SUPPLEMENTARY APPENDIX

APPENDIX 1

In this appendix we provide a topological characterization of the graph of Nash equilibria. This characterization allows us to numerically verify that there is a unique equilibrium for *every* γ (see Section 2.1), not just for $\gamma = 0.35$ and $\gamma = 0.49$. Likewise, we can also easily verify that the competition effect holds for *every* γ (i.e., total turnout rate is increasing in γ for every $\gamma < 0.5$) and that the size effect holds for several values of N.

Recall that in the general model we have $N \geq 2$ agents who may cast a vote for one of two parties $K = A, B$. (To simplify notation we assume that N is even.) Each agent has private information about her preferred party and her voting cost. Every agent's preferred party is A with probability γ and B with probability $1-\gamma$. The payoff to a K-supporter if party K wins the election is π, otherwise her payoff is zero. Agent i's voting cost is independently drawn from the common cost distribution F. To have an interesting problem, we assume $F(\pi/2) > 0$.

The winner of the election is the party with the most votes and a fair coin is tossed in case of a tie. Agents only have two strategies, they either cast a vote for their preferred party or they abstain.[1] If an agent votes, then she has to pay her voting cost. The benefits of voting come from either making or breaking a tie. If a is the probability that a randomly selected voter votes for A and b is the probability that a randomly selected voter votes for B, then the probability that an A-supporter can make or break a tie in favor of party A by choosing to vote is:

$$P_A(a,b,N) := \sum_{k=0}^{\frac{N}{2}-1} \frac{(N-1)!}{k!(k+1)!(N-2-2k)!} a^k b^{k+1} (1-a-b)^{N-2-2k} + \sum_{k=0}^{\frac{N}{2}-2} \frac{(N-1)!}{k!k!(N-1-2k)!} a^k b^k (1-a-b)^{N-1-2k}.$$

The analogous expression holds for B-supporters. Therefore, given a, b and N, a K-supporter with voting cost c votes if and only if

$$\frac{\pi}{2} P_K(a,b,N) \geq c.$$

[1] Of course, voting for the party they do not support is a weakly dominated strategy.

As it is customary in this model, we focus on type symmetric equilibria. Thus, the equilibrium conditions characterizing voting shares a and b are:

$$a = \gamma F\left[\frac{\pi}{2}P_A(a,b,N)\right], \tag{1}$$

$$b = (1-\gamma)F\left[\frac{\pi}{2}P_B(a,b,N)\right]. \tag{2}$$

Taylor and Yildirim (2010) show that if $a < b$ then $P_A(a,b,N) > P_B(a,b,N)$, and the probability functions P_A and P_B are both strictly decreasing in b.

Assume that the cost distribution F is continuous and fix the population size N and the utility parameter π. We define the equilibrium correspondence

$$\mathrm{E} : [0,.5] \to [0,1]\times[0,1]$$

that assigns to any $\gamma \le .5$ the set $\mathrm{E}(\gamma)$ of pairs (a,b) that satisfy the equilibrium conditions (1) and (2). Let its graph be $\mathrm{Graph(E)} := \{(\gamma,a,b) : (a,b) \in \mathrm{E}(\gamma)\}$. From Börgers (2004) we know that if $\gamma = .5$ then there is a unique equilibrium (a^*,b^*) where, of course, $a^* = b^* \le .5$.

Theorem 1. *The projection mapping* $\Pi : \mathrm{Graph(E)} \to [0,a^*]$ *defined by* $\Pi(\gamma,a,b) = a$ *is a homeomorphism.*

Proof. Define the functions:

$$G_A(a,b) = \frac{a}{F\left[\frac{\pi}{2}P_A(a,b,N)\right]} \qquad \text{and} \qquad G_B(a,b) = \frac{b}{F\left[\frac{\pi}{2}P_B(a,b,N)\right]} \tag{3}$$

and notice that a pair $(a,b) \ge (0,0)$ is an equilibrium for some γ if and only if

$$G(a,b) = G_A(a,b) + G_B(a,b) = 1. \tag{4}$$

In particular, if $(a,b) \ge (0,0)$ satisfies (4) then it is an equilibrium for $\gamma = G_A(a,b)$.

