## Appendix A Proofs

*Characteristics of $F(w)$*

The following propositions characterize the set of allocations chosen by the clearing house with positive probability after the offer profile $w$, and are used in later proofs.

**Proposition 1** *For any $y \in F(w)$, if $y_b > 0$ and $y_s < 0$ for a buyer $b$ and a seller $s$, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

*Proof* Suppose $\widetilde{p}_b < \widetilde{p}_s$. Consider an alternative allocation $y'$ such that $y'_i = y_i$ if $i \neq b, s$ and $y'_b = y_b - 1$, $y'_s = y_s + 1$. Since $y \in F(w) \subseteq Y(w)$, we have $y' \in Y(w)$, and

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_s - \widetilde{p}_b) > \sum_{i \in I} y_i \widetilde{p}_i.$$

Then $y \notin F(w)$, contradicting the assumption.

□

**Proposition 2** *Given an offer profile $w$ and a buyer $b$ and a seller $s$ such that $\widetilde{p}_b \geq \widetilde{p}_s$, there cannot be an allocation $y \in F(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$.*

*Proof* For a given offer profile $w$ such that there is a buyer $b$ and a seller $s$ that $\widetilde{p}_b \geq \widetilde{p}_s$, suppose there is an allocation vector $y \in Y(w)$ such that $y_b < \widetilde{q}_b$ and $y_s > \widetilde{q}_s$. We can show that $y \notin F(w)$. Take an alternative allocation vector $y'$, let $y'_i = y_i$ if $i \neq b, s$, and $y'_b = y_b + 1$, $y'_s = y_s - 1$. We have $y' \in Y(w)$. The arbitrage profit for the clearinghouse by allocating $y'$ is

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_s) \geq \sum_{i \in I} y_i \widetilde{p}_i,$$

where the last term is the clearing house's profit if it allocates $y$. Therefore, if $\widetilde{p}_b > \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i < \sum_{i \in I} y'_i \widetilde{p}_i$ and $y \notin \Pi(w)$; and if $\widetilde{p}_b = \widetilde{p}_s$, then $\sum_{i \in I} y_i \widetilde{p}_i = \sum_{i \in I} y'_i \widetilde{p}_i$ but $y$ is ray dominated by $y'$. Either way we have $y \notin F(w)$.

□

**Proposition 3** *If, for a given offer profile $w$, seller $a$ and seller $b$ offer $\widetilde{p}_a < \widetilde{p}_b$, and seller $b$ is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$. Symmetrically, if in a given offer profile $w$, buyer $a$ and buyer $b$ offer $\widetilde{p}_a > \widetilde{p}_b$, and buyer $b$ is an active trader, then for all $y \in F(w)$ we have $y_a = \widetilde{q}_a$.*

*Proof* We will show the proof for the sellers' case, since the buyers' case is symmetric. By definition, if seller $b$ is an active trader, there exists an allocation $y^* \in F(w)$ such that $y^*_b < 0$. First we show that $y^*_a = \widetilde{q}_a$.

Suppose $y^*_a > \widetilde{q}_a$. Take an alternative allocation vector $y'$ given by $y'_a = y^*_a - 1$, $y'_b = y^*_b + 1$, and $y'_i = y^*_i$ for $i \neq a, b$. It is easy to see that $y' \in Y(w)$. The profit for the clearing house by allocating $y'$ equals

$$\sum_{i \in I} y'_i \widetilde{p}_i = \sum_{i \in I} y^*_i \widetilde{p}_i + (\widetilde{p}_b - \widetilde{p}_a) > \sum_{i \in I} y^*_i \widetilde{p}_i.$$

The last term is the profit of the clearing house if $y^*$ is allocated. Hence $y^* \notin \Pi(w)$, so $y^* \notin F(w)$. Therefore, if $y^*_b < 0$ and $y^* \in F(w)$, we must have $y^*_a = \widetilde{q}_a$. By the same argument, we must have $y_a = \widetilde{q}_a$ for every allocation $y \in F(w)$ such that $y_b < 0$.

Now suppose there is an allocation $\hat{y} \in F(w)$ such that $\hat{y}_b = 0$ and $\hat{y}_a > \widetilde{q}_a$. According to the result in the first part of the proof, for any seller $h$ that offers $\widetilde{p}_h > \widetilde{p}_a$, $\hat{y}_h = 0$, otherwise $\hat{y} \notin F(w)$. Hence

$$\sum_{\{h \in S: \widetilde{p}_h > \widetilde{p}_a\}} \hat{y}_h \;=\; 0 \;>\; y^*_b \;\geq\; \sum_{\{h \in S: \widetilde{p}_h > \widetilde{p}_a\}} y^*_h.$$

According to the first part of the proof, $y^*_i = \widetilde{q}_i$ for $i \in S$ if $\widetilde{p}_i < \widetilde{p}_b$. Since $\widetilde{p}_b > \widetilde{p}_a$, we have

$$\sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} \hat{y}_h \;\geq\; \sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} \widetilde{q}_h \;=\; \sum_{\{h \in S: \widetilde{p}_h \leq \widetilde{p}_a\}} y^*_h.$$

Therefore,

$$\sum_{i \in B} y^*_i = -\sum_{i \in S} y^*_i > -\sum_{i \in S} \hat{y}_i = \sum_{i \in B} \hat{y}_i.$$

Since $\sum_{i \in B} \hat{y}_i < \sum_{i \in B} y^*_i$, there exists at least one buyer, say buyer $e$, such that $0 \leq \hat{y}_e < y^*_e \leq \widetilde{q}_e$. Since $y^*_e > 0$ and $y^*_a < 0$, from proposition 1 we have $\widetilde{p}_e \geq \widetilde{p}_a$. Therefore $\widetilde{p}_e \geq p_a$, $\hat{y}_a > \widetilde{q}_a$, and $\hat{y}_e < \widetilde{q}_e$, violating proposition 2. □

**Proposition 4** *Given an offer profile w, if buyer $b \in AB(w)$ and seller $s \in AS(w)$, then $\widetilde{p}_b \geq \widetilde{p}_s$.*

*Proof* Since $b \in AB(w)$ and $s \in AS(w)$, there must be some $y, y' \in F(w)$ such that $y_b > 0$ and $y'_s < 0$. If $y = y'$, the desired result follows from Proposition 1. Suppose $y \neq y'$. Since $y, y' \in \Pi(w)$, we have

$$\sum_{i \in AB(w)} y_i \widetilde{p}_i + \sum_{i \in AS(w)} y_i \widetilde{p}_i = \sum_{i \in AB(w)} y'_i \widetilde{p}_i + \sum_{i \in AS(w)} y'_i \widetilde{p}_i.$$

Suppose $\widetilde{p}_b < \widetilde{p}_s$. From Proposition 1, we have $y_s = 0$ and $y'_b = 0$. From proposition 3, then, there is no active seller submitting a price higher than $p_s$, and no active buyer submitting a price lower than $p_b$. Denote by $\overline{AB}$ the set of active buyers that offer $p_b$, and $\underline{AS}$ the set of active sellers that offer $p_s$. From proposition 3, for $i \in AB(w) \backslash \overline{AB}$ and $i \in AS(w) \backslash \underline{AS}$,

$$y'_i = y_i = \widetilde{q}_i.$$

Thus,

$$\sum_{i \in \overline{AB}} y_i p_b + \sum_{i \in \underline{AS}} y_i p_s = \sum_{i \in \overline{AB}} y'_i p_b + \sum_{i \in \underline{AS}} y'_i p_s,$$

which is equivalent to

$$p_b \cdot \left( \sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i \right) = p_s \cdot \left( \sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i \right).$$

Given $\widetilde{p}_b < \widetilde{p}_s$, the equation above implies either

$$\sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i = \sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i = 0$$

or

$$\sum_{i \in \overline{AB}} y_i - \sum_{i \in \overline{AB}} y'_i > \sum_{i \in \underline{AS}} y'_i - \sum_{i \in \underline{AS}} y_i.$$

Since $y_b > 0$, in the first case there must be some buyer $c$ such that $y'_c > 0$ and $\widetilde{p}_c = \widetilde{p}_b < \widetilde{p}_s$. But since $y'_s < 0$, proposition 1 implies $\widetilde{p}_c \geq \widetilde{p}_s$, a contradiction.

