Electronic Supplementary Material
for
Cognitive Sophistication and Deliberation Times

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Appendix A Additional Analyses

Appendix A.1 Beauty Contest

To further investigate the relation between deliberation times and levels of reasoning beyond the linear relationship documented in the main text, we ran additional regressions with a full set of level dummies. The results of these regressions are reported in Table A.1. In these regressions we added a dummy variable k.level for each observed level $k = 1, \ldots, 5$ taking the value 1 if the choice was classified as level-k and zero otherwise. Overall, 38 subjects were classified as level-1, 12 as level-2, 11 as level-3, 5 as level-4, and 2 as level-5. We observe that all level dummies have positive coefficients. The coefficients for levels 1 and 2 are not significant, whereas those for levels 3 to 5 are significant and positive. With the exception of level 4 the coefficients are larger for higher-level dummies, suggesting that deliberation times are increasing for each step. These observations are unaffected when we add controls for cognitive ability (model 2) and other controls (model 3). However, these results should be treated with caution since our data is concentrated on lower levels, and as pointed out above beyond level-1 there are 12 data points or fewer per level.

Two subjects chose the Nash equilibrium strategy at 0. Since those choices cannot be attributed to any finite level, we included an additional "Nash" dummy. Its coefficient in the regression models is negative, although not significant in model 1. It becomes marginally significant when adding controls (models 2, 3).

Appendix A.2 11-20 Games (BASE and FLAT)

Figure A.1 displays the absolute choice frequencies across all 8 instances of the BASE and FLAT versions. Table A.2 reports the results of an additional random-effects probit regression on a binary variable that takes the value 1 if level is larger or equal to 1 and 0 otherwise. Table A.3 is a restricted regression for FLAT restricted to levels 0 and 1.

Table A.1: Linear regression on DT with level dummies for the beauty contest game.

DT	1	2	3
1.level	1.9982	0.9484	0.4935
	(3.0023)	(3.0865)	(3.1766)
2.level	3.4889	2.6039	2.7875
	(4.5621)	(4.5903)	(4.6714)
3.level	19.5983***	18.4856***	18.1668***
	(4.7308)	(4.7818)	(4.8409)
4.level	12.9473*	11.5615*	9.7953
	(6.7049)	(6.7552)	(6.8416)
5.level	35.4133***	33.2763***	32.3046***
	(10.3459)	(10.4234)	(10.4798)
Nash	−15.8717	−18.0087*	−18.3342*
	(10.3459)	(10.4234)	(10.4302)
HighCRT		3.7558	2.9075
		(2.7237)	(2.9064)
Constant	19.1637***	17.5448***	16.9674**
	(1.8889)	(2.2181)	(7.6604)
Controls	No	No	Yes
Observations	128	128	128

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Omitted controls are gender, strategic uncertainty, and swiftness.

