Supporting Information:

How to Improve the Substantive Interpretation of Regression Results when the Dependent Variable is logged

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SI.1 Literature Review

Article	only direction?	% increase?	QoI?	QoI-plot on original scale?	QoI plot on log-scale?
Kriner and Reeves (2015)	✗	✓	First Difference	✓	✗
Dynes and Huber (2015)	✗	✓	First Difference*	✗	✗
Xu and Yao (2015)	✗	✓	✗	✗	✓
Rogowski (2016)	✗	✓	✗	✗	✗
Laitin and Ramachandran (2016)	✓	✗	✗	✗	✗
Gulzar and Pasquale (2017)	✗	✓	✗	✗	✗
Hainmueller, Hangartner and Pietrantuono (2017)	✗	✓	✗	✗	✗
Nellis and Siddiqui (2018)	✓	✗	✗	✗	✗
Hyytinen et al. (2018)	✗	✓	First Difference*	✗	✗
Grossman and Michelitch (2018)	✓	✗	✗	✗	✗
Szakonyi (2018)	✗	✓	✗	✗	✗
Gordon and Simpson (2018)	✗	✓	✗	✗	✗
Guardado (2018)	✗	✓	First Difference*	✗	✓
Ch et al. (2018)	✗	✓	✗	✗	✓
Kim (2018)	✓	✗	✗	✗	✗
Li (2018)	✗	✓	Predicted Values*	✓*	✗
Fouka (2019)	✗	✓	✗	✗	✗
Lipscy (2015)	✗	✓	✗	✗	✗
Shepherd and You (2020)	✗	✓	First Difference*	✗	✗
Grumbach and Sahn (2020)	✗	✓	✗	✗	✗
Gulzar, Haas and Pasquale (2020)	✗	✓	First Difference*	✗	✗
Earle and Gehlbach (2015)	✗	✓	✗	✗	✗
Coleman and Mwangi (2015)	✗	✓	✗	✗	✗
Beazer and Woo (2016)	✗	✓	✗	✗	✗
Berry and Fowler (2016)	✗	✓	✗	✗	✗
Zhu (2017)	✗	✓	✗	✗	✗
Carnegie and Marinov (2017)	✗	✓	First Difference	✗	✗
Goldstein and You (2017)	✗	✓	✗	✗	✗
Fouirnaies and Hall (2018)	✗	✓	First Difference*	✗	✗
Fouirnaies (2018)	✗	✓	✗	✗	✗
Hollibaugh and Rothenberg (2018)	✗	✗	Expected Values	✓	✗
Beazer and Blake (2018)	✓	✗	✗	✗	✗
Distelhorst and Locke (2018)	✗	✓	First Difference	✗	✓
Jiang (2018)	✗	✗	✗	✗	✗
Paglayan (2019)	✗	✓	✗	✗	✗
Mohr et al. (2019)	✗	✓	First Difference*	✗	✓
Schultz and Mankin (2019)	✗	✓	First Difference*	✗	✗
Pond and Zafeiridou (2020)	✗	✓	✗	✗	✗
Jensen, Findley and Nielson (2020)	✓	✗	✗	✗	✗

* without uncertainty

Table A1: Results of a content analysis of all research articles published in the *American Political Science Review* and the *American Journal of Political Science* between 2015 and 2020.

SI.2 Coverage Rate: Monte Carlo Evidence

We present Monte Carlo evidence to show that our approach to simulate confidence intervals produces confidence intervals with a proper coverage rate. The Monte Carlo study is set up as follows: We simulate data based on the data generating process described in the main paper, and introduced by Rainey (2017). We restate the DGP here:

$$\ln(\textsc{Income}) = \beta_{\text{cons}} + \beta_{\text{edu}}\textsc{Education} + \epsilon, \text{ and } \epsilon \sim N(0, \sigma^2) \tag{2}$$

The true values of the coefficients are given by $\beta_{\text{cons}} = 2.5$, $\beta_{\text{edu}} = 0.1$, and $\sigma^2 = 1$. Further, $N = 10$ with the set of observed values for $\textsc{Education} \in \{10, 11, 12, 13, 14, 16, 17, 18, 19, 20\}$. We set up a Monte Carlo algorithm that generates data, fits a linear regression, and derives the 95%-confidence intervals for a range of quantities of interest, including conditional median values, conditional mean values, first differences of mean values, first differences of median values, the ratio of median values, and the ratio of mean values. Precisely, the Monte Carlo algorithm takes the following steps:

1. Draw $\epsilon \sim N(0, \sigma^2)$ and compute $\ln(\textsc{Income})$ using equation 2 for all $N = 10$ observations.
2. Regress $\ln(\textsc{Income})$ on $\textsc{Education}$ via OLS.
3. Use the regression result to simulate the 95%-Confidence Intervals of
   - $\text{Med}(y|\text{edu} = 1)$
   - $\text{Med}(y|\text{edu} = 20)$
   - $E(y|\text{edu} = 1)$
   - $E(y|\text{edu} = 20)$
   - $\text{Med}(y|\text{edu} = 20) - \text{Med}(y|\text{edu} = 1)$
   - $E(y|\text{edu} = 20) - E(y|\text{edu} = 1)$
   - $\text{Med}(y|\text{edu} = 20)/\text{Med}(y|\text{edu} = 1)$
   - $E(y|\text{edu} = 20)/E(y|\text{edu} = 1)$

   following the simulation algorithm presented in the main text with 10,000 simulations.
4. Report whether the true quantity is within the 95%-Confidence Interval.

We repeat this simulation algorithm 10, 000 times. Table A2 reports the coverage rate of the quantities of interest. As expected, the coverage rate of all quantities is close to 95%.

Estimand	Coverage
$\mathrm{Med}(y \mid \mathrm{edu} = 1)$	0.9475
$\mathrm{Med}(y \mid \mathrm{edu} = 20)$	0.9479
$E(y \mid \mathrm{edu} = 1)$	0.9478
$E(y \mid \mathrm{edu} = 20)$	0.9447
$\mathrm{Med}(y \mid \mathrm{edu} = 20) - \mathrm{Med}(y \mid \mathrm{edu} = 1)$	0.9489
$E(y \mid \mathrm{edu} = 20) - E(y \mid \mathrm{edu} = 1)$	0.9507
$\mathrm{Med}(y \mid \mathrm{edu} = 20)/\mathrm{Med}(y \mid \mathrm{edu} = 1)$	0.9482
$E(y \mid \mathrm{edu} = 20)/E(y \mid \mathrm{edu} = 1)$	0.9482

Table A2: Monte Carlo Results for the coverage rate of simulated 95%-Confidence Intervals.

SI.3 A simple example with code

```
install_github("mneunhoe/simloglm")
library(simloglm)

# Estimating the model
df <- simloglm:::example_df(n = 10)

m1 <- lm(log(income)~educ, data = df)

# Calculating QoI and simulating Confidence Intevals with
   simloglm
set.seed(220609)
res1 <- simloglm(m1, scenario = list(educ = c(1, 20)))

# Summarize results for median
get_summary(res1, which_qoi = "median")

# Summarize results for mean
get_summary(res1, which_qoi = "mean")

# Or get first difference between the two scenarios
get_first_difference(res1, which_qoi = "median")



# Calculating QoI and simulating Confidence Intervals by hand

# Function to sample from inverse gamma distribution
rinvgamma <- function (n,
                       shape,
```

```r
                       rate = 1,
                       scale = 1 / rate)
{
  if (missing(rate) && !missing(scale))
    rate <- 1 / scale
  1 / stats::rgamma(n, shape, rate)
}



# Set up informal posterior of coefficients

# Set number of draws
nsim <- 1000

beta_hat <- coef(m1)
sigma_hat <- summary(m1)$sigma
X_prime_X <- summary(m1)$cov.unscaled

set.seed(220609)
# First sigma^2
sigma2_tilde <- rinvgamma(
  nsim,
  shape = m1$df.residual / 2,
  rate = (sigma_hat ^ 2 * m1$df.residual) / 2
)



# Then the betas
```

```r
beta_tilde <- matrix(NA, nrow = nsim, ncol = length(beta_hat)
  )

for (sim in 1:nsim) {
  beta_tilde[sim, ] <-
    MASS::mvrnorm(1, beta_hat,  X_prime_X * sigma2_tilde[sim
       ])
}

# Set your scenarios as a matrix (don't forget the intercept)
X_c <- rbind(c(1, 1),
             c(1, 20))

# Calculate the linear predictor on the log scale
X_beta <- beta_tilde %*% t(X_c)

# Now transform back to original scale using the appropriate
   formula

# Expected Values/Conditional Mean
# First add the draws of 1/2*sigma2_tilde to each column
X_beta_sigma_tilde <- apply(X_beta, 2, function(x) x + 1/2*
   sigma2_tilde)

# Transform
E_Y_c <- exp(X_beta_sigma_tilde)

# Summarize to get Confidence Intervals
CI_E_Y_c <- apply(E_Y_c, 2, quantile, c(0.025, 0.975))
```

```r
# Use beta_hat and sigma_hat for point estimates
X_beta_hat <- beta_hat %*% t(X_c)
X_beta_sigma_hat <- X_beta_hat + 1/2*sigma_hat^2

# Point estimate
E_Y_c_hat <- exp(X_beta_sigma_hat)

# Conditional Median
# First add the draws of 1/2*sigma2_tilde to each column

# Transform
Med_Y_c <- exp(X_beta)

# Summarize to get Confidence Intervals
CI_Med_Y_c <- apply(Med_Y_c, 2, quantile, c(0.025, 0.975))

# Point estimate
Med_Y_c_hat <- exp(X_beta_hat)



# Or get first difference of the medians between the two
   scenarios

# Point estimate
fd_Med_hat <- Med_Y_c_hat[,2] - Med_Y_c_hat[,1]

# Confidence Intevals
fd_Med <- Med_Y_c[,2] - Med_Y_c[,1]
```

```
CI_fd_Med <- quantile(fd_Med, c(0.025, 0.975))
```