In the second case we have

$$\sum_{i\in\overline{AB}} y_i + \sum_{i\in\underline{AS}} y_i > \sum_{i\in\underline{AS}} y'_i + \sum_{i\in\overline{AB}} y'_i,$$

which implies

$$\sum_{i\in I} y_i > \sum_{i\in I} y'_i.$$

But since $y, y' \in F(w) \subseteq Y(w)$, we have $\sum_{i\in I} y_i = \sum_{i\in I} y'_i = 0$, a contradiction.

□

**Proposition 5** *If $y \in F(w)$, then either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$ or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$. Furthermore, if there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AS(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AS(w)$. Symmetrically, if there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AB(w)$, then $y_i = \widetilde{q}_i$ for all $y \in F(w)$ for all $i \in AB(w)$.*

*Proof* For the first part, from proposition 4, if buyer $b$ and seller $s$ are active, we have $p_b \geq p_s$. Thus, from proposition 2, there is no $y \in F(w)$ such that $y_s > \widetilde{q}_s$ and $y_b < \widetilde{q}_b$. Therefore, for every $y \in F(w)$, either $y_i = \widetilde{q}_i$ for all $i \in AS(w)$, or $y_i = \widetilde{q}_i$ for all $i \in AB(w)$.

For the second part, we show the proof for the active sellers' case, since the buyers' case is symmetric. Suppose there is $y^* \in F(w)$ such that $y^*_i = \widetilde{q}_i$ for all $i \in AS(w)$, and $y' \in F(w)$ such that $y'_a > \widetilde{q}_a$ for some $a \in AS(w)$. Then

$$\sum_{i\in B} y^*_i = -\sum_{i\in S} y^*_i > -\sum_{i\in S} y'_i = \sum_{i\in B} y'_i.$$

Therefore there must be an active buyer, say $b$, such that $y'_b < y^*_b \leq \widetilde{q}_b$. From proposition 4, we have $\widetilde{p}_b \geq \widetilde{p}_a$. But, from proposition 2, $y'_a > \widetilde{q}_q$ and $y'_b < \widetilde{q}_b$ imply $\widetilde{p}_b < \widetilde{p}_a$, a contradiction.

□

**Proposition 6** *If seller a offers $(p, \widetilde{q}_a)$ in offer profile w and $a \in AS(w)$, then seller b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b < 0$ is also an active seller. Symmetrically, if buyer a offers $(p, \widetilde{q}_a)$ in strategy profile w and $a \in AB(w)$, then buyer b who offers $(p, \widetilde{q}_b)$ where $\widetilde{q}_b > 0$ is also an active buyer.*

*Proof* We show the proof for the sellers' case, since the buyers' case is symmetric. Suppose $y_b = 0$ for all $F(w)$. Provided seller $a$ is an active seller, there exists $y \in F(w)$ such that $y_a > 0$. Consider an allocation vector $y'$ that $y'_i = y_i$ for $i \neq a, b$ and $y'_a = y_a - 1$, $y'_b = 1$. It's easy to see that $y' \in Y(w)$. The profit for the clearing house if $y'$ is allocated is equal to

$$\sum_{i\in I} y'_i \widetilde{p}_i = \sum_{i\in I} y_i \widetilde{p}_i + p - p = \sum_{i\in I} y_i \widetilde{p}_i,$$

so that $y' \in \Pi(w)$. Thus, either $y' \in F(w)$ or there is $y'' \in F(w)$ such that $y'' \neq y'$ and $|y''_i| \geq |y'_i|$ for all $i$, so that in either case $b$ is an active trader, contradicting the assumption. Therefore, as long as seller $a$ is an active seller, so is seller $b$.

□

*Proof of Theorem 1*

Suppose $(p,q)$ is a competitive equilibrium. First we claim that the offer profile $w=((p,q_i))$ induces the same outcome with probability one. To see this, since $\widetilde{p}_i = p$ for all $i$, the arbitrage profit for the clearing house is 0 for each $y \in Y(w)$, so that $\Pi(w) = Y(w)$. Clearly $q \in Y(w)$ since by definition of a competitive equilibrium $q_i \in Q_i$ and $\sum_i q_i = 0$. Moreover, by definition of $Y(w)$, for every $y \in Y(w)$ we have $|y_i| \leq |q_i|$. Hence $q$ ray-dominates any other allocation in $\Pi(w)$ and is the unique element of $F(w)$. Thus, $w$ induces the outcome $(q,(-pq_i))$ with probability one. This is precisely the outcome induced by the competitive equilibrium.

Next, we show that no individual $i$ has an incentive to deviate from the offer profile $w=((p,q_i))$. We consider deviations for buyers, since the proof for sellers is symmetric. We classify possible individual deviations for $i \in B$ from $w$ into three categories, and show that none of them is profitable.

(i) Consider $w_i' = (p,q_i')$ such that $Q_i \ni q_i' \neq q_i$. In any outcome with positive probability after that deviation, the utility for $i$ is $u_i(y,-py)$ for some $y \in Q_i$. Since $q_i \in \arg\max_{q \in Q_i} u_i(q,-pq)$, we have that the expected utility after the deviation cannot be larger.

(ii) Consider $w_i' = (p_i',q_i')$ such that $q_i' \in Q_i$ and $p_i' < p$. Since every seller $s \in S$ is asking $\widetilde{p}_s = p$, by proposition 1 we must have that in any outcome with positive probability after that deviation $y_i = 0$. But then the expected utility after the deviation is 0, and since $q_i \in \arg\max_{q\in Q_i} u_i(q,-pq)$, we have $u_i(q_i,-pq_i) \geq u_i(0,-p\times 0) = 0$.

(iii) Consider $w_i' = (p_i',q_i')$ such that $q_i' \in Q_i$ and $p_i' > p$. Denote by $w'$ the new offer profile. For any $y' \in F(w')$, buyer $i$ gets a payoff of $u_i(y_i',-p_i'y_i')$. Note that $u_i(y_i',-p_i'y_i') < u_i(y_i',-py_i') \leq u_i(q_i,-pq_i)$, where the first inequality follows from $p_i' > p$ and the fact that $u_i(q,-pq)$ is decreasing in $p$, and the second from $q_i \in \arg\max_{q\in Q_i} u_i(q,-pq)$. It follows that $Eu_i(w') < Eu_i(w)$.

*Proof of Theorem 2*

The proof comes in several steps. Note that the 'thick market' condition (at least two active traders in the Nash equilibrium) is invoked only in the last step.

**Lemma 1** *In each Nash equilibrium, all active sellers offer the same price, and all active buyers offer the same price.*

*Proof* We prove the result for the sellers; the proof for the buyers is analogous. Take any offer profile $w^*$ such that two active sellers offer different prices $\overline{p}$ and $\underline{p}$, with $\overline{p} > \underline{p}$. We can claim that the seller, say trader $l$, who offers $(\underline{p},\widetilde{q}_l)$ would be better off submitting $(p',\widetilde{q}_l)$ such that $\underline{p} < p' < \overline{p}$.

