Online Appendix for "The Political Violence Cycle"

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This appendix contains supplementary information to back up several claims made in the main text, and a full description of the procedure for the simulated comparative statics on the difference between violence levels with and without elections.

Discussion of the case where $\Delta_t^* < 0$ for some t

Recall that if $\Delta_t^* < 0$, then there will be no violence in period $t-1$ as the opposition would rather remain the opposition in period t. Intuitively, this can occur if the probability of taking over the government in period t is very high but then violence becomes very ineffective. Figure 1 illustrates such a case, where violence becomes extremely effective in period 6 and is nearly ineffective for all periods after that. The solid black line plots the violence level in each period, and the grey line plots Δ_t^*. Since $\Delta_t^* < 0$ for periods 4-6, there is no violence in periods 3-5. However, moving back to period 3, Δ_t^* is positive again, and there is a positive level of violence in period 2. This is because for each period with no violence the incumbent collects ψ with no chance of being removed from office. So, in effect the tradeoff in period 3 is between capturing the incumbency for two periods followed by nearly surely losing office, while in period 2 the opposition can capture the incumbency for three periods before being ousted. For these sets of parameters, it is worth committing violence in period 2 but not period 3.

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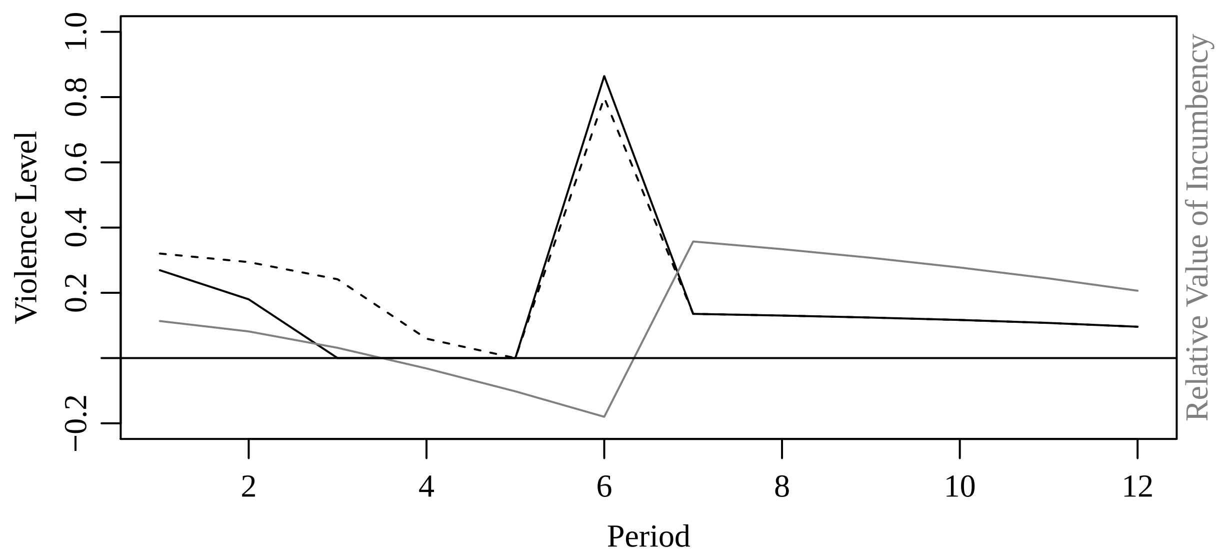
Figure 1: Example with no violence in some periods



The dashed line shows what happens when violence becomes slightly less effective in period 6. This has no effect on the level of violence in period 5, as the opposition would rather remain as opposition in either case. However, when violence is less effective in period 6, there is only one period where it is better to enter as opposition, and hence there is a positive violence level in period 4, and a higher level in all periods prior to period 6. So, in this case, making violence more effective in period t only *weakly* increases violence in period $t-1$, as it may be zero no matter what. However, when this is the case the effect of violence on periods before $t-1$ appears to be larger than in the main examples we consider. So, it does not appear that allowing for periods where it is better to be the opposition affects the overall conclusions (and the simulations with random parameter draws allow this).

