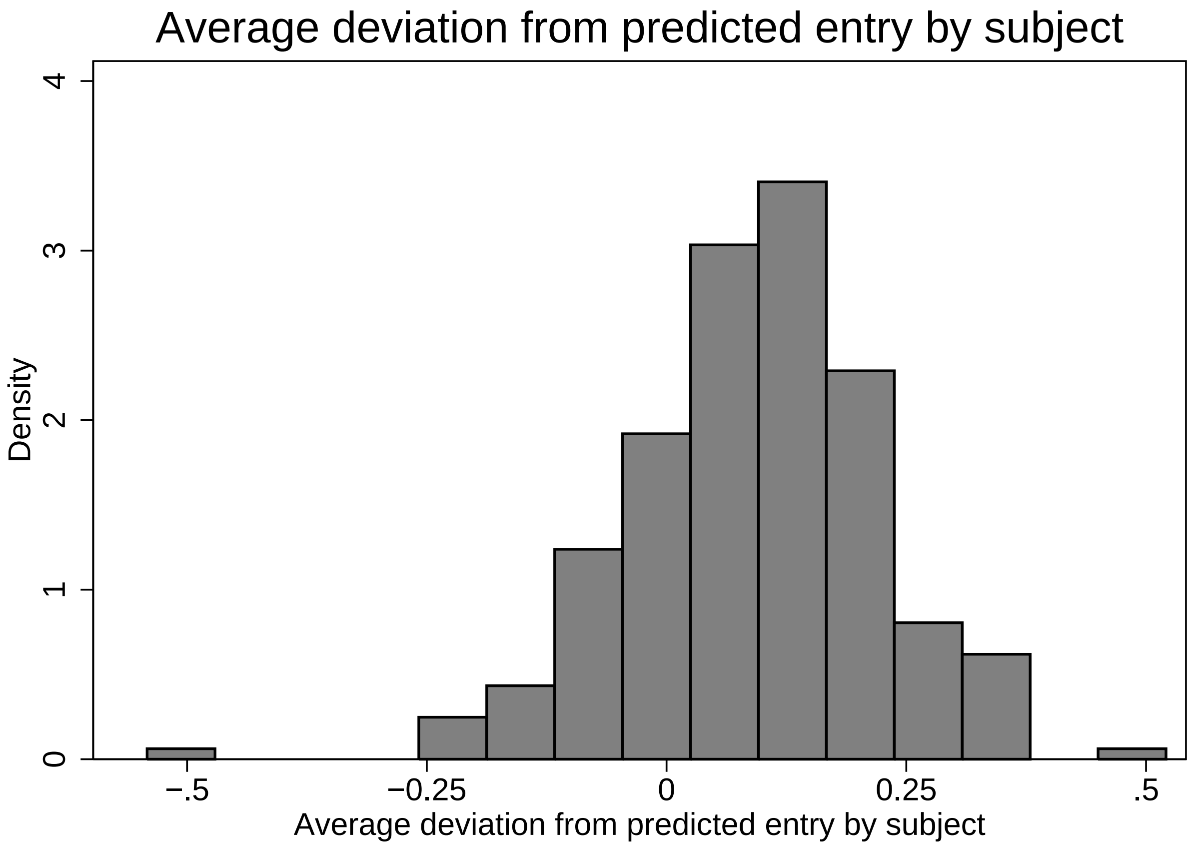

Figure 2: Average deviation from predicted entry by subject. Over-entry is denoted as one, predicted entry as zero, and under-entry as negative one.

2 Results for payoffs

As mentioned in the results section, predictions and results for payoffs closely mirror those of revenue. We first consider payoffs for the entire game (as opposed to only those of bidders). When the auction format is English clock, payoffs are higher. This is true both when bidders are informed (sign test, $w = 8, p = 0.0195$) and when bidders are uninformed (sign test, $w = 10$, $p = 0.001$). As with revenue, the effect of information structure on payoffs differs by auction format. When bidders are informed, payoffs are higher in first-price auctions (robust rank order test, $\acute{U} = 5.367$, $p < 0.001$) and lower in English clock auctions (robust rank order test, $\acute{U} = 1.474$, $p < 0.10$).

Relative to theory, payoffs are lower than predicted in first-price auctions, both when bidders are informed (sign test, $w = 9$, $p = 0.002$) and when they are uninformed (sign test, $w = 10$, $p = 0.0010$). However, in English clock auctions we are unable to reject that payoffs are equal to their predictions both when bidders are informed (sign test, $w = 5$, n.s.) and when they are uninformed (sign test, $w = 5$, n.s.). Since we see both overbidding and over-entry in first-price auctions, this is not surprising. In English clock auctions, the reduction in payoffs resulting from over-entry is offset by the slight reduction in bidding. One might expect that higher payoffs in English clock auctions would result in higher entry such that payoffs (and consequently revenue) are equalized between the two formats. However, recall that, even when we restrict attention to the second half of the experiment, there is no difference in entry behavior. This suggests that either potential bidders have a difficult time discerning the differences in expected payoffs between the formats, or that their entry decisions are not solely driven by financial considerations. To analyze this question in more detail, we would need cleaner measurements of willingness to pay for each auction format. This extension is considered in Aycinena et al. (2017).