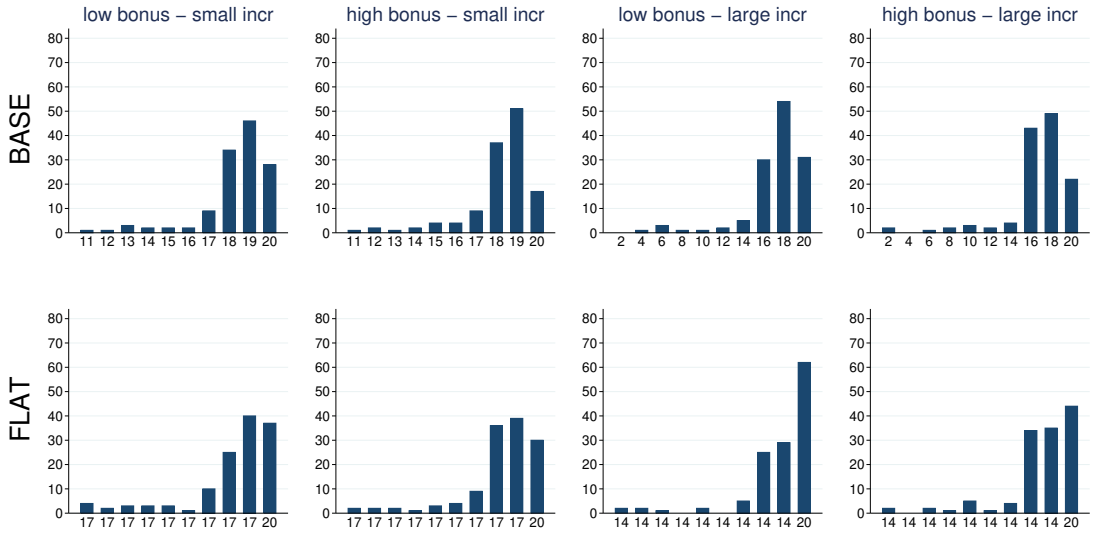

Figure A.1: Histograms of choices for each of the 11-20 games, BASE and FLAT versions.

Table A.2: Random effects Probit regressions of $\text{Level}_{\geq 1}$ (dummy) with controls for bonus and increment

$\text{Level}_{\geq 1}$	BASE	FLAT
HighBonus	0.4947***	0.4008***
	(0.1689)	(0.1443)
LargeIncr	−0.2262	−0.6373***
	(0.1660)	(0.1480)
HighCRT	0.6014*	0.3163
	(0.3096)	(0.2714)
Period	0.0380**	−0.0272*
	(0.0171)	(0.0155)
Constant	0.4593	1.3706*
	(0.8365)	(0.7113)
Controls	Yes	Yes
Observations	512	512
Subjects	128	128

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. $\text{Level}_{\geq 1}$ (dummy) takes the value 1 if level is larger or equal to 1 and 0 otherwise. Models are restricted to subsamples including only the four decisions in BASE and FLAT, respectively. Omitted controls are gender, strategic uncertainty, and swiftness.

Table A.3: Panel regressions of DT with bonus and increment for FLAT restricted to levels 0 and 1.

DT	FLAT
HighBonus	1.7681**
	(0.7914)
LargeIncr	0.7391
	(1.1185)
Level	0.0861
	(1.1169)
Level × LargeIncr	2.9628*
	(1.6777)
Period	−0.9944***
	(0.1114)
Constant	15.8161***
	(4.9493)
Controls	Yes
R^2 (overall)	0.3123
Observations	316
Subjects	128

Notes: Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Models are restricted to subsamples including only level-0 and level-1 choices in FLAT. Omitted controls are cognitive ability, gender, strategic uncertainty, and swiftness.

Appendix A.3 11-20 Games (EXTR and SOCP)

Figure A.2 displays the absolute choice frequencies across all 8 instances of the EXTR and SOCP versions.

