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Online Appendix: Is the Allais Paradox Due to Appeal of Certainty or Aversion to Zero?

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A Detailed results

In this appendix, we present the full results for all of our treatments. For detailed descriptions, see the main article.

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Pattern		Frequency
SSSSSS (EU)		11
RRRRRR (EU)		7
RSSSSS (ZE)		4
SRRRRR		1
RSSSRR		1
RSSRSS		1
RSRRRR		1
RRSSRS		1
RRRSRR		1
ISRRSS		1
IRSISS		1
IRRRRR		1
IRRRRI		1
Consistent with EU		
	All observations	18^{***}
	Excluding indifferences	18^{***}
Consistent with ZE		
	All observations	4^{***}
	Excluding indifferences	4^{***}
Consistent with CE		0

Table 1: Results: Small main treatment, real incentives, table presentation. $n = 32$, 28 excluding indifferences. ${}^{*}p < 0.1, {}^{**}p < 0.05, {}^{***}p < 0.01$

Pattern		Frequency
SSSSSS (EU)		4
RRRRRR (EU)		7
RSSSSS (ZE)		3
ISSSSS (ZE)		1
RSRRSS		2
SSRRSS		1
SRRRRR		1
RSSRSS		1
RSSRRS		1
RSRSRS		1
RRRSSS		1
RRRSRR		1
RRRSRI		1
RRRRSS		1
IRSRSS		1
IRRRRS		1
IIRRSS		1
Consistent with EU		
	All observations	11^{***}
	Excluding indifferences	11^{***}
Consistent with ZE		
	All observations	4^{***}
	Excluding indifferences	3^{***}
Consistent with CE		0

Table 2: Results: Small main treatment, real incentives, narrative presentation. $n = 29$, 24 excluding indifferences. $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$

Pattern	Frequency	
SSSSSS (EU)		1
RRRRRR (EU)		20
RSSSSS (ZE)		4
ISSSSS (ZE)		1
SSSRSR		1
SRSSRR		1
SRRRRR		1
RSSSRS		1
RSSRSS		1
RISIIR		1
RIIRRR		1
RIIIIR		1
ISSRRR		1
Consistent with EU		
	All observations	21^{***}
	Excluding indifferences	21^{***}
Consistent with ZE		
	All observations	5^{***}
	Excluding indifferences	4^{***}
Consistent with CE		0

Table 3: Results: Small main treatment, hypothetical incentives, table presentation, $n = 35$, 30 excluding indifferences. $^{*}p < 0.1,^{**} p < 0.05,^{***} p < 0.01$

Pattern		Frequency
SSSSSS (EU)		18
RRRRRR (EU)		9
IIIIII (EU)		1
RSSSSS (ZEP)		1
ISSSSS (ZEP)		2
ISRSSS		2
SSSSRR		1
SSSRSS		1
SSRRRR		1
SRSSRS		1
SRRSSS		1
RSRSSS		1
RSRRSS		1
RSIRSS		1
RRSSSS		1
RRRSSS		1
RRRRSS		1
RRRRRS		1
RIRRSS		1
ISSRSS		1
IRSSSS		1
IRSRSR		1
IIRRRR		1
Consistent with EU		
	All observations	28^{***}
	Excluding indifferences	27^{***}
Consistent with ZEP		
	All observations	3^{**}
	Excluding indifferences	1
Consistent with CE		0

Table 4: Results: Small robust c treatment, real incentives, table presentation, $\mathbf{c_1} = \$\mathbf{1}$.

$n = 50$, 39 excluding indifferences. $^{*}p < 0.1,^{**}p < 0.05,^{***}p < 0.01$

Pattern		Frequency
SSSSSS (EU)		11
RRRRRR (EU)		16
RSSSSS (ZEP)		1
ISSSSS (ZEP)		1
RRSRRR (CE)		2
SSSSRS		2
RRRSRR		2
SSRRRR		1
SRSSSR		1
SRSIRR		1
SRRSRR		1
SRIRIR		1
SIRSSS		1
SIIIIR		1
RSSRRR		1
RSRSSS		1
RSRSRS		1
RRRRSR		1
RIRSRS		1
ISISSI		1
ISIRRR		1
IIIISS		1
Consistent with EU		
	All observations	27^{***}
	Excluding indifferences	27^{***}
Consistent with ZEP		
	All observations	2
	Excluding indifferences	1
Consistent with CE		
	All observations	2
	Excluding indifferences	2

Table 5: Results: Small robust ℓ treatment, hypothetical incentives, table presentation, $\ell = \$\mathbf{1}$. $n = 50$, 41 excluding indifferences. $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$

