Appendices

A Appendix: Theoretical analysis

A.1 The theoretical analysis based on Cerreia-Vioglio et al. (2015) and Klibanoff et al. (2005)

To accommodate the potential that a decision-maker might not be fully confident about her choices, we assume an individual has multiple utility functions that we call multiple selves, with each self representing one particular way to trade off conflicting objectives in choices. Such a modelling technique has been used in models of incomplete preferences (see e.g., Bewley, 2002; Dubra et al., 2004; Cerreia-Vioglio et al., 2015).

Specifically, let u_τ denote the utility function of the self τ, and $\mathcal{T}$ denote the set of selves. Let π denote the subjective probability distribution over $\mathcal{T}$, which, similar to the modelling technique of Loomes and Sugden (1982), represents "the individual's degree of belief or confidence in the occurrence of the corresponding states" (Loomes and Sugden, 1982, p. 807). This belief could come from introspection or experiences with similar options. Given a utility function u_τ, we follow the standard assumption that the self behaves according to EUT. Let $U_\tau(l)$ denote the expected utility of an option $l \in L$.[1] We further assume that the individual dislikes disagreement among selves. This is because, to arrive at a choice when there are multiple selves with different preferences is, in essence, similar to situations where a group of people with different opinions tries to reach a consensus. The more strongly group members disagree with each other, the harder it is for the group to make compromises and agree on a single opinion. Hence, aversion to disagreement among selves can be interpreted as the cost of forcing different selves to reach a consensus. With

[1] The function $U(\cdot)$ could be made more general to allow for non-EUT preferences to incorporate unsureness about how strongly to weight the extra factor, such as probability weighting or loss aversion, in a non-EU model.

the above assumptions, we can write the individual's preference over an option l as:

$$V(l) = \int_{\mathcal{T}} \phi \left[U_{\tau}(l) \right] d\pi, \tag{1}$$

where concave $\phi(\cdot)$ implies an aversion to disagreement - deviations from the mean expected utility - among different selves. Similar to the connection between the concavity of the utility function and risk aversion, the concavity of $\phi(\cdot)$ implies that the individual places more weight on the selves who have lower value for l. Such a cautious attitude is consistent with Levitt (2021) who showed that subjects who have difficulties making a decision are often excessively cautious in the sense of preferring to maintain the status quo.

Equation 1 extends directly from Klibanoff et al. (2005) and Cerreia-Vioglio et al. (2015). It can be seen as a smooth version of the cautious expected utility model (Cerreia-Vioglio et al., 2015). It is also a parallel of the smooth ambiguity model of Klibanoff et al. (2005). Indeed, in the smooth ambiguity model, an individual is unsure about the probability distribution of the states of nature, and she has a subjective belief over these probability distributions. Likewise, in this model, an individual is unsure about her utility function, and she has a subjective belief over her multiple selves. Note that this does not mean this model only applies to decision-making under risk. If there is preference uncertainty under risk (or even under certainty, e.g., over options about experience goods) because individuals have difficulties evaluating options, this uncertainty is also likely to be present in more complex situations of decision-making under ambiguity. In this sense, this model complements the smooth model of ambiguity and general models about uncertainty in beliefs. Ultimately, the lack of decision confidence arises from the difficulties in evaluating options, which may be due to uncertainty in both beliefs and preferences. A general model accommodating both sources of uncertainty could be written as:

$$V(a) = \int_{\mathcal{M}} \int_{\mathcal{T}} \phi \left[U_{\tau,\mu}(a) \right] d\pi d\mu,$$

where a represents an act, and μ is a subjective probability distribution over M, the set of probability distributions of the states of nature.

We are now ready to establish the link between decision confidence and the randomization probability in the randomized choices. Specifically, recall that in our mechanism, the individual chooses a randomization probability $\lambda \in [0,1]$ and builds a lottery $(\lambda, x; (1-\lambda), y)$: She receives x with probability λ and y with probability $1-\lambda$. Since for any given self τ, the individual's preference over the lottery $(\lambda, x; (1-\lambda), y)$ satisfies EUT, we have $U_\tau[\lambda x + (1-\lambda)y] = \lambda U_\tau(x) + (1-\lambda)U_\tau(y)$. The individual's decision is then to maximize her utility by choosing the optimal randomization probability $0 \leq \lambda \leq 1$:

$$Max_\lambda \; V[\lambda x + (1-\lambda)y] = \int_{\mathcal{T}} \phi \left[\lambda U_\tau(x) + (1-\lambda)U_\tau(y)\right] d\pi.$$

In the experiment, y is a sure payment. Sure monetary payments are probably the easiest options to evaluate, hence we assume the individual is always confident about her evaluation of a sure payment: $U_\tau(y) = u(y), \; \forall \tau \in \mathcal{T}$. Applying the Taylor expansion to the above equation at y, we can derive the optimal λ as:[2]

$$\lambda^* \approx \frac{1}{-\frac{\phi''[u(y)]}{\phi'[u(y)]}} \times \frac{E_\pi\left[U_\tau(x)\right] - u(y)}{\sigma_x^2} \tag{2}$$

where $\sigma_x^2 = E_\pi\left[U_\tau(x) - E_\pi(U_\tau(x))\right]^2$ is the standard deviation of the valuation of the lottery across multiple selves and approximates how strongly different selves disagree with each other. Similar to decision-making under risk, $-\frac{\phi''(u(y))}{\phi'(u(y))}$ can be interpreted as a metric of attitudes towards disagreement among selves. Thus, the randomization probability aggregates the three important determinants of decision confidence: preference uncertainty, the utility difference between the two options, and her attitude toward preference uncertainty. It is in this sense we argue that the randomization probability captures decision confidence.

Deriving the hypotheses

To see how the individual may randomize for sure payments that yield similar utility as

[2]More precisely, since $0 \leq \lambda \leq 1$, $\lambda^* \approx min\left\{max\left\{0, \frac{1}{-\frac{\phi''[u(y)]}{\phi'[u(y)]}} \times \frac{\Delta_u}{\sigma_x^2}\right\}, 1\right\}$. The detailed derivation can be found below.

the lottery, notice that the certainty equivalent of the lottery is

$$\begin{aligned} u(CE_x) &= \int \phi\left[U_\tau(l)\right] d\pi \\ &\approx E_\pi \left\{ E_\pi\left[U_\tau(x)\right] + \phi'\left(E_\pi\left[U_\tau(x)\right]\right)\left[U_\tau - E_\pi\left[U_\tau(x)\right]\right] + \frac{\phi''\left(E_\pi\left[U_\tau(x)\right]\right)}{2}\left[U_\tau - E_\pi\left[U_\tau(x)\right]\right]^2 \right\} \\ &= E_\pi\left[U_\tau(x)\right] + \frac{\phi''\left(E_\pi\left[U_\tau(x)\right]\right)}{2}\sigma_x^2. \end{aligned}$$

The optimal randomization probability at the sure payment which is equal to the certainty equivalent of the lottery ($u(y) = u(CE_x) = E_\pi\left[U_\tau(x)\right] + \frac{\phi''(E_\pi[U_\tau(x)])}{2}\sigma_x^2$) is

$$\lambda^* \approx \frac{1}{-\frac{\phi''[u(CE_x)]}{\phi'[u(y)]}} \times \frac{E_\pi\left[U_\tau(x)\right] - u(CE_x)}{\sigma_x^2} = \frac{1}{2} \times \frac{\phi'\left[u(CE_x)\right]\phi''\left(E_\pi\left[U_\tau(x)\right]\right)}{\phi''\left[u(CE_x)\right]}.$$

When $\phi'\left[u(CE_x)\right]$ is close to one and the function $\phi(\cdot)$ is smoothly concave, which is likely to hold for options with moderate payoffs, the randomization probability is around 0.5. This implies that the individual would choose randomization probabilities close to 0.5 when two options yield similar utilities. Furthermore, the smallest sure payment that the individual chooses $\lambda^* < 1$ (the lower bound), and the largest sure payment that the individual chooses $\lambda^* > 0$ (the upper bound) are defined by $u(\underline{y_x}) = E_\pi\left[U_\tau(x)\right] - \frac{-\phi''[u(y)]}{\phi'[u(y)]}\sigma_x^2$, and $u(\bar{y_x}) = E_\pi\left[U_\tau(x)\right]$. The range of sure payments that the individual randomizes strictly is

$$u(\bar{y_x}) - u(\underline{y_x}) = \frac{-\phi''\left[u(y)\right]}{\phi'\left[u(y)\right]}\sigma_x^2,$$

which varies with preference uncertainty (σ_x^2).

Relating these results to our experiment, we expect subjects to have more preference uncertainty about a complex lottery than a simple lottery, as the individual may find it harder to evaluate a complex lottery. She considers relevant a larger set of utility functions and the subjective belief π becomes flatter. This translates into larger preference uncertainty (δ_x increases). Experience with a lottery, on the other hand, reduces preference uncertainty about the lottery because the individual attains clearer preferences about the lottery when she gains more experience (the set of utility functions becomes smaller and

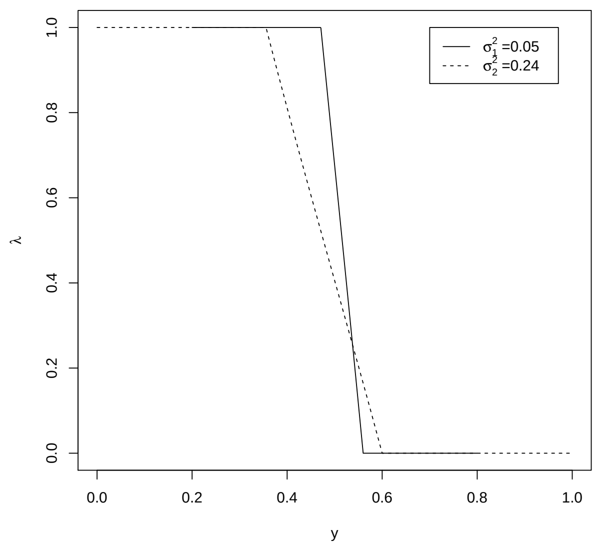


Figure A.1: The relationship between the randomization probability λ and the sure payment y. The figure is produced by assuming $\phi(U_\tau) = 1 - e^{-U_\tau}$, $\pi(u_1) = 0.6$, $\pi(u_2) = 0.4$, $U_1(x_1) = 0.8$ and $U_2(x_1) = 0.2$, $U_1(x_2) = 1.0$ and $U_2(x_2) = 0$, and $U_1(y) = U_2(y) = y$.

δ_x decreases). These lead to the hypotheses in the main text.

As a concrete illustration, consider the following numerical example: the individual has two selves $\tau = 1, 2$, and $\pi(u_1) = 0.6$, $\pi(u_2) = 0.4$. The individual's preference over the lottery x_1 is such that $U_1(x_1) = 0.8$ and $U_2(x_1) = 0.2$. Her preference over the lottery x_2 is such that $U_1(x_2) = 1.0$ and $U_2(x_2) = 0$. Thus, the individual perceives more preference uncertainty about the lottery x_2 than the lottery x_1 ($\sigma_{x_1} = 0.05 < \sigma_{x_2} = 0.24$). Option y is a sure payment, and $u_1(y) = u_2(y) = y$. The function $\phi(U_\tau) = 1 - e^{-U_\tau}$. Simple calculation shows that $\lambda_{x_1} = -\frac{1}{0.8-0.2} ln(\frac{0.4}{0.6} \times \frac{y}{1-y})$ and $\lambda_{x_2} = -\frac{1}{1-0} ln(\frac{0.4}{0.6} \times \frac{y}{1-y})$, subject to $0 \le \lambda \le 1$. Figure A.1 shows the relationship between the optimal λ and sure payment y. The Figure shows that the randomization probability decreases with y, and approaches to 0.5 for y that yields similar decision utility as the lottery ($y = 0.515$ for x_1 and $y = 0.476$ for x_2). Furthermore, the individual randomizes over a wider range of y for x_2 which she perceives higher preference uncertainty compared to x_1.

Derivation of the optimal λ^*

Taking the first order derivative of the optimization equation gives:[3]

$$\frac{dV\left[\lambda x+(1-\lambda)y\right]}{d\lambda}=\int_{\mathcal{T}}\phi'\left[\lambda U_\tau(x)+(1-\lambda)u(y)\right]\times\left[U_\tau(x)-u(y)\right]d\pi=0.$$

Note that $U_\tau(x)$ is a random variable governed by the subjective probability distribution π. Let $X=U_\tau(x)$, and $\Delta_\tau=X-u(y)$. With these notations, we have

$$\phi'\left[\lambda U_\tau(x)+(1-\lambda)u(y)\right]=\phi'\left[u(y)+\lambda\Delta_\tau\right].$$

We are most interested in scenarios where the individual finds it difficult to choose between x and y, i.e., when the two options are close and Δ_τ is small relative to X and $u(y)$. When this is the case, we can use the Taylor expansion at y and obtain

$$\phi'\left[u(y)+\lambda\Delta_\tau\right]=\phi'(u(y))+\phi''(u(y))\lambda\Delta_\tau+O\left(\lambda\Delta_\tau\right)\approx\phi'(u(y))+\phi''(u(y))\lambda\Delta_\tau,$$

where $O\left(\lambda\Delta_\tau\right)$ is the sum of the terms that have $\lambda\Delta_\tau$ with a power of two or higher. The above first order condition can be written as

$$\begin{aligned}\frac{dV[\lambda x+(1-\lambda)y]}{d\lambda} &= \int_{\mathcal{T}}\phi'\left[u(y)+\lambda\Delta_\tau\right]\Delta_\tau d\pi,\\ &\approx \int_{\mathcal{T}}\left[\phi'(u(y))+\phi''(u(y))\lambda\Delta_\tau\right]\Delta_\tau d\pi\\ &= E_\pi\left[\phi'(u(y))\Delta_\tau\right]+\lambda E_\pi\left[\phi''(u(y))\Delta_\tau^2\right]=0,\end{aligned}$$

where $E_\pi(\cdot)$ is the expectation operator with respect to the distribution π. Solving for λ, we have:

$$\lambda^*\approx min\left\{max\left\{0,\frac{1}{-\frac{\phi''[u(y)]}{\phi'[u(y)]}}\times\frac{\Delta_u}{\sigma_x^2-\Delta_u^2}\right\},1\right\}\approx min\left\{max\left\{0,\frac{1}{-\frac{\phi''[u(y)]}{\phi'[u(y)]}}\times\frac{\Delta_u}{\sigma_x^2}\right\},1\right\},$$

where $\Delta_u=E_\pi\left[U_\tau(x)\right]-u(y)$ is the (expected) utility difference of x and y, $\sigma_x^2=$

[3]The second-order derivative is $\frac{d^2V[\lambda x+(1-\lambda y)]}{d\lambda^2}=\int_{\mathcal{T}}\phi''\left[\lambda U_\tau(x)+(1-\lambda)u(y)\right]\times\left[U_\tau(x)-u(y)\right]^2d\pi$. Since $\phi(\cdot)$ is concave, $\phi''(\cdot)$ is negative. We are interested in situations where options x and y are not the same, i.e., $U_\tau(x)\neq u(y)$ for some $\tau\in\mathcal{T}$. Together we have $\phi''\left[\lambda U_\tau(x)+(1-\lambda)u(y)\right]\times\left[U_\tau(x)-u(y)\right]^2\leq 0$, and the inequality is strict for some $\tau\in\mathcal{T}$. Consequently, $\frac{d^2V[\lambda x+(1-\lambda y)]}{d\lambda^2}=\int_{\mathcal{T}}\phi''\left[\lambda U_\tau(x)+(1-\lambda)u(y)\right]\times\left[U_\tau(x)-u(y)\right]^2d\pi<0$. This ensures we are indeed seeking for the maximum.

