Eliciting utility curvature in time preference

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Appendices: Not for publication

A Theoretical predictions

A.1 Discounted utility

Assume an additively-separable intertemporal utility function of the form of equation 3 in the text. Let $\left(x_t^S, x_{t+k}^S\right)$ denote the smaller-sooner payoff vector presented as Option A, and let $\left(x_t^L, x_{t+k}^L\right)$ be the larger-later one presented as Option B. In rows 1 to 9 of each choice list,[‡] a subject chooses Option B as:

$$D(t)\,v\left(x_t^S\right) + D(t+k)\,v\left(x_{t+k}^S\right) < D(t)\,v\left(x_t^L\right) + D(t+k)\,v\left(x_{t+k}^L\right)$$

$$\frac{D(t)}{D(t+k)} < \frac{v\left(x_{t+k}^L\right) - v\left(x_{t+k}^S\right)}{v\left(x_t^S\right) - v\left(x_t^L\right)} \tag{12}$$

The quantity on the left of equation 12 represents the relative weight of utility on date t compared to $t+k$. Moving down the rows of a choice list, as k falls toward 0 weeks, $D(t+k)$ increases toward $D(t)$ and so $D(t)/D(t+k)$ falls toward 1. The quantity on the right is the ratio of the utility gained at date $t+k$ to utility lost at date t should the subject choose Option B. Within a given choice list, the term on the right is constant because all payoffs are fixed. A subject's "switch point" from smaller-sooner to larger-later is thus defined by the value of k at which $D(t)/D(t+k)$ first falls below this fixed ratio of utility differences.

For the case of linear utility, the switch point is defined by:

$$\frac{D(t)}{D(t+k)} < \frac{x_{t+k}^L - x_{t+k}^S}{x_t^S - x_t^L} \tag{13}$$

where, by design of the experiment, the ratio of *payoff* differences is constant and equal to 1.125 in all choice lists. Thus, *a subject with linear instantaneous utility is predicted to switch from Option A to Option B at the same row (and thus make the same number of sooner choices) in all choice lists.*

[‡] In row 10, $k = 0$ and $D(t)\,v\left(x_t^S + x_{t+k}^S\right) < D(t)\,v\left(x_t^L + x_{t+k}^L\right)$ such that the subject should always choose Option B.

In the case of concave utility, for given-sized differences in the magnitudes of the payoffs, the utility difference in the numerator of equation 12 shrinks as the later payoffs are shifted further up the utility function, while the utility difference in the denominator grows as the sooner payoffs are shifted further down. For example, comparing switch points for the AB and CA choice lists, concave utility implies that:

$$\frac{v(38)-v(20)}{v(17)-v(1)} < \frac{v(20)-v(2)}{v(33)-v(17)}$$

since the numerator of the term on the left is smaller than that of the one on the right, while the denominator is larger. It follows that a subject with concave instantaneous utility is predicted to make a later switch from small-sooner to larger-later in the AB choice list compared to CA.

Note that *these predictions hold regardless of the shape of the discount function* $D(t)$, provided it satisfies the standard assumptions that $D(t) > 0$ and $D'(t) < 0$. For example, they hold not only for exponential discounting, but also any conventional model of hyperbolic discounting (see Andersen et al., 2014, for a review), as well as the models of time-dependent probability weighting mentioned in the introduction.

A.2 Intertemporal risk aversion

For choice situations where subjects face risks at different points in time, Andersen et al. (2018) and Cheung (2015) consider the possibility that the intertemporal utility function may not be additively separable, contrary to what is assumed in equation 3 in the text. The basic idea is that subjects may be averse not only to the atemporal risk they face at a single point in time, but also the intertemporal risk inherent in their overall payoff stream. A simple specification that captures this idea of intertemporal risk aversion (or "correlation aversion") is to replace equation 3 by:

$$IU(x_t,t;x_{t+k},t+k) = U\left[D(t)\cdot v(x_t) + D(t+k)\cdot v(x_{t+k})\right] \tag{14}$$