Figure 2: Examples where equation 1 does not hold.

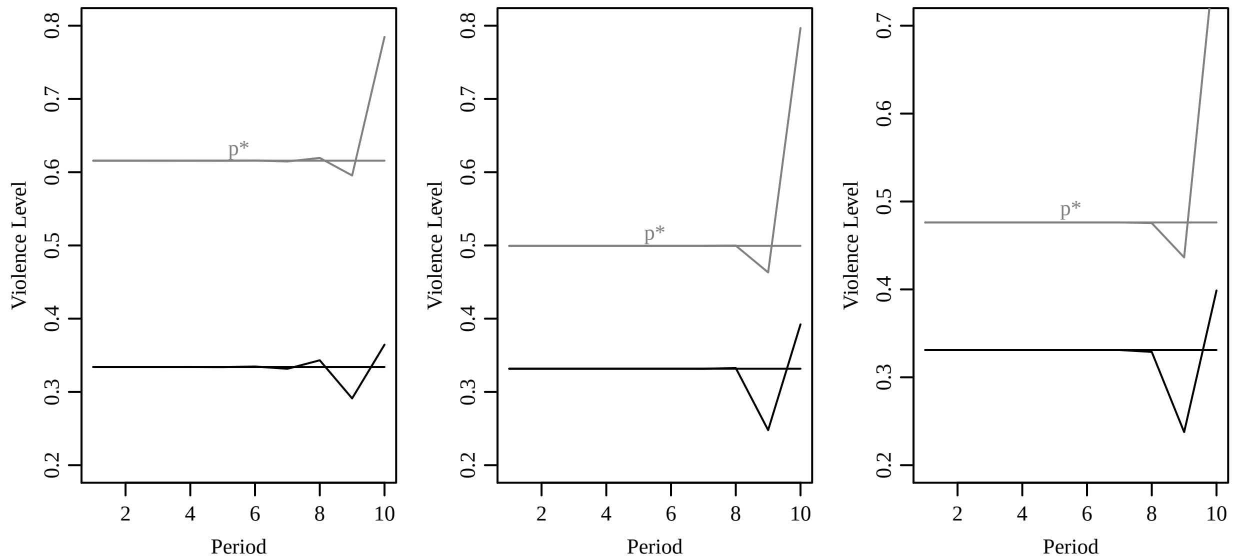


Illustration where increasing future k_t' increases v_t

Recall the condition for future violence effectiveness to decrease violence in period t is:

$$\delta(1 - 2p(v_t^*; k_t)) > -\frac{\partial v_t^*}{\partial \Delta_{t+1}^*} c'(v_t^*) \tag{1}$$

When equation 1 does not hold, increasing the effectiveness of violence in the future can increase violence in period t. Violations o this equation are generally possible when $p(v_t^*; k_t)$ is large. Intuitively, if the opposition is likely to remain the opposition in the future, anything that increases the relative value of incumbency tomorrow also increases the relative likelihood of incumbency today. The right hand side represents the indirect effect that increasing the relative value of incumbency tomorrow leads to more violence today, which in turn lowers the relative value of incumbency today. In all of our simulations this indirect effect is quite small, so as long as $p(v_t^*; k_t)$ is generally low, increasing the effectiveness of violence in period t reduces violence in *every* previous period.

The left two panels of figure 2 shows cases where condition 1 does not hold. In the left panel, the probability of the opposition taking control of the government is above 0.6, and the black curve shows the effect of making violence even more effective in the final period. Compared to the horizontal line (where the effectiveness of violence is constant), there is more violence in the last period, less in the second to last, but more in the last to last. Iterating back, the level of violence alternates between being above and below the constant effectiveness benchmark. The intuition behind this is that when the incumbency is likely to switch hands every period, making violence more effective in period 10 helps the opposition in period 8 as they expect to lose the incumbency in period 9 and regain it in period 10. Similarly, this helps the opposition in every even period.