We now restrict attention to payoffs of bidders. Table 6 contains summary statistics of bidder payoffs and predicted bidder payoffs, both total, and net of c. We find that payoffs are higher in English clock auctions, both when bidders are informed (sign test, $w = 9$, $p = 0.002$) and uninformed (sign test, $w = 10$, $p = 0.001$). In first-price auctions, bidders are better off when they are informed (robust rank order test, $\acute{U} = 6.706$, $p < 0.001$). In English clock auctions, we cannot reject that bidder payoffs are equal (robust rank order test, $\acute{U} = 1.297$, n.s.).

Bidder payoffs are higher than predicted in English clock auctions in both the informed (sign test, $w = 8$, $p = 0.0195$) and uninformed case (sign test, $w = 8$, $p = 0.0547$). This reflects the slight underbidding we observe in English clock auctions.

Not surprisingly, when bidders are uninformed in first-price auctions payoffs are lower than predicted (sign test, $w = 10$, $p = 0.001$). However, we cannot reject that payoffs are equal to predictions in FPI auctions (sign test, $w = 5$, n.s.). This result implies that the reduction in overall payoffs in FPI auctions is due to over-entry, rather than by bidding behavior.

Theory predicts that the expected payoff of a potential bidder with $v_i = v_c$ is c. If $v_i > v_c$, then the expected payoff of entering the auction is greater than c. As such, determining whether bidder payoffs exceed c is of interest. It particular, do bidders, on average, earn payoffs that merit entry? We find that in FPU auctions, payoffs bidders are not statistically different from c (sign test, $w = 5$, n.s.). However, if we restrict attention to the second half of the experiment, payoffs of such bidders are greater than c (sign test,

Table 6: Summary statistics for payoffs

Treatment	Observed payoffs of bidders	Predicted payoffs of bidders	Observed payoffs of bidders less c	Predicted payoffs of bidders less c
FPI	11.913	12.017	2.431	2.535
	(22.943)	(22.406)	(21.896)	(21.343)
FPU	9.574	17.178	-0.075	7.529
	(15.409)	(11.042)	(15.248)	(10.097)
ECI	17.231	13.345	7.493	3.607
	(27.263)	(22.927)	(26.555)	(22.164)
ECU	18.461	17.141	8.776	7.456
	(27.768)	(10.957)	(27.034)	(10.079)

Notes: Table contains means with standard deviations in parentheses of observed and predicted payoffs of participating bidders. Predicted payoffs are payoffs predicted by Nash bidding, conditional on observed value, outside option and entry.

$w = 9$, $p = 0.011$). In the remaining treatments, bidder payoffs are significantly higher than c.[1] Thus, no bidders are worse off for having entered the auction, on average.

3 Results for efficiency

In most of the literature on single unit auctions with independent private values, allocative efficiency is predicted to be perfect, since, in equilibrium, the bidder with the highest value will always obtain the good, and there is no opportunity cost of bidding in the auction. However, efficiency considerations are more complex when entry is endogenous with opportunity costs of participation. In particular, if no potential bidders enter the auction, then the good remains with the auctioneer (who is assumed to have a value of zero). In equilibrium this occurs if all potential bidders have values less than v_c. As such, efficiency is not always predicted to be perfect.

We consider three notions of efficiency. The first, which we call selection efficiency, is equal to one if the potential bidder with the highest value enters the auction, and is otherwise equal to zero. This is of practical concern in auction design, since whether a particular environment successfully attracts the bidder with the highest value may affect revenue.

We also consider allocative efficiency, which is the percentage of possible surplus actually realized, neglecting efficiency concerns about c. This measure is important, as an auction designer may not be concerned about the efficiency consequences of c, since the cost of entry does not go to the seller.[2] The measure of allocative efficiency that we use is given by

$$\frac{v_{winner}}{v_{max}} \tag{1}$$

where v_{winner} is the value of the person who obtained the good (possibly the auctioneer), and v_{max} is the

[1] The corresponding test statistics are FCI: sign test, $w = 9$, $p = 0.002$; ECU: sign test, $w = 10$, $p = 0.001$; ECI: sign test, $w = 9$, $p = 0.002$.

[2] This would not be the case if c represented an entry fee for the auction.

highest value from among the potential bidders.

Lastly, we call the normalized efficiency measure which accounts for efficiency losses due to potential bidders forgoing c and entering the auction total efficiency. It is measured by

$$\frac{(v_{winner} - mc) - min(v_{min} - nc, 0)}{max\,(v_{max} - c, 0) - min(v_{min} - nc, 0)}. \tag{2}$$

Note that if $v_{max} < c$, then the efficient outcome is for no potential bidder to enter. If $v_{max} \geq c$, then the efficient outcome is for only the potential bidder with the highest value to enter, and to obtain the good. Each additional bidder causes an efficiency loss of c, with no gain in allocative efficiency. In equilibrium, of course, any potential bidder with a value weakly above v_c is predicted to enter. As such, predicted total efficiency is likely to be lower than allocative efficiency. Note that, as the number of potential bidders increases, expected total efficiency will decrease, while predicted allocative efficiency will increase. Note further that this efficiency measure is normalized to take into account the observed efficiency relative to the worst possible outcome: the allocation to v_{min} with all potential bidders entering the auction, as long as $v_{max} < nc$ -if $v_{max} > nc$, the worst possible outcome would be that no bidder enters.[3]