where $U(\cdot)$ is an increasing concave function (cf. equation 4 in Andersen et al., 2018, and equation 2 in Cheung, 2015). However, in the experiment in this paper, subjects only face choices in which payments on both dates are sent with certainty, so there is only a single possible realization of intertemporal utility and choices cannot be affected by intertemporal risk aversion. That is, $IU\left(x_t^S,t;x_{t+k}^S,t+k\right) < IU\left(x_t^L,t;x_{t+k}^L,t+k\right)$ as $DU\left(x_t^S,t;x_{t+k}^S,t+k\right) < DU\left(x_t^L,t;x_{t+k}^L,t+k\right)$, and the switch point defined by equation 12 remains unchanged. Similarly, by standard textbook arguments for increasing monotonic transformations, the slope of an indifference curve for the specification in equation 14 is identical to that of the additive specification, as given by equation 4 in the text, since terms involving $U'(\cdot)$ cancel out.

In short, intertemporal risk aversion loses its bite when risk and time are not interacted in the experiment design. The same is true for recursive utility models: since risk is absent, the certainty equivalent function is not invoked and only the time aggregator is involved in evaluating the alternatives.

A.3 Discounted incremental utility

Assuming discounted incremental utility given by equation 10 in the text, a subject chooses Option B as:

$$\frac{D(t)}{D(t+k)} < \frac{V\left(x_t^L + x_{t+k}^L\right) - V\left(x_t^S + x_{t+k}^S\right)}{V\left(x_t^S\right) - V\left(x_t^L\right)} + 1 \tag{15}$$

Equation 12 for discounted utility described a trade-off between the utility of a larger payoff at $t+k$ against that of a smaller one at t. Equation 15 for discounted incremental utility simply describes an analogous trade-off between the utility of a larger *cumulative* payoff at $t+k$ against that of a smaller one at t. As before, the right-hand side is constant within a given choice list because the payoffs are fixed. Moreover, when $V(\cdot)$ is linear this simplifies to equation 13, which implies a fixed switch point in all lists.

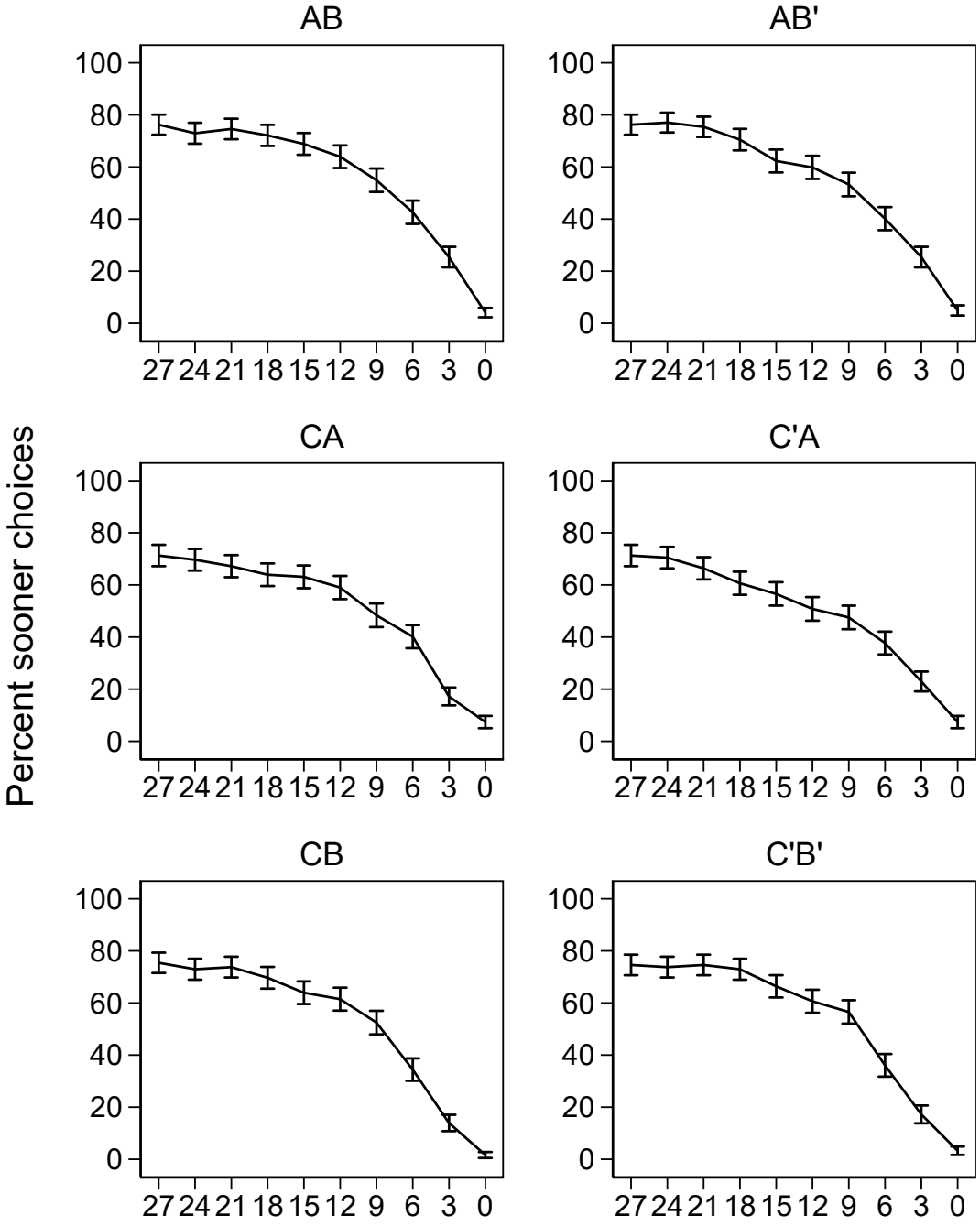
For concave $V(\cdot)$, comparing AB to CA the later cumulative payoffs are increased, and thus shifted further up the utility function, while the sooner ones are decreased, and thus shifted further down. It follows that:

$$\frac{V(1+38) - V(17+20)}{V(17) - V(1)} < \frac{V(17+20) - V(33+2)}{V(33) - V(17)}$$

which again results in in a later switch point in AB compared to CA.

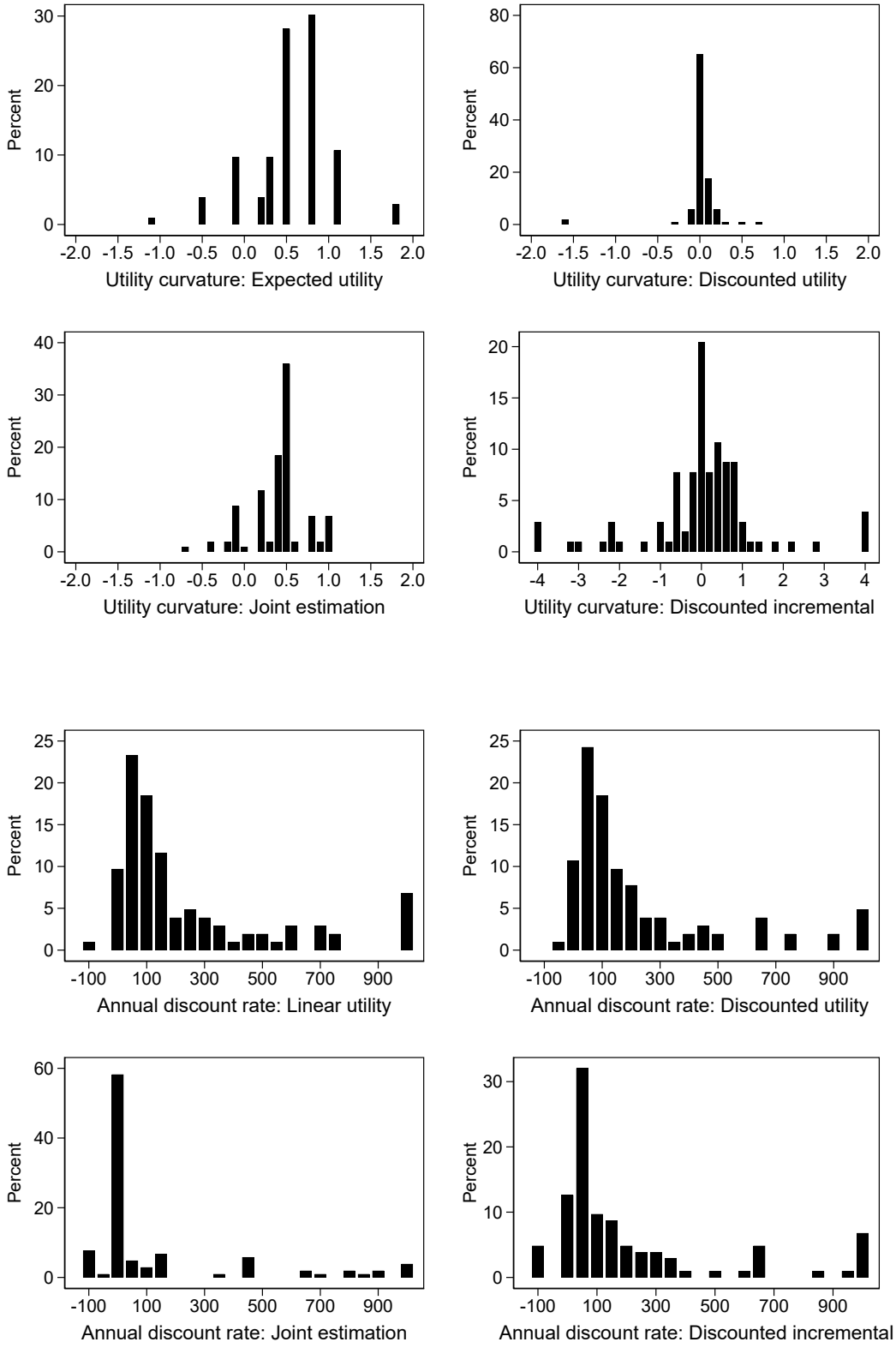
B Supplementary analyses

B.1 Choice behavior in the time preference tasks, by choice list

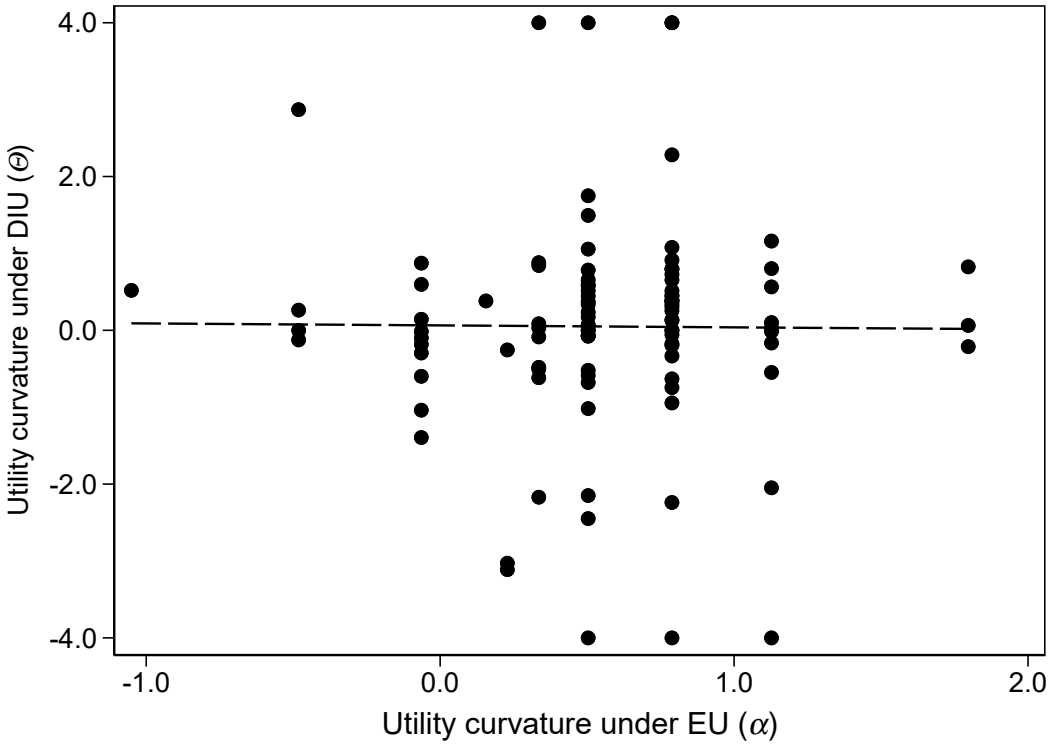


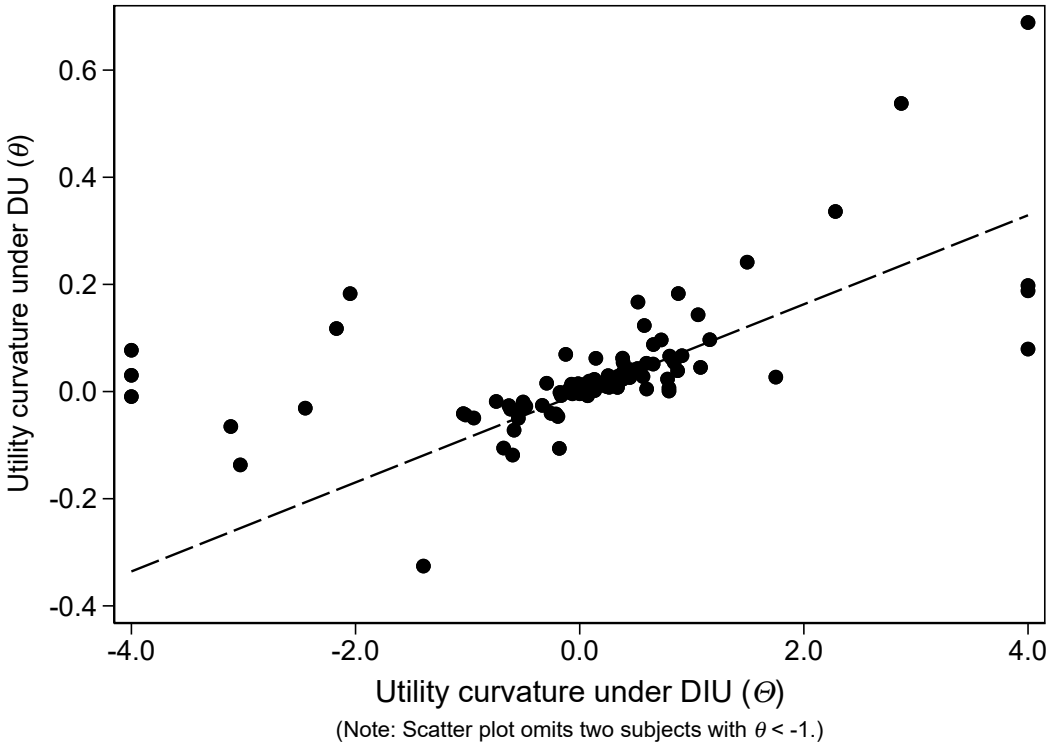
Note: Error bars represent +/- one SEM.

B.2 Histograms of individual utility curvature and discount rate estimates