We now show that Π has a continuous inverse. Such an inverse satisfies $\Pi^{-1}(a^*) = (.5,a^*,a^*)$ and, letting b^0 be the unique solution to $b^0 = F\left[\frac{\pi}{2}(1-b^0)^{N-1}\right]$, it also satisfies $\Pi^{-1}(0) = (0,0,b^0)$. For any $a \in (0,a^*)$ we have $G(a,a) < 1$ because a is not part of an equilibrium for $\gamma = .5$, and $G(a,1-a) > 1$ by definition of the function G. Furthermore, if $b \ge a$ then $G_A(a,b)$ is non-decreasing in b and $G_A(a,b)$ is strictly increasing in b because, under such condition, both $P_A(a,b)$ and $P_B(a,b)$ are non-increasing in b. Thus, if $a \in (0,a^*)$ then there is a unique b such that (a,b) satisfies (4) and $\Pi^{-1}(a) = (G_A(a,b),a,b)$. For similar reasons, we know that if $a > a^*$ then a cannot be in the image of Π because $G(a,a) > 1$. Finally, Π^{-1} must be a continuous function because G is continuous. Therefore, Π is a homeomorphism of Graph(E) into $[0,a^*]$. □

The last theorem does not imply that there is a unique equilibrium for every γ, but it does say that the global structure of equilibria as a function of γ is very simple, i.e., it is a closed interval. In particular, it allows us to easily compute the set of equilibria for every value of

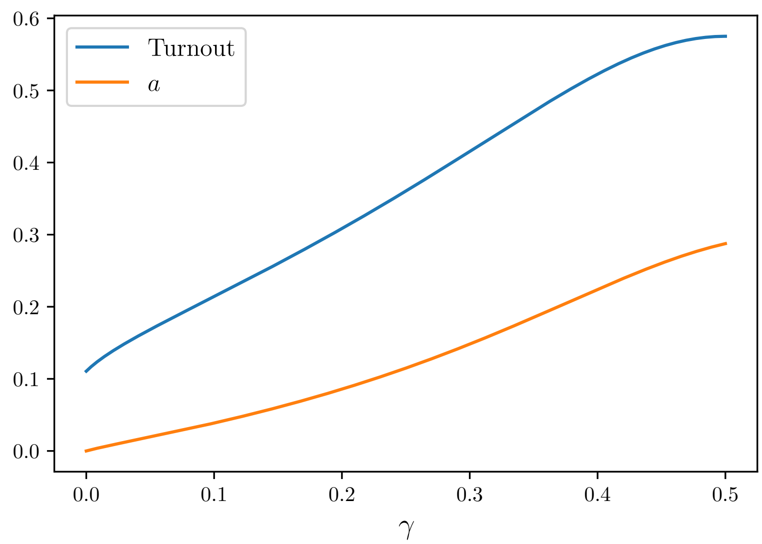


FIGURE 1. Equilibrium graph represented by the inverse of $\hat{\Pi}^{-1}$ (bottom curve), and total turnout rate as a function of γ (top curve). Cost distribution half-normal with $\sigma = 0.6$ and $N = 30$.

γ by following the value of the function Π^{-1} as a traverses the interval $[0, a^*]$. This can be efficiently done using this path-following algorithm:[2]

(1) Let $t = 0$. Take $a^0 = 0$ and find b^0 that solves $b^0 = F\left[\frac{\pi}{2}(1-b^0)^{N-1}\right]$.
(2) Let $t = t+1$. Add a small increment to a^{t-1} to obtain a^t.
(3) Find the unique b^t (which is close to b^{t-1}) that satisfies $G(a^t, b^t) = 1$.
(4) If $\gamma^t < .5$ go to Step 2 again. If $\gamma^t \geq .5$ stop.

Let $\hat{\Pi}^{-1} : [0, a^*] \to [0, .5]$ be the function that, for every a projects the value of $\Pi^{-1}(a)$ into the dimension corresponding to the value of γ. By Theorem 1, the function $\hat{\Pi}^{-1}$ is surjective, but if it is also injective (i.e. invertible) then the equilibrium is unique for every value of γ. Put differently, the equilibrium is unique for every value of γ if $\hat{\Pi}^{-1}$ is a strictly increasing function. (In terms of the previous algorithm, this implies that $\{\gamma^t\}$ must be an increasing sequence.)