Table 7 contains summary statistics regarding selection, allocative and total efficiency. Since we observe over-entry, one might expect selection efficiency to be higher than predicted, since this would reduce the number of cases in which no potential bidder enters the auctions. However, cases in which potential bidders with lower values enter while the potential bidder with the highest value does not are common enough that the reverse is true. This difference is significant in all but FPU auctions.[4] Further, selection efficiency is not affected by information structure in first-price auctions(robust rank order test, $\acute{U} = 0.407$, n.s.) or English clock auctions (robust rank order test, $\acute{U} = -1.020$, n.s.). Likewise, it is not affected by auction format when bidders are informed (sign test, $w = 6$, n.s.) or uninformed (sign test, $w = 5$, n.s.).

Note that predicted allocative efficiency is low relative to the case of exogenous entry. This indicates that there are a non-negligible number of auctions in which no potential bidders are predicted to enter. This is not surprising given that there are three potential bidders for any given auction, and that c can be as high as 20. Also note that predicted total efficiency is not dramatically different from predicted allocative efficiency. This is also a result of the low number of potential bidders. The higher the number of potential bidders, the larger the efficiency losses from entry.

Notice that allocative efficiency is, on average, higher than predicted in all four treatments. However, in all cases, this is not statistically significant.[5] This is driven by the excess entry observed in all treatments; in particular, over-entry reduces the number of cases in which there are no entrants. Total efficiency will account for such efficiency gains from excess entry, while also accounting for the efficiency losses from additional potential bidders forgoing c.

Contrary to theory allocative efficiency is greater in English clock auctions than in first-price auctions

[3]In our design, the number of potential bidders is constant. An interesting question that we leave for future research is the effect on total and allocative efficiency of increasing the number of potential bidders.

[4]The corresponding test statistics are FPI: sign test, $w = 9$, $p = 0.002$; FPU: sign test, $w = 6$, n.s.; ECI: sign test, $w = 7$, $p = 0.035$; ECU: sign test, $w = 9$, $p = 0.011$.

[5]The corresponding test statistics are FPI: sign test, $w = 5$, n.s.; FPU: sign test, $w = 7$, n.s.; ECI: sign test, $w = 8$, n.s.; ECU: sign test, $w = 7$, n.s..

Table 7: Summary statistics for efficiency

Treatment	Observed selection efficiency	Predicted selection efficiency	Observed allocative efficiency	Predicted allocative efficiency	Observed total efficiency	Predicted total efficiency
FPI	0.792	0.855	0.861	0.855	0.830	0.854
	(0.406)	(0.352)	(0.347)	(0.352)	(0.183)	(0.179)
FPU	0.806	0.854	0.879	0.854	0.846	0.852
	(0.395)	(0.353)	(0.327)	(0.353)	(0.170)	(0.182)
ECI	0.814	0.853	0.899	0.853	0.851	0.850
	(0.390)	(0.354)	(0.302)	(0.354)	(0.168)	(0.184)
ECU	0.792	0.854	0.885	0.854	0.850	0.852
	(0.406)	(0.353)	(0.319)	(0.353)	(0.168)	(0.182)

Notes: Table contains means with standard deviations in parentheses.

when bidders are informed, although the result is only marginally significant (sign test, $w = 7$, $p = 0.090$). Since first-price auctions are typically observed to have lower efficiency when entry is exogenous and the number of bidders is known, this is in line with the existing literature. However, when bidders are uninformed we cannot reject that allocative efficiency is equal between first-price and English clock auctions (sign test, $w = 6$, n.s.). Further, we cannot reject that information structure does not affect allocative efficiency for both first-price auctions (robust rank order test, $\acute{U} = 0.707$, n.s.) and English clock auctions (robust rank order test, $\acute{U} = 0.913$, n.s.).

In first-price auctions total efficiency is higher when bidders are uninformed (robust rank order test, $\acute{U} = 2.096$, $p < 0.05$). However, information structure does not affect total efficiency in English clock auctions (robust rank order test, $\acute{U} = 0.388$, n.s.). When bidders are informed total efficiency is greater in English clock auctions (sign test, $w = 8$, p=0.0195). Yet, when they are uninformed we are unable to reject that total efficiency is equal between the two formats (sign test, $w = 5$, n.s.). We are also unable to reject that total efficiency is in line with predictions for all treatments except FPI auctions, where total efficiency is significantly less than predicted.[6]

4 Derivations of Nash Equilibrium

A set of players $N \equiv \{1, ..., n\}$ are potential bidders in an auction for a single unit of an indivisible good. The seller's valuation of the good is 0, and this is common knowledge. Potential bidder i's value of obtaining the good is denoted as v_i, and is an independently drawn realization of the random variable V, with continuous and differentiable distribution F, density f and support on $[0, v_H]$. There is an opportunity cost of entering an auction, $c \in (0, v_H)$. This opportunity cost is common to all potential bidders and is common knowledge. Each potential bidder i must decide, after observing both v_i and c, whether or not to enter the auction. We denote as m the number of potential bidders who forgo c and enter, and refer to them as bidders.