SI.4 A Reanalysis of Shepherd and You (2020)

We present here a second reanalysis of a prominently published article to show that correctly interpreting effects in regression models is substantively important. Shepherd and You (2020) study the influence of career paths of congressional staffers on the legislative output. In particular they are interested in what happens when staffers later become lobbyists. Shepherd and You (2020, 270) conclude: "Using comprehensive data on congressional staffers, we find that employing staffers who later become lobbyists is associated with higher legislative productivity for members of Congress, especially in staffers final terms in Congress."

They run several OLS models with transformed dependent variables.[8] One of the three dependent variables to measure legislative productivity is the legislative effectiveness score (LES) introduced by Volden and Wiseman (2014, 2018). We focus on the LES since this is the dependent variable that Shepherd and You (2020) offer a substantive interpretation for. With the provided replication data we can reproduce the results in the original regression table exactly. Yet, following our guidelines the substantive results using quantities of interest differ. Again, this difference is due to an erroneous transformation of the transformed quantities of interest back to the original scale.

Shepherd and You (2020, Corrigendum 1) report the following substantive effect: "Given that our outcome variables are log-transformed, a one standard deviation increase in the number of future lobbyist staff (0.34) is associated with 1.8% increase in a members Legislative Effectiveness Score (LES) ($\exp(\log(1.7) + 0.0317 \times 0.34) - 1.7 = 0.0184$), if we evaluate the effect [...] at the mean level of LES." Where 1.7 is the mean of the LES in their sample, 0.0317 is the coefficient for the number of future lobbyists in their model and 0.34 is supposed to be a

[8]The log-transformation of the variables is motivated by "Given that all outcome variables have highly skewed distributions, we use log-transformed variables in the estimation" (Shepherd and You, 2020, 276). Unfortunately, this commonly used motivation is wrong, the decision of transforming the dependent variable should only be based on the distribution of $Y|X$, i.e. the error distribution. Moreover, since all of the variables contain 0s, they decide to calculate the log as $\log(x + 1)$. This has to be taken into account when transforming the quantities of interest back to the original scale.

residualized standard deviation, as proposed in Mummolo and Peterson (2018), in the number of future lobbyist staff.

According to our replication analysis the mean of LES is at 1 and the residualized standard deviation of the number of future lobbyists ≈ 0.74. However, even correcting those two values would still yield an erroneous transformation. Note, that this interpretation neglects the multiplicative nature of a model with a log-transformed dependent variable, the substantive effect depends on a scenario for all other independent variables.[9].

Using the results from the model in Shepherd and You (2020, 276, Table 2, Model 4) and following the steps outlined in section 3 we get that a one standard deviation increase in the number of future lobbyist staff (0.74) is associated with a 5.6% increase in a member's LES for an average member of Congress,[10] with a 95% Confidence Interval from 2.8% to 8.3%. Thus, on average, the effect is more than 3 times as large as reported by Shepherd and You (2020).

Using the 95% Confidence Interval to test the classical two-sided hypothesis whether a one standard deviation increase in future lobbyist staff has an effect (different from 0) on a member's LES, we can reject the Null hypothesis of no effect. This highlights the importance to communicate the uncertainty surrounding quantities of interest and not only relying on a point estimate.