B.3 Scatter plots of individual utility curvature estimates, discounted incremental utility





(Note: Scatter plot omits two subjects with $\theta < -1$.)

C Details of experiment design

C.1 Parameters of the risk preference choice list

Row	Option A				Option B				EV_A	EV_B	CRRA at indifference
	x_b	$1-p_g$	x_g	p_g	x_b	$1-p_g$	x_g	p_g			
1	17	0.9	20	0.1	1	0.9	38	0.1	17.3	4.7	–1.936
2	17	0.8	20	0.2	1	0.8	38	0.2	17.6	8.4	–1.095
3	17	0.7	20	0.3	1	0.7	38	0.3	17.9	12.1	–0.594
4	17	0.6	20	0.4	1	0.6	38	0.4	18.2	15.8	–0.222
5	17	0.5	20	0.5	1	0.5	38	0.5	18.5	19.5	0.087
6	17	0.4	20	0.6	1	0.4	38	0.6	18.8	23.2	0.367
7	17	0.3	20	0.7	1	0.3	38	0.7	19.1	26.9	0.643
8	17	0.2	20	0.8	1	0.2	38	0.8	19.4	30.6	0.948
9	17	0.1	20	0.9	1	0.1	38	0.9	19.7	34.3	1.354
10	17	0.0	20	1.0	1	0.0	38	1.0	20.0	38.0	n/a

Note: Values in the final column are the value of the CRRA coefficient α for risk at which a subject is indifferent between Options A and B, assuming expected utility and zero background consumption. Values in the final three columns were not presented to subjects.