Pattern	Frequency
SSS (EU)	24
RRR (EU)	51
RSS (ZE)	30
RSR (CE)	10
SRR	16
RRS	13
SRS	5
SSR	4
Consistent with EU	75^{***}
Consistent with ZE	30^{***}
Consistent with CE	10

Table 6: Results: Large main treatment, original Allais experiment with an additional task. $n = 153$. $^{*}p < 0.1,^{**}p < 0.05,^{***}p < 0.01$

Pattern	Frequency
SSS (EU)	25
RRR (EU)	50
RSS (ZE)	17
RSR (CE)	6
SRS	8
SSR	8
RRS	8
SRR	7
Consistent with EU	75^{***}
Consistent with ZE	17
Consistent with CE	6

Table 7: Results: Large robust c treatment, modified Allais experiment. $n = 129$. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$

	Including violations		Excluding violations	
	n=198		n=160	
	With zeros	Without Zeros	With zeros	Without zeros
Consistent with EU	96^{***} (48%)	96^{***} (48%)	89^{***} (56%)	86^{***} (54%)
Consistent with ZE	25^{**} (13%)	26^{**} (13%)	25^{***} (16%)	17 (11%)
Consistent with CE	16 (8%)	21 (11%)	12 (8%)	18^{*} (11%)
Reverse ZE (SRR)	22 (11%)	20 (10%)	13 (8%)	20^{**} (13%)
Reverse CE (SRS)	11 (6%)	11 (6%)	4 (3%)	5 (3%)
RRS	11	5	5	3
SSR	17	19	12	11

Table 8: Results: Triangle CCE main and robust, within-subject analysis full results: $n = 198$ including FOSD violations, $n = 160$ excluding FOSD violations. $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$.

	$\ell = 0$	$\ell = 1$
Consistent with EU	$80\%^{***}$	$78\%^{***}$
Common Ratio Pattern	13%	11%
Reverse Common Ratio Pattern	7%	11%
Common Ratio / (Common Ratio or Reverse)	$65\%^{**}$	50%
n	198	198

Table 9: Results, Triangle common ratio effect (CRE) treatments, lowest outcome 0 vs. lowest outcome $A\$1$. $^{*}p < 0.1, ^{**}p < 0.05, ^{***}p < 0.01$.

B Marschak-Machina Triangle

In Figure 1, we show the difference in the predictions of the certainty effect (on the left) and the zero effect (on the right). The black lines near the three corners of the triangle are parallel. Their endpoints correspond to lottery pairs that differ only in whether their common consequence is 0, \$8, or \$10.

The dashed lines represent the indifference curves an experimenter might estimate after observing the certainty effect or the zero effect, respectively. In both diagrams, the estimated indifference curve through the origin is steeper than the estimated indifference curve along the bottom edge of the triangle. This would arise because both the zero effect and the certainty effect would be consistent with a decision maker preferring lottery A (the certain lottery with a positive outcome) to B, while preferring lottery B' to A'. This pattern of behavior suggests indifference curves fanning out along the x-axis.

Under the certainty effect, the indifference curve through the origin is special in this respect. No other lottery is certain, and as the figure shows, the indifference curve running through A'' would also be expected to be flatter than the one running through A. The estimated indifference curves along the vertical axis, therefore, would appear to fan in, compared with the indifference curve through the origin.

By contrast, under the zero effect, what makes A special is not that it is at the origin, but rather that it is on the vertical axis. An estimated indifference curve through A'', therefore, would not necessarily be expected to fan in, compared with the indifference curve through the origin.

Under the additional assumption of no other departures from expected utility, we can say a bit more. The certainty effect would predict that the indifference curves through A' and A'' would have the same slope, and the one through A would be steeper. For the zero effect, under the further assumption that the decision maker's aversion to zero does not depend on other aspects of a lottery, there would likewise be stronger implications. In this case, the zero effect would predict that the indifference curves through A and A'' would have the same slope, and the one through A' would be flatter.

An experimenter willing to make these additional assumptions would therefore have an alternative way to compare whether participants behave according to the certainty effect or the zero effect. The experiment would involve finding a lottery off the vertical axis which the participant views as equally valuable as the certain lottery at the origin. A line between these two lotteries would provide a linearly interpolated indifference curve. Next, the experimenter would pick one or more lotteries on the vertical axis, and for each, would find a lottery off the vertical axis which the participant views as equally valuable. Lines between each of these pairs would give linearly interpolated indifference curves along the vertical axis. Finally, the experimenter would pick one or more pairs of lotteries for which neither is on the vertical axis (and neither of which is part of any of the other lottery pairs). Interpolate the indifference curves for these as well. Under the certainty effect, the interpolated indifference curve through the origin would be steeper than all the other interpolated indifference curves. Under the zero effect, the interpolated indifference curves along the vertical axis would be steeper than all the other indifference curves.

