Preference Discovery — **Online Appendix**

Appendix A: Threshold Derivation for Full Voluntary Discovery

What does full voluntary discovery look like?

- Subject believes that all untried goods are not worth trying, b/c they are sufficiently inferior to numeraire good.
- For each good i and time-point t, there is a threshold X_i^t such that the cost of trying the good exceeds the potential benefits if and only if $\beta_i^0 \leq X_i^t$.

Possible ways of implementing that:

(I) Highest possible threshold: $X_i^t = z = 65$, based on myopic optimization. This reflects the behavior of a *minimally experimental* agent.

(II) Lowest possible threshold: in non-myopic setting, with numeraire as only available alternative in all future periods. This corresponds to a *maximally experimental* agent.

Both settings are extreme, so that we can be sure that the true threshold lies somewhere in between these two estimates.

How to implement (II)??

- Hereby, X_i^t is defined as the value of β_i^0 such that the expected cost of trying m units of good i equals the expected benefit of the option (without obligation) to consume good i in the future.
- If $\beta_i^0 \geq 65$, then the expected cost of trying good i is negative (or 0), so that the subject should try good i for sure. Therefore, we focus our analysis on the cases where $\beta_i^0 < 65$.
- The expected cost is the same in each period, but the expected benefit declines over time, since fewer periods remain for the good to be consumed. Therefore, X_i^t is increasing in t.

To make the calculation of X_i^t feasible, we make the following assumptions:

(a) The agent is risk-neutral.

(b) No discounting, that is all periods have the same value.

(c) The agent knows the true probability q_i that the good appears in any given period.

(d) The agent knows the distribution of the true parameter value, i.e. she knows that $\beta_i \sim U(\{50, 51, \ldots, 80\})$.

Some background work to determine the conditional expectations

Recall the underlying distributional assumptions in our experiment:

- True parameter value $\beta_i \sim U(\{50, 51, \ldots, 80\})$.
- Prior $\beta_i^0 \sim U(\{\beta_i - \sigma, \ldots, \beta_i + \sigma\})$, with $\sigma > 15$.

Therefore, the conditional probability of β_i, given the prior β_i^0 (for $\beta_i^0 < 65$) is given by:

$$\mathbb{P}\left(\beta_i \mid \beta_i^0\right) = \begin{cases} \frac{1}{\beta_i^0+\sigma-49} & , \quad \text{if } \beta_i^0 \in \{50-\sigma, \ldots, 79-\sigma\} \text{ and } b \in \{50, \ldots, \beta_i^0+\sigma\} \\ \frac{1}{31} & , \quad \text{if } \beta_i^0 \in \{80-\sigma, \ldots, 64\} \text{ and } b \in \{50, \ldots, 80\} \\ 0 & , \quad \text{otherwise, for } \beta_i^0 \leq 64 \end{cases} .$$

This implies that for $\beta_i^0 \in \{50-\sigma, \ldots, 79-\sigma\}$:

$$\begin{aligned} \mathbb{E}\left[\beta_i \mid \beta_i^0\right] &= \sum_{b=50}^{80} b \cdot \mathbb{P}\left(b \mid \beta_i^0\right) = \sum_{b=50}^{\beta_i^0+\sigma} b \cdot \frac{1}{\beta_i^0+\sigma-49} \\ &= \frac{1}{\beta_i^0+\sigma-49} \cdot \left(50+\beta_i^0+\sigma\right) \cdot \frac{\beta_i^0+\sigma-50-1}{2} = \frac{50+\beta_i^0+\sigma}{2}, \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}\left[(\beta_i - 65)_+ \mid \beta_i^0\right] &= \sum_{b=66}^{80} (b-65) \cdot \mathbb{P}\left(b \mid \beta_i^0\right) = \sum_{b=66}^{\beta_i^0+\sigma} (b-65) \cdot \frac{1}{\beta_i^0+\sigma-49} \\ &= \frac{1}{\beta_i^0+\sigma-49} \cdot \sum_{x=1}^{\beta_i^0+\sigma-65} x = \frac{1}{\beta_i^0+\sigma-49} \cdot \left(\beta_i^0+\sigma-64\right) \cdot \frac{\beta_i^0+\sigma-65}{2}, \end{aligned}$$

as long as $\beta_i^0 \in \{65-\sigma, \ldots, 79-\sigma\}$, while $\mathbb{E}\left[(\beta_i - 65)_+ \mid \beta_i^0\right] = 0$ for $\beta_i^0 \in \{50-\sigma, \ldots, 64-\sigma\}$.