[6]The corresponding test statistics are FPI: sign test, $w = 9$, $p = 0.002$, FPU: sign test, $w = 5$, n.s., ECI: sign test, $w = 5$, n.s., ECU: sign test, $w = 5$, n.s..

We consider two auction formats: first-price auctions and English clock auctions. In a first-price auction, all bidders simultaneously submit bids, the highest of which wins the auction. The price paid is the bid submitted. In an English clock auction the price begins at zero, and continues to increase if excess demand exists. Bidders indicate their bid by abandoning the auction at the corresponding price. When there is only one bidder remaining in the auction it ends, and the remaining bidder wins. The price paid is the price at which the last bidder abandoned the auction. In both auction formats the payoff of all bidders who do not win the auction is zero.

Additionally, we consider environments where m is made common knowledge before bids are placed, and environments where it is not. When m is revealed we say that bidders are informed; when it is not we say that bidders are uninformed.

In what follows we refer to first-price auctions with informed bidders as FPI auctions, and first-price auctions with uniformed bidders as FPU auctions. Analogously, we refer to English clock auctions with informed bidders as ECI auctions, and English clock auctions with uninformed bidders as ECU auctions.

Following Menezes and Monteiro (2000) we consider symmetric equilibria in which risk-neutral potential bidders use a threshold entry rule, and the subsequent equilibrium bidding functions are monotonically increasing and differentiable.[7] In such an equilibrium, potential bidders only enter the auction if their value is (weakly) greater than some threshold. When the opportunity cost of entry is c, we denote the associated entry threshold as v_c. We will show that, in equilibrium, this entry threshold is the same in all the environments we study.

Since, in equilibrium, bid functions are monotonically increasing, the only way a potential bidder with a value of v_c can win the auction with positive probability is to be the sole entrant. This would result in a payoff of v_c since she would obtain the good at a price of zero. The probability of being the only bidder is given by $F(v_c)^{n-1}$. Thus, her expected payoff of entering the auction is $v_c F(v_c)^{n-1}$. Since the entry threshold is the value for which a potential bidder is indifferent between entering the auction or not, v_c must satisfy $c = v_c F(v_c)^{n-1}$. Crucially, notice that this condition is the same for both auction formats and both information structures.

Thus, conditional on having entered the auction, each bidder's value is an independent draw from

$$G_c(v) \equiv F(v \mid v \geq v_c) = \frac{F(v) - F(v_c)}{1 - F(v_c)},$$

with positive density on $[v_c, v_H] \subset [0, v_H]$. We denote the density function associated with $G_c(v)$ as $g_c(v)$.

Note that v_c also allows potential bidders to form beliefs regarding how many others will enter the auction. In particular, the probability that $r \leq n-1$ other potential bidders enter is the same as the probability that r of them have values such that $v_i \geq v_c$, and the remaining $n-r-1$ potential bidders have values such that $v_i < v_c$. There are $\frac{(n-1)!}{(n-r-1)!r!}$ ways in which this could occur. Thus, the corresponding probability is given by $\left(\frac{(n-1)!}{(n-r-1)!r!}\right) F(v_c)^{n-r-1}(1 - F(v_c))^r$.

[7]Equilibrium in a model in which symmetric potential bidders are risk averse would involve more aggressive bidding in first-price auctions, and a higher entry threshold. See Menezes and Monteiro (2000) for proof of this assertion. As will be discussed in the results section, this is not consistent with our data. A model in which potential bidders have heterogeneous risk preferences may be able to explain our data.

4.1 First-price auctions with informed bidders

Consider the case of FPI auctions. Since m is common knowledge and all potential bidders only participate if $v_i \geq v_c$, this auction is a standard independent private values auction with values being drawn from $G_c(v)$.

Since, when $m = 1$ the sole bidder can win with a bid of zero, we need only solve the case where $m \geq 2$. Denote the symmetric equilibrium bidding function as β_m. Assuming that the other $m-1$ bidders bid according to β_m, the expected payoff of bidder i with $v_i \geq v_c$ who bids $b \neq \beta_m(v_i)$, with $b \geq \beta_m(v_c)$, is given by

$$\pi_i^{FPI}(b, v_i|m) = G_c\left(\beta_m^{-1}(b)\right)^{m-1}(v_i - b).$$

To maximize this expected payoff, we take the partial derivative with respect to b, set it equal to zero. Since β_m is an equilibrium bid function, the bid which maximizes bidder i's profit must be $b = \beta_m(v_i)$. Rearranging yields the differential equation

$$\frac{d}{dv_i}\beta_m(v_i)(G_c(v_i))^{m-1} = (m-1)(G_c(v_i))^{m-2}(g_c(v_i))(v_i).$$

Integrating both sides and using the initial condition $\beta_m(v_c) = v_c$ yields

$$\beta_m(v_i) = \frac{1}{G_c(v_i)^{m-1}}\int_{v_c}^{v_i}(m-1)\,G_c(t)^{m-2}g_c(t)t\,dt.$$

Note that this is simply the expected value of highest of the other bidder's values, conditional on bidder i's value being the highest.