$E_\pi \left[U_\tau(x) - E_\pi(U_\tau(x)) \right]^2$ is the standard deviation of $U_\tau(x)$.

A.2 The theoretical analysis based on Fudenberg et al. (2015)

Below, we perform a theoretical analysis of our experiment based on Fudenberg et al. (2015) to demonstrate the links between randomization probabilities and decision confidence.[4] Fudenberg et al.'s (2015) original representation concerns final outcomes. To apply their model to our experiments with lotteries, we write the individual's preference over randomizing between lottery x and sure payment y as:[5]

$$V(\lambda, x; 1-\lambda, y) = \lambda U(x) - c(\lambda) + (1-\lambda)u(y) - c(1-\lambda),$$

where $U(x)$ is the expected utility of the lottery x and $c(\lambda)$ is a weak cost function with finite steepness (the first order derivative of the cost function at the limit of 0 is not infinite). Using the weak cost function allows the model to accommodate zero choice probability that is present in our experiment. The cost function captures the implementation costs of making the desired choice, such as time and cognitive resources. In the Fudenberg et al.'s (2015) main representation, the cost function is independent of the option and the choice set. In an earlier version of their paper (Fudenberg et al., 2014), they proposed two extensions (item-invariant and menu-invariant APU) in which the cost function may depend on the preference uncertainty over options or the choice problem. We consider these two extensions to examine the effects of our treatments (increasing the complexity of the lottery or increasing subjects' experience with the lottery) on the cost function.

When $c(\lambda)$ is strictly convex, there exists an optimal randomization probability λ^* which maximizes the individual's utility, as defined by the equation $c'(\lambda^*) - c'(1-\lambda^*) = U(x) - u(y)$, where $c'(\lambda^*) - c'(1-\lambda^*)$ measures the convexity of the cost function $c''(\cdot)$. While

[4] Cerreia-Vioglio et al. (2019) predict preference for randomization when the individual faces non-degenerated lotteries. However, when one of the two options is a sure payment, as in our experiment, the individual has no preference for randomization. This follows directly from the axiom of Weak Stochastic Certainty Effect.

[5] Cerreia-Vioglio et al. (2019, Footnote 22, p.2437) proposed an alternative approach in which the individual integrates the lottery and the sure payment into a compound lottery, applies the reduction of the compound lottery, and implements the cost function to each outcome. We illustrate their approach and point out the differences between the two below. In particular, that approach predicts that the optimal randomization probability for the pair of the lottery and the sure payment that the individual is indifferent with depends on the number of outcomes in the lottery.

the exact value of the optimal randomization probability depends on the cost function, some observations are in order. First, the optimal randomization probability approaches 0.5 when $U(x)$ is close to $u(y)$. Second, for the same utility difference between the two options, the individual chooses a randomization closer to 0.5 when the cost function is more convex. More generally, as Proposition 3 in Fudenberg et al. (2015) demonstrates, the individual becomes less selective and randomizes more when $c''(\cdot)$ increases. Third, simple calculations show that the largest sure payment that the individual chooses $\lambda^* = 0.9$ (the lower bound) is $u(\underline{y}) = U(x) - \Delta$, and the smallest sure payment she chooses $\lambda^* = 0.1$ (the upper bound) is $u(\bar{y}) = U(x) + \Delta$, where $\Delta = c'(0.9) - c'(0.1) > 0$.[6] Thus, the individual randomizes over a larger range of sure payments when the cost function is more convex $(u(\bar{y}) - u(\underline{y})) = 2\Delta)$. According to Fudenberg et al. (2015), the cost function may depend, among other things, on the individual's perceived preference uncertainty over the options and her attitude towards uncertainty. Using this interpretation of the cost function, the three properties of randomization probabilities correspond to the three properties of decision confidence we outlined in the main body of the paper. It is in this sense that we say randomization probabilities measure decision confidence.

If we are willing to make more specific assumptions about the cost function, we can obtain a direct solution of the optimal randomization probability. For example, when the cost function takes the form of $c(\lambda) = \eta\lambda log(\lambda)$, we can derive the familiar logit/logistic choice rule:

$$\lambda^* = \frac{e^{U(x)/\eta}}{e^{U(x)/\eta} + e^{u(y)/\eta}}. \tag{3}$$

As shown by Holman and Marley, the parameter η can be linked to the variance of the i.i.d. Gumbel preference shocks in a random utility representation (Luce and Suppes, 1965, p.338). In the context of our study, η can be interpreted as the individual's preference uncertainty about lottery x. Figure A.2 depicts the relationship between the optimal randomization probability λ^* and the sure payments y. As we can see, randomization probabilities decrease with the value of y and approach 0.5 when the two options have

[6] The values of 0.1 and 0.9 were chosen to accommodate experimental data.

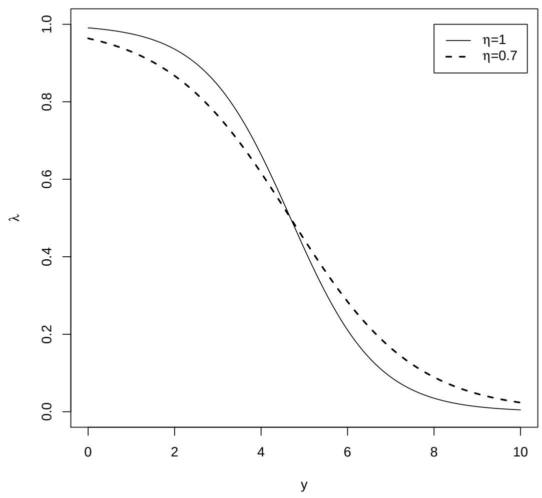
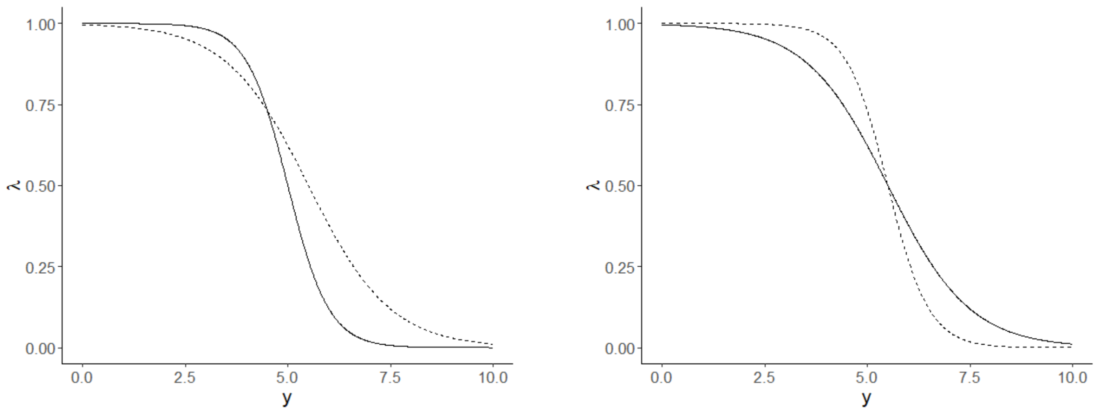


Figure A.2: The relationship between the optimal randomization probability λ^* and the sure payments y. The figure is produced according to the logit/logistic choice rule $\lambda^* = \frac{e^{U(x)/\eta}}{e^{U(x)/\eta}+e^{u(y)/\eta}}$. The parameter η captures the preference uncertainty over lottery x, with a larger η implying more convexity in the cost function and thus more preference uncertainty.

similar utilities. Furthermore, when η increases, the cost function becomes more convex and the individual's randomization probabilities become more compressed (the dashed line) and closer to 0.5.

Individuals may perceive more preference uncertainty over the complex lottery than over the simple lottery ($\Delta_c > \Delta_s$, where c denotes the complex lottery and s denotes the simple lottery), and experience with the lottery may reduce preference uncertainty about the lottery ($\Delta_e < \Delta_n$, where e denotes experience and n denotes no experience). In light of our analysis above, we expect that subjects' randomization probabilities are closer to 0.5 and that they randomize strictly over a wider range of sure payments when they make decisions about the complex lottery than when they make decisions about the simple lottery. In addition, compared to the no-experience treatment, randomization probabilities of subjects in the experience treatments are stretched away from 0.5, and subjects randomize strictly over a smaller range of sure payments. Figure A.3 demonstrates the effects.



(a) Simple versus complex

(b) No experience versus full experience

Figure A.3: The effects of complexity and experience on the lower bound, the upper bound, and the size of randomization range.

Cerreia-Vioglio et al. (2019)'s approach

Cerreia-Vioglio et al. (2019, Footnote 22, p.2437) proposed an alternative approach to apply Fudenberg et al.'s (2015) model to lotteries. We illustrate their approach with the following example. Consider an individual who faces a choice between a sure payment y and a lottery $x = 9_{0.5}1$ which pays 9 or 1 with equal likelihood. Cerreia-Vioglio et al. (2019) treat the randomized choice as a compound lottery. With the reduction of the compound lottery, the randomization of $(\lambda, x; 1-\lambda, y)$ becomes $9_{0.5\lambda}1_{0.5\lambda}y$, and the individual' preference over $9_{0.5\lambda}1_{0.5\lambda}y$ is

$$\begin{aligned} V(\lambda, x; 1-\lambda, y) &= 0.5\lambda u(9) - c(0.5\lambda) + 0.5\lambda u(1) - c(0.5\lambda) + (1-\lambda)u(y) - c(1-\lambda) \\ &= \lambda U(x) - 2c(0.5\lambda) + (1-\lambda)u(y) - c(1-\lambda) \end{aligned}$$

This formulation predicts an optimal randomization probability of 2/3 when the expected utility of the lottery is close to the utility of the sure payment ($c'(0.5\lambda) - c'(1-\lambda) = U(x) - u(y) = 0 \Rightarrow \lambda = 2/3$). The intuition is that the above formulation rewards the individual for randomizing over *more outcomes*, and thus the individual assigns a higher randomization probability to lotteries with more outcomes. It can be shown that, when the lottery x has four outcomes which are equally likely, the optimal randomization probability is $\lambda = 4/5$ when $U(x) = u(y)$. These predictions are different from those obtained based on Fudenberg et al. (2015)'s approach.

A.3 The asymmetric treatment effects on the lower and upper bound of randomization range

We illustrate the asymmetric treatment effects on the lower bound and the upper bound of randomization range in this section. Recall that $\underline{y}$ denotes the largest sure payment that the individual chooses $\lambda^* < 1$ (the lower bound) and $\bar{y}$ denotes the smallest sure payment she chooses $\lambda^* > 0$ (the upper bound).

In the model extended from Cerreia-Vioglio et al. (2015) and Klibanoff et al. (2005),

$$\begin{aligned} u(\bar{y_x}) &= E_\pi \left[U_\tau(x)\right], \\ \underline{y} &= E_\pi \left[U_\tau(x)\right] - \frac{-\phi'' \left[u(y)\right]}{\phi' \left[u(y)\right]} \sigma_x^2. \end{aligned}$$

The changes in the upper and lower bounds depend on both $E_\pi \left[U_\tau(x)\right]$ and σ_x^2. We observe that subjects on average valued the complex lottery higher than the simple lottery (mean CE of 4.68 for the simple lottery versus 4.98 for the complex lottery in Experiment 2, p<0.01). Since the complex lottery has a larger σ_x^2 and the average valuation of the lottery is $E_\pi \left[U_\tau(x)\right] - \frac{-\phi''(E_\pi[U_\tau(x)])}{2}\sigma_x^2$, this implies an increase in $E_\pi \left[U_\tau(x)\right]$ for the complex lottery. The increase in $E_\pi \left[U_\tau(x)\right]$ increases both the upper bound and the lower bound, while the increase in σ_x^2 decreases only the lower bound. Together, they imply that the treatment effect on the upper bound could be larger than on the lower bound. Similarly, we observe an increase, albeit small, in the valuation of the complex lottery in the full-experience treatment (mean CE of 4.98 in the no-experience treatment versus 5.07 in the full-experience treatment in Experiment 2, p>0.10). The increase in $E_\pi \left[U_\tau(x)\right]$ increases both the upper and lower bounds, and the decrease in σ_x^2 increases the lower bound further. Consequently, the treatment effect could be stronger on the lower bound than on the upper bound.

The analysis based on Fudenberg et al. (2015) follows similarly. In Fudenberg et al. (2015), $\bar{y} = EU(x) + \Delta$, $\underline{y} = EU(x) - \Delta$. The changes in the upper and lower bounds depend on both $EU(x)$ and Δ. Since the average valuation of the lottery is $EU(x)$, the higher average valuation of the complex lottery implies higher $EU(x)$ of the complex lottery compared to the simple lottery. Higher $EU(x)$ and Δ imply a stronger treatment effect on the upper bound than on the lower bound. Likewise, an increase in experience level is associated with an increase in $EU(x)$ and a decrease in Δ, which jointly imply a stronger treatment effect on the lower bound than the upper bound.