Moreover, for $\beta_i^0 \in \{80-\sigma, \ldots, 64\}$

$$\mathbb{E}\left[\beta_i \mid \beta_i^0\right] = \sum_{b=50}^{80} b \cdot \mathbb{P}\left(b \mid \beta_i^0\right) = \sum_{b=50}^{80} b \cdot \frac{1}{31} = 65 \,,$$

and

$$\begin{aligned} \mathbb{E}\left[(\beta_i - 65)_+ \mid \beta_i^0\right] &= \sum_{b=66}^{80} (b-65) \cdot \mathbb{P}\left(b \mid \beta_i^0\right) = \sum_{b=66}^{80} (b-65) \cdot \tfrac{1}{31} \\ &= \tfrac{1}{31} \cdot \sum_{x=1}^{15} x = \tfrac{1}{31} \cdot (15+1) \cdot \tfrac{15}{2} = \tfrac{120}{31} \,. \end{aligned}$$

Cost-Benefit Comparison

To experimentally consume good i, the agent needs to purchase $m_i = 1$ unit of good i at a price of $p_i = 1$, thereby foregoing the consumption of one unit of the numeraire good, worth $z = 65$. Therefore, the (conditional) expected cost of trying good i in period t is

$$EC_i^t(\beta_i^0) = 65 - \mathbb{E}\left[\beta_i \mid \beta_i^0\right] \,.$$

On the other hand, if the agent discovers her true β_i at time t, she will be able to consume y units of the good in each of the future $T-t$ periods where the good appears, that is with probability q_i. However, she has no obligation to consume good i in the future and will only do so if $\beta_i > z = 65$. Her benefit per consumed unit in this case is therefore $\beta_i - 65$ (since she gives up the consumption of an equal quantity of the numeraire good). Therefore, the agent's (conditional) expected benefit from potential future consumption is given by

$$EB_i^t(\beta_i^0) = q_i \cdot y \cdot (T-t) \cdot \mathbb{E}\left[(\beta_i - 65)_+ \mid \beta_i^0\right] \,.$$

Let's first consider the case of $\beta_i^0 \in \{65-\sigma, \ldots, 79-\sigma\}$. Here, the threshold value for full voluntary discovery, X_i^t, is determined by the equation

$$\begin{aligned} EC_i^t(X_i^t) &= EB_i^t(X_i^t) \\ 65 - \tfrac{50 + X_i^t + \sigma}{2} &= q_i \cdot y \cdot (T-t) \cdot \tfrac{1}{\beta_i^0 + \sigma - 49} \cdot (\beta_i^0 + \sigma - 64) \cdot \tfrac{\beta_i^0 + \sigma - 65}{2} \,, \end{aligned}$$

which solves for

$$X_i^t \;=\; 64.5 - \sigma + \sqrt{240.25 - 240 \cdot \frac{q_i \cdot y \cdot (T-t)}{1 + q_i \cdot y \cdot (T-t)}}\;. \qquad (1)$$

We can see from Equation (1) that the threshold is decreasing in the agent's income, the likelihood of the good appearing in future periods, and the number of periods remaining. This is to be expected because in each of these cases, there is an increased upside to preference discovery.

Now we show that for all other priors β_i^0, this definition of X_i^t is also suitable.

- For $\beta_i^0 \in \{80-\sigma, \ldots, 64\}$, $EC_i^t(\beta_i^0) = 65 - 65 = 0$, so that

$$EB_i^t(\beta_i^0) \;=\; q_i \cdot y \cdot (T-t) \cdot \frac{120}{31} \;>\; 0 \;=\; EC_i^t(\beta_i^0)\,,$$

  that is the (maximally experimental) agent should try good i at any time. Since here, $\beta_i^0 \geq 80 - \sigma > X_i^t$ for all t, according to Equation (1), our definition of X_i^t still works here.

- For $\beta_i^0 \in \{50-\sigma, \ldots, 64-\sigma\}$, we have

$$EB_i^t(\beta_i^0) \;=\; q_i \cdot y \cdot (T-t) \cdot 0 \;=\; 0 \;<\; EC_i^t(\beta_i^0)\,,$$

  and therefore the agent can get no possible benefit out of this good (due to its exceptionally low prior). Thus, the good should not be discovered. This is also reflected in the definition of X_i^t from Equation (1), since here $\beta_i^0 \leq 64 - \sigma < X_i^t$ for all t.

Therefore:

To sum up . . .

Compute X_i^t based on Equation (1). At time t, the agent has achieved *maximally experimental full voluntary discovery* if she has learned her preferences for all goods i for which $\beta_i^0 > X_i^t$.