C.2 Parameters of the time preference choice lists

CA

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	33	1	2	28	17	1	20	28	25.46
2	33	1	2	25	17	1	20	25	29.07
3	33	1	2	22	17	1	20	22	33.86
4	33	1	2	19	17	1	20	19	40.53
5	33	1	2	16	17	1	20	16	50.43
6	33	1	2	13	17	1	20	13	66.59
7	33	1	2	10	17	1	20	10	97.49
8	33	1	2	7	17	1	20	7	177.54
9	33	1	2	4	17	1	20	4	670.27
10	33	1	2	1	17	1	20	1	∞

C'A

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	25	1	11	28	17	1	20	28	25.46
2	25	1	11	25	17	1	20	25	29.07
3	25	1	11	22	17	1	20	22	33.86
4	25	1	11	19	17	1	20	19	40.53
5	25	1	11	16	17	1	20	16	50.43
6	25	1	11	13	17	1	20	13	66.59
7	25	1	11	10	17	1	20	10	97.49
8	25	1	11	7	17	1	20	7	177.54
9	25	1	11	4	17	1	20	4	670.27
10	25	1	11	1	17	1	20	1	∞

AB'

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	17	1	20	28	9	1	29	28	25.46
2	17	1	20	25	9	1	29	25	29.07
3	17	1	20	22	9	1	29	22	33.86
4	17	1	20	19	9	1	29	19	40.53
5	17	1	20	16	9	1	29	16	50.43
6	17	1	20	13	9	1	29	13	66.59
7	17	1	20	10	9	1	29	10	97.49
8	17	1	20	7	9	1	29	7	177.54
9	17	1	20	4	9	1	29	4	670.27
10	17	1	20	1	9	1	29	1	∞

Note: Interest rates in final column of each table were not presented to subjects. Delay lengths are in weeks.

AB

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	17	1	20	28	1	1	38	28	25.46
2	17	1	20	25	1	1	38	25	29.07
3	17	1	20	22	1	1	38	22	33.86
4	17	1	20	19	1	1	38	19	40.53
5	17	1	20	16	1	1	38	16	50.43
6	17	1	20	13	1	1	38	13	66.59
7	17	1	20	10	1	1	38	10	97.49
8	17	1	20	7	1	1	38	7	177.54
9	17	1	20	4	1	1	38	4	670.27
10	17	1	20	1	1	1	38	1	∞

CB

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	33	1	2	28	1	1	38	28	25.46
2	33	1	2	25	1	1	38	25	29.07
3	33	1	2	22	1	1	38	22	33.86
4	33	1	2	19	1	1	38	19	40.53
5	33	1	2	16	1	1	38	16	50.43
6	33	1	2	13	1	1	38	13	66.59
7	33	1	2	10	1	1	38	10	97.49
8	33	1	2	7	1	1	38	7	177.54
9	33	1	2	4	1	1	38	4	670.27
10	33	1	2	1	1	1	38	1	∞

Note: Interest rates in final column of each table were not presented to subjects. Delay lengths are in weeks.

C'B'

Row	Option A				Option B				Annual
	x_t	t	x_{t+k}	$t+k$	x_t	t	x_{t+k}	$t+k$	Rate
1	25	1	11	28	9	1	29	28	25.46
2	25	1	11	25	9	1	29	25	29.07
3	25	1	11	22	9	1	29	22	33.86
4	25	1	11	19	9	1	29	19	40.53
5	25	1	11	16	9	1	29	16	50.43
6	25	1	11	13	9	1	29	13	66.59
7	25	1	11	10	9	1	29	10	97.49
8	25	1	11	7	9	1	29	7	177.54
9	25	1	11	4	9	1	29	4	670.27
10	25	1	11	1	9	1	29	1	∞

Note: Interest rates in final column of each table were not presented to subjects. Delay lengths are in weeks.