If $\hat{\Pi}^{-1}$ is a strictly increasing function then it has an inverse. Such an inverse is represented as the bottom curve in Figure 1 for the cost distribution half-normal with $\sigma = 0.6$ when $N = 30$. Figure 1 also represents total turnout rate $a + b$ as a function of γ, and shows that it is non-decreasing (i.e., the competition effect is satisfied). We computed the equilibrium graph using the normal distribution with every combination of values for the mean and

[2] A Python script implementing this algorithm is available at:
`http://research.economics.unsw.edu.au/cpimienta/turnout/algorithm.py`.

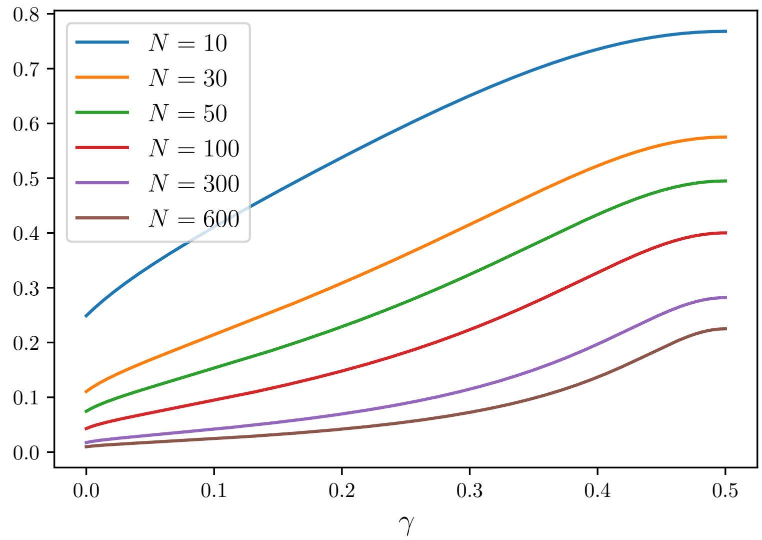


FIGURE 2. Total turnout rate as a function of γ using different values of N. Cost distribution half-normal with $\sigma = 0.6$.

standard deviation between 0.1 and 5 with 0.1 increments and, in every case, we found equilibrium uniqueness and competition effect. We performed a similar exercise truncating the normal distributions at zero and for the chi-squared, exponential, half-normal, log-normal, Pareto, and uniform distributions, with the same result.

Furthermore, we used size values $N = 10, 30, 50, 100, 300, 600$, which allowed us to confirm that, for those values and every cost distribution mentioned above, the size effect is satisfied. That is, for a fixed value of γ, the total turnout rate is decreasing in N. As an example, Figure 2 shows the inverse of $\hat{\Pi}^{-1}$ for $N = 10, 30, 50, 100, 300, 600$ using the cost distribution half-normal with parameter $\sigma = 0.6$.

Appendix 2

Table 1. Voting and Non-voting Completion Times Using Device Driven Variation

	Non-Voters	Voters
ChromeOS	−11.71***	−27.49***
	(4.19)	(6.68)
MacOS	−9.85***	−28.99***
	(2.70)	(4.20)
Windows	−9.62***	−29.91***
	(2.13)	(3.80)
iPad	−5.16	−32.36***
	(3.34)	(6.58)
iPhone	−9.14**	−19.50***
	(3.61)	(6.67)
Linux/Other	−10.19**	−25.93***
	(4.41)	(7.72)
Number of pixels	-5×10^{-6}	-1.2×10^{-6}***
	(0.00)	(0.00)
Pixels squared	1×10^{-6}	2×10^{-6}**
	(0.00)	(0.00)
Constant	33.62***	88.35***
	(3.35)	(4.83)
R2	0.063	0.119
Observations	546	654

Notes: The dependent variable is the number of minutes it took the participant to complete the entire experiment. OLS regressions are used to estimate the number of minutes taken to complete the experiment. Android, CromeOS, MacOS, Windows, iPad, iPhone and Linux/Other are categorical variables indicating operating systems/device types used by the participant. The omitted dummy variable for operating system/device types is the variable Android, for handheld devices that use the Android operating system. Number of pixels is a variable that measures the number of pixels on the device used by the participant, e.g. a 640 by 480 display has a total number of $640x480 = 307,200$ pixels. Pixels squared is calculated as (number of pixels / 1000)^2. Standard errors are in parentheses. * $p < 0.10$; ** $p < 0.05$; *** $p < 0.01$.