B Additional figures and tables

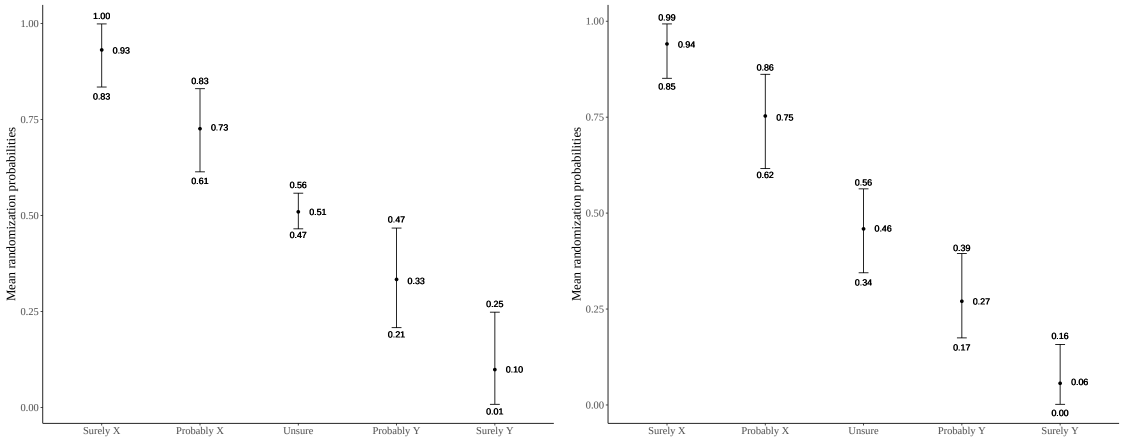
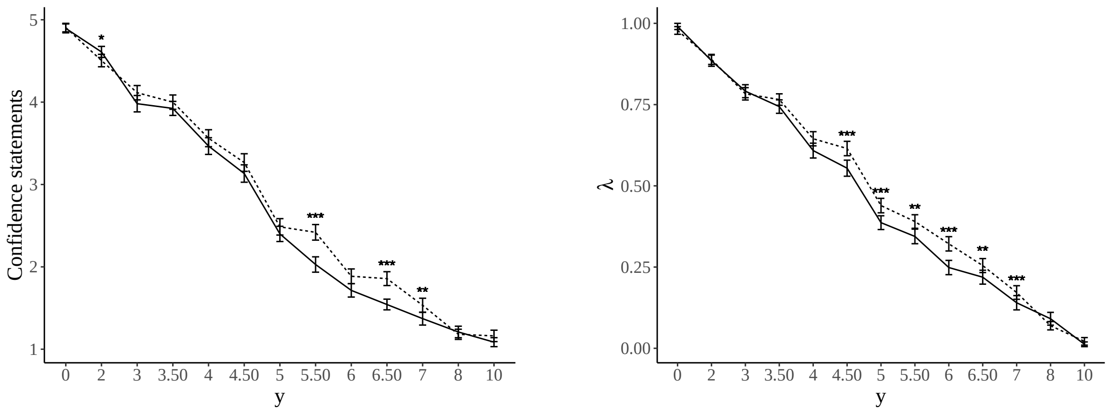


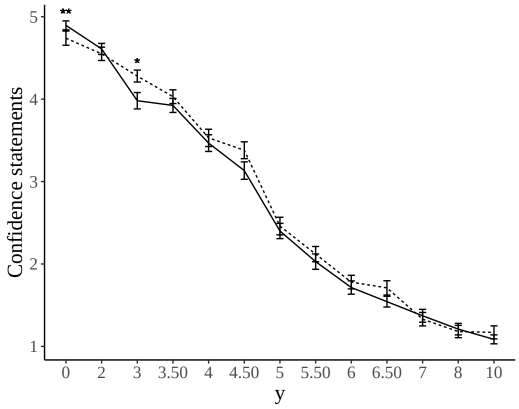
Figure B.1: The mean randomization probabilities at each confidence statement. The bars show the average minimum and maximum values. The values show the aggregate values for the baseline treatment – simple lottery, no-experience – in Experiment 1 (left) and Experiment 2 (right). The mean, minimum, and maximum values for the separate treatments in each of the experiments can be found in Table B.2. These values are broadly consistent with the cutoff probabilistic confidence levels of each confidence statement reported in Vanberg (2008, Footnote 10, p.1472: the probabilistic confidence level of 0.85 as the cutoff between surely and probably, 0.68 as the cutoff between probably and unsure, and 0.50 as the mean value for unsure). The minimum randomization probabilities were 0.83 and 0.85 for "Surely x" and 0.61 and 0.62 for "Probably x," and the mean randomization probabilities were 0.51 and 0.46 for "Unsure" in Experiments 1 and 2 respectively.



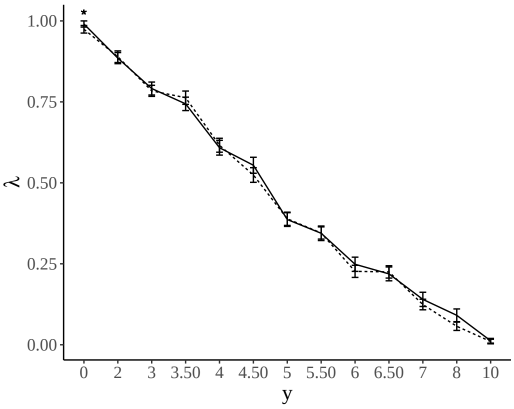
(a) Confidence statements

(b) Randomization probabilities

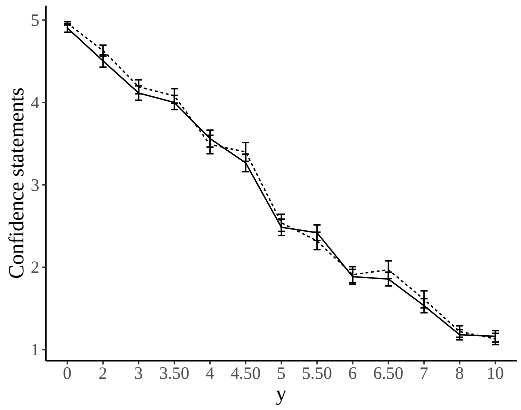
Figure B.2: The mean self-reported decision confidence and randomization probabilities for each value of y obtained from decisions about the simple lottery (solid line) and decisions about the complex lottery (dashed line) in Experiment 1. Wilcoxon signed-rank tests were performed to test the difference between the simple lottery and the complex lottery for each value of y: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.



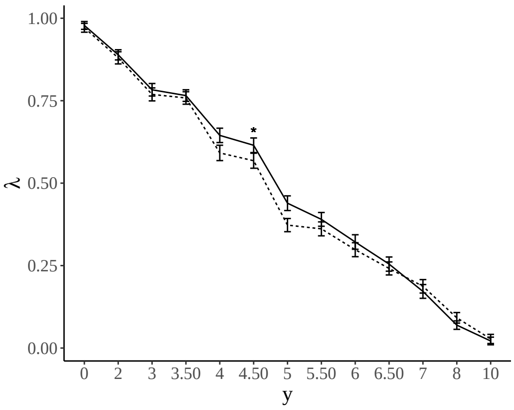
(a) Confidence statements, simple lottery



(b) Randomization probabilities, simple lottery

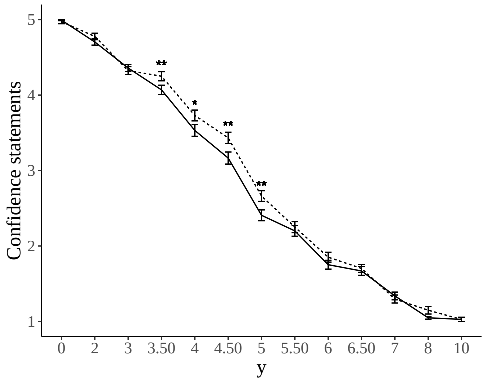


(c) Confidence statements, complex lottery

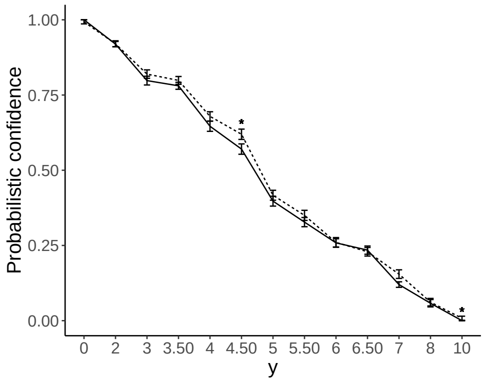


(d) Randomization probabilities, complex lottery

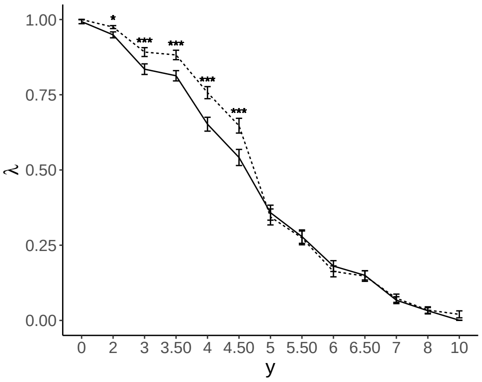
Figure B.3: The mean self-reported decision confidence and randomization probabilities for each value of y in the no-experience treatment (solid line) and the partial-experience treatment (dashed line) in Experiment 1. Wilcoxon rank-sum tests were performed to test the difference between the partial-experience treatment and no-experience treatment for each value of y: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.



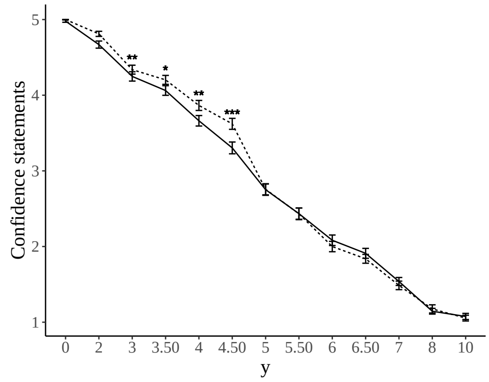
(a) Confidence statements, simple lottery



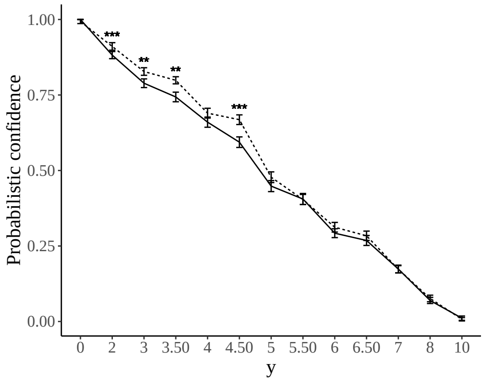
(b) Probabilistic confidence, simple lottery



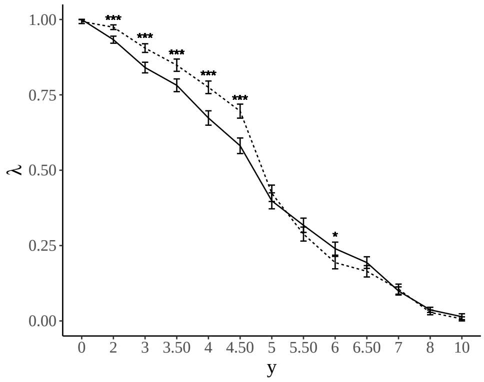
(c) Randomization probabilities, simple lottery



(d) Confidence statements, complex lottery



(e) Probabilistic confidence, complex lottery



(f) Randomization probabilities, complex lottery

Figure B.4: The mean self-reported decision confidence and randomization probabilities for each value of y in the no-experience treatment (solid line) and the full-experience treatment (dashed line) in Experiment 2. Wilcoxon rank-sum tests were performed to test the difference between full-experience treatment and no-experience treatment for each value of y: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Lottery	Treatment		Correlation between randomization probabilities and confidence statements		prob. confidence
			Experiment 1	Experiment 2	Experiment 2
Simple	No experience	10th percentile	0.60	0.71	0.73
		25th percentile	0.78	0.82	0.82
		median	0.91	0.89	0.90
		75th percentile	0.95	0.94	0.96
		90th percentile	0.97	0.96	0.98
	Experience	10th percentile	0.60	0.78	0.77
		25th percentile	0.85	0.85	0.85
		median	0.93	0.90	0.91
		75th percentile	0.96	0.95	0.96
		90th percentile	0.97	0.97	0.99
Complex	No experience	10th percentile	0.69	0.67	0.64
		25th percentile	0.83	0.77	0.83
		median	0.90	0.88	0.89
		75th percentile	0.94	0.93	0.95
		90th percentile	0.97	0.96	0.97
	Experience	10th percentile	0.62	0.69	0.77
		25th percentile	0.81	0.80	0.84
		median	0.88	0.90	0.90
		75th percentile	0.94	0.94	0.95
		90th percentile	0.96	0.97	0.97

Table B.1: Nonparametric Spearman correlation between randomization probabilities and the two self-reported confidence measures at the 10th percentile, 25th percentile, median, 75th percentile, and 90th percentile in the two experiments for each lottery and experience treatment group.

Treatment	Lottery		Surely x	Probably x	Unsure	Probably y	Surely y
Experiment 1							
No-experience	Simple	Mean	0.93 (0.011)	0.73 (0.016)	0.51 (0.023)	0.33 (0.018)	0.10 (0.010)
		Min	0.83 (0.027)	0.61 (0.022)	0.47 (0.027)	0.21 (0.019)	0.01 (0.005)
		Max	1 (0.001)	0.83 (0.015)	0.56 (0.025)	0.47 (0.026)	0.25 (0.025)
	Complex	Mean	0.92 (0.012)	0.72 (0.015)	0.56 (0.015)	0.35 (0.017)	0.09 (0.011)
		Min	0.82 (0.024)	0.62 (0.019)	0.49 (0.023)	0.24 (0.020)	0.01 (0.005)
		Max	0.99 (0.006)	0.82 (0.016)	0.63 (0.020)	0.46 (0.018)	0.22 (0.026)
Partial-experience	Simple	Mean	0.90 (0.018)	0.68 (0.017)	0.52 (0.021)	0.33 (0.018)	0.10 (0.012)
		Min	0.79 (0.0129)	0.57 (0.021)	0.45 (0.022)	0.24 (0.021)	0.01 (0.008)
		Max	0.98 (0.013)	0.79 (0.019)	0.59 (0.026)	0.42 (0.021)	0.24 (0.027)
	Complex	Mean	0.89 (0.015)	0.69 (0.016)	0.51 (0.023)	0.31 (0.016)	0.11 (0.013)
		Min	0.77 (0.029)	0.55 (0.023)	0.44 (0.027)	0.21 (0.018)	0.01 (0.008)
		Max	0.98 (0.009)	0.82 (0.016)	0.58 (0.028)	0.42 (0.022)	0.22 (0.024)
Experiment 2							
No-experience	Simple	Mean	0.94 (0.010)	0.75 (0.018)	0.46 (0.023)	0.27 (0.020)	0.06 (0.008)
		Min	0.85 (0.022)	0.62 (0.027)	0.34 (0.026)	0.17 (0.020)	0 (0.001)
		Max	0.99 (0.007)	0.86 (0.017)	0.56 (0.029)	0.39 (0.026)	0.16 (0.022)
	Complex	Mean	0.95 (0.008)	0.73 (0.019)	0.46 (0.025)	0.22 (0.017)	0.05 (0.009)
		Min	0.88 (0.018)	0.59 (0.028)	0.34 (0.027)	0.13 (0.015)	0 (0)
		Max	1 (0)	0.86 (0.017)	0.58 (0.029)	0.34 (0.026)	0.13 (0.021)
Full-experience	Simple	Mean	0.95 (0.008)	0.79 (0.017)	0.51 (0.026)	0.22 (0.019)	0.06 (0.010)
		Min	0.87 (0.022)	0.68 (0.026)	0.39 (0.031)	0.12 (0.018)	0.01 (0.007)
		Max	1 (0.002)	0.89 (0.015)	0.62 (0.031)	0.34 (0.026)	0.16 (0.023)
	Complex	Mean	0.95 (0.009)	0.78 (0.018)	0.50 (0.025)	0.24 (0.019)	0.51 (0.010)
		Min	0.87 (0.022)	0.65 (0.027)	0.39 (0.030)	0.15 (0.019)	0.01 (0.007)
		Max	1 (0.001)	0.89 (0.015)	0.62 (0.028)	0.36 (0.026)	0.13 (0.021)

Table B.2: The mean, minimum, and maximum randomization probabilities that correspond to each confidence statement for all treatments in the two experiments. The values in parentheses are the standard errors of the mean.