To see this, from proposition 3, since there is another active seller offering the price $\overline{p}$, seller $l$ sells $|\widetilde{q}_l|$ units when she offers $w_l^* = (\underline{p},\widetilde{q}_l)$. We can show that seller $l$ sells $|\widetilde{q}_l|$ units as well when she offers $w' = (p',\widetilde{q}_l)$. Suppose there is $y' \in F(w')$ such that $y_l' > \widetilde{q}_l$. As in the last step of the proof of proposition 3, for any $y \in F(w^*)$ we have

$$\sum_{i\in B} y_i = -\sum_{i\in S} y_i > -\sum_{i\in S} y_i' = \sum_{i\in B} y_i'.$$

Therefore there must be an active buyer at $w^*$, say $b$, such that $y_b' < y_b \leq \widetilde{q}_b$. From proposition 4, we have $\widetilde{p}_b \geq \overline{p}$. But, from proposition 2, at profile $w'$ we have $y_l' > \widetilde{q}_l$ and $y_b' < \widetilde{q}_b$ implying $\widetilde{p}_b < p' < \overline{p}$, a contradiction.

Thus, by offering $(p',\widetilde{q}_l)$, seller $l$ gets

$$u_l(\widetilde{q}_l,-p'\widetilde{q}_l) = -p'\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj} > -\underline{p}\widetilde{q}_l - \sum_{j=1}^{|\widetilde{q}_l|} r_{lj},$$

where the last term is the payoff seller $l$ gets by offering $(\underline{p}, \widetilde{q}_l)$. Hence the seller gets better off by offering $(p', \widetilde{q}_l)$, so that $w^*$ cannot be a Nash equilibrium.

□

**Lemma 2** *In each Nash equilibrium, all active traders offer the same price.*

*Proof* Consider an offer profile $w^*$ such that there is trade and such that all active buyers offer the same price, say $p_b$, and all active sellers offer the same price, say $p_s$. From lemma 1, we know that only such profiles, or some profiles such that there is no trade, can be Nash equilibria. By proposition 4, we have $p_s \leq p_b$. We will show that if $p_s < p_b$, at least one active trader has an incentive to deviate, so that $w^*$ cannot be a Nash equilibrium.

If $p_s < p_b$, following proposition 5, we have that either $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$, or $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AB(w^*)$. Suppose $y_i = \widetilde{q}_i$ for all $y \in F(w^*)$ for all $i \in AS(w^*)$ (the argument for the other case is analogous). Note that if there are inactive sellers in $w^*$, for any such seller $h$ we have $\widetilde{p}_h > p_s$ or $\widetilde{q}_h = 0$. If $\widetilde{p}_h < p_s$, then following proposition 3 we have $y_h = \widetilde{q}_h$ for all $y \in F(w^*)$, so the seller can be inactive only if $\widetilde{q}_h = 0$. If $\widetilde{p}_h = p_s$, according to proposition 6 there must be some $y \in F(w^*)$ that $y_h < 0$ unless $\widetilde{q}_h = 0$. Denote by $p_{\neg s}$ the lowest price offered with a non-zero quantity by inactive sellers, if there is any, and note that in that case $p_{\neg s} > p_s$.

We claim that if $p_s < p_b$, an active seller, say $a$, would have an incentive to deviate from $w^*_a = (p_s, \widetilde{q}_a)$ to $w'_a = (p', \widetilde{q}_a)$, where $p' \in (p_s, \min\{p_b, p_{\neg s}\})$ if there are non-zero quantity inactive sellers, and $p' \in (p_s, p_b)$ otherwise.

To prove the claim, we argue first that for any $y' \in F(w')$ we have $y'_a = \widetilde{q}_a$. Suppose there is a $y' \in F(w')$ such that $y'_a > \widetilde{q}_a$. Then, from proposition 3, inactive sellers at $w^*$ remain so at $w'$ since $p_{\neg s} > p'$. Therefore $\sum_{i \in S} y'_i > \sum_{i \in S} y_i$ for every $y \in F(w^*)$. Thus, for any $y \in F(w^*)$,

$$\sum_{i \in B} y'_i = -\sum_{i \in S} y'_i < -\sum_{i \in S} y_i = \sum_{i \in B} y_i.$$

Therefore there must be an active buyer at $w^*$, say $h$, who offers $p_b$ and gets $y'_h < y_h \leq \widetilde{q}_h$. But, from proposition 2, at profile $w'$ we have $y'_a > \widetilde{q}_a$ and $y'_h < \widetilde{q}_h$ implying $p_b < p'$, a contradiction.

From the previous argument, by offering $w'$ instead of $w^*$, seller $a$ is allocated $\widetilde{q}_a$, and gets a utility of

$$u_a(\widetilde{q}_a, -p'\widetilde{q}_a) = -p'\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} > -p_s\widetilde{q}_a - \sum_{j=1}^{|\widetilde{q}_a|} r_{aj} = u_a(\widetilde{q}_a, -p_s\widetilde{q}_a).$$

Hence seller $a$ gets better off by offering $w'$, so that $w^*$ cannot be a Nash equilibrium.

□

**Lemma 3** *In each Nash equilibrium, every trader is indifferent between all outcomes that occur with positive probability.*

*Proof* Consider an offer profile $w^*$ such that all active traders, if there is any, offer the same price, say $p$. From lemma 2, we know that only such profiles can be Nash equilibria if trades happen with positive probability. Take trader $a$, a seller, for example. If seller $a$ is inactive, then her utility is 0 for any positive probability outcome. Suppose $a$ is active, and moreover there are $y, y'' \in F(w^*)$ that $u_a(y_a, -py_a) > u_i(y''_a, -py''_a)$. We can show that in this case, $w$ cannot be a Nash equilibrium.

Since $F(w^*)$ is finite, there is some $y^* \in F(w^*)$ such that $u_a(y^*_a, -py^*_a) \geq u_a(y_a, -py_a)$ for all $y \in F(w^*)$ and moreover $u_a(y^*_a, -py^*_a) > u_a(y''_a, -py''_a)$. Since $y''$ has positive probability, $u_a(y^*_a, -py^*_a) > Eu_a(w)$. By continuity, there is some $p' < p$ such that $u_a(y^*_a, -py^*_a) > u_a(y^*_a, -p'y^*_a) > Eu_a(w)$.

We claim that if seller $a$ offers $w'_a = (y^*_a, p')$, then $y'_a = y^*_a$ for every $y' \in F(w')$, so that the seller obtains $u_a(y^*_a, -p'y^*_a)$ which is a profitable deviation from $w^*$ by the inequality above. The claim implies that $w^*$ cannot be a Nash equilibrium. To verify the claim, suppose first that there is another seller $h$ that is active at $w'$; since seller $h$ offers the price $p > p'$, the claim follows from proposition 3. Suppose that no other seller is active at $w'$, then if $y'_a > y^*_a$ we get for any $y' \in F(w')$,

$$\sum_{i\in B} y'_i = -\sum_{i\in S} y'_i < -\sum_{i\in S} y^*_i = \sum_{i\in B} y^*_i.$$

Then there must be some buyer, say $b$, such that $y'_b < y^*_b \leq \tilde{q}_b$. Since there is also a seller, seller $a$, such that $y'_a > y^*_a$ and moreover this seller offers a price $p'$ below the price offered by the buyer, we get a contradiction with proposition 2.