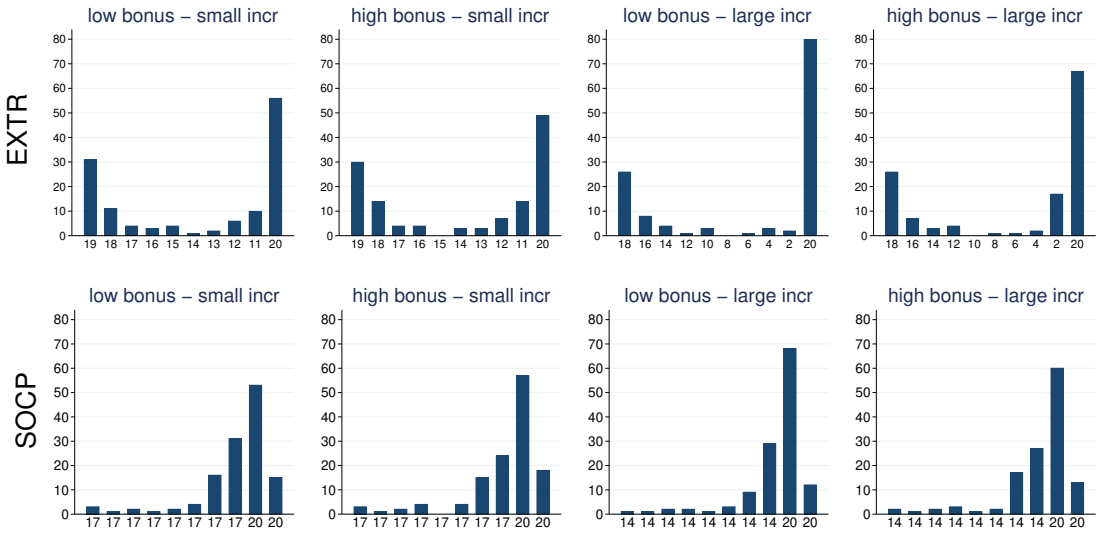

Figure A.2: Histograms of choices for each of the 11-20 games, EXTR and SOCP versions.

Appendix B Other Level-0 Specifications in the 11-20 Game

Arad and Rubinstein (2012) argue that choosing 20 in the 11-20 game is a natural anchor for an iterative reasoning process. However, Hargreaves Heap et al. (2014) show that level-0 behavior might depend on the payoff structure of the game. This might be less problematic in our setting because a further appeal of the original 11-20 game, which essentially corresponds to BASE, is that it is fairly robust to the level-0 specification. Specifically, choosing 19 in the original 11-20 game, or box 1 in BASE, is the level-1 strategy for a wide range of level-0 specifications. Still, this robustness depends on the particular payoff structure of the game and hence might be different across the various versions used in our experiment. In this appendix we explore the robustness of BASE and FLAT to the level-0 specification.

Let $\sigma_0 = (p_0, \ldots, p_9)$ denote a level-0 specification that assigns probability p_i to box i. Recall that box 0 always contains the salient amount of 20. We want to study the range of σ_0 such that choosing box 1 is still the unique level-1 strategy, that is, $BR(\sigma_0) = \{1\}$. A necessary condition is that $p_0 > \underline{p_0}$, where $\underline{p_0} = (20 - A_1)/R$, which is derived from the condition that the expected payoff of box 1 exceeds that of box 0, i.e. $A_1 + p_0 R > 20$. Table B.1 gives an overview over the values of $\underline{p_0}$ across BASE and FLAT for each combination of bonus and increment size, and already provides a first

intuition: the condition is very mild for BASE and also for FLAT although to a lesser extent, especially for the low bonus, large increment version.

Table B.1: Lower bounds on p_0

	BASE		FLAT	
bonus	low	high	low	high
small increment	5%	2.5%	15%	7.5%
large increment	10%	5%	30%	15%

The condition $p > \underline{p_0}$ is in general not sufficient. It is straightforward to show that, as long as box 1 contains the second-highest sure amount, that is, $A_1 \geq A_j$ for all $j \neq 0, 1$, and $p_0 > \underline{p_0}$, a sufficient condition is that no box $j \neq 0$ is assigned a probability larger than p_0. This holds in BASE as long as box 0 is most probable under σ_0 (note that this implies $p_0 > 10\%$, hence $p_0 > \underline{p_0}$). Hence, choosing box 1 is the unique level-1 strategy in BASE under fairly weak requirements, in particular even if σ_0 is taken to be uniform randomization as usually assumed in games without a salient strategy (e.g. the beauty contest game).[1]

For FLAT, the sufficient condition holds if box 0 is most likely under σ_0 and $p_0 > \underline{p_0}$ (similarly to BASE, this latter condition is void for high bonus and small increment). This is a slightly stronger condition, because the lower bounds $\underline{p_0}$ are tighter. In particular, in the extreme case of uniform randomization the level-1 strategy is still to choose box 1 only for high bonus and small increments, while it prescribes to stay with box 0 for the other conditions. Overall, however, the requirement remains mild and amounts to assuming a small degree of salience for box 0.