It may be useful to compare how each of these effects predicts indifference curves change with respect to a small shift in the location of a lottery pair. The certainty effect has implications only for a lottery pair in which one lottery is at the origin. A small shift, in any direction, should lead to a flattening of indifference curves. By contrast, the zero effect has implications for lottery pairs in which one lottery is located on the y-axis. There would not be any prediction about the effects of a vertical shift. However, a small rightward shift should lead to a flattening of indifference curves.

Additional tests in a triangle design would be possible but would require further assumptions. For example, increasing the smallest outcome in a triangle from 0 (say, to $1, as we discuss in the main paper) potentially creates an ability to compare across triangles. In a triangle with a zero lowest outcome, the certainty effect is a special case of the zero effect. In a triangle with a $1 lowest outcome, the zero effect cannot occur, but the certainty effect still can. A reduction in the frequency of the certainty effect across the triangles would, therefore, provide indirect evidence for the zero effect.

However, the change in the lowest outcome improves every lottery off the vertical axis in the sense of first-order stochastic dominance, introducing a confound. Moreover, this type of test would be an indirect measure of the importance of the zero effect, through its impact on the apparent certainty effect, and involves changing two aspects of the lotteries simultaneously. We view direct tests as more appropriate. Introducing a fourth outcome avoids these difficulties, and enables us to test the two effects on an equal footing. We lose some of the beauty of the triangle representation, but in exchange we obtain a direct test that varies only one part of each lottery at a time.

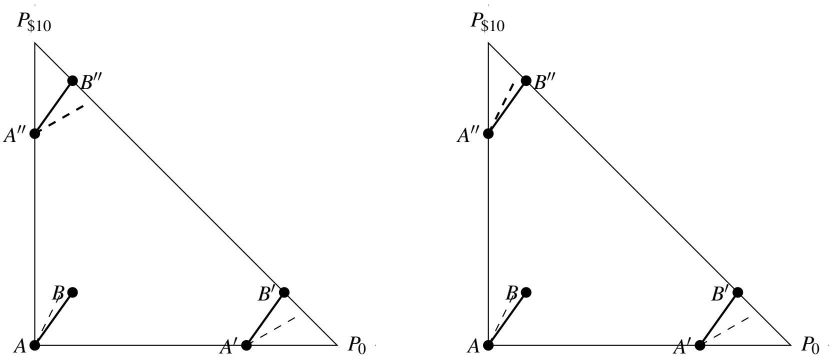

Fig. 1 The solid lines connect pairs of lotteries. Under the certainty effect (triangle on the left), the interpolated indifference curves (dashed lines) are flatter away from the origin than through the origin. Under the zero effect (triangle on the right), indifference curves are flatter away from the vertical edge than through the vertical edge.

To illustrate, we found four studies that examine lottery pairs in the three corners of the triangle. We show summary statistics in Table 10. The percentage of safe choices provides a proxy for the slope of the indifference curves. By examining the three corners, these studies give us a way to estimate the slope of an indifference curve at the origin, the slope of one away from the origin but on the vertical axis (the top corner), and the slope of one that is away from the vertical axis (the right corner).

Two of the prior studies, Camerer's (1989) Study 1 and Sopher and Gigliotti (1993), find a larger share of safe choices for the origin lottery pairs, compared with the pairs near the right corner, which is evidence for the common consequence effect. The other two studies, (Camerer's (1989) Study 2 and Starmer, 1992) do not find the common consequence effect, and cannot provide us with much direction. Among the two studies that find the common consequence effect, the proxies for slopes of indifference curves provide support for the zero effect.

In Camerer, Study 1, the share of safe choices for the top corner lottery pairs is very similar to the share of safe choices for the origin lottery pairs, and much higher than the share of safe choices for the right corner lottery pairs. This is consistent with zero effect, but not with certainty effect. Similarly, Sopher and Gigliotti (1993) finds a larger share of safe choices on the origin lottery pairs compared with the right corner lottery pairs, again implying common consequence effect. Furthermore, they find that the share of safe lotteries for the top corner lottery pairs, like the share at the origin, is higher than the share of safe choices for the right corner lottery pairs.

	Camerer Study 1 (1989)	Camerer Study 2 (1989)	Starmer (1992)	Sopher and Gigliotti (1993)
Top corner	73%	59%	44%	40%
Origin	71%	48%	49%	62%
Right corner	27%	46%	41%	9%

Table 10: Fraction of safe choices around the three edges of a triangle diagram

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