□

**Lemma 4** *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all non-active traders are allocated utility-maximizing quantities.*

*Proof* In the proof of this and the following lemmas, let $w^*$ be a Nash equilibrium with active trading, and (invoking lemma 2) let $p^*$ be the price offered by all active traders. We focus on sellers; the proof for the buyers is analogous.

As shown in the second paragraph of lemma 2, non-active sellers offer $\tilde{p}_i > p^*$ or $\tilde{q}_i = 0$. Therefore, they get $y_i = 0$ for all $y \in F(w^*)$, and thus obtain $Eu_i(w^*) = 0$. We claim that for inactive sellers, $y_i = 0$ is utility-maximizing given price $p^*$. Equivalently, we claim that $r_{i1} \geq p^*$.

To see this, suppose trader $i$ is an inactive seller and $r_{i1} < p^*$. Consider a deviation for $i$ to $w'_i = (p^*, -1)$. By proposition 6, if seller $i$ is inactive under the offer profile $w'$, so is every seller in $AS(w^*)$ under the offer profile $w'$, and by proposition 3 so is every seller. But this would violate proposition 2, since there are trades in each side of the market active under $w^*$ and thus offering $p^*$ should induce positive probability to trade. Hence, there exists $y' \in F(w')$ such that $y'_i = -1$. Since $u_i(y'_i, -p^*y'_i) = p^* - r_{i1} > 0$, by deviating to offer $(p^*, -1)$, agent $i$ would have $Eu_i(w') > 0$, so that $w^*$ would not be a Nash equilibrium.

□

**Lemma 5** *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, all active traders are allocated quantities that are either utility-maximizing or involve less in absolute value than the utility-maximizing trade.*

*Proof* For a given active seller, say $s$, let $\underline{\delta}_s$ and $\overline{\delta}_s$ be the minimal and the maximal element, respectively, of the set $\arg\max_{q_s \in Q_s} u_s(q_s, -p^*q_s)$, so that $-k \leq \underline{\delta}_s \leq \overline{\delta}_s \leq 0$. From the utility maximization problem, it follows that every $x \in Q_s$ such that $\underline{\delta}_s \leq x \leq \overline{\delta}_s$ is also a utility maximizer.

We claim that for every $y \in F(w^*)$ we have $y_s \geq \underline{\delta}_s$ so that either the seller is allocated an optimal trade or a smaller (in absolute value) than optimal trade. For suppose there is $y \in F(w^*)$ such that $y_s < \underline{\delta}_s$ so that $u_s(y_s, -p^*y) < u_s(\underline{\delta}_s, -p^*\underline{\delta}_s)$. If $\underline{\delta}_s = 0$ or $p^* = 0$, it follows that $u_s(y_s, -p^*y) < 0$, and by lemma 3, $Eu_s(w^*) < 0$. But then trader $s$ can deviate to $(p^*, 0)$ and guarantee an expected utility of zero, so that $w^*$ cannot be a Nash equilibrium. Suppose instead that $\underline{\delta}_s < 0$ and $p^* > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^*y) < u_s(\underline{\delta}_s, -p'\underline{\delta}_s) < u_s(\underline{\delta}_s, -p^*\underline{\delta}_s).$$

Now consider a deviation by $s$ to $w'_s = (p', \underline{\delta}_s)$. We show that such deviation guarantees $y'_s = \underline{\delta}_s$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(\underline{\delta}_s, -p'\underline{\delta}_s) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that

$y'_s > \underline{\delta}_s$. Since $p' < p^*$, and all other sellers offer a price equal or larger than $p^*$ or a quantity equal to zero, it follows from proposition 3 that for all other $i \in S$ we have $y'_i = 0$. Therefore

$$\sum_{i\in B} y_i = -\sum_{i\in S} y_i \geq -y_s > -\underline{\delta}_s = -\sum_{i\in S} y'_i = \sum_{i\in B} y'_i.$$

But then there must be a buyer, say $a$, such that $y'_a < y_a \leq \tilde{q}_a$ offering price $p^* > p'$, contradicting proposition 2.

□

**Lemma 6** *In every positive probability outcome of a Nash equilibrium with active trading, given the price offered by all active traders, if there are two or more active traders on the same side of the market, then all traders on this side of the market are allocated utility-maximizing quantities.*

*Proof* We claim that if there are at least two active sellers, then every $y \in F(w^*)$ satisfies $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$ and is therefore a utility maximizer.

In lemma 5 we have shown in every positive probability allocation $y$, active sellers are allocated quantities that are either utility-maximizing given the price or involve less trade ($\underline{\delta}_s \leq y_s \leq 0$) so we need only focus on active sellers.

Now suppose there are two active sellers, say $s$ and $h$. If $y_s < \underline{\delta}_s$ for any $y \in F(w^*)$, we have that $w^*$ cannot be a Nash equilibrium by the previous step. If $\underline{\delta}_s \leq y_s \leq \overline{\delta}_s$, the claim follows from lemma 3. In the last part of this proof, we show that if there is a $y \in F(w^*)$ such that $y_s > \overline{\delta}_s$, $w^*$ cannot be a Nash equilibrium.

Since $|y_s| < |\overline{\delta}_s|$, from the utility maximization problem we must have $r_{|y_s|+1} < p^*$. Hence $u_s(y_s - 1, -p^*(y_s - 1)) - u_s(y_s, -p^* y_s) = p^* - r_{|y_s|+1} > 0$. By continuity, there is some $p' < p^*$ such that

$$u_s(y_s, -p^* y) < u_s(y_s - 1, -p'(y_s - 1)) < u_s(y_s - 1, -p^*(y_s - 1))).$$

Also, for any $y, y'' \in F(w^*)$, we have $y_s = y''_s$. Suppose there exists $y, y'' \in F(w^*)$ such that $y_s < y''_s$, then

$$u_s(y''_s, -p^* y''_s) - u_s(y_s, -p^* y_s) = -p^*(y''_s - y_s) + \sum_{j=|y''_s|+1}^{|y_s|} r_{sj} < 0,$$

contradicting lemma 3.

Now consider a deviation by $s$ to $w'_s = (p', y_s - 1)$. We show that such deviation guarantees $y'_s = y_s - 1$ for all $y' \in F(w')$, so that by Lemma 3, $Eu_s(w') = u_s(y_s - 1, -p'(y_s - 1)) > Eu_s(w^*)$. To see this, suppose there is some $y' \in F(w')$ such that $y'_s > y_s - 1$. Since $p' < p^*$, and all other sellers offer a price equal or larger than $p^*$ or a quantity equal to zero, it follows from proposition 3 that for all other $i \in S$ we have $y'_i = 0$. Therefore, take any $y'' \in F(w^*)$ such that $y''_h < 0$,

$$\sum_{i\in B} y''_i = -\sum_{i\in S} y''_i \geq -y_s - y''_h \geq -y_s + 1 > -y'_s = -\sum_{i\in S} y'_i = \sum_{i\in B} y'_i$$

But then there must be a buyer, say $a$, such that $y'_a < y''_a \leq \tilde{q}_a$ offering price $p^* > p'$, contradicting proposition 2.

□

Since the market clearing condition in the equilibrium definition is satisfied by any allocation induced by any offer profile, theorem 2 follows from lemma 6.