APPENDIX 3

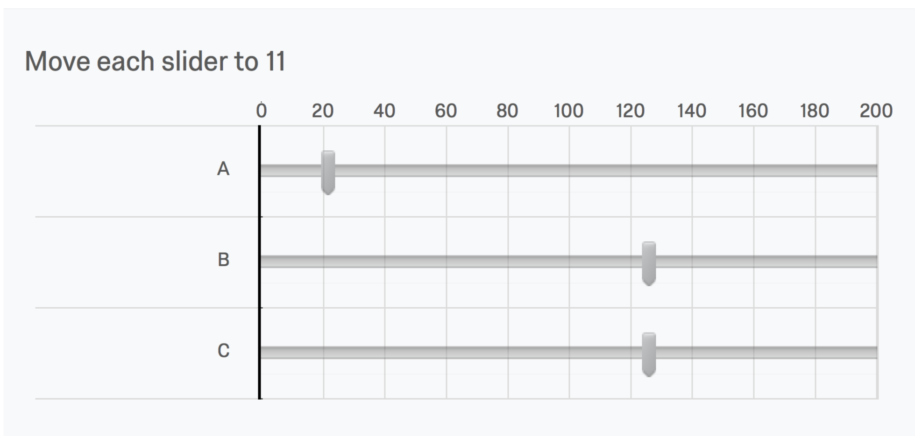
Instructions are identical for all treatments, except for the values taken by N and γ. As an example, we report the instructions for a member of group A in the large close election.

Welcome. This HIT consists of two stages. Stage 1 is made of three steps. First, we will ask you to complete a short slider task (5 minutes). Second, we will provide you with the instructions for Stage 2 (3 minutes). Third, we will require you to successfully answer a set of quiz questions (5 minutes) about these instructions. You will be paid $1.5 for successfully completing Stage 1. If you fail to correctly answer the quiz questions you will not receive any payment. Next, you will have the potential to earn an additional $5 bonus in Stage 2. Stage 2 will take either 0 minutes or 30 minutes to complete, depending on your choice. Your Stage 2 bonus will depend on your decision and the decisions of others.

* * *

We are now going to ask you to perform 20 sets of 3 slider tasks.

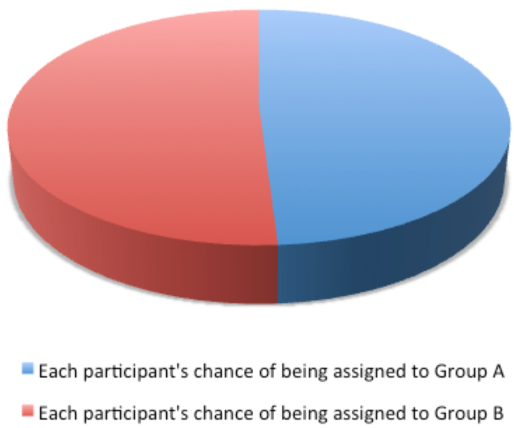
* * *



[*Followed by 19 others*]

* * *

Read carefully these instructions. Your payment today will depend on your understanding of them. 300 people, including yourself, are participating in this HIT. These people are divided into two groups: A and B. Each person is randomly assigned to either Group A or Group B. For each person the computer chooses a number between 1 and 100, each number being equally likely to be drawn. If the number drawn for that person falls between 1 and 49 then the person is assigned to Group A. Therefore, a person has a 49% chance of being in Group A. If the number drawn for that person falls between 50 and 100 then that person is assigned to Group B. Therefore, a person has a 51% chance of being in Group B. When you click Next, the computer will generate a random number for you.