4.2 First-price auctions with uninformed bidders

Now consider a FPU auction. In this case, bidders are no longer able to condition their bids on m, and form their beliefs regarding the number of bidders they face based on v_c.

The expected payoff of bidder i with value $v_i \geq v_c$ who bids $b \neq \gamma(v_i)$, such that $0 < b < \gamma(v_c)$ would only obtain the good if there were no other bidders in the auction. In this event, a bid of $b = 0$ would result in a strictly higher expected payoff. The expected payoff of a bidder who bids $b \neq \gamma(v_i)$ such that $b \geq \gamma(v_c)$ is given by

$$\pi_i^{FPU}(b, v_i) = F\left(\gamma^{-1}(b)\right)^{n-1}(v_i - b).$$

Maximizing this expected payoff, we take the partial derivative with respect to b, set it equal to zero and use the fact that, if γ is an equilibrium, then the bid which maximizes bidder i's profit must be $b = \gamma(v_i)$, so that $\gamma^{-1}(b) = v_i$. This leaves us with the differential equation

$$(n-1)\,F(v_i)^{n-2}f(v_i)\frac{1}{\gamma'(v_i)}(v_i - \gamma(v_i)) - F(v_i)^{n-1} = 0.$$

Solving, and using the initial condition $\gamma(v_c) = v_c$, we find that

$$\gamma(v_i) = \frac{1}{F(v_i)^{n-1}} \int_{v_c}^{v_i} (n-1) F(t)^{n-2} f(t)(t)\, dt.$$

This equilibrium function closely resembles that of the analogous first-price auction with exogenous entry in which all n potential bidders bid in the auction without forgoing c. In particular, rather than integrating from 0 to v_i as with the exogenous entry case, the lower limit of integration is v_c. This accounts for the fact that any bidder with a value less than v_c will not enter the auction. Note that this implies that bidders are shading their bids more in the case of exogenous entry.

4.3 English clock auctions with informed bidders

In the English clock auction with informed bidders, bidders abandon the auction once the price reaches their value, as this is the weakly dominant bidding strategy. That is, the symmetric equilibrium bid function is given by $\rho(v_i) = v_i$. Note that this bid function does not depend on m. In the event that $m = 1$ the sole bidder employs the same equilibrium bid function. However, the bidder would obtain the good at a price of zero since the auction would end immediately.

4.4 English clock auctions with uninformed bidders

In the English clock auction with uninformed bidders the symmetric equilibrium bid function is also $\rho(v_i) = v_i$. This is because in English clock auctions, regardless of how many bidders there are in the auction, abandoning the auction at a price equal to your value is weakly dominant. As such, whether or not m is common knowledge is irrelevant to equilibrium bidding behavior.

4.5 Equilibrium payoffs and revenue

We know that threshold entry, in all four environments we study must satisfy $c = v_c F(v_c)^{n-1}$.

4.5.1 FPI auctions

The equilibrium bid function when $m \geq 2$ is given by

$$\beta_m(v_i) = \frac{1}{\left(\frac{F(v_i)-F(v_c)}{1-F(v_c)}\right)^{m-1}} \int_{v_c}^{v_i} (m-1) \left(\frac{F(t)-F(v_c)}{1-F(v_c)}\right)^{m-2} \left(\frac{f(t)}{1-F(v_c)}\right)(t)\, dt$$

for $m \geq 2$. Integrating by parts and simplifying yields

$$\beta_m(v_i) = v_i - \frac{\int_{v_c}^{v_i} (F(t) - F(v_c))^{m-1}\, dt}{(F(v_i) - F(v_c))^{m-1}}.$$

Notice that if $m = 1$, the sole bidder can obtain the good with a bid of zero. Plugging the equilibrium bid function into the payoff function shows that the equilibrium payoff of a bidder with value $v_i \geq v_c$ and $m > 1$ is given by

$$\pi_i^{FPI}\left(\beta_m\left(v_i\right), v_i | m\right) = \int_{v_c}^{v_i} \left(\frac{F\left(t\right) - F\left(v_c\right)}{1 - F\left(v_c\right)}\right)^{m-1} dt.$$

The expected payoff of entering the auction for a potential bidder who observes an opportunity cost of c, and has value $v_i \geq v_c$ is

$$\pi_i^{FPI}\left(\beta_m\left(v_i\right), v_i\right) = v_i F\left(v_c\right)^{n-1} + \sum_{m=2}^{n} \left(\frac{(n-1)!}{(n-m)!(m-1)!}\right)\left(v_i - \beta_m\left(v_i\right)\right)\left(F\left(v_i\right) - F\left(v_c\right)\right)^{m-1} F\left(v_c\right)^{n-m}.$$

Plugging in the equilibrium bid function leaves us with

$$\pi_i^{FPI}\left(\beta_m\left(v_i\right), v_i\right) = v_i F\left(v_c\right)^{n-1} + \sum_{m=2}^{n} \left(\frac{(n-1)!}{(n-m)!(m-1)!}\right)\left(\int_{v_c}^{v_i} \left(F\left(t\right) - F\left(v_c\right)\right)^{m-1} dt\right) F\left(v_c\right)^{n-m}.$$