*Properties of $v_b$ and $v_s$*

**Lemma 7** *In every competitive equilibrium $(p,q) \in \xi(r)$ that contains the smallest number of transactions, the lowest reservation value of buyers' traded unit(s) is equal to $v_b$, and the highest reservation value of sellers' traded unit(s) is equal to $v_s$.*

*Proof* We show the proof for $v_b$; the proof for $v_s$ is analogous. By definition of $v_b$, there is a competitive equilibrium $(\hat{p},\hat{q})$ such that every unit bought has a buyer's valuation greater than or equal to $v_b$. Suppose there is a competitive equilibrium $(\tilde{p},\tilde{q})$ such that a buyer, say $i \in B$, buys a unit with valuation strictly below $v_b$. Then it must be the case that $\tilde{p} < v_b$. But then we have that $\tilde{q}_i > \hat{q}_i$ and for every $j \in B \setminus \{i\}$, $\tilde{q}_j \geq \hat{q}_j$, so that strictly more units are traded in $(\tilde{p},\tilde{q})$ than in $(\hat{p},\hat{q})$.

□

*Proof of Theorem 3*

First we prove the condition in the statement of the theorem is sufficient. Suppose $w^*$ is a Nash equilibrium with active trading, and suppose there are at least two inframarginal sellers and at least two weakly inframarginal buyers. (The other case is analogous.) From lemma 1 and lemma 2, all active traders offer the same price, say $p^*$. Denote $\underline{\delta}_i$ and $\overline{\delta}_i$ the minimal and maximal element, respectively, of the set $\arg\max_{q_i \in Q_i} u_i(q_i, -p^* q_i)$. From lemma 5, for any $y \in F(w^*)$, we have $\underline{\delta}_i \leq y_i \leq 0$ for every active seller $i$, and $0 \leq y_i \leq \overline{\delta}_i$ for every active buyer $i$, and moreover from lemma 4, non-active traders acquire utility-maximizing quantities given $p^*$. That is, no one trades in excess of their utility-maximizing quantity.

Consider first the case $p^* > v_s$. We claim that every inframarginal seller must be active. For suppose an inframarginal seller $i$ is not active; then the seller is making a payoff equal to zero in every allocation $y \in F(w^*)$. But by deviating unilaterally to $w_i'(p,1)$ for any $v_s < p < p^*$, the seller can guarantee herself a positive payoff $u_i(-1,p) = -r_{i1} + p > -v_s + p^* > 0$ in every allocation with positive probability given the new offer profile. Hence, two or more sellers are active in $w^*$. If two or more buyers are active in $w^*$, then applying theorem 2, $p^*$ is a competitive price and all the outcomes from the Nash equilibrium are competitive.

If only one buyer is active in $w^*$, say buyer $a$, we must have that at least one weakly inframarginal buyer, say buyer $c$, who is not active in $w^*$. Since $c$ is not active in $w^*$, we must have $p^* \geq r_{c1}$; otherwise $c$ has a profitable deviation. Therefore $p^* \geq r_{c1} \geq \underline{p}$. If $p^* > \overline{p}$, then for every $y \in F(w^*)$,

$$\sum_{i \in B} y_i \leq \sum_{i \in B} \overline{\delta}_i < -\sum_{i \in S} \underline{\delta}_i \leq -\sum_{i \in S} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 6 which implies that for all the active sellers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them, and from lemma 5 which implies that for any active buyer $y_i \leq \overline{\delta}_i$. The strict inequality in the middle is a result of the price being higher than any competitive price. Hence $\underline{p} \leq p^* \leq \overline{p}$ so that $p^*$ is a competitive price.

Now suppose that there is an allocation $y \in F(w^*)$ such that $y_a < \underline{\delta}_a$. Since $p^*$ is competitive, in any competitive equilibrium allocation $(q_i)$, we have $-\sum_{i \in S} q_i \geq \underline{\delta}_a$. Thus in every competitive equilibrium at price $p^*$, there exists at least one seller $s$ that has $q_s < y_s$. Since $y_s, q_s \in [\underline{\delta}_s, \overline{\delta}_s]$, we have $r_{s,|q_s|} = p^*$. Hence for any competitive equilibrium at $p^*$, there is at least a traded unit with reservation value $p^*$ for a seller. By definition of $v_s$ this implies $p^* \leq v_s$, a contradiction to the assumption. Therefore for the only active buyer $a$, $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$. Hence, all traders obtain utility-maximizing quantities given $p^*$, and every outcome $y \in F(w^*)$ is competitive.

Consider the remaining case $p^* \leq v_s$. Since $p^* < r_{1i}$ for every weakly inframarginal buyer, it follows that there are at least two active buyers in Nash equilibrium and moreover every buyer chooses utility-maximizing quantities given $p^*$. As in the previous proof, if there are two or more active sellers, then, from theorem 2, all outcomes in $F(w^*)$ are competitive. Similarly, if there is a unique active seller $a$ and $y_a \in [\underline{\delta}_a, \overline{\delta}_a]$ for every $y \in F(w^*)$, then all traders obtain utility-maximizing quantities given $p^*$, and every outcome $y \in F(w^*)$ is competitive. The remaining case is that there is a unique active seller $a$ and $\overline{\delta}_a < y_a < 0$, so that $\sum_{i \in B} y_i = -y_s < -\overline{\delta}_s$.

Suppose $p^* = v_s = \underline{p}$. Since $p^*$ is a competitive price, in every competitive equilibrium allocation $(q_i)$, we have $\sum_{i \in B} q_i \geq -\overline{\delta}_s$; i.e. aggregate demand should be able to meet an individual seller's supply. Thus in every competitive equilibrium at $p^*$, there exists at least one buyer $b$ that has $q_b > y_b$. Since $y_b, q_b \in [\underline{\delta}_b, \overline{\delta}_b]$, we have $r_{b,q_b} = p^*$. Hence in every competitive equilibrium at $p^*$, there is at least one traded unit with reservation value $p^*$ for a buyer. By definition of $v_b$, this implies $p^* \geq v_b$. Using $v_b > v_s$ we get a contradiction to the assumption $p^* = v_s$.

Finally, suppose $p^* = v_s < \underline{p}$ or $p^* < v_s$. In either case, $p^* < \underline{p}$, and

$$-\sum_{i \in S} y_i \leq -\sum_{i \in S} \overline{\delta}_i < \sum_{i \in B} \underline{\delta}_i \leq \sum_{i \in B} y_i,$$

violating the allocation rule of the clearing house. The first and the last inequality comes from lemma 5 which implies that for any active seller $y_i \geq \underline{\delta}_i$ and from lemma 6 which implies that for all the active buyers $y_i \in [\underline{\delta}_i, \overline{\delta}_i]$ since there are at least two of them. The strict inequality in the middle is a result of the price being lower than any competitive price.

This finishes the proof of sufficiency. We now prove that the condition is necessary. Since at least two units are traded in every competitive equilibrium, there is at least one inframarginal trader on each side of the market. Possible violations of the condition in the theorem are that, among the remainder of traders, either (a) there is no additional weakly inframarginal trader on one side of the market, or (b) there is no additional inframarginal trader in either side.