* * *

Your number is 10, therefore you have been assigned to Group A.

* * *

In Stage 2, you are asked to choose whether you want to perform a task. The task consists of successfully completing 120 sets of 3 slider tasks like the ones you previously trialled. Your bonus payment will depend on what you and the other 299 people participating in this HIT choose to do: If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

* * *

Before advancing to the slider task, we need you to answer a few questions about this HIT. You will not be able to progress if you answer incorrectly any of these questions.

- How many people, yourself included, are participating in this HIT?
- What is the percentage chance that a given participant belongs to Group A?
- What is the percentage chance that a given participant belongs to Group B?

* * *

We will now test your understanding of how your bonus payment is determined under different scenarios. Note that the numbers used in these scenarios are hypothetical and have been chosen for practice purposes only. There are 5 questions in total and you will only be paid if you answer them correctly, so read them very carefully. If you make one mistake, you will be taken back to the first question for another attempt. You are only allowed 4 attempts.

* * *

Remember: You were assigned to Group A. If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

Imagine that, excluding yourself, 0 people from Group A and 2 people from Group B perform the task in Stage 2.

- ☐ If I choose not to perform the task I will get a bonus of $0 and if I choose to perform the task I will get a bonus of $0.
- ☐ If I choose not to perform the task I will get a bonus of $0 and if I choose to perform the task I will get a bonus of $2.50.
- ☐ If I choose not to perform the task I will get a bonus of $0 and if I choose to perform the task I will get a bonus of $5.
- ☐ If I choose not to perform the task I will get a bonus of $2.50 and if I choose to perform the task I will get a bonus of $0.
- ☐ If I choose not to perform the task I will get a bonus of $2.50 and if I choose to perform the task I will get a bonus of $2.50.
- ☐ If I choose not to perform the task I will get a bonus of $2.50 and if I choose to perform the task I will get a bonus of $5.
- ☐ If I choose not to perform the task I will get a bonus of $5 and if I choose to perform the task I will get a bonus of $0.
- ☐ If I choose not to perform the task I will get a bonus of $5 and if I choose to perform the task I will get a bonus of $2.50.
- ☐ If I choose not to perform the task I will get a bonus of $5 and if I choose to perform the task I will get a bonus of $5.

[*For each of the following 4 scenarios we provide the same set of 9 options, so we omit them from now on.*]

* * *

Remember: You were assigned to Group A. If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

Imagine that, excluding yourself, 1 person from Group A and 1 person from Group B perform the task in Stage 2.

* * *

Remember: You were assigned to Group A. If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

Imagine that, excluding yourself, 0 people from Group A and 1 person from Group B perform the task in Stage 2.

* * *

Remember: You were assigned to Group A. If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

Imagine that, excluding yourself, 2 people from Group A and 0 people from Group B perform the task in Stage 2.

* * *

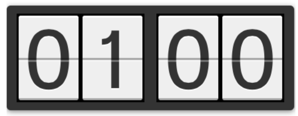
Remember: You were assigned to Group A. If more people from Group A than people from Group B perform the task your bonus is $5. If more people from Group B than people from Group A perform the task your bonus is $0. Finally, if an equal number of people from Group A and Group B perform the task your bonus is $2.50. This bonus payment rule applies to everyone.

Imagine that, excluding yourself, 1 person from Group A and 0 people from Group B perform the task in Stage 2.

* * *

Remember that 300 people, including yourself, are taking this HIT and like you have to decide whether to perform the task.

Do you want to perform the task and complete 120 sets of 3 slider tasks?
Wait for 60 seconds before making your decision.



* * *

Do you want to perform the task and complete 120 sets of 3 slider tasks?

☐ Yes

☐ No

[*If the subject answers Yes she faces 120 sets of 3-slider tasks, then, after short demographic questionnaire she receives an MTurk code for the payment. If she answers No she is taken straight to the questionnaire and then is provided with the MTurk code.*]

References

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TAYLOR, C., AND H. YILDIRIM (2010): "A Unified Analysis of Rational Voting with Private Values and Group-specific Costs," *Games and Economic Behavior*, 70(2), 457–471.