	Lottery	Randomization probabilities	Confidence statements	Probabilistic confidence
		Experiment 1		
Lower bound	Simple	2.99	2.95	
	Complex	2.84*	2.94	
Upper bound	Simple	6.61	6.30	
	Complex	6.90***	6.56**	
Range size	Simple	3.63	3.36	
	Complex	4.06***	3.62	
		Experiment 2		
Lower bound	Simple	3.16	3.03	2.63
	Complex	3.19	2.99	2.59
Upper bound	Simple	6.18	6.19	7.00
	Complex	6.38***	6.5***	7.21***
Range size	Simple	3.03	3.15	4.37
	Complex	3.19*	3.58***	4.63***

Table B.3: Comparisons of the lower bound, the upper bound, and the range size between the simple lottery and complex lottery in the no-experience treatment in the two experiments. The lower bound, the upper bound, and the range sizes are defined by randomization probabilities ($0.10 \leq \lambda \leq 0.90$), confidence statements ("Probably x", "Unsure", "Probably y") and probabilistic confidence (between "90% x, 10% y" and "10% x, 90% y"). Wilcoxon signed-rank tests were performed to test the difference between the simple lottery and the complex lottery for each measure: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	Self-reported probabilistic confidence									
	100% x 0% y	90% x 10% y	80% x 20% y	70% x 30% y	60% x 40% y	40% x 60% y	30% x 70% y	20% x 80% y	10% x 90% y	0% x 100% y
	Simple lottery, no-experience treatment									
Rand. prob.	0.98 (0.008)	0.86 (0.021)	0.82 (0.022)	0.73 (0.022)	0.58 (0.027)	0.36 (0.024)	0.23 (0.023)	0.18 (0.022)	0.09 (0.018)	0.02 (0.004)
	Complex lottery, no-experience treatment									
Rand. prob.	0.97 (0.007)	0.85 (0.024)	0.76 (0.026)	0.73 (0.024)	0.59 (0.025)	0.33 (0.024)	0.21 (0.021)	0.18 (0.024)	0.08 (0.016)	0.04 (0.012)
	Simple lottery, full-experience treatment									
Rand. prob.	0.97 (0.007)	0.92 (0.016)	0.85 (0.024)	0.76 (0.025)	0.63 (0.026)	0.37 (0.025)	0.23 (0.024)	0.16 (0.020)	0.07 (0.015)	0.03 (0.009)
	Complex lottery, full-experience treatment									
Rand. prob.	0.98 (0.007)	0.92 (0.017)	0.81 (0.025)	0.71 (0.028)	0.62 (0.027)	0.35 (0.023)	0.20 (0.024)	0.16 (0.027)	0.06 (0.015)	0.02 (0.008)

Table B.4: The mean randomization probabilities at each self-reported probabilistic confidence level in Experiment 2 for each lottery and experience treatment group. The standard errors of the mean are reported in the parentheses. We compute the mean randomization probability at each level of probabilistic confidence for each subject before taking its mean across subjects.

Lottery		Experience	Randomization probabilities	Confidence statements	Probabilistic confidence
			Experiment 1		
Simple	Lower bound	No	2.99	2.95	
		Partial	2.75	2.80	
	Upper bound	No	6.61	6.30	
		Partial	6.60	6.25	
	Range size	No	3.63	3.36	
		Partial	3.85	3.45	
Complex	Lower bound	No	2.84	2.94	
		Partial	2.74	3.03	
	Upper bound	No	6.90	6.56	
		Partial	6.90	6.52	
	Range size	No	4.06	3.62	
		Partial	4.16	3.49	
			Experiment 2		
Simple	Lower bound	No	3.16	3.03	2.63
		Full	3.44**	3.26*	2.71
	Upper bound	No	6.18	6.19	7.00
		Full	6.18	6.30	7.10
	Range size	No	3.03	3.15	4.37
		Full	2.74	3.04	4.38
Complex	Lower bound	No	3.19	2.99	2.59
		Full	3.60***	3.29**	2.73*
	Upper bound	No	6.38	6.57	7.21
		Full	6.27	6.45	7.31
	Range size	No	3.19	3.58	4.63
		Full	2.67**	3.16**	4.58

Table B.5: Comparisons of the lower bound, the upper bound, and the range size between the no-experience treatment and experience treatments by lottery type in the two experiments. The lower bound, the upper bound, and the range sizes are defined by randomization probabilities ($0.10 \leq \lambda \leq 0.90$), confidence statements ("Probably x", "Unsure", "Probably y") and probabilistic confidence (between "90% x, 10% y" and "10% x, 90% y"). Wilcoxon rank-sum tests were performed to test the difference between experience treatment and no-experience treatment for each measure: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

B.1 Order effects in experiment 2

We selected three different orders and randomly assigned subjects to each order: Order 1) binary choices and confidence statements → randomized choices → probabilistic confidence, Order 2) randomized choices → binary choices and confidence statements → probabilistic confidence, Order 3) probabilistic confidence → binary choices and confidence statements → randomized choices. Order 1 is similar to the task order in Experiment 1, allowing us to assess the robustness of the findings in Experiment 1. Order 2 removes the potential priming effects of the self-reported confidence measures on randomized choices. Order 3 preserves the potential of the priming effects, but allows us to look at the probabilistic confidence measure when it is elicited first.

Given our proposal to capture decision confidence with randomization probabilities, the question that is most relevant to us is whether the subjects randomized differently when they made randomization choices prior to and after they completed the self-reported confidence measures.

We find that subjects were less likely to randomize strictly ($0 < \lambda < 1$) when randomization probabilities were elicited before self-reported decision measures (Order 2). Table B.6(a) compares the proportion of decisions in which subjects randomized strictly when the randomized choices were made first versus when they were made later across orders and treatments. On average, subjects in Order 2 randomized strictly in fewer choices compared to subjects in the other two orders. This difference is statistically significant in three out of eight comparisons, but it is not significant when we aggregate across all treatments. Table B.6(b) shows that the above order effects were stronger in choices with high values ($y \geq 4$) than with low values ($y \leq 3.5$) of sure payments. Consistent with these findinds, Figure B.5 shows that the proportion of subjects with strict randomization probabilities tends to be lower at each value of y in Order 2 (the red lines) than in the other two orders, and more so for high values of sure payments.

Apart from the lower tendency to randomize, randomization probabilities corresponded to the probabilistic confidence less well at some probabilistic confidence values (e.g., 30% x

	Simple		Complex	
	No experience	Experience	No experience	Experience
Order 1	0.456	0.368	0.452	0.373
Order 2	0.345	0.331	0.369	0.328
Order 3	0.471	0.412	0.498	0.400
Wilcoxon rank-sum tests:				
Order 1 vs Order 2	$p < 0.10$	$p = 0.625$	$p = 0.217$	$p = 0.541$
Order 2 vs Order 3	$p < 0.05$	$p = 0.192$	$p < 0.05$	$p = 0.194$

Table (a)

	$y = 0$ to 3.5	$y = 4$ to 6	$y = 6.5$ to 10
Order 1	0.303	0.622	0.267
Order 2	0.272	0.514	0.197
Order 3	0.339	0.650	0.296
Wilcoxon rank-sum tests:			
Orders 2 vs 1	$p = 0.258$	$p < 0.05$	$p < 0.05$
Orders 2 vs 3	$p < 0.05$	$p < 0.01$	$p < 0.01$

Table (b)

Table B.6: Panel (a) reports the proportion of strict randomization choices ($0 < \lambda < 1$) across treatments in each order. Panel (b) the average proportions of strict randomiztion at different ranges of sure payments aggregated across treatments and lotteries in each order.

and 40% x) in Order 2 compared to the other two orders. Figure B.7 reports this result. Finally, we find that the cumulative distributions of the correlation between randomization probabilities and a self-reported decision confidence measure in Order 2 tend to be on the left of the other two orders (see Figure B.6). This implies that there were more subjects with lower correlation between randomization probabilities and self-reported decision confidence in Order 2 than in the other two orders. As a whole, the results suggest that randomized choices were affected by priming.

Despite the presence of the priming effects on randomized choices, we find support for our hypotheses when we restrict our analyses to subjects in Order 2. First, we find high correlations between randomization probabilities and self-reported confidence in Order 2, consistent with H1: the median correlation between randomization probabilities and confidence statement as well as the median correlation between randomization probabilities and probabilistic confidence in Order 2 ranges from 0.87 to 0.89 across treatments respectively. Second, subjects in Order 2 reported low decision confidence for choices around

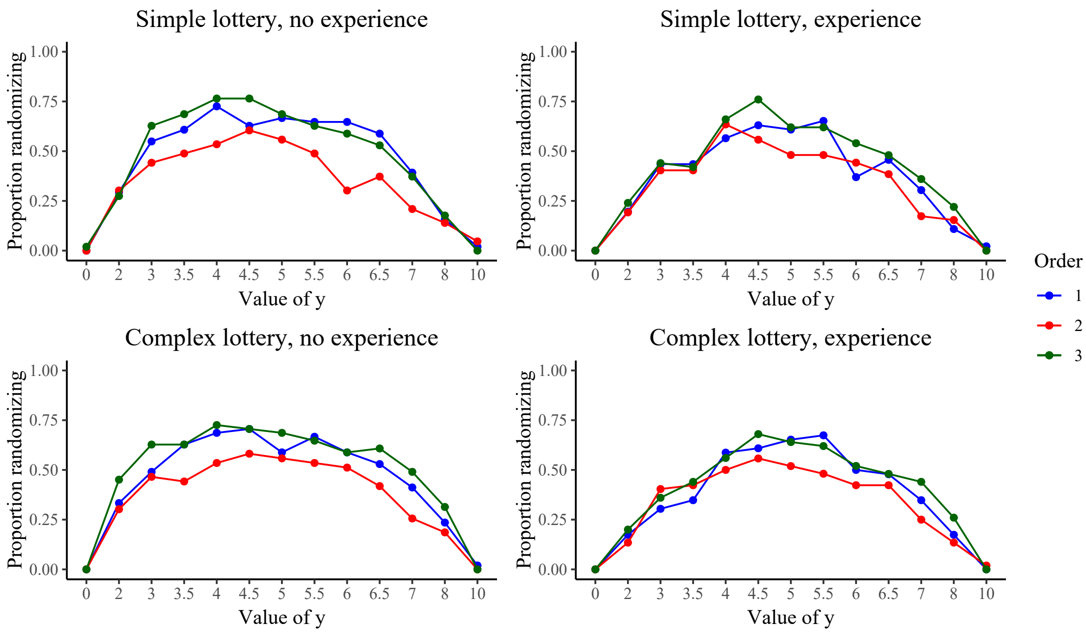


Figure B.5: The proportion of strict randomization choices ($0 < \lambda < 1$) at each value of y in the three orders.

the switching range and chose randomization probabilities close to 0.5 in these decisions, supporting H2. Table B.7 shows that subjects' median randomization probability for all choices that falls within the switching range is between 0.48 and 0.50 across treatments, consistent with their median confidence statement of "Unsure," and their median probabilistic confidence which ranges from 40% x to 60% x across treatments.[7] Third, like in the full sample, we find significant treatment effects in Order 2. Table B.8 reports the range of sure payments over which subjects expressed less than full decision confidence in their decisions. The ranges based on self-reported decision measures and randomization choices suggest that subjects in Order 2 had lower decision confidence for decisions involving the complex lottery than for decisions involving the simple lottery, consistent with H3. We find similar support for H4 in Order 2 as in the full sample. Among subjects in Order 2, those in the full-experience treatment reported less than full decision confidence in smaller ranges of sure payments for decisions involving the complex lottery, but not for decisions

[7]The median randomization probability at the upper bound ($\bar{y_b}$) of the switching range is lower in Order 2 compared to the other two orders in most treatments. This is consistent with our earlier finding of the order effects that there were fewer strict randomization choices in Order 2, and more so for choices involving larger sure payments.

involving the simple lottery. In terms of randomization probabilities, these ranges are 2.30 vs 2.89 in Order 2. In comparison, they are 2.81 vs 3.09 in Order 1 and 2.90 vs 3.51 in Order 3. The difference is statistically significant in aggregate (2.67 vs 3.19, $p < 0.05$), but not when considering any order separately ($p > 0.10$).

Taken together, priming effects could have strengthened some of our aggregate findings. However, the findings that subjects in Order 2 randomized in ways that were broadly consistent with our hypotheses suggest that randomization probabilities and self-reported decision confidence measures are likely to share common psychological foundations.

It is worth noting that the aforementioned order effects do not automatically imply that randomization probabilities are a poorer proxy for decision confidence than self-reported decision confidence. Confidence statements and probabilistic confidence are also noisy proxies of decision confidence, and there is no objective criterion for the "right" amount of strict randomization. For example, it is possible that subjects may have not randomized too little in Order 2, but that they have randomized excessively in the other two orders due to the priming effects. Since there is no obvious benchmark to compare decision confidence measures, and decision confidence is not directly observable, we consider the value of decision confidence on the basis of its correspondence with actual choices. When decision confidence corresponds perfectly with choices, having decision confidence of $p\%$ for x would imply that x is chosen $p\%$ of time. Figure B.8 shows, across subjects and choices, the proportion of choices in which x was chosen based on subjects' binary choices by randomization probabilities as well as the probabilistic confidence in the three orders.[8] On average, both measures of decision confidence closely trace the proportions of x chosen in binary choices. Importantly, randomization probabilities exhibit a closer correspondence to binary choices than probabilistic confidence in all three orders. This alignment is particularly evident in Order 2, with randomization probabilities exhibiting a significantly closer match to these proportions than probabilistic confidence at nine levels of randomization probabilities/probabilistic confidence versus five levels in Order 1 and six levels in Order 3.

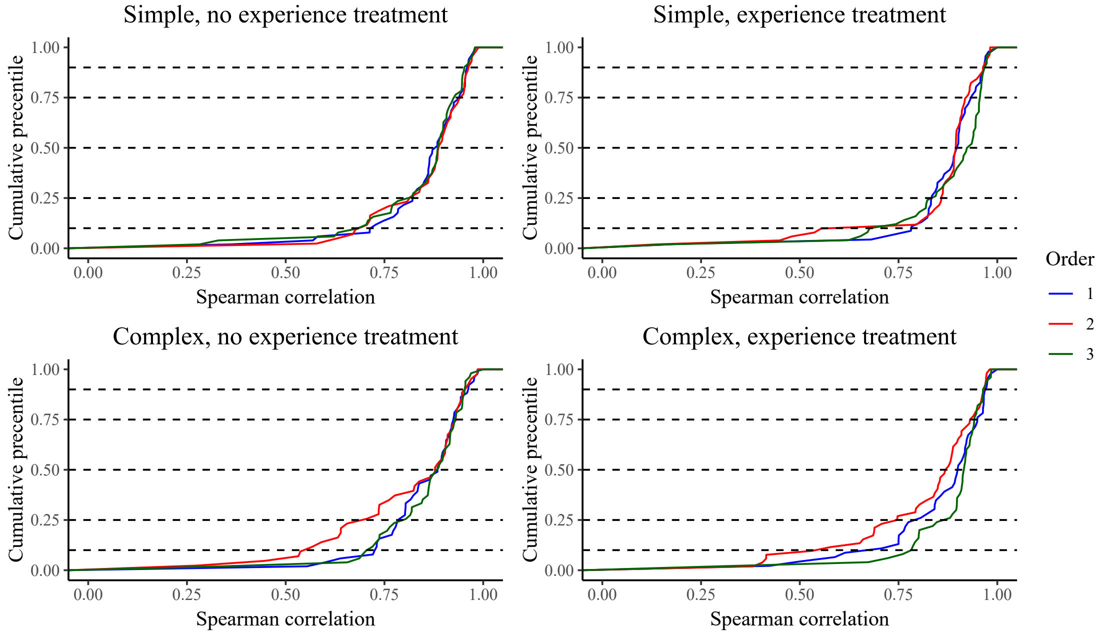
[8]We did not consider confidence statements here, because they were elicited on the same decision screen as binary choices.