Consider case (a), and suppose without loss of generality that trader 1 is the unique weakly inframarginal seller, so that every seller $i \in S \setminus \{1\}$ is such that either $r_{i1} \geq v_b$ or $r_{i1} > \overline{p}$; recall that each of these conditions imply $r_{i1} > v_s$. Take a competitive equilibrium that has the smallest number of units traded, and denote the allocation by $\hat{q} = (\hat{q}_i)$. From lemma 7, $\hat{q}_i = 0$ for every seller $i \in S \setminus \{1\}$. From lemma 7 as well, a unit of value $v_b$ is bought by at least one buyer, say without loss of generality buyer 2, and moreover for every buyer $j$ such that $q_j > 0$ we must have $r_{j,\hat{q}_j} \geq v_b$. Recall that the highest equilibrium price $\overline{p}$ satisfies $\overline{p} \leq v_b$, and moreover $(\overline{p}, \hat{q})$ is a competitive equilibrium.[1] Suppose first that $\overline{p} = v_b$. Consider the offer profile $w$ such that $w_1 = (v_b, \hat{q}_1 + 1)$ (seller 1 sells one fewer unit than in the competitive equilibrium), $w_2 = (v_b, \hat{q}_2 - 1)$ (buyer 2 buys one fewer unit), and $w_i = (v_b, \hat{q}_i)$ for every $i \neq 1, 2$. It is easy to check that no trader has a profitable deviation; buyer 2 in particular is indifferent between buying one more unit or not.

Now suppose that $\overline{p} < v_b$. Define

$$\tilde{p} = \begin{cases} \min\left\{\min_{i \in S \setminus \{1\}} r_{i1}, v_b\right\} & \text{if } S \setminus \{1\} \neq \emptyset \\ v_b & \text{if } S \setminus \{1\} = \emptyset \end{cases},$$

and consider the offer profile $\tilde{w}$ such that $\tilde{w}_i = (\tilde{p}, \hat{q}_i)$ for all $i \in S \cup B$. It is easy to check that no trader has a profitable deviation. But the induced outcome is not competitive since $\tilde{p} > \overline{p}$.

Consider case (b), and suppose without loss of generality that trader 1 is the unique inframarginal seller and that trader 2 is the unique inframarginal buyer, so that for every seller $i \in S \setminus \{1\}$ and buyer $j \in B \setminus \{2\}$, $r_{i1} > v_s$

[1] In quasilinear economies, the set of competitive equilibria is the product of the set of competitive allocations and the set of competitive equilibrium prices.

and $r_{j1} < v_b$. Take a competitive equilibrium $(\hat{p}, \hat{q})$ that has the smallest number of units traded. Since $v_s \leq \hat{p} \leq v_b$, traders 1 and 2 are the only traders who are trading in $\hat{q}$. Consider the offer profile $w_1 = (\hat{p}, -1)$, $w_2 = (\hat{p}, 1)$, and $w_k = (\hat{p}, 0)$ for every $k \in S \cup B \setminus \{1, 2\}$. No trader has a profitable deviation, but this offer profile induces an allocation which is not competitive under the assumption that at least two units are traded in competitive equilibrium.

## Appendix B Instructions and quizzes

### B.1 Instructions for CH treatments

### Instructions

**Welcome to today's experiment! You have earned \$5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.**

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. Your group and your role will remain the same throughout the experiment. Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

### Payoffs

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of \$4 from buying the first unit, and \$3 from buying the second. If Buyer A obtains 2 units at the unit price of \$2, then

$$\text{Buyer A's payoff} = \underbrace{(\$4 + \$3)}_{\text{Values}} - \underbrace{(\$2 + \$2)}_{\text{Payment}} = \$3$$

Suppose Seller A sells 1 unit at the price of \$5.6, and her cost is \$1 for selling the first unit and \$3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

Since Seller A does not sell the second unit, only the cost of the first unit incurs.

If a participant does not trade in a round, her payoff from that round is $0.

The payoffs from different rounds do not accrue. At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the $5 show-up bonus.

**How to trade**

Each group trades in its own market. In each round, the market opens for 2 minutes, during which each participant can submit an offer. In a buying offer, a buyer submits a unit price, together with how many units (1 or 2) she would like to buy for that price. In a selling offer, a seller submits a unit price, and how many units (1 or 2) she would like to sell for that price. The offer you submit will NOT be shown to any other participant.

Please note that you can submit only ONE offer in each round, and you cannot revise your offer once you submit it.

After two minutes, or once every participant has submitted a unit price and quantity, transactions will be determined under the rules below, as demonstrated in the following example.

**Example**

Suppose the submitted offers are as follows.
Buyer A: buying offer for 1 unit, at the unit price of $3
Buyer B: buying offer for 2 units, at the unit price of $1
Seller A: selling offer for 1 unit, at the unit price of $4
Seller B: selling offer for 1 unit, at the unit price of $2.

Please note that this example is only for demonstration of the procedure, the submitted offers will NOT be shown to any participant in the experiment.

- **Sort Orders** Firstly, buying offers and selling offers will be sorted separately. If an offer contains two units (eg. Buyer B's offer), it will be split into TWO IDENTICAL offers, each containing one unit. Buying offers for each unit will be queued in descending order, and selling offers for each unit will be queued in ascending order, as the following table shows.

| Buying offers for one unit (high to low) | Selling offers for one unit (low to high) |
|---|---|
| $3 (from Buyer A) | $2 (from Seller B) |
| $1 (from Buyer B) | $4 (from Seller A) |
| $1 (from Buyer B) | |

In case of tied buying offers or tied selling offers, the order of them will be randomly determined.

- **Trade Units** After the orders are sorted, each buying offer and selling offer at the same position in the queues will be compared. As long as the buying price is no lower than the selling price, the corresponding buyer and seller make a trade.
The first buying offer in the queue ($3 from Buyer A) and the first selling offer ($2 from Seller B) make a trade since 3>2. The second buying offer and selling offer cannot trade since the buying price ($1 from Buyer B) is lower than the selling price ($4 from Seller A). The third buying offer cannot be fulfilled since there is not a selling offer corresponding to it. By this procedure, the buying offer with higher price is more likely to be fulfilled, and so is the selling offer with lower price.

– **Prices** When a trade happens, the buyer will pay the price she offered and get one unit of the good, and the seller will receive the price she asked for and sell one unit of the good. In this example, one unit of the good is traded. Buyer A pays $3 for the unit she bought, as she offered to. Seller B gets $2 for the unit she sells, as she asked for.

In each round, a participant who does not submit any offer will not make any trade. To prevent losing money, a buyer/seller cannot submit an offer that could cause a loss for her.

**Summary of Each Round**

The market for each group opens at the beginning of each round. After each participant in your group submits an offer or when the market closes, you will be informed of how many units you trade, and your payoff in the current round. Also, the price(s) for each traded unit in your market will be revealed anonymously to all participants in your group. You will NOT be informed of the buying/selling offers that do not result in trade.

**This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be a practice round, which does not count towards payment.**
**Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.**

### B.2 Quiz for CH treatments

## Quiz

1. True or False. Circle your answers.

| | | |
|---|---|---|
| Your role (buyer or seller) will remain the same in all of the rounds. | T | F |
| Your group does not change throughout the experiment. | T | F |
| In each round, you can revise your offer after you submit it. | T | F |
| Your costs or values will not change between rounds. | T | F |
| Your offer in each round will not be shown to other participants. | T | F |

2. Suppose the offers submitted are as follows.

Buyer A: buying offer for 2 units, at the unit price of $3
Buyer B: buying offer for 1 unit, at the unit price of $5
Seller A: selling offer for 2 units, at the unit price of $1
Seller B: selling offer for 1 unit, at the unit price of $2.