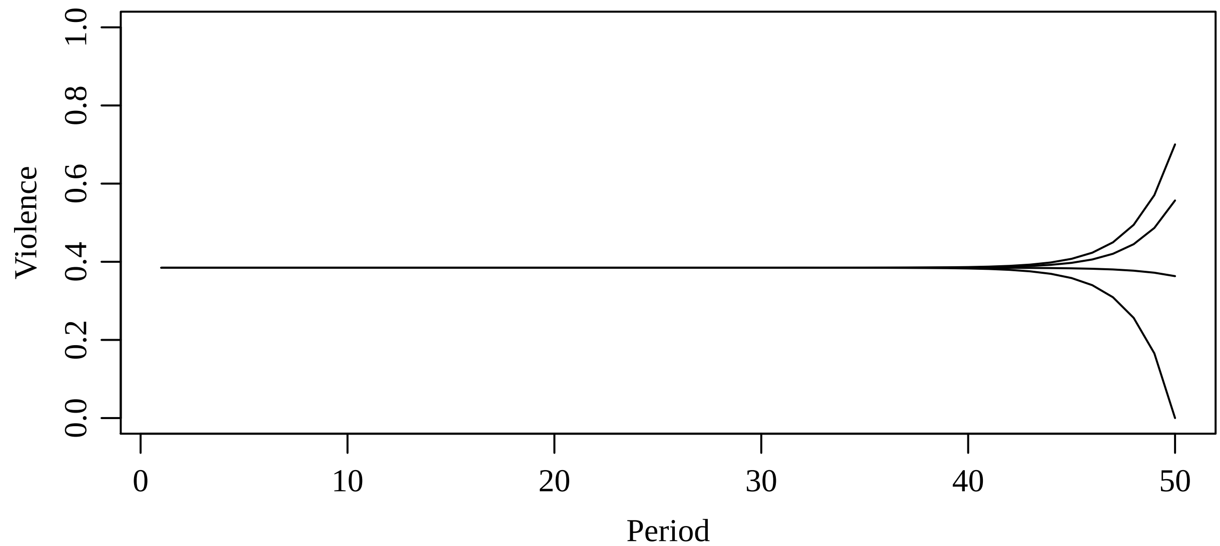
The center panel shows this "bouncing" pattern can hold – if almost imperceptibly – when $p^*(v_t^*, k_t)$ is at $1/2$, as the right-hand side of equation 1 is negative. However, as long as $p^*(v_t^*; k_t)$ is below around $.48$ (right panel), making violence more effective in the last period reduces violence in every previous period. In the context of our examples with elections, as long as the probability of taking over the incumbency in non-electoral periods is not too high, increasing the effectiveness of violence in an electoral period will decrease the levels of violence for each period of the preceding non-electoral spell as long as the chance of taking office by force is not too high without an election.

Stationary continuation values

In all of our simulations, we set the continuation value at the end of the game such that with no elections (and hence a constant k_t) the violence level is stationary. This is found by finding the level of Δ^*_{T+1} which makes $\Delta^*_T = \Delta^*_{T+1}$, which by induction means that when k_t is constant Δ^*_t is constant and hence the level of violence is constant. Referring back to the proof of proposition 2, this is given by the solution to:

$$\Delta^* = \psi - c(v_t^*(\Delta^*)) + \delta(1 - 2p(v_t^*(\Delta^*); k_t))\Delta^*$$

Figure 3: Illustration of the effect of changing Δ^* from 0 to 9 when $\psi = 1$.



In all of our simulations there is a unique Δ^* solving this equation. Figure 3 shows how changing Δ^* affects violence throughout a 50 period model. Changing the continuation value has a large effect on violence choices in the last period, and some effect for the preceding periods. However, by period 40 the differences are undetectable.