Confidence statements						
			No experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
Order 2	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
Order 3	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
			Experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
Order 2	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
Order 3	Probably x	Probably y	Unsure	Probably x	Probably y	Unsure
Probabilistic confidence						
			No experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	60% x	40% x	40% x	60% x	40% x	60% x
Order 2	70% x	40% x	60% x	60% x	40% x	40% x
Order 3	60% x	40% x	60% x	60% x	30% x	40% x
			Experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	60% x	40% x	50% x	60% x	40% x	40% x
Order 2	70% x	40% x	60% x	60% x	40% x	40% x
Order 3	60% x	40% x	60% x	60% x	40% x	60% x
Randomization probabilities						
			No experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	0.65	0.46	0.50	0.68	0.40	0.50
Order 2	0.68	0.27	0.50	0.63	0.30	0.48
Order 3	0.60	0.46	0.50	0.58	0.21	0.40
			Experience			
Order	Simple			Complex		
	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$	$\underline{y_b}$	$\bar{y_b}$	$[\underline{y_b}, \bar{y_b}]$
Order 1	0.68	0.30	0.55	0.60	0.33	0.50
Order 2	0.63	0.23	0.50	0.66	0.25	0.49
Order 3	0.70	0.43	0.51	0.61	0.40	0.50

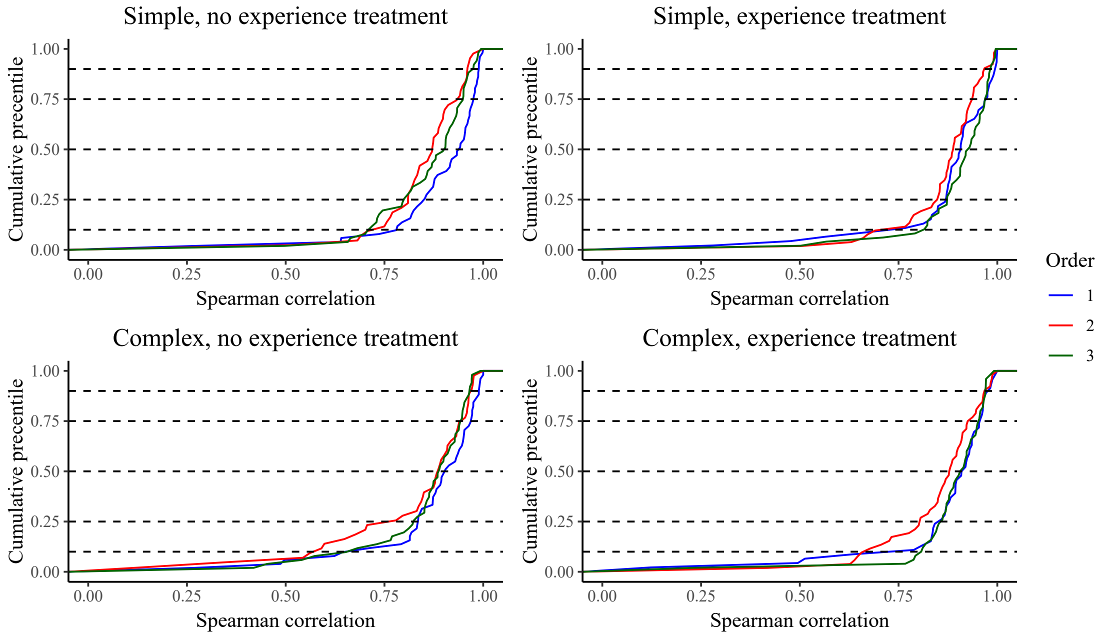
Table B.7: Median behavior around the switching choices ($\underline{y_b}$ and $\bar{y_b}$) and within the switching range ($[\underline{y_b}, \bar{y_b}]$) aggregated across lotteries and treatments for each order.

	Lottery	Treatment	Combined	Order 1	Order 2	Order 3
Confidence Statements	Simple	No experience	3.15	2.96	2.77	3.68
		Experience	3.04	2.84	2.92	3.35
	Complex	No experience	3.58***	3.25*	3.56***	3.93
		Experience	3.16_{**}	3.23	3.03_{*}	3.23_{*}
Probabilistic Confidence	Simple	No experience	4.37	4.28	4.06	4.72
		Experience	4.38	4.24	4.40	4.50
	Complex	No experience	4.63***	4.34	4.61***	4.93*
		Experience	4.58	4.53	4.50	4.70
Randomization Probabilities	Simple	No experience	3.03	3.17	2.67	3.14
		Experience	2.74	3.00	2.34	2.93
	Complex	No experience	3.19*	3.09	2.89*	3.51**
		Experience	2.67_{**}	2.81	2.30	2.90

Table B.8: The mean size of the range of sure payments over which subjects express that they are not fully confident about their decision based on each of the confidence measures, by the lottery and experience treatments in aggregate and in each order separately (randomization probabilities ($0.10 \leq \lambda \leq 0.90$), confidence statements ("Probably x", "Unsure", "Probably y") and probabilistic confidence (between "90% x, 10% y" and "10% x, 90% y")). Stars in the upper right corners of a cell denote statistical significance of Wilcoxon signed-rank tests between the simple lottery and the complex lottery, while stars in the lower right corner denote statistical significance of Wilcoxon rank-sum tests between the no experience and experience treatment: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.



(a) Confidence statements



(b) Probabilistic confidence

Figure B.6: ECDF for the correlation with randomization probabilities in each order across treatments.

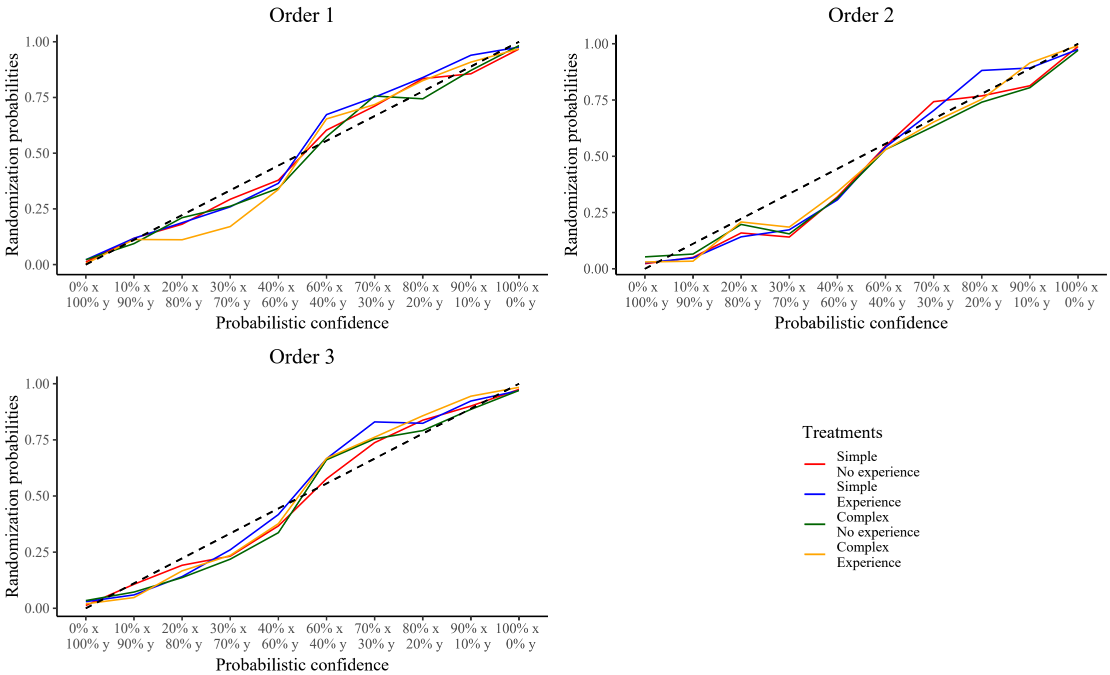


Figure B.7: The mean randomization probabilities at each self-reported probabilistic confidence level in Experiment 2 for each lottery and experience treatment in each order separately.

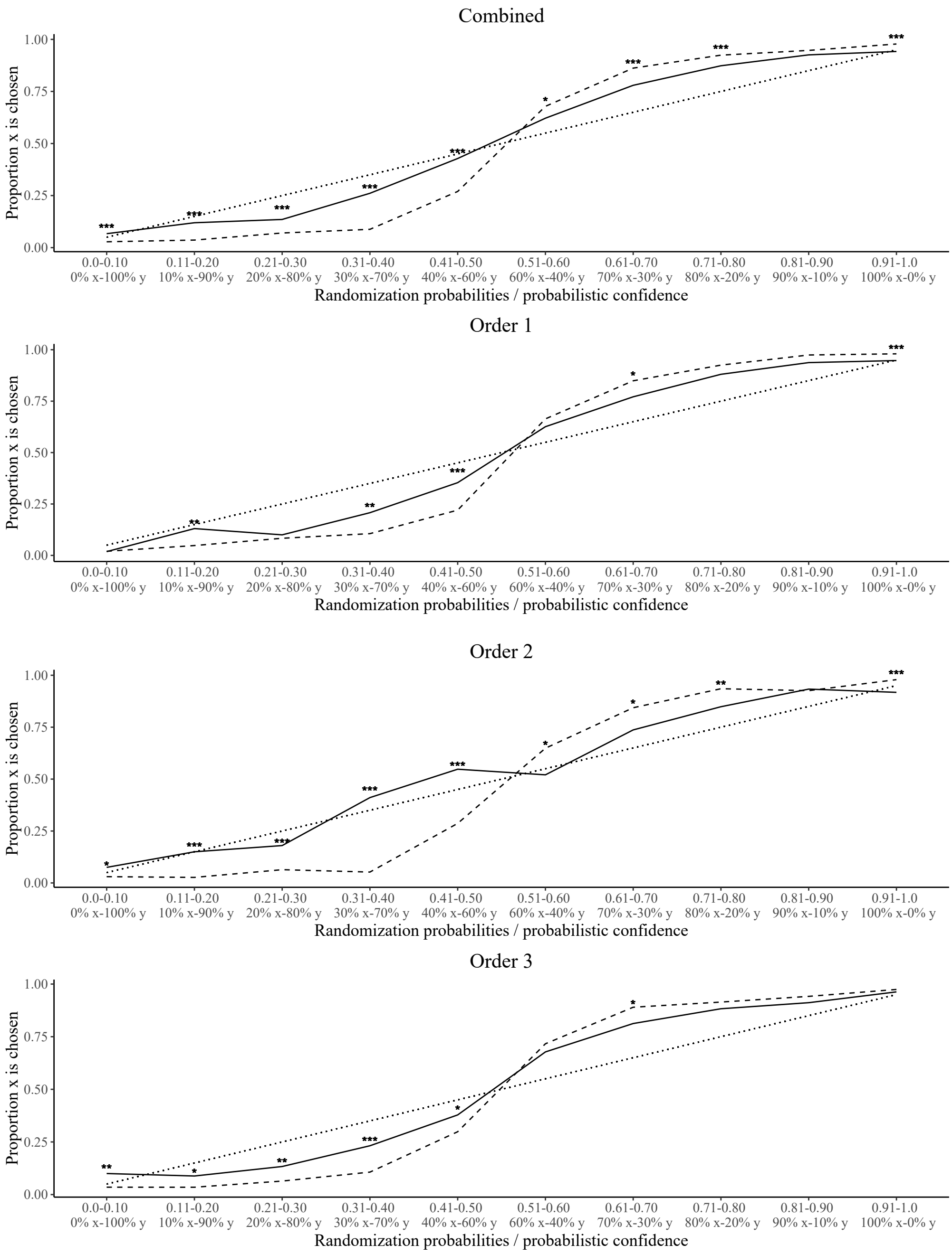


Figure B.8: Correspondence of randomization probabilities (solid line) and probabilistic confidence (dashed line) with the proportion of binary choices in which x is chosen (y-axis) across lotteries and treatments for each decision order. The dotted line represents a 45-degree line. Fisher's exact tests were performed to test the difference in choice proportions between the randomization probabilities and probabilistic confidence: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B.2 Alternative interpretations of randomization probabilities

In the main text, we have shown that indifference and random errors cannot be the driving force behind subjects' randomization behavior. We elaborate why utility difference alone cannot explain randomization here.

Butler et al. (2014) call utility difference the strength of preferences: "the relative degree of difference between the two options as perceived by the decision maker" (Butler et al., 2014, p.538). For example, we can write this explicitly as a Fechnerian utility model $p = \phi\left[U(L) - u(y)\right]$, where $\phi : R \Rightarrow [0, 1]$ is a cumulative distribution function with $\phi(0) = 0.5$ (Luce and Suppes, 1965, p.334). The lower bound and the upper bound of randomization are then $u(\underline{y}) = U(L) - \phi^{-1}(0.90)$ and $u(\bar{y}) = U(L) - \phi^{-1}(0.10)$. If randomization probabilities depend only on utility differences, when $U(L)$ increases: (1) the randomization probability should increase for each value of sure payment; and (2) the lower bound and the upper bound of randomization should increase equally. Our results clearly reject these two predictions. Subjects' randomization probabilities did not shift horizontally but were instead compressed towards 0.5 when they faced the complex lottery compared to the simple lottery, and were stretched away from 0.5 in the full-experience treatment compared to the no-experience treatment. In addition, Table B.3 shows that while $\bar{y}$ for the complex lottery was significantly higher than $\bar{y}$ for the simple lottery in both experiments (Experiment 1: 6.90 vs 6.61, Wilcoxon signed-rank test, $p < 0.01$; Experiment 2: 6.38 vs 6.18, Wilcoxon signed-rank test $p < 0.01$), $\underline{y}$ for the complex lottery was significantly lower than $\underline{y}$ for the simple lottery in Experiment 1 at 10% significance level (2.84 vs 2.99, Wilcoxon signed-rank test $p < 0.10$), but was not significantly different in Experiment 2 (3.19 vs 3.16, Wilcoxon signed-rank test $p = 0.646$).