(a) Use the procedure demonstrated in the instructions, fill out the buying and selling offers in the table.

| Buying offers for one unit (high to low) | Selling offers for one unit (low to high) |
|---|---|
| $5 (from Buyer B) | $1 (from Seller A) |
| $_____ (from Buyer _____) | $1 (from Seller A) |
| $3 (from Buyer A) | $_____ (from Seller _____) |

(b) How many units does Buyer A buy? ______ unit(s)

(c) How much does Buyer A pay for the unit(s) she buys <u>in total</u> ? $______

(d) Suppose the first unit Buyer A obtains will generate a value of $5 to her, and the second unit she obtains will generate $4. What is Buyer A's payoff here?

$$\text{Buyer A's payoff} = \underbrace{\$\_\_\_\_\_\_\_\_\_\_\_\_}_{\text{Value(s)}} - \underbrace{\$\_\_\_\_\_\_\_\_\_\_\_\_}_{\text{Payment}} = \$\_\_\_\_\_\_\_\_\_\_\_\_$$

(e) Suppose the first unit Seller B sells will cost her $0.5, and the second unit she sells will cost $2.5. What is Seller B's payoff here?

$$\text{Seller B's payoff} = \underbrace{\$\_\_\_\_\_\_\_\_\_\_\_\_}_{\text{Revenue}} - \underbrace{\$\_\_\_\_\_\_\_\_\_\_\_\_}_{\text{Cost(s)}} = \$\_\_\_\_\_\_\_\_\_\_\_\_$$

### B.3 Instructions for DA treatments

**Instructions**

**Welcome to today's experiment! You have earned $5 for showing up on time. The following instructions will explain how you can make decisions and earn more money, so please read them carefully. During the experiment, please keep your cell phone turned off, and refrain from talking to other participants. If at some point you have a question, please raise your hand, and we will address it with you privately.**

In the experiment, you will be grouped anonymously with three other participants, whose identities will not be revealed. Two of the participants in your group will be buyers, and the other two will be sellers. <u>Your group and your role will remain the same throughout the experiment.</u> Your role will be revealed to you at the beginning of the experiment.

There will be 20 formal rounds. In each round, each of the two buyers has the opportunity to buy up to 2 units of the good from the two sellers in the same group, and each of the two sellers has the opportunity to sell up to 2 units of the good to the two buyers in the same group.

Obtaining each unit of the good generates a value for the buyer, and selling each unit of the good incurs a cost to the seller. The values to a buyer and the costs to a seller may vary by unit. Values may vary between buyers and costs may vary between sellers.

Your own values (if you are a buyer) or costs (if you are a seller) will be revealed to you at the beginning of the experiment. Your values/costs remain constant throughout the experiment. The values/costs of other participants will NOT be revealed to you.

**Payoffs**

The values and costs are in US Dollars. A buyer's payoff in one round equals the value she obtains from the unit(s) she buys minus the total price she pays for her purchase. A seller's payoff in one round equals the revenue she gets from the sale minus the cost incurred for the unit(s) she sells.

Buyer's payoff = value obtained from purchase – payment for purchase

Seller's payoff = revenue from sale – cost incurred for sale

For example, suppose Buyer A generates a value of $4 from buying the first unit, and $3 from buying the second. If Buyer A obtains the first unit at the price of $2 and the second unit at the price of $1, then

$$\text{Buyer A's payoff} = \underbrace{(\$4 + \$3)}_{\text{Values}} - \underbrace{(\$2 + \$1)}_{\text{Payment}} = \$4$$

Suppose Seller A sells 1 unit at the price of \$5.6, and her cost is \$1 for selling the first unit and \$3 for selling the second. Then

$$\text{Seller A's payoff} = \underbrace{\$5.6}_{\text{Revenue}} - \underbrace{\$1}_{\text{Cost(s)}} = \$4.6$$

Since Seller A does not sell the second unit, only the cost of the first unit incurs.

If a participant does not trade in a round, her payoff from that round is \$0.

The payoffs from different rounds do not accrue. At the end of the experiment, one of the 20 formal rounds will be randomly chosen. Your total earnings in this experiment will be your payoff from the chosen round, plus the \$5 show-up bonus.

**How to trade**

Each group trades in its own market. In each round, the market opens for a maximum of two minutes, during which each participant can submit offers. In a buying offer, a buyer submits a price she is willing to buy a unit at. In a selling offer, a seller submits a price she is willing to sell a unit at. For each participant, only after her first unit is traded can she trade her second unit.

The timer on the screen counts down the time remaining for the current round. The timer starts from two minutes at the beginning of each round, then jumps to 20 seconds once a participant attempts to submit an offer, and restarts from 20 seconds every time a participant attempts to submit an offer. The round finishes if two minutes elapse, or if no new attempt occurs within 20 seconds of the last attempt, whichever occurs first.

The attached pages are screen shots of the interface for a seller and a buyer in the same market. Screen shot 1 is for the seller. Screen shot 2 is for the buyer.

From left to right in the upper part of the interface are the ***Submit Your Offer*** section, where you can enter the price for each of your offers; the section for ***general information***, where you can see the number of rounds, your role, time remaining in the current round, and your real-time payoff in the current round; ***Your Values/Costs*** section, where you can see the values or costs for your units and whether they are traded or not.

On the lower part of the interface, from left to right are the ***Selling Offers*** section, which lists the selling offers from low to high; the ***Buying Offers*** section, which lists the buying offers from high to low; the ***Transactions*** section, which displays all transactions in your market in the current round. Your own offers and transactions will be highlighted on the lists.

- **How to Sell**
  - **Offer to Sell**
    You can offer to sell one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be lower than the lowest selling offer at the time, which is the top one on the ***Selling Offers*** list. If you make a new offer, it will replace your previous offer.
    As shown in the screen shots, the lowest selling offer is \$3, so if any of the sellers wants to make a new offer, she has to offer a price lower than \$3.
    To prevent losing money, you cannot submit an offer that could cause a loss for you.
  - **Accept A Buying Offer**
    You can sell one unit by submitting a price equal to the highest buying offer, which is the top one on the ***Buying Offers*** list. By doing so, you sell the unit to the buyer and incur the cost, the buyer pays you the price you submitted. (If you submit a price lower than the highest buying offer, you sell the unit at the price you submit.) In the example from the screen shots, the highest buying offer is \$2, if a seller submits an offer of \$2, she sells the unit to the buyer, and the buyer pays her \$2.

  - **Transactions**
    There are two ways you sell one unit. Your selling offer is accepted by a buyer, or you accept a buying offer. When you sell one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to sell your second unit or accept another buying offer on the list. The rules are the same as for the first unit.

- **How to Buy**
  - **Offer to Buy**
    You can offer to buy one unit by submitting a price in the ***Submit Your Offer*** section. When you make an offer, the price has to be higher than the highest buying offer at the time, which is the top one on the ***Buying Offers*** list. If you make a new offer, it will replace your previous offer.
    As shown in the screen shots, the highest buying offer is $2, so if any of the buyers wants to make a new offer, she has to offer a price higher than $2.
    To prevent losing money, you cannot submit an offer that could cause a loss for you.
  - **Accept A Selling Offer**
    You can buy one unit by submitting a price equal to the lowest selling offer, which is the top one on the ***Selling Offers*** list. By doing so, you buy the unit from the seller and obtain the value, and pay the seller the price you submitted. (If you submit a price higher than the lowest selling offer, you buy the unit at the price you submit.) In the example from the screen shots, the lowest selling offer is $3, if a buyer submits an offer of $3, she buys the unit from the seller, and pays the seller $3.
  - **Transactions**
    There are two ways you buy one unit. Your buying offer is accepted by a seller, or you accept a selling offer. When you buy one unit, your offer for that unit will be removed from the list, the transaction will be recorded, and your payoff will be updated. Then you may offer to buy your second unit or accept another selling offer on the list. The rules are the same as for the first unit.