C.3 Order of presentation of choice lists

Decisions	1-10	11-20	21-30	31-40	41-50	51-60	61-70
Order 1A	AB	AB'	CA	C'A	CB	C'B'	Risk
Order 1B	AB'	AB	C'A	CA	C'B'	CB	Risk
Order 2A	CA	C'A	AB	AB'	CB	C'B'	Risk
Order 2B	C'A	CA	AB'	AB	C'B'	CB	Risk

Note: Choice lists were not described using these labels in the decision sheets presented to subjects.

D Experiment instructions (Order 1A)

ELIGIBILITY TO PARTICIPATE

Welcome to today's session, and thank you for coming here on time. Please do not talk to the other participants while the session is in progress. Mobile phones must also be turned off. If you have a question at any time, please raise your hand, and someone will come to assist you in private.

IN THIS STUDY, YOU WILL RECEIVE SOME OR ALL OF YOUR EARNINGS IN THE FUTURE.

Therefore, to be eligible to participate, you must be willing to receive payments by cheque, to be written to you by Dr Stephen Cheung, a Lecturer in the School of Economics. These cheques will be drawn on the University of Sydney branch of the National Australia Bank.

You will receive two cheques. The first cheque will arrive one week from today. The second cheque will arrive between one and twenty-eight weeks from today. You will be fully informed of the amount of each cheque before you leave the lab at the end of this session. There is also a chance that you may receive some payment in cash at the end of the session.

Cheques will be delivered by Express Post, to your own nominated mailing address in Sydney. Australia Post guarantees next business day delivery for mail sent by Express Post to addresses within the Sydney metropolitan region.

Therefore to take part in this study, you must be willing to provide your mailing address in Sydney. This will only be seen by Dr Cheung and his assistants. After payment has been sent, your address will no longer be retained and your identity will not be a part of the subsequent data analysis.

Finally, you must be willing to stay for the full duration of today's session, and to comply with the instructions of the experimenter; otherwise you will not receive any payment at all.

If you do not agree to all of these points, please raise your hand now.

IF YOU AGREE, PLEASE TURN OVER THIS PAGE TO SIGN THE CONSENT FORM,
AND HAND IT IN WHEN ASKED TO DO SO.

GENERAL INFORMATION AND EARNINGS

In this study you will make a total of 70 decisions involving amounts of money that differ with respect to the dates on which the money will be received, or the chances of receiving the money.

In each of these decisions you will be presented with two alternatives, labelled "Option A" and "Option B", and you will be asked to indicate which of the two options you would prefer to receive.

Afterwards, we will ask you to complete a brief questionnaire about yourself. Both your choices and your responses to the questionnaire will be anonymous and not linked to your identity in any way.

At the end of the session, one of the 70 decisions will be randomly selected by drawing a numbered ball from the bingo cage at the front of the lab. We will do this separately for each participant. Your earnings will be determined by the choice that you made – Option A or Option B – for that decision.

You will also receive a participation fee of $10.00 in return for submitting valid responses for all 70 decisions, as well as every item of the questionnaire.

Your participation fee will be paid by cheque, in two equal instalments of $5 each. The first cheque will arrive one week from today. The second cheque will arrive between one and twenty-eight weeks from today. In particular:

- **If the decision randomly selected as the one that counts is one of Decisions 1 through 60,** then your earnings from the choice that you made will be added to your two cheques of $5 each, and sent to you on the dates indicated as part of that decision.
- **If the decision randomly selected as the one that counts is one of Decisions 61 through 70,** then your earnings from the choice that you made will be paid to you in cash at the end of the session. In this case, you would still receive two cheques of $5 each: the first would be sent one week from today, and the second would be sent sixteen weeks from today.

The selection of the decision that counts is entirely random, and your choices have no influence over whether you receive payment in the form of cash as well as cheque. What your choices do affect are the amounts of money you receive, in the event that a decision is chosen to count for payment. You will always receive two cheques by mail, and the value of each cheque will always be at least $5.

The decisions are not designed to test you – the only correct answers are the ones you really think are best for you. Since every decision has an equal chance to be selected as the one that determines your earnings, you should consider each one carefully, treating it as if it may be the one that counts.