Importantly, although the difference in the mean valuation of the lottery with or without experience was similar to that of the complex lottery versus the simple lottery, Table B.5 shows that $\underline{y}$ were significantly lower for subjects in the full-experience treatment than in the no experience treatment (Simple lottery: 3.16 vs 3.44, Wilcoxon rank-sum test $p < 0.05$; Complex lottery: 3.19 vs 3.60, Wilcoxon rank-sum test $p < 0.01$) but not $\bar{y}$ (Simple lottery: 6.18 vs 6.18, Wilcoxon rank-sum test $p = 0.789$; Complex lottery: 6.27

Randomization Interval	The number of subjects who chose randomization			
	0 times	1 time	2 times or more	3 times or more
Experiment 1: Simple lottery, no-experience				
$0 < \lambda < 1$	2	6	97	95
$0.10 \leq \lambda \leq 0.90$	3	6	96	93
$0.40 \leq \lambda \leq 0.60$	22	26	57	35
Experiment 1: Complex lottery, no-experience				
$0 < \lambda < 1$	4	1	100	98
$0.10 \leq \lambda \leq 0.90$	4	2	99	98
$0.40 \leq \lambda \leq 0.60$	21	15	69	46
Experiment 1: Simple lottery, partial-experience				
$0 < \lambda < 1$	6	1	93	89
$0.10 \leq \lambda \leq 0.90$	6	1	93	88
$0.40 \leq \lambda \leq 0.60$	13	24	63	38
Experiment 1: Complex lottery, partial-experience				
$0 < \lambda < 1$	3	5	92	89
$0.10 \leq \lambda \leq 0.90$	3	7	90	89
$0.40 \leq \lambda \leq 0.60$	14	19	67	46
Experiment 2: Simple lottery, no-experience				
$0 < \lambda < 1$	25	8	112	106
$0.10 \leq \lambda \leq 0.90$	26	7	112	105
$0.40 \leq \lambda \leq 0.60$	42	24	79	44
Experiment 2: Complex lottery, no-experience				
$0 < \lambda < 1$	26	6	113	100
$0.10 \leq \lambda \leq 0.90$	26	7	112	98
$0.40 \leq \lambda \leq 0.60$	37	38	70	42
Experiment 2: Simple lottery, full-experience				
$0 < \lambda < 1$	32	11	105	98
$0.10 \leq \lambda \leq 0.90$	34	11	103	96
$0.40 \leq \lambda \leq 0.60$	55	36	57	32
Experiment 2: Complex lottery, full-experience				
$0 < \lambda < 1$	35	11	102	91
$0.10 \leq \lambda \leq 0.90$	35	12	101	90
$0.40 \leq \lambda \leq 0.60$	56	25	67	38

Table B.9: The distribution of subjects who chose $0 < \lambda < 1$, $0.10 \leq \lambda \leq 0.90$, and $0.40 \leq \lambda < 0.60$ zero times, one time, two times or more, and three times or more across treatments in the two experiments.

Probabilistic confidence associated with each confidence statement						
Levels	Median	10th	30th	70th	90th	SD
Surely (min)	85%	70%	80%	90%	100%	16.31%
Probably (max)	80%	70%	80%	90%	99%	11.10%
Probably (min)	55%	25%	50%	60%	65%	16.59%
Unsure (max)	54%	25%	50%	60%	64%	17.58%
Unsure (min)	35%	0%	0%	40%	50%	21.12%
Confidence statements associated with each probabilistic confidence level						
Levels	Median	10th	30th	70th	90th	SD
100%	Surely x	Surely x	Surely x	Surely x	Surely x	0.17
90%	Surely x	Probably x	Surely x	Surely x	Surely x	0.54
80%	Probably x	Probably x	Probably x	Surely x	Surely x	0.54
70%	Probably x	Probably x	Probably x	Probably x	Probably x	0.34
60%	Unsure	Unsure	Unsure	Surely x	Surely x	0.54

Table B.10: The median, 10th, 30th, 70th, 90th percentile, and standard deviation of probabilistic confidence associated with each confidence statement and the median, 10th, 30th, 70th, 90th percentile, and standard deviation of confidence statements associated with each probabilistic confidence level. Consistent with Result 1, we code confidence statements of surely x, probably x, unsure, probably y, and surely y as 5, 4, 3, 2, and 1 respectively. Standard deviations are calculated accordingly.

vs 6.38, Wilcoxon rank-sum test $p = 0.369$). Further, randomization probabilities were larger at low sure payments but smaller at high sure payments when we compare the full-experience treatment with the no-experience treatment. These results highlight the central role of preference uncertainty beyond utility difference in affecting decision confidence.

One may perceive that self-reported decision confidence measures are easier to interpret than randomization probabilities because they ask about decision confidence explicitly. We show that self-reported decision confidence measures can be just as difficult to interpret by analyzing how subjects associate the two self-reported decision measures in the post-experiment questionnaire of Experiment 2. We asked subjects which confidence statement best described their probabilistic confidence p% in choosing x and 100-p% in choosing y for values $p = 60, 70, 80, 90, 100$. In a separate session, we asked subjects to state the minimum level of probabilistic confidence for "Surely", and the minimum and maximum levels of probabilistic confidence for "Probably" and "Unsure" on a scale from 0% to 100%.

Table B.10 summarizes the subjects' responses to the two questions. The top panel shows the range of probabilistic confidence levels associated with each confidence statement. Although the first column shows that the median probabilistic confidence thresholds are

Probabilistic confidence associated with each confidence statement						
For subjects: Unsure (max) $\geq$ 50% and Unsure(min) $>$ 0%						
Levels	Median	10th	30th	70th	90th	S.D
Surely (min)	85%	75%	80%	90%	100%	14.80%
Probably (max)	85%	75%	80%	90%	99%	9.13%
Probably (min)	60%	41%	55%	60%	70%	13.26%
Unsure (max)	60%	50%	55%	60%	65%	9.62%
Unsure (min)	40%	30%	40%	45%	50%	10.46%
For subjects: Unsure (max) $<$ 50%						
Levels	Median	10th	30th	70th	90th	S.D
Surely (min)	80%	50.3%	75%	85.5%	99%	20.22%
Probably (max)	80%	60%	75%	84%	95%	14.01%
Probably (min)	40%	20%	30%	50%	60%	17.57%
Unsure (max)	30%	10%	20%	35.4%	40%	12.94%
Unsure (min)	0%	0%	0%	0%	10%	8.46%

Table B.11: The median, 10th, 30th, 70th, 90th percentile, and standard deviation of probabilistic confidence associated with each confidence statement for subjects who fit the criteria specified in the table.

well-ordered (the median maximum probabilistic confidence of a lower ordered statement was always smaller than the median minimum probabilistic confidence of a higher ordered statement), the standard deviations reported in the last column as well as minimum and maximum probabilistic confidence assigned to each confidence statement at different percentile levels show the presence of substantial heterogeneity in the probabilistic confidence associated with each confidence statement.

Further, we find two different interpretations of the confidence statement “Unsure”. A large group of subjects (n=172) reported probabilistic confidence higher than 50% as the maximum of “Unsure” and higher than 0% as the minimum of “Unsure,” while another group of subjects (n=84) reported a probabilistic confidence level lower than 50% as the maximum of “Unsure” and close 0% as the minimum of “Unsure.” Table B.11 shows how different these two groups were in their associations of probabilistic confidence and other confidence statements. For example, the maximum level of probabilistic confidence for the statement “Probably” ranges from 75% to 99% among subjects who reported a probabilistic confidence level higher than 50% as the maximum of “Unsure,” and it ranges from 60% to 95% among subjects who reported a probabilistic confidence level lower than 50% as the maximum of “Unsure.”

B.3 Reasons to randomize

At the end of the session on randomized choices in Experiment 2, we asked the subjects who had chosen to randomize at least once in the post-experiment questionnaire, what their reasons for randomizing were. Of the 120 subjects who provided an answer to this question, 22% stated that they randomized because they were unsure about their choice or found it difficult to compare the two options. Here are a few examples:

- "Because I was not completely sure whether I wanted to choose A or B.'
- "I was not sure exactly what the consequences of my decision was going to be and I was not 100% confident in choosing either A or B."
- "Its difficult to make a decision for sure, so a combination feels more safe."

Another group of subjects (22.5%) randomized for reasons related to hedging. Here are a few examples:

- "Even though the certain option was less valued, certainty is nice and preferred over risky options. Therefore, I chose to combine them some of the time."
- "To hedge my bets when the expected gains of A and B were similar, gaining a small chance for big gains or loses in option A, adding some suspense."
- "For example when I preferred A but B felt a little safer so I thought it wouldn't hurt adding a bit more security since a B amount for sure isn't bad."

Around 18% stated that they chose to randomize when the sure payment amount was close to the expected value of the lottery but did not explain why randomizing is better. In contrast, most of the subjects who did not to randomize at all stated that they did not randomize because they did not want to pay the cost of 0.10 euro for randomizing and/or that they made their choices solely based on the computation of the expected value of the lottery.

B.4 Results for the loss lottery and the mixed lottery in Experiment 1

Treatment	Lottery		Surely x	Probably x	Unsure	Probably y	Surely y
No-experience	Loss	Mean	0.94	0.73	0.50	0.29	0.07
			(0.008)	(0.015)	(0.021)	(0.019)	(0.011)
		Min	0.84	0.61	0.40	0.19	0
			(0.022)	(0.023)	(0.025)	(0.019)	(0.002)
		Max	1	0.85	0.60	0.41	0.15
			(0.003)	(0.016)	(0.025)	(0.026)	(0.024)
	Mixed	Mean	0.90	0.71	0.53	0.32	0.10
			(0.015)	(0.015)	(0.025)	(0.020)	(0.016)
		Min	0.75	0.56	0.44	0.17	0.02
			(0.035)	(0.025)	(0.029)	(0.018)	(0.011)
		Max	0.99	0.85	0.61	0.46	0.23
			(0.003)	(0.016)	(0.030)	(0.031)	(0.031)
Partial-experience	Loss	Mean	0.91	0.70	0.55	0.34	0.07
			(0.014)	(0.018)	(0.019)	(0.019)	(0.011)
		Min	0.81	0.55	0.42	0.23	0.01
			(0.026)	(0.026)	(0.027)	(0.021)	(0.006)
		Max	0.98	0.82	0.66	0.46	0.14
			(0.012)	(0.016)	(0.022)	(0.025)	(0.022)
	Mixed	Mean	0.91	0.71	0.57	0.33	0.11
			(0.014)	(0.019)	(0.025)	(0.023)	(0.014)
		Min	0.78	0.57	0.47	0.23	0.01
			(0.031)	(0.027)	(0.032)	(0.026)	(0.006)
		Max	0.99	0.83	0.67	0.46	0.28
			(0.006)	(0.018)	(0.026)	(0.030)	(0.035)

Table B.12: The mean, minimum, and maximum randomization probabilities that correspond to each confidence statement for the loss and mixed lottery in both treatments in Experiment 1. The values in parentheses are the standard errors of the mean.

Treatment		Correlation between randomization probabilities and confidence statements	
		The loss lottery	The mixed lottery
No experience	10th percentile	0.67	0.35
	25th percentile	0.81	0.64
	median	0.90	0.85
	75th percentile	0.95	0.91
	90th percentile	0.97	0.96
Experience	10th percentile	0.65	0.47
	25th percentile	0.80	0.66
	median	0.87	0.81
	75th percentile	0.94	0.92
	90th percentile	0.97	0.95

Table B.13: Nonparametric Spearman correlation at the 10th percentile, 25th percentile, median, 75th percentile, and 90th percentile for the loss lottery and mixed lottery in both treatments in Experiment 1.

C Experimental materials

Experiment 1

Welcome

You are invited to participate in an experiment in which we examine how individuals make decisions. Your decisions in the experiment are about choices between different options. There are no right or wrong answers. The whole experiment will take approximately 20 minutes.

You will receive a participation fee of €1 for completing the survey. In addition you will receive monetary compensation up to €10 based on the decisions you make in the experiment. Specifically, one of the questions will be randomly selected. The decision you made in this question will determine your additional compensation.
You will receive the payment (the participation fee of €1 and the additional compensation) via bank transfer. For this we will ask your IBAN number. This information will only be used for payment, and will be permanently deleted afterwards.

Thank you for your participation!

Sincerely yours,

Associate Professor Dr. Jianying Qiu and PhD student Sara Arts
The Institute of Management Research
Radboud University Nijmegen.

(a)

Voluntary participation

Your participation in this research is voluntary. This means that you can withdraw your participation and consent at any time during the survey, without giving a reason. All data we have collected from you will be deleted permanently. If you desire to withdraw, please simply close your internet browser. After completion of the survey it will not be possible to withdraw your data form the research.

What will happen to the data?

The research data we collect during this study will be used by scientists as part of data sets, articles and presentations. The anonymized research data is accessible to other scientists for a period of at least 10 years. When we share data with other researchers, these data cannot be traced back to you.

More information?

Should you want more information on this research study, please contact Sara Arts (email: s.arts@fm.ru.nl)

CONSENT:

Please select your choice below.

Checking "Agree" below indicates that:
you voluntarily agree to participate.

- (●) I agree with the provided information, and I would like to proceed to the survey
- ( ) I do not agree with the above.

(b)

Figure C.1: Welcome screen (a) and informed consent (b) of the experiment.

The following questions are about the options below:

Option A:
Gain €9,75 with a chance of **20%**,
gain €7,50 with a chance of **30%**,
gain €2,50 with a chance of **30%**, and
gain €0,25 with a chance of **20%.**

Option B:
Gain a **sure** amount (which varies across questions).

You will be asked to choose between the two options, and to describe how confident you feel about your choice.

(a)

The following questions are about the options below:

Option A:
Gain €9 with a chance of **50%,** and
gain €1 with a chance of **50%**.

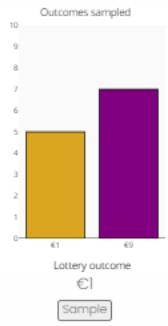
Option B:
Gain a **sure** amount (which varies across questions).

You will be asked to choose between the two options, and to describe how confident you feel about your choice.

Before you are asked to make the decisions we want to give you the opportunity to **experience the different outcomes of option A**. For this, you can click the button below. Each time you click the button a possible outcome of option A will be shown. You will get to sample **20** outcomes.