In their paper Shepherd and You (2020, 273) formulate their hypothesis as a directed hypothesis: "Hiring a future revolving-door staffer should be associated with increases in member legislative effectiveness and bill sponsorship activity." Another advantage of the simulation approach is that we can easily calculate the probability that the % Change in Legislative Effectiveness Score that is associated with a one standard deviation increase in future lobbyist staff is greater than 0, a test that is better suited to the directed hypothesis. With our simulation results

[9]The approach of Shepherd and You (2020) would give the same results only if *all* of the other coefficients were exactly 0 or if the intercept is 0 and they chose a scenario where all other covariates are set to 0.

[10]Note that this observation does not exist. Other approaches for scenarios based on actual observations have been proposed (e.g. Hanmer and Kalkan, 2013). Using our `R` package `simloglm` it is easy to also calculate the percent increase using the observed value approach.

we find that this is the case in all of our $1,000$ simulations, thus, yielding a p-value of $< \frac{1}{1000}$ for the one-sided hypothesis test.

Figure A1 displays the correctly calculated results for the scenarios outlined in Shepherd and You (2020). Since the sampling distribution of quantities of interest on the original scale is not necessarily symmetric we advise applied scholars to communicate as much information as possible on the shape of the distribution. This can be achieved, for instance, by reporting multiple confidence intervals.

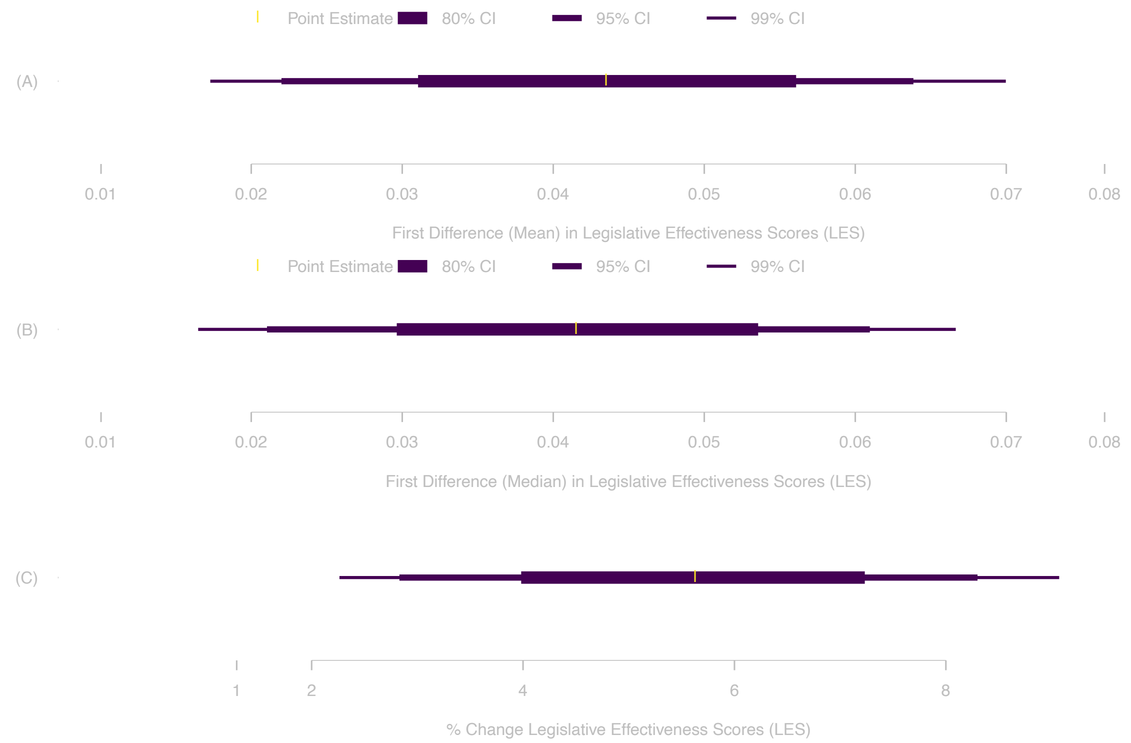

Figure A1: Correct Quantities of Interest based on the results in Shepherd and You (2020). Results for the effect of a one (residualized) standard deviation increase (0.74) in the number of future lobbyists on the Legislative Effectiveness Score (LES), where all other covariate values are set to their means. Panel (A) shows the First-Difference between Expected Values. Panel (B) shows the First-Difference between Medians. Panel (C) shows the % Change.

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