One business day before each payment date, your cheque will dispatched for delivery by Express Post. Australia Post guarantees next business day delivery for mail sent by Express Post to addresses within the Sydney metropolitan region.

Attached to your Participation Information Statement is Dr Cheung's business card. Please keep this in a safe place. If you do not receive a cheque on the designated date, please contact Dr Cheung.

On your desk are two envelopes: one for each of your two cheques. Please take the time now to address these to yourself at your own mailing address in Sydney.

DECISION TABLES 1 TO 6 (Decisions 1 to 60)

Decision Tables 1 to 6 each show ten choices between two options labelled "A" and "B". Each decision involves a choice between receiving different amounts of money on two different dates, which will be clearly indicated for each decision. You will make 60 of these choices in total.

For example: In Decision 1, Option A pays $17 in 1 week from today, **and** it also pays $20 in 28 weeks from today. Option B pays $1 in 1 week from today, **and** it also pays $38 in 28 weeks from today.

The other decisions in Decision Table 1 are similar, except that as you move down the table, the date of the second payment in each option becomes sooner. In fact, for Decision 10 in the bottom row, both payments will arrive on the same date in one week from today. So in Decision 10, your choice is between $17 **and** $20 in Option A, or $1 **and** $38 in Option B, all arriving one week from today.

Decision Tables 2 to 6 also have the same format. It is only the amounts of money in Option A and Option B that change from one table to the next.

For each decision, you are asked to choose either Option A or Option B by marking an "X" in the appropriate box.

You may choose A for some decisions and B for others, and you may make your choices in any order.

Please note that the decision sheets will be double-sided, and that you should complete all decisions on both the front and back of each page.

If one of these decisions is randomly selected as the one that counts for payment, then your earnings will be determined by the choice that you made – either Option A or Option B – for that decision.

These earnings would be added to the two instalments of your participation fee, and mailed to you by cheque on the two dates indicated as part of that decision.

For example: Suppose that one of the choices in Decision Table 1 is selected to count for payment.

- Then, if you chose Option A, $17 would be added to the first instalment of your participation fee and sent to you on the first date. **In addition,** $20 would also be added to the second instalment of your participation fee and sent to you on the second date.
- Otherwise, if you chose Option B, $1 would be added to the first instalment of your participation fee and sent to you on the first date. **In addition,** $38 would also be added to the second instalment of your participation fee and sent to you on the second date.

At the time you make each choice, you will not know if it will be selected for payment. Since each choice is equally likely to be selected, you should treat each one as if it may be the one that counts.

PLEASE DO NOT BEGIN MAKING YOUR DECISIONS UNTIL YOU ARE INSTRUCTED TO DO SO.

DECISION TABLE 7 (Decisions 61 to 70)

Decision Table 7 shows ten choices between two options labelled "A" and "B". These choices involve different chances to receive different amounts of money at the end of today's session.

We will use a ten-sided die to determine these chances; the faces are numbered from 1 to 10 (the "0" face of the die will serve as 10).

For example: In Decision 61, Option A pays $20 if the roll of the ten-sided die is 1, and it pays $17 if the roll is 2 to 10. Option B pays $38 if the roll of the die is 1, and it pays $1 if the roll is 2 to 10.

The other decisions are similar, except that as you move down the table, the chances of the higher payment in each option increase. In fact, for Decision 70 in the bottom row, the die will not be needed since each option pays the higher payoff for sure, so your choice here is between $20 or $38.

For each decision row, you are asked to choose either Option A or Option B by marking an "X" in the appropriate box.

You may choose A for some decisions and B for others, and you may make your choices in any order.

If one of these decisions is randomly selected as the one that counts for payment, then we will roll the ten-sided die to determine your earnings according to the choice that you made – either Option A or Option B – for that decision.

This amount would be paid to you in cash at the end of today's session. In this case, you would still receive your participation fee by cheque in two instalments of $5 each: the first would be sent one week from today, and the second would be sent sixteen weeks from today.

At the time you make each choice, you will not know if it will be selected for payment. Since each choice is equally likely to be selected, you should treat each one as if it may be the one that counts.

PLEASE DO NOT BEGIN MAKING YOUR DECISIONS UNTIL YOU ARE INSTRUCTED TO DO SO.