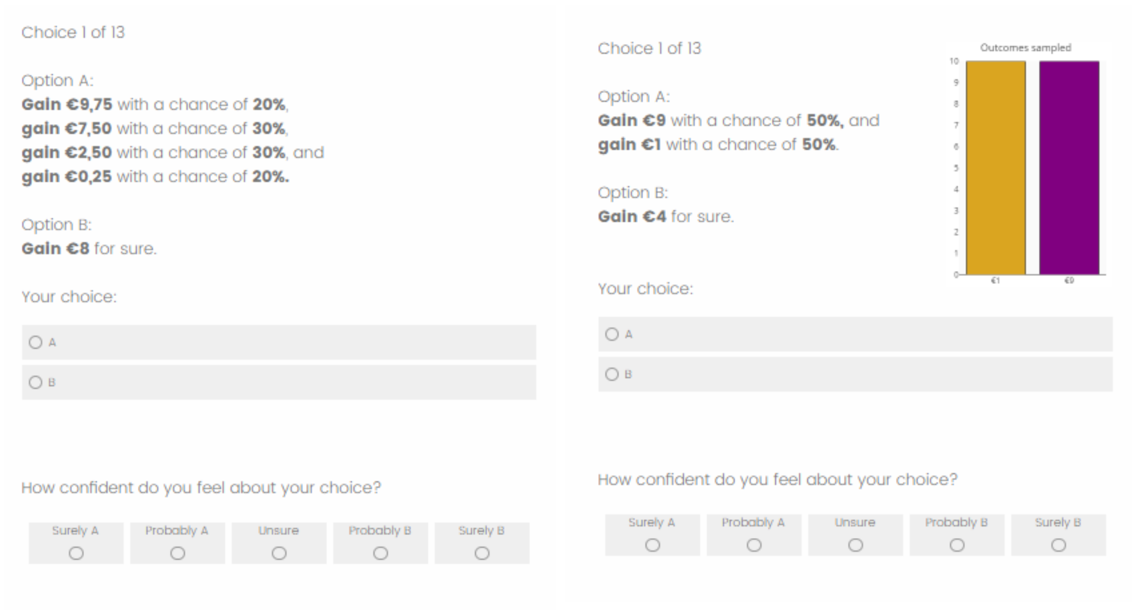
The outcomes that you obtain by clicking the button do not influence your payoff but are only presented to make you experience the possible outcomes.

To keep track of the sampled outcomes, they will be presented in a bargraph.



(b)

Figure C.2: The introduction of the binary choices and confidence statements for the complex lottery in the no-experience treatment (a) and the simple lottery in the partial-experience treatment (b).



(a) (b)

Figure C.3: Examples of the decision screens for the binary choices and confidence statements for the complex lottery in the no-experience treatment (a) and the simple lottery in the partial-experience treatment (b).

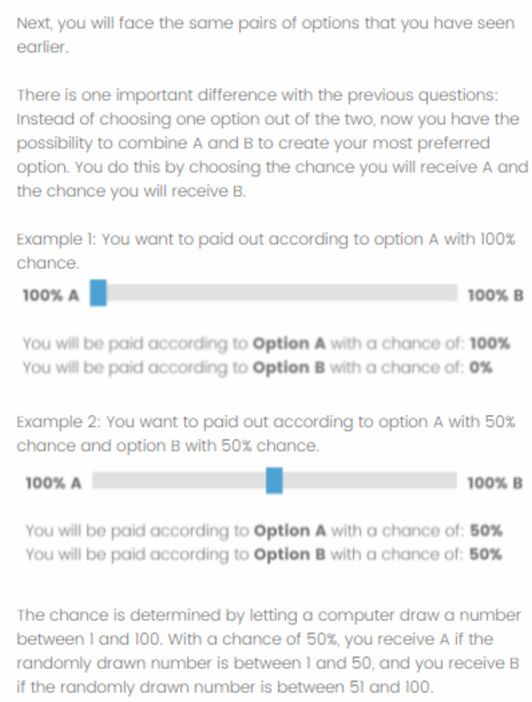
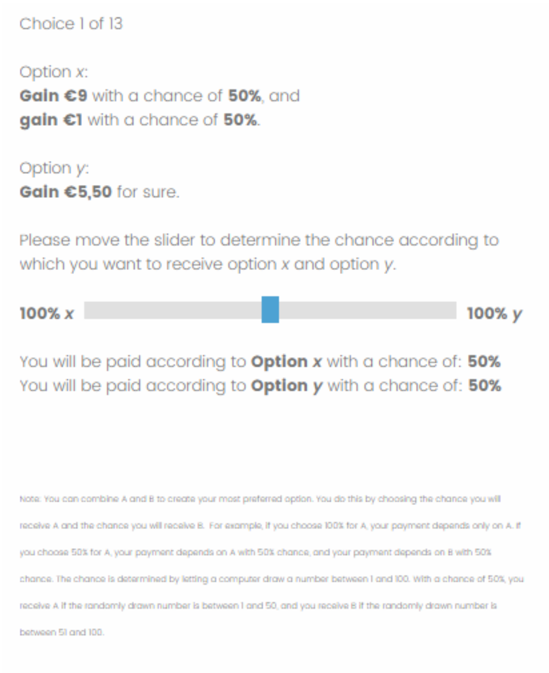
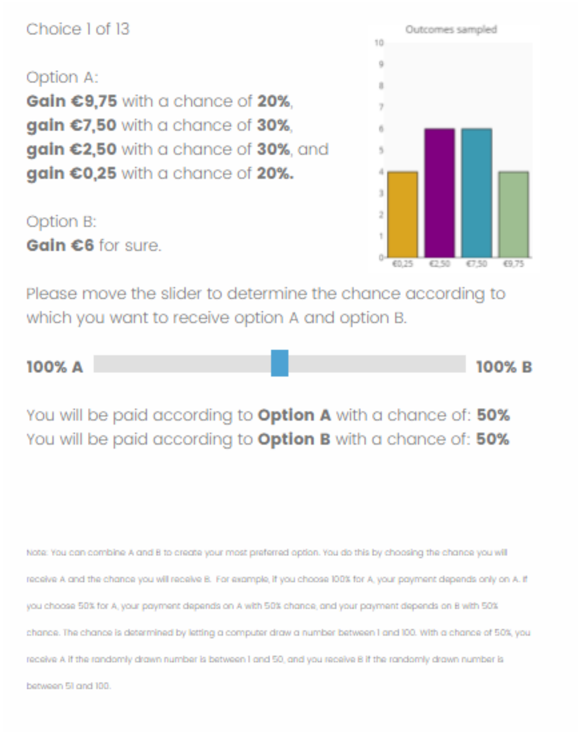


Figure C.4: Explanation of the randomized choices.

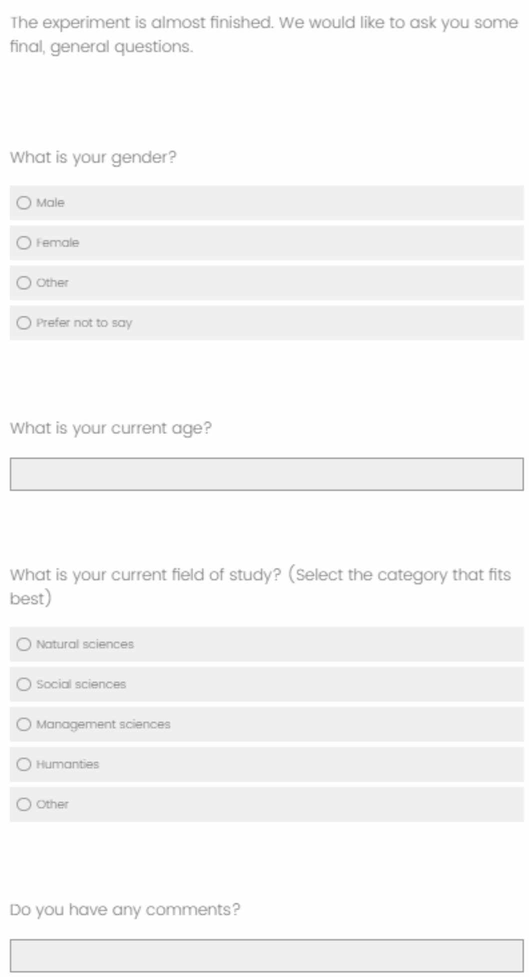


(a)



(b)

Figure C.5: Examples of the decision screens for the randomized choices for the simple lottery in the no-experience treatment (a) and the complex lottery in the partial-experience treatment (b).



The experiment is almost finished. We would like to ask you some final, general questions.

What is your gender?

- Male
- Female
- Other
- Prefer not to say

What is your current age?

What is your current field of study? (Select the category that fits best)

- Natural sciences
- Social sciences
- Management sciences
- Humanties
- Other

Do you have any comments?

Figure C.6: Demographic questions asked at the end of the experiment.

Experiment 2

Welcome to our experiment!

You are invited to participate in an experiment in which we examine how individuals make decisions. Your decisions in the experiment are about choices between different options. There are no right or wrong answers. The experiment is split up in 3 parts that will be distributed one week apart, each part will take approximately 10 minutes.

For each part you will receive a participation fee of €1. In addition you will receive monetary compensation up to €10 based on the decisions you make in the experiment. Specifically, one of the questions will be randomly selected. The decision you made in this question will determine your additional compensation.

You will receive the payment (the participation fees and the additional compensation) via bank transfer. For this we will ask your name, IBAN number and address. This information will only be used for payment, and will be permanently deleted afterwards.

You will only be eligible for payment after you have completed all three parts.

Thank you for your participation!

Sincerely yours,
Dr. Jianying Qiu, Dr. Qiyan Ong, and Sara Arts.
The Institute of Management Research
Radboud University Nijmegen.

(a)

The following information applies to all three parts of the experiment:

Voluntary participation
Your participation in this research is voluntary. This means that you can withdraw your participation and consent at any time during the survey, without giving a reason. All data we have collected from you will be deleted permanently. If you desire to withdraw, please simply close your internet browser. After completion of the survey it will not be possible to withdraw your data form the research.

What will happen to the data?
The research data we collect during this study will be used by scientists as part of data sets, articles and presentations. The anonymized research data is accessible to other scientists for a period of at least 10 years. When we share data with other researchers, these data cannot be traced back to you.

More information?

Should you want more information on this research study, please contact Sara Arts (email: s.arts@fm.ru.nl)

CONSENT:

Please select your choice below.

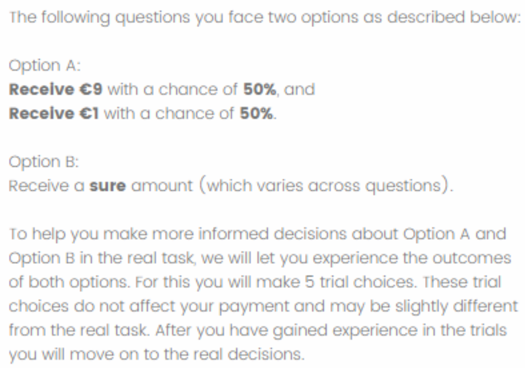
Checking "Agree" below indicates that:
you voluntarily agree to participate.

○ I agree with the provided information, and I would like to proceed to the survey

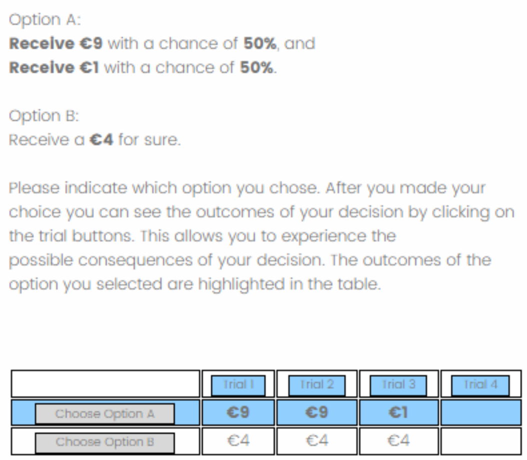
○ I do not agree with the above.

(b)

Figure C.7: Welcome screen (a) and informed consent (b) of the experiment.

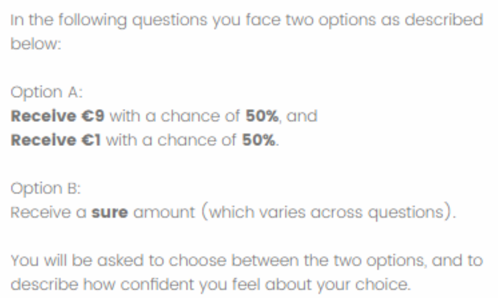


(a)

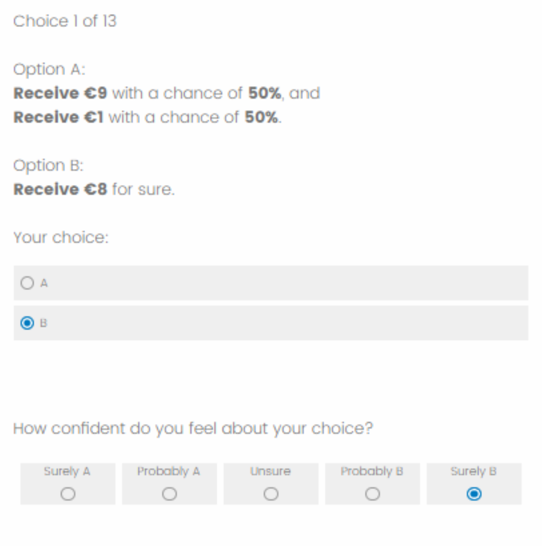


(b)

Figure C.8: The introduction (a) and an example of the hypothetical decision screens (b) of the full-experience treatment.

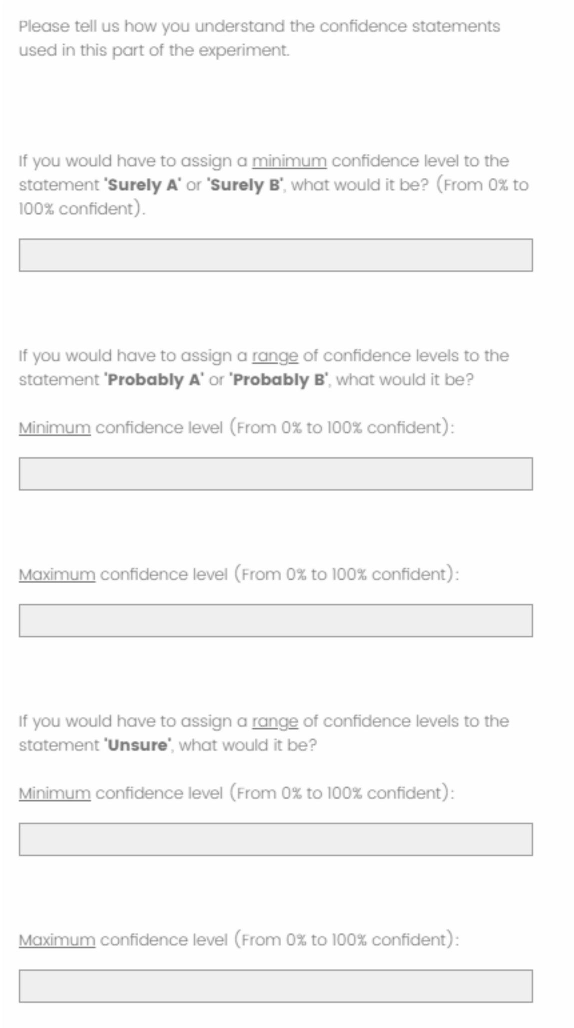


(a)



(b)

Figure C.9: The introduction (a) and an example of the decision screens (b) of binary choices and confidence statements for the simple lottery.


Please tell us how you understand the confidence statements used in this part of the experiment.