In the main text, we assumed that the starting point in the 11-20 game for our model of iterative thinking was to choose the rightmost box containing the salient amount of 20. As just illustrated, the best-reply structure in BASE and FLAT is robust for a wide range of alternative level-0 specifications. Thus, even if, contrary to our level-0 assumption, the starting point does not assign probability one to choosing the rightmost box, the best-reply structure and hence our results are unaffected as long as p_0 is not too small.

Hence, we conclude that the results presented in the main text cannot be explained by differences in the robustness to the level-0 specification between treatments.

Robustness of EXTR to Level-0 Specification. Analogously to the analysis above, we now explore the robustness of EXTR to the level-0 specification. Table B.2 shows the lower bounds for the necessary condition $p_0 > \underline{p_0}$ for EXTR, and we see immediately

[1]For large increment and low bonus, in the extreme case of uniform randomization choosing box 1 is a best reply, but not a unique one, because it ties with choosing the rightmost box.

Table B.2: Lower bounds on p_0

	EXTR	
bonus	low	high
small increment	45%	22.5%
large increment	90%	45%

that those restrictions are rather tight compared the the mild conditions for BASE and FLAT.

The sufficient conditions identified for BASE and FLAT are not sufficient for EXTR because box 1 contains the lowest sure amount, hence the probability assigned to the rightmost box has to exceed the probability of any box j by more than $(A_j - A_1)/R$. This condition together with $p_0 > \underline{p_0}$ is sufficient to make box 1 the unique best response in EXTR. This is a relatively demanding condition, as is the lower bound $p > \underline{p_0}$ in this case. In particular, choosing the leftmost box that grants the second highest sure payoff is the level-1 strategy for a relatively wide range of specifications that include uniform randomization. Hence, the best-reply structure of EXTR is less robust to changes in the level-0 specification, and there is a clear alternative best-reply structure where the leftmost box is the level-1 strategy.

To check for robustness in the case of EXTR we consider an alternative best-reply structure by assuming that the level-0 specification is mixed and the best reply is to choose the leftmost box, which we then classify as the level-1 strategy. The best reply to that is to choose the rightmost box containing the salient amount of 20, now classified as level 2. From there the best-reply structure follows the familiar pattern from right to left. We repeated the complete analysis of EXTR presented above for this alternative classification, and found no qualitative difference with the previous analysis.[2]

Appendix C Sequence of Games

To control for order effects we counterbalanced the order of the different 11-20 games using the following four randomized sequences. We denote the small increment – low bonus version of BASE, FLAT, EXTR, and SOCP by B, F, E, and S, respectively. Similarly for $X \in \{B, F, E, S\}$ we use the notation $+X$ to indicate large increments, and $X+$ to indicate high bonus, e.g. $+B+$ denotes BASE with large increments and high bonus.

[2]The alternative regressions are available upon request.

Table 1: Pseudo-randomized sequences of the 11-20 games used in the experiment.

Sequence 1	B	F	E	S	E+	B+	S+	F+	+F	+S	+B	+E	+S+	+E+	+F+	+B+
Sequence 2	+E	+B	+S	+F	+B+	+F+	+E+	+S+	S	E	F	B	F+	S+	B+	E+
Sequence 3	+F+	+S+	+B+	+E+	B+	F+	E+	S+	+S	+E	+F	+B	E	B	S	F
Sequence 4	S+	E+	F+	B+	F	S	B	E	+E+	+B+	+S+	+F+	+B	+F	+E	+S

Appendix D Translated Instructions

These are the instructions given to subjects during the experiment. Instructions for each part were presented separately on screen, at the beginning of each part. The original instructions were in German. Text in brackets [...] was not displayed to subjects.

General Instructions

Welcome to this economic experiment. Thank you for supporting our research.