4.5.2 FPU auctions

The equilibrium bid function is given by

$$\gamma\left(v_i\right) = \frac{1}{F\left(v_i\right)^{n-1}} \int_{v_c}^{v_i} (n-1)\, F\left(t\right)^{n-2} f\left(t\right)\left(t\right) dt.$$

Plugging the equilibrium bid function into the expected payoff function and integrating by parts shows that the equilibrium payoff of a bidder with value $v_i \geq v_c$ is given by

$$\pi_i^{FPU}\left(\gamma\left(v_i\right), v_i\right) = v_c F\left(v_c\right)^{n-1} + \int_{v_c}^{v_i} F\left(t\right)^{n-1} dt.$$

Note that this is also the expected payoff of a potential bidder who enters the auction when she observes an opportunity cost of c, and has value $v_i \geq v_c$, since bidders are uninformed.

4.5.3 ECI and ECU auctions

Whether or not bidders are informed, equilibrium bidding does not change: $\rho\left(v_i\right) = v_i$.

The equilibrium expected payoff of a bidder in an ECI auction with $v_i \geq v_c$ who observes that there are $m > 1$ bidders is given by

$$\pi_i^{ECI}\left(\rho\left(v_i\right), v_i | m\right) = G\left(v_i\right)\left(v_i - \left(\frac{1}{G\left(v_i\right)}\right)\int_{v_c}^{v_i} tg\left(t\right) t\right).$$

Integrating by parts, plugging in G_c, and simplifying yields

$$\pi_i^{ECI}\left(\rho\left(v_i\right), v_i | m\right) = \int_{v_c}^{v_i} \left(\frac{F\left(t\right) - F\left(v_c\right)}{1 - F\left(v_c\right)}\right)^{m-1} dt.$$

Thus, the equilibrium expected payoff of entering an ECI auction for a potential bidder with value $v_i \geq v_c$ is given by

$$\pi_i^{ECI}\left(\rho\left(v_i\right), v_i\right) = v_i F\left(v_\theta\right)^{n-1} + \sum_{m=2}^{n} \left(\frac{(n-1)!}{(n-m)!(m-1)!}\right) \left(\int_{v_c}^{v_i} \left(F\left(t\right) - F\left(v_c\right)\right)^{m-1} dt\right) F\left(v_c\right)^{n-m}.$$

Note that the equilibrium expected payoff of a potential bidder in an ECU auction with value $v_i \geq v_c$ is given by

$$\pi_i^{ECU}\left(\rho\left(v_i\right), v_i\right) = v_i F\left(v_c\right)^{n-1} + \int_{v_c}^{v_i} \left(v_i - t\right)\left(n-1\right) F\left(t\right)^{n-2} f\left(t\right) dt.$$

Integrating by parts and simplifying leaves us with

$$\pi_i^{ECU}\left(\rho\left(v_i\right), v_i\right) = v_c F\left(v_c\right)^{n-1} + \int_{v_c}^{v_i} F\left(t\right)^{n-1} dt.$$

5 Instructions

This appendix contains the instructions, translated from the original Spanish, when the number of bidders who chose to enter the auction in not revealed to bidders before they place their bids.

SLIDE No.1
Introduction

- The following instructions explain to you how you can earn money. The amount of money that each participant earns may vary considerably depending on the decisions made.
- Participants will interact only through computers. If anyone disobeys the rules, we will terminate the experiment and will ask you to leave without any earnings.

SLIDE No.2
Earnings in the experiment

- The amounts in the experiment are denominated in Experimental Pesos (E$).
- Each participant will start the experiment with a balance of E$75. The profits (or losses) are added (or subtracted) to the balance.
- At the end of the experiment, we will convert your accumulated balance to Quetzales (Q1 = E$7.5), and we will pay it in cash.

SLIDE No.3
Overview

- The experiment will have 48 rounds. In each round, you and 2 other participants will be potential buyers in an auction (in some rounds in Auction A and in other rounds in Auction B). Each participant will decide between getting a FIXED AMOUNT or participating in the auction.
- If you participate, you will not receive the FIXED AMOUNT, but you could earn money if you buy the good at a price lower than what it actually is worth.
- If you do not participate in the action, you will receive the FIXED AMOUNT as payment for not participating.

SLIDE No.4
Value

- Each potential buyer will know his value of the good to be auctioned, but the potential buyer will not know how much the good is valued by the other 2 potential buyers.

- The VALUE of the good for each buyer will be between E$0 and E$100, and it will be determined randomly. (All the values between 0 and 100 have the same probability in being designated).
- The VALUE of each buyer shall be independent from the others; the VALUE is not related (and probably will be different) to the VALUE of the others.

SLIDE No.5

- The earnings you can get (if you purchase the good in the auction) depend on its VALUE, and the Price that you pay for the good. If the Price you pay is lower than its VALUE, you will earn the difference.
- VALUE - Price = Earnings
- If the Price you pay is greater, you will lose money. If you do not buy the good, you will not earn or lose any money.

SLIDE No.6
Fixed Amount

- In each round, participants can choose between receiving the FIXED AMOUNT or participating in the auction.
- At the beginning of each round, a FIXED AMOUNT between E$1 and E$20 will be randomly determined. (All amounts between E$1 and E$20 are equally likely to be designated).
- The FIXED AMOUNT will be the same for all participants. That is, in each round all potential buyers will have the same FIXED AMOUNT, but probably a different VALUE.

