Online Appendix for "The Making of the Boy Who Cried Wolf: Fake News and Media Skepticism"

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In the main text, we argue that Proposition 1 still holds when D observes another piece of news after updating her belief. This can be proved if we can show that the variance in the posterior expectation of the incumbent's characteristic after observing another piece of news is decreasing in f. Suppose that D updated her belief after observing the first piece of news, denoted by x_1. Let $c|x_1$ denote D's updated belief about c. Then, we have

$$c|x_1 \sim \mathcal{N}\left(f\mu + (1-f)\frac{\tau^2 x + \sigma^2\mu}{\tau^2+\sigma^2}, f\tau^2 + (1-f)\frac{\tau^2\sigma^2}{\tau^2+\sigma^2} + f(1-f)\frac{\tau^4(\mu-x)^2}{(\tau^2+\sigma^2)^2}\right),$$

which now serves as a new prior belief of D. When D receives the second piece of news x_2, given D still does not know whether x_2 is fake or true, her posterior expectation of the incumbent's characteristic is

$$\mathbb{E}(c|x_2) = f\left(f\mu + (1-f)\frac{\tau^2 x_1 + \sigma^2\mu}{\tau^2+\sigma^2}\right) + (1-f)\left(\frac{\tau^2 x_2 + \sigma^2\left(f\mu + (1-f)\frac{\tau^2 x_1+\sigma^2\mu}{\tau^2+\sigma^2}\right)}{\tau^2+\sigma^2}\right).$$

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Still, $\mathbb{E}(c|x_2)$ is a random variable such that

$$\mathbb{E}(c|x_2) \sim \mathcal{N}\left(\mu, \sigma^2\left(\frac{\tau^2(1-f)[(\tau^2+\sigma^2)(1+f)+\sigma^2(1-f)]}{(\tau^2+\sigma^2)^2}\right)^2\right).$$

To determine the relationship between f and this variance, we should take the first derivative of the variance with respect to f, and we need to focus only on the numerator because σ^2 and τ^2 are positive. Using the chain rule, we have

$$\begin{aligned}&\frac{\partial}{\partial f}(\tau^2(1-f)[(\tau^2+\sigma^2)(1+f)+\sigma^2(1-f)])^2\\&=\underbrace{2(\tau^2(1-f)[(\tau^2+\sigma^2)(1+f)+\sigma^2(1-f)])}_{(1)}\underbrace{(-2f\tau^2(\tau^2+\sigma^2)-\sigma^2)}_{(2)}.\end{aligned}$$

Since $0 < f < 1$ and $\sigma^2, \tau^2 > 0$, (1) is always positive, but (2) is always negative. Therefore, we have

$$\frac{\partial}{\partial f}\sigma^2\left(\frac{\tau^2(1-f)[(\tau^2+\sigma^2)(1+f)+\sigma^2(1-f)]}{(\tau^2+\sigma^2)^2}\right)^2 < 0,$$

which implies that the variance of $\mathbb{E}(c|x_2)$ is decreasing in f.

Finally, if we set

$$c|x_t \sim \mathcal{N}\left(f\mu_{t-1}+(1-f)\frac{\tau^2 x+\sigma^2\mu_{t-1}}{\tau^2+\sigma^2}, f\tau^2+(1-f)\frac{\tau^2\sigma^2}{\tau^2+\sigma^2}+f(1-f)\frac{\tau^4(\mu_{t-1}-x)^2}{(\tau^2+\sigma^2)^2}\right),$$

where $\mu_{t-1} = \mathbb{E}(c|x_{t-1})$, $\mu_o = \mu$, $t \in \{1, 2, ..., n-1, n\}$, and n denotes the number of pieces of news D can read, we can see that, for all t, the variance in $\mathbb{E}(c|x_t)$ is always decreasing in f, which implies that D's posterior expectations at t (i.e., $\mathbb{E}(c|x_t)$) would be more densely distributed around μ_{t-1} as more pieces of fake news are spread. □