**Summary of Each Round**

The market for each group opens at the beginning of each round. A seller can make selling offers, or accept buying offers, by submitting prices on the interface. A buyer can make buying offers, or accept selling offers, by submitting prices on the interface. When an offer is accepted, a transaction happens. Offers, transactions and your payoff in the current round will be displayed on your screen.

**This is the end of the instructions. We now proceed to a quiz to ensure everyone understands the instructions. The experiment will begin after everyone gives a correct answer to each question. Before the formal rounds begin, there will be one practice round, which does not count towards payment.**

**Again, if you have any question at any point of the experiment, please raise your hand and an experimenter will assist you.**

### B.4 Quiz for DA treatments

## Quiz

1. True or False. Circle your answers.
2. Suppose you are a **buyer**, and the lists of offers are as follows. Your offer is highlighted.

| Selling Offers | Buying Offers |
|---|---|
| $4 | $3 |
| $5 | $1 |

| | | |
|---|---|---|
| Your role (buyer or seller) will remain the same in all of the rounds. | T | F |
| Your group does not change throughout the experiment. | T | F |
| Your costs or values will not change between rounds. | T | F |
| You can submit offers for both of your units at the same time. | T | F |

(a) Which of the following prices can you submit as a new offer? Circle your answer.
A. 2 B. 0.5 C. 3.7 D. 1.5

(b) Which of the following prices can you submit to accept the selling offer of $4? Circle you answer.
A. 4 B. 2.5 C. 1.2 D. 3

(c) If you accept the lowest selling offer on the list, and your values for the first and second unit are $7 and $6 respectively, what is your payoff?

$$\text{Your payoff} = \$\underbrace{\rule{3cm}{0.4pt}}_{\text{Value(s)}} - \$\underbrace{\rule{3cm}{0.4pt}}_{\text{Payment}} = \$\rule{3cm}{0.4pt}$$

3. Suppose you are a **seller**, and the lists of offers are as follows. Your offer is highlighted.

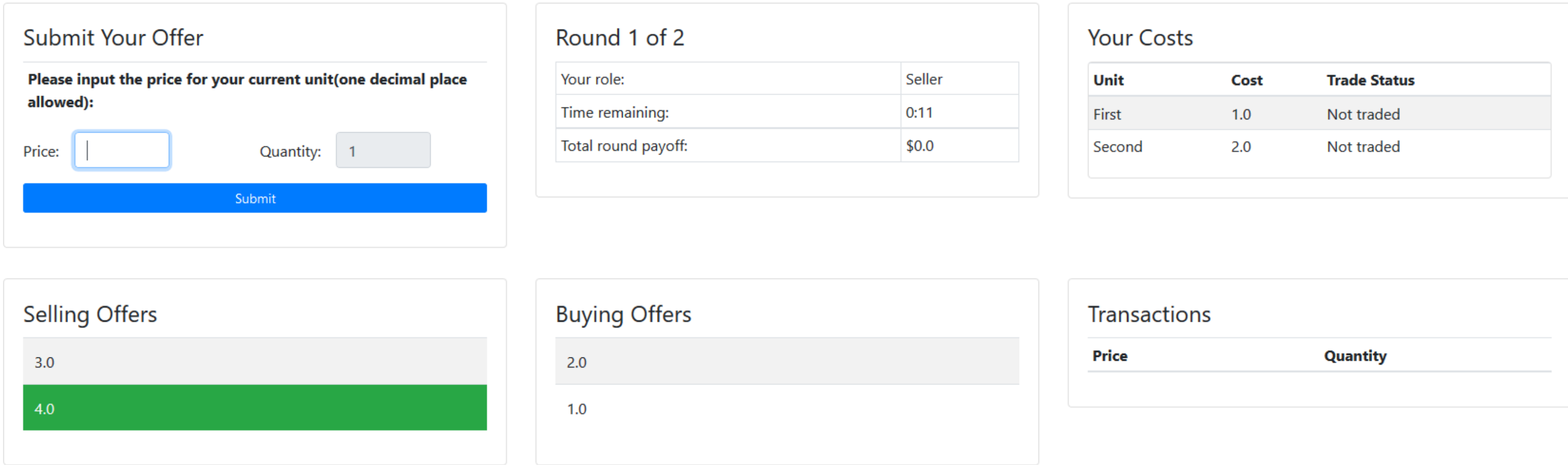
| Selling Offers | Buying Offers |
|---|---|
| $4 | $3 |
| $5 | $1 |

(a) Which of the following prices can you submit as a new offer? Circle your answer.
A. 6 B. 2.1 C. 4 D. 5

(b) Which of the following prices can you submit to accept the buying offer of $3? Circle you answer.
A. 3.5 B. 4.1 C. 5 D. 3

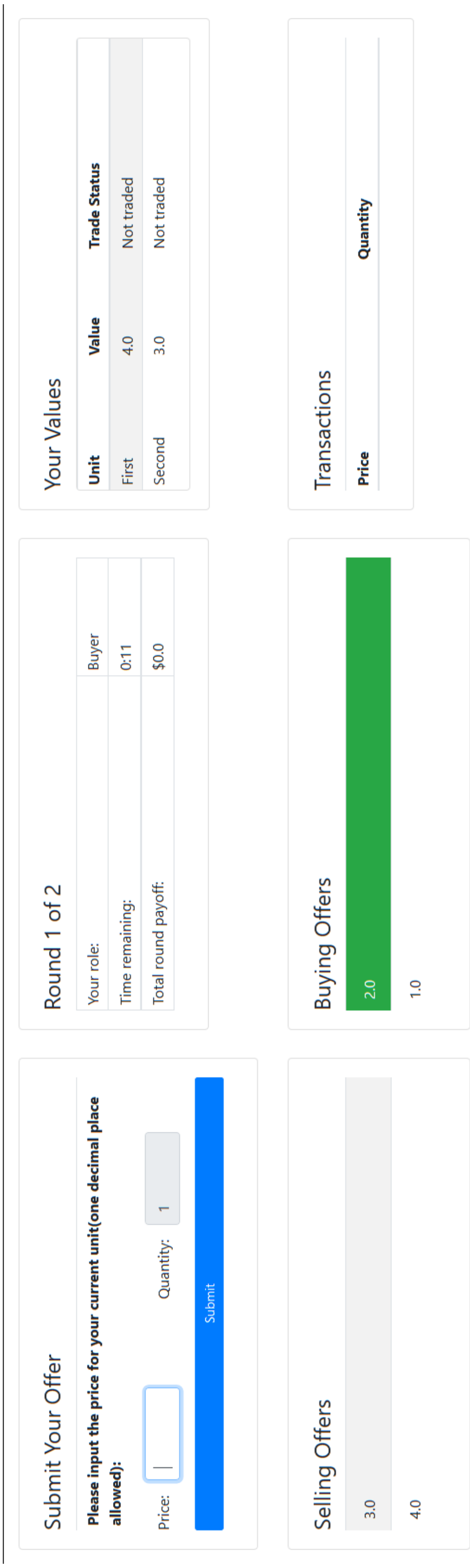
(c) Suppose the first and second unit you sell will cost $0.1 and $0.4 respectively, and you accept both buying offers on the list. What is your payoff?

$$\text{Your payoff} = \$\underbrace{\rule{3cm}{0.4pt}}_{\text{Revenue}} - \$\underbrace{\rule{3cm}{0.4pt}}_{\text{Cost(s)}} = \$\rule{3cm}{0.4pt}$$




**Screen shot 1: Interface for a seller**

Note that numbers here are for demonstration purpose.




**Screen shot 2: Interface for a buyer**

Note that numbers here are for demonstration purpose.