SLIDE No.7
Participation Decision

- At the beginning of the round, each potential buyer will see how much the good is worth to him (its VALUE) and the FIXED AMOUNT. Then the potential buyer will decide whether to participate or not in the auction.
- If you choose not to participate, you will receive the FIXED AMOUNT.
- If you choose to participate, you will have the option to earn money if you buy the good at a lower price than its VALUE. You will NOT know how many participants are in the auction before it starts.

SLIDE No.8
Auctions

- In some rounds, the good is sold in Auction A, and in others in Auction B. At the beginning of each round, all participants will know which auction will be used to determine who buys the good. (The type of auction used in each round will be the same for all).

SLIDE No.9

Auction A

- Each potential buyer that participates in the auction will see the starting price of E$0, which will increase by E$1 every 0.65 seconds, and may indicate at each price if he is willing to continue in the auction and buy the good, or if he wishes to abandon the auction and not buy the good.
- The person who has not abandoned the auction after everybody else has will buy the good. The Price which the buyer will pay will be equal to the price at which the last person abandoned the auction.
- If you are the only participant in the auction, you will automatically buy the good at a price of 0.

SLIDE No.10

Example for Auction A

- Example: Suppose that your value is 65. If the other two people abandon the auction at 27 and 60 and you are still in the auction, you will buy the good and pay the price (60). Your profit in this round would be (65 - 60 =) 5.
- If you leave the auction you do not buy the good, and you will have neither earnings nor losses.

SLIDE No.11

Auction B

- Each potential buyer that participates in the auction will make an Price Offer.
- The person that submits the highest Price Offer will buy the good. (In case of a tie between two or more offers, the buyer will be determined randomly). The buyer will pay the Price equal to his Offer.
- If you are the only participant in the auction, you will buy the good with any offer you submit, even with an offer of 0. However, you will NOT know if you are the only participant in the auction; you will not know if there are 0, 1 or 2 other participants besides yourself).

SLIDE No.12

Example for Auction B

- Example: Suppose that your value is 65. If your offer is 61 and the other participants submit offers of 28 and 59, you will buy the good and pay the price (61). Your profit in this round would be (65 - 61 =) 4.

- If your offer is not the highest one, you will not buy the good, and you will have neither earnings nor losses.

SLIDE No.13
Earnings in the Auction

- The earnings of the buyer is the difference between the VALUE and the Price:
- VALUE - Price = Earnings.
- Note that you will lose money if you buy the good at a Price higher than its VALUE.
- Those who do not buy the good will have earnings of 0.

SLIDE No.14
Not Participating in the Auction

- If you choose not to participate you will obtain the FIXED AMOUNT.
- While the auction is being held, you can automatically make use of a pastime: Tic-tac-toe.
- You will play against the computer and you will win if you can place 3 of the symbols (X) in a straight line (horizontal, vertical or diagonal).
- Your results in this pastime will not affect your earnings.

SLIDE No.15
Rounds

- The experiment will have 48 rounds. In each round, the participants will be randomly reassigned in groups of 3; that is, you will NOT be participating with the same people in all rounds.

SLIDE No.16
Summary

- The experiment consists of a series of rounds, where you and other 2 people will be potential buyers of a good. Before each round, everyone will know the auction being used to determine who will buy the good (Auction A or Auction B). In each round, you will decide:
    1. If you will participate or not.
    2. If you decide to participate, you will have to determine the price that you are willing to pay or offer.

SLIDE No.17

Summary

- If you decide not participate in the auction, you will obtain the FIXED AMOUNT.
- If you participate in the auction, you can earn money by buying the good at a price lower than its value.
- Earnings (if you buy the good) = VALUE **-** Price.
- Earnings (if you do not buy the good)=E$0.
- Earnings (if you do not participate) = FIXED AMOUNT.

Once the participants finish watching the video which contains the instructions, they are asked to answer the following questions to ensure understanding:

1. Suppose that in a round is the VALUE AMOUNT FIXED 50 and is 14. How much is the FIXED AMOUNT VALUE and for 2 other potential bidders in that round?
2. Suppose that in a period your VALUE is 22 and the FIXED AMOUNT is 6. You buy the good and the price is 38. What are your earnings in this round?
3. Suppose your VALUE in a round is 78 and the FIXED AMOUNT is 12. You do not participate in the auction, and the price is 60. What are your earnings?
4. Suppose your VALUE in a round is 47 and the FIXED AMOUNT is 7. You participate in the auction, the price is 60, but you do not buy the good. What are your earnings?

5.1 Instructions for the risk elicitation task

This appendix contains the instructions, translated from the original Spanish, for the risk elicitation task. The decision sheet provided to subjects can be found in Figure 3.

SLIDE No.1

Welcome. You will be participating in a decision-making experiment. These instructions will explain to you how you may earn money. If you have any questions during these instructions, please raise your hand and we will address them in private. As of right now, it is very important not to talk or try to communicate in any way with the other participants. If you disobey the rules, we will have to end the experiment and ask you to leave without any payment.