Please note the following rules:

1. From now on until the end of the experiment, you are not allowed to communicate with each other.
2. If you have questions, please raise your hand and one of the instructors will answer your question individually.
3. Please refrain from using any features of the computer that are not part of the experiment.

The experiment consists of five parts and a questionnaire. The experiment involves a series of decisions which will affect your payoff at the end of the experiment. In this experiment you will earn points. At the end of the experiment the points you have earned in each part will be added up and the sum will be exchanged into Euros according to the following exchange rate:

$$10 \textbf{ points} = 25 \textbf{ Eurocents.}$$

Independently of your decisions, you will receive an additional 4 EUR for your participation in the experiment.

Part 1: [11-20 Games]

In this part you will make a series of 16 decisions. For each decision you will be randomly paired with another participant in the experiment. You will not meet the same participant more than once.

For each decision you will see 10 boxes in line on your screen. Each box contains a certain amount of points.

You have to choose one of the boxes.

Each participant will receive the amount in the box he/she selected. In addition, a participant may get a bonus if the selected box is exactly **one to the left** of the box that the other participant chooses.

The amount of points contained in each box may change from one round to another round. Below you can see an example for such a decision. Note that the amount contained in each box as well as the size of the bonus in the experiment will differ from this example. The size of the bonus and the amount of points contained in each box will be displayed in the following way:

Possible bonus: **30 points**

14	13	12	11	19	18	17	16	15	20

Part 2 [Beauty Contest Game]

In this part you and all other participants in this session will make one decision. You and all other participants each have to choose an integer between 0 and 100.

The participant who chose the number closest to the target number wins. All other participants do not win anything.

To determine the target number, the average of all chosen numbers will be computed and multiplied by 2/3 (in words: two thirds).

Target number= (2/3) * (Average of the numbers chosen by all participants)

The participant who chose the number closest to the target number wins and receives 500 points.

In case there is a tie between two or more participants (because all their numbers are equally close to the target number) the points are split equally among all winners.

Part 3 [Cognitive Reflection Test]

In this part you are asked to answer a series of questions. For each question there is exactly one correct answer.

If you answer the questions correctly, you can earn additional points. In total you have to answer 9 questions. For each correct answer you will receive 5 points.

Part 4 [Social Value Orientation]

In this part you have to make a series of decisions about allocating points between you and another randomly selected participant.

Henceforth, we will refer to this randomly selected participant simply as the "other."

In each of the following 6 decisions, you can choose how many points you would like to allocate to yourself and how many points you would like to allocate to the other.

Please select for each decision exactly one of the 9 available allocations. All amounts are displayed in points. Please take all decisions seriously, since each of the 6 decisions has the same probability of being selected.

You can receive additional points in case a decision of another participant is selected, where he has allocated points to you.

Part 5 [Strategic Uncertainty]

In this part you have to make 10 decisions for different decision situations. Each situation is independent of the other.

In each situation you can decide between A and B. The amount of points you will earn in this part depends on these decisions.

In this part, you and 3 other randomly selected participants will form a group.

There will be 10 decision situations displayed on your screen in a table. In each of the situations you can choose between option A and option B. At the end of this part, 1 out of the 10 situations will be chosen randomly. Your payment will be according to the situation picked and is determined as follows:

- If you choose option A, you will receive the sure payment given in the second column.
- If you choose option B, your payment will depend on how many members of your group (including yourself) chose B.
  - If 3 or more out of the 4 members of your group chose B, you will receive 50 points.
  - If 2 or less of the 4 members of your group chose B, you will receive 0 points.

References

Arad A, Rubinstein A (2012) The 11–20 Money Request Game: A Level-k Reasoning Study. American Economic Review 102(7):3561–3573

Hargreaves Heap S, Rojo Arjona D, Sugden R (2014) How Portable Is Level-0 Behavior? A Test of Level-k Theory in Games with Non-Neutral Frames. Econometrica 82(3):1133–1151