If you would have to assign a minimum confidence level to the statement **'Surely A'** or **'Surely B'**, what would it be? (From 0% to 100% confident).

If you would have to assign a range of confidence levels to the statement **'Probably A'** or **'Probably B'**, what would it be?

Minimum confidence level (From 0% to 100% confident):

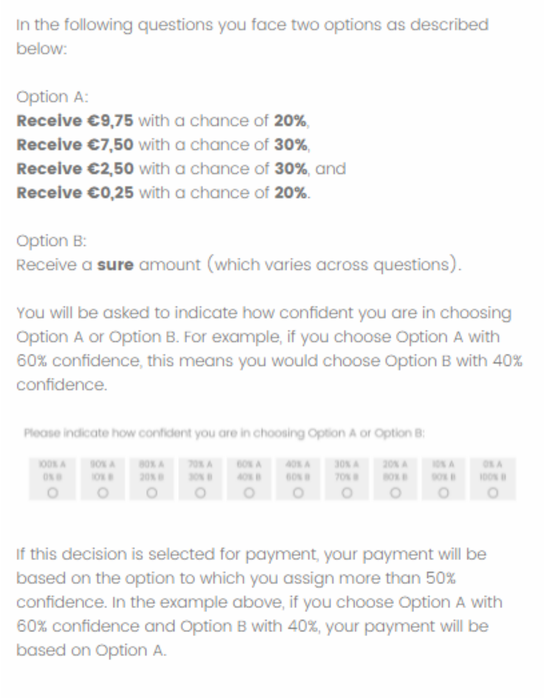
Maximum confidence level (From 0% to 100% confident):

If you would have to assign a range of confidence levels to the statement **'Unsure'**, what would it be?

Minimum confidence level (From 0% to 100% confident):

Maximum confidence level (From 0% to 100% confident):

Figure C.10: post-experiment questionnaire after the binary choices and confidence statements.



In the following questions you face two options as described below:

Option A:
Receive €9,75 with a chance of **20%**,
Receive €7,50 with a chance of **30%**,
Receive €2,50 with a chance of **30%**, and
Receive €0,25 with a chance of **20%**.

Option B:
Receive a **sure** amount (which varies across questions).

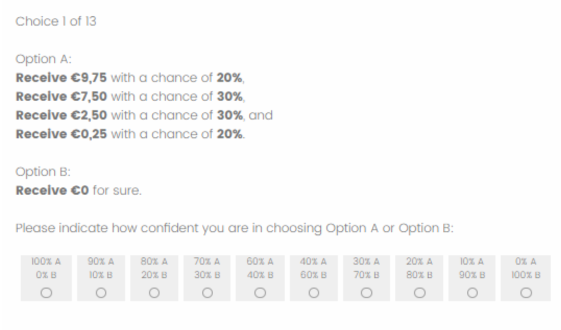
You will be asked to indicate how confident you are in choosing Option A or Option B. For example, if you choose Option A with 60% confidence, this means you would choose Option B with 40% confidence.

Please indicate how confident you are in choosing Option A or Option B:

100% A 0% B	90% A 10% B	80% A 20% B	70% A 30% B	60% A 40% B	40% A 60% B	30% A 70% B	20% A 80% B	10% A 90% B	0% A 100% B
○	○	○	○	○	○	○	○	○	○

If this decision is selected for payment, your payment will be based on the option to which you assign more than 50% confidence. In the example above, if you choose Option A with 60% confidence and Option B with 40%, your payment will be based on Option A.

(a)



Choice 1 of 13

Option A:
Receive €9,75 with a chance of **20%**,
Receive €7,50 with a chance of **30%**,
Receive €2,50 with a chance of **30%**, and
Receive €0,25 with a chance of **20%**.

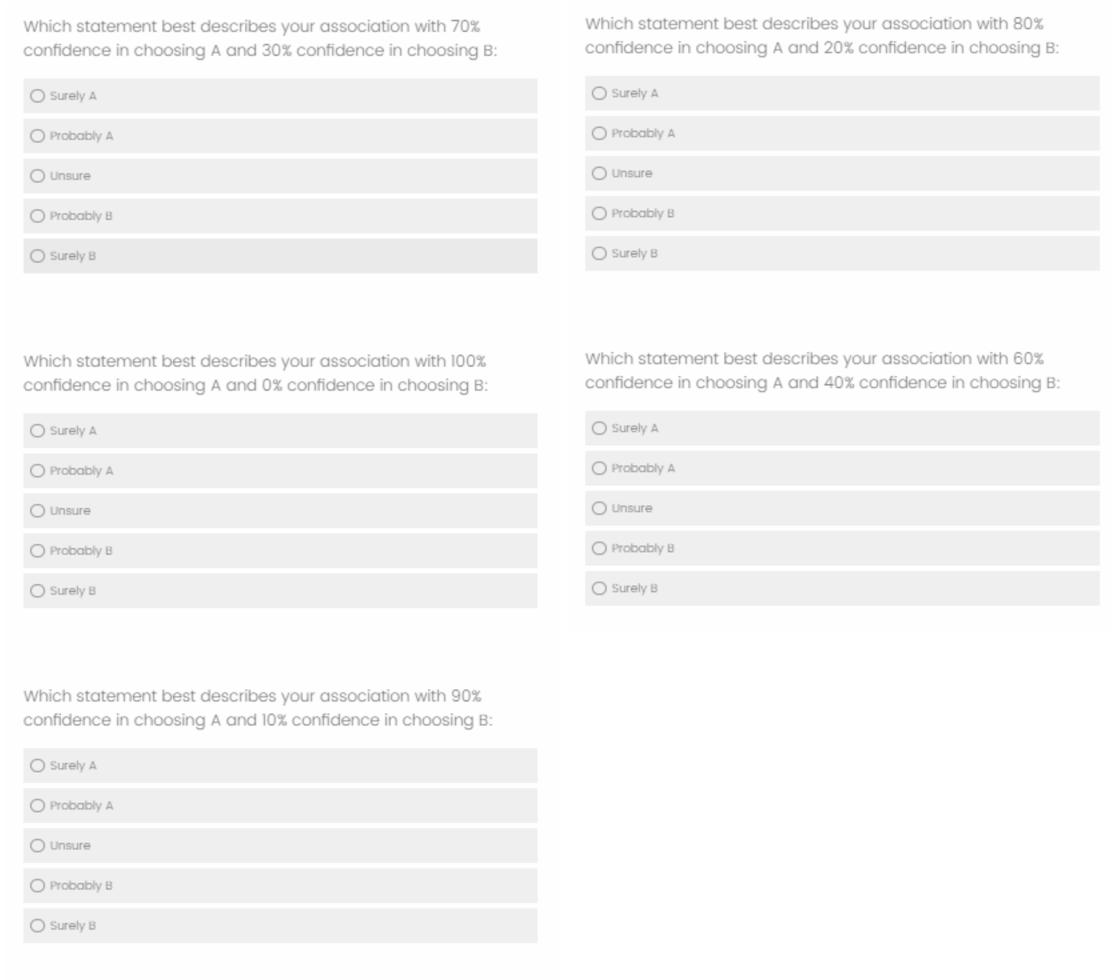
Option B:
Receive €0 for sure.

Please indicate how confident you are in choosing Option A or Option B:

100% A 0% B	90% A 10% B	80% A 20% B	70% A 30% B	60% A 40% B	40% A 60% B	30% A 70% B	20% A 80% B	10% A 90% B	0% A 100% B
○	○	○	○	○	○	○	○	○	○

(b)

Figure C.11: The introduction (a) and an example of the decision screens (b) of probabilistic confidence choices for the complex lottery.



Which statement best describes your association with 70% confidence in choosing A and 30% confidence in choosing B:

- Surely A
- Probably A
- Unsure
- Probably B
- Surely B

Which statement best describes your association with 80% confidence in choosing A and 20% confidence in choosing B:

- Surely A
- Probably A
- Unsure
- Probably B
- Surely B

Which statement best describes your association with 100% confidence in choosing A and 0% confidence in choosing B:

- Surely A
- Probably A
- Unsure
- Probably B
- Surely B

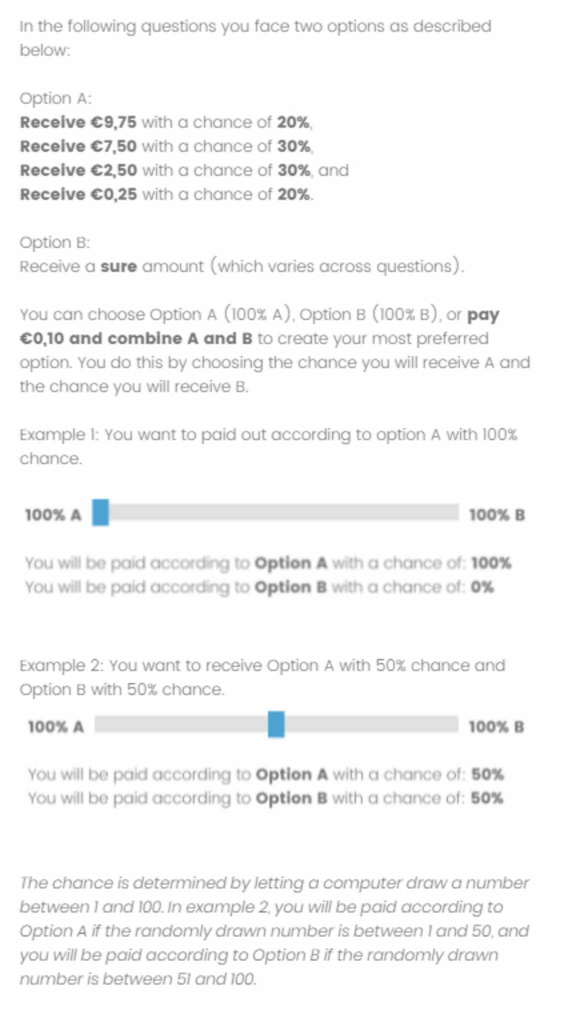
Which statement best describes your association with 60% confidence in choosing A and 40% confidence in choosing B:

- Surely A
- Probably A
- Unsure
- Probably B
- Surely B

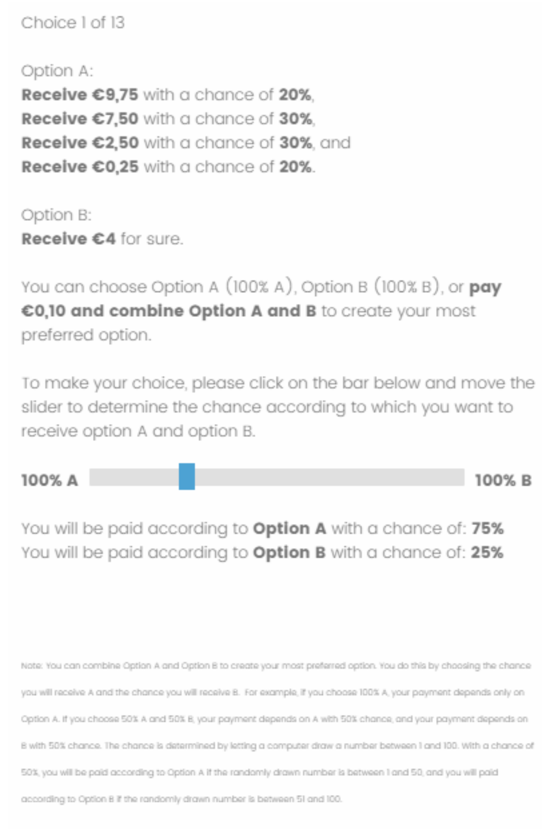
Which statement best describes your association with 90% confidence in choosing A and 10% confidence in choosing B:

- Surely A
- Probably A
- Unsure
- Probably B
- Surely B

Figure C.12: post-experiment questionnaire after the probabilistic confidence choices.

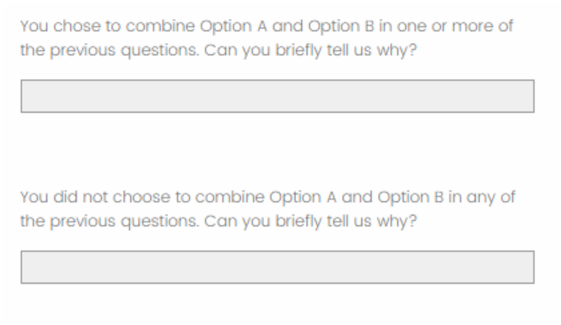


(a)



(b)

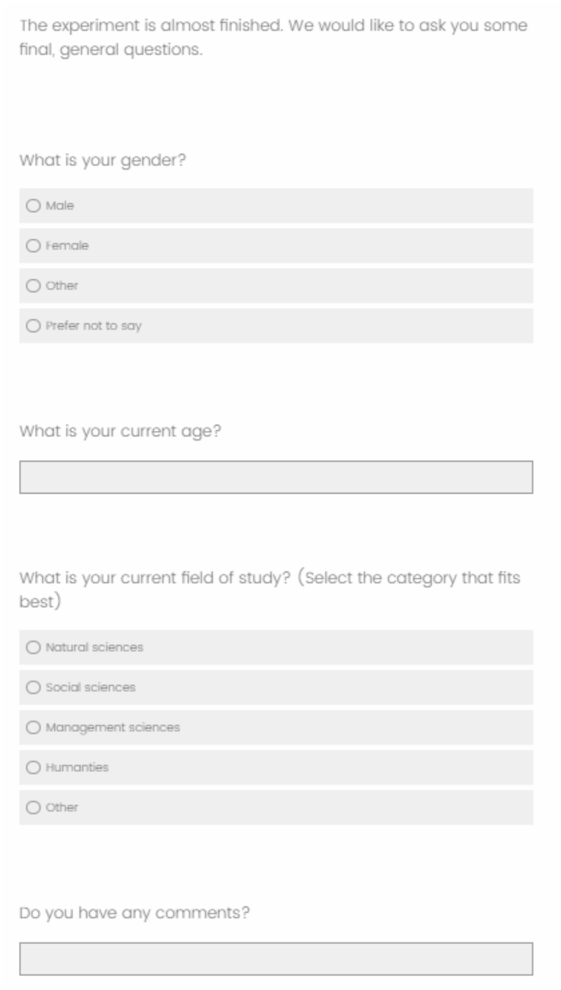
Figure C.13: The introduction (a) and an example of the decision screens (b) of randomized choices for the complex lottery.



You chose to combine Option A and Option B in one or more of the previous questions. Can you briefly tell us why?

You did not choose to combine Option A and Option B in any of the previous questions. Can you briefly tell us why?

Figure C.14: post-experiment questionnaire after the randomized choices. The first question was asked if a subject chose randomization probabilities other than 0 or 1 in at least 1 choice. The second question was asked if a subject only chose randomization probabilities of 0 or 1.



The experiment is almost finished. We would like to ask you some final, general questions.

What is your gender?

- Male
- Female
- Other
- Prefer not to say

What is your current age?

What is your current field of study? (Select the category that fits best)

- Natural sciences
- Social sciences
- Management sciences
- Humanties
- Other

Do you have any comments?

Figure C.15: Demographic questions asked at the end of the experiment.