SLIDE No.2

Your decision sheet shows 10 rows of decisions. Each of them is a selection between two options, Option A and Option B. Option A represents a fixed payment; unlike Option B, whose payment depends on the throw of a 10-sided die.

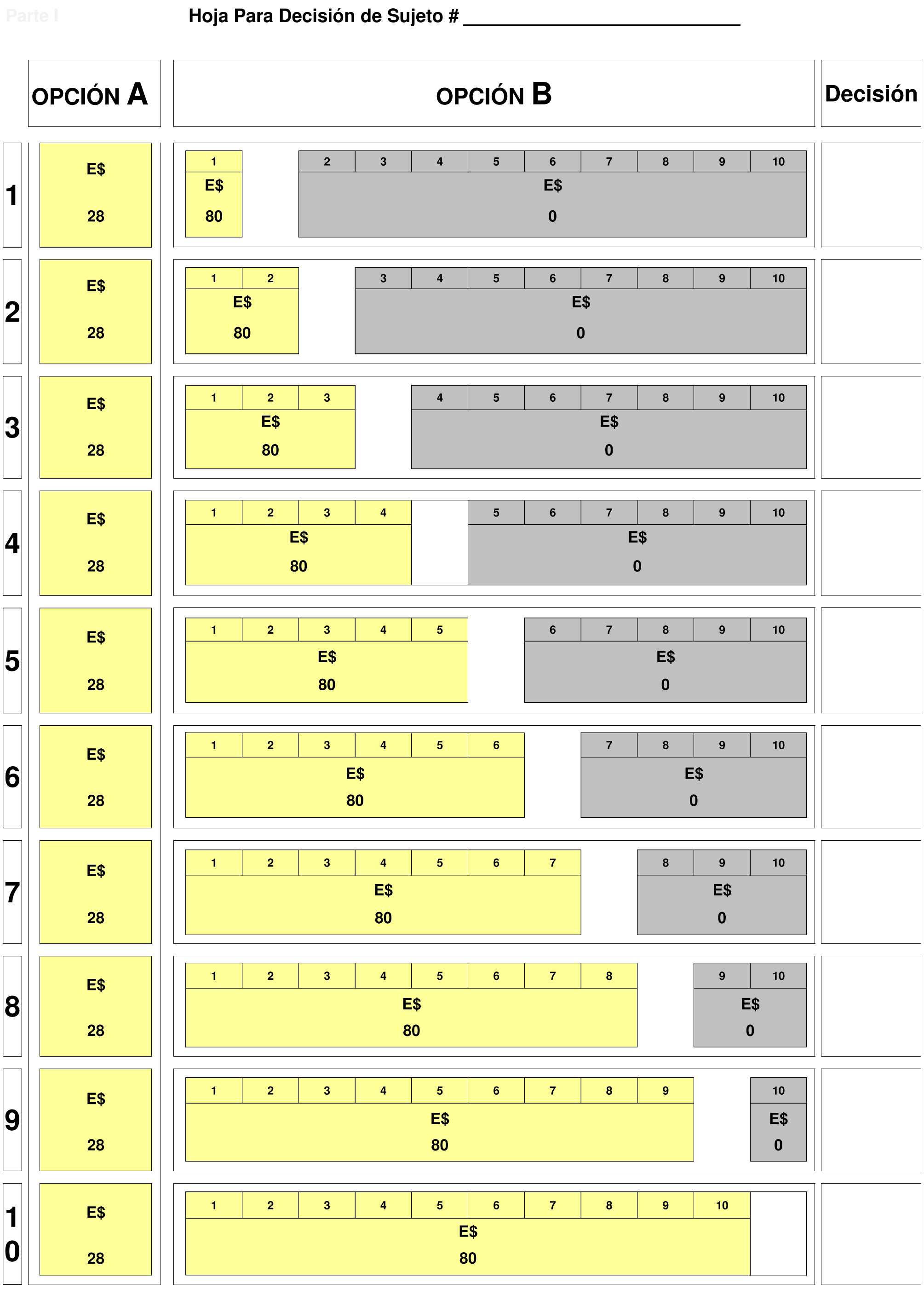

Figure 3: Decision sheet used in the risk elicitation task

SLIDE No.3 Now, please look at the first row at the top of the decision sheet. Option A pays $E\$28.00$. Option B pays $E\$80.00$ if the die lands on the number 1, but if the dice lands in any number between 2 and 10 it pays $E\$0.00$. The other decisions are similar, except that as you move down the table, the probability of the higher payment for Option B increases. In fact, for row 10, the last one, the option pays $E\$80.00$ with certainty so that you will have to choose between $E\$28.00$ and $E\$80.00$. Only one of the 10 rows determines your earnings. You will choose one option for each of the 10 rows and write it in the right column.

SLIDE No.4

After you have made all your selections, we will throw the 10-sided die to select the row that will determine your earnings. (Obviously, each decision row has the same probability of being chosen.)

SLIDE No.5

If for the decision row that will determine your earnings you chose option A, you will earn $E\$28.00$. If for that row you chose option B, we will throw the die a second time to determine your earnings. Remember you have to choose an option for each decision row. Now, please write your name and student ID number on the decision sheet.

References

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