Microfoundation of the Effectiveness of Violence During Elections

One way to microfound the assumption that violence is more effective during elections is to let the opposition choose two different types of violence, one of which is only effective during electoral periods. Formally, let $v_t^n \geq 0$ be the amount of violence not related to elections chosen in each period, and $v_t^e \geq 0$ be the amount of electoral violence. Let $E_t = 1$ be an indicator for period t being an election period, where $E_t = 0$ in non-electoral periods. Suppose the probability of taking office is given by $p(v_{n,t} + E_t v_{e,t})$, where $p' > 0$ and $p'' < 0$. (We drop the k_t parameter for reasons which will become apparent.) The cost to choosing $v_{n,t}$ is given by $c(v_{n,t})$ as in the main model and the cost to choosing $v_{e,t}$ is $\beta_e^{-1}c(v_{e,t})$ for some $\beta_e > 0$ (i.e., β_e is the relative cost of electoral violence). Let $v_t = v_{n,t} + v_{e,t}$.

To make the connection to the baseline model more clear, it is useful to interpret the opposition action not as picking violence levels, but first deciding what cost to pay for violence and then deciding how to allocate between electoral and non-electoral violence for that cost. In non-electoral periods this is straightforward: when paying cost c_t the resulting level of violence is $v_t(c_t) = c^{-1}(v_t)$. So in non-electoral periods the opposition picks c_t that meets:

$$\delta \frac{\partial p(v_{n,t})}{\partial c_t} \Delta^*_{t+1} = \delta \frac{\partial p(v_{n,t})}{\partial v_t} \frac{\partial v_t}{\partial c_t} \Delta^*_{t+1} = 1 \tag{2}$$

By the assumptions placed on c, it must be the case that v_t' is continuous and decreasing with $v_t'(0) = \infty$, so there is a unique solution to this equation. This is also the solution to electoral periods as $\beta_e \to 0$. In electoral periods:

$$v_t(c_t) = \max_{v_{n,t}, v_{e,t} : c(v_{n,t}) + \beta_e^{-1} c(v_{e,t}) = c_t} v_{n,t} + v_{e,t}$$

The allocations must be at a point where the marginal return to each level of violence is equal. So the increase in the sum amount of violence with the optimal allocation as the cost paid increases is:

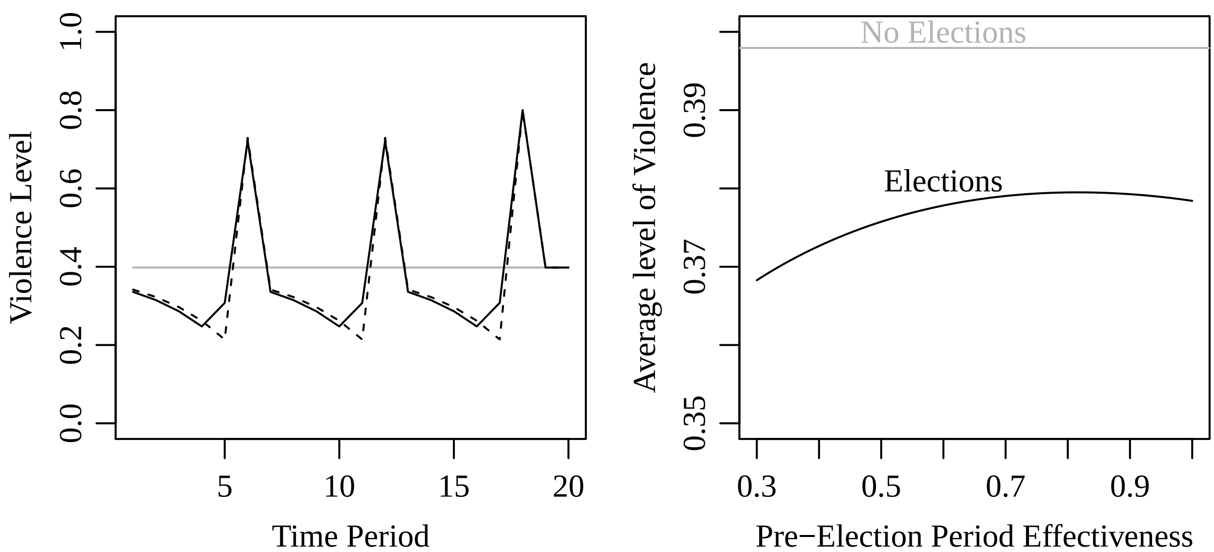
$$\frac{\partial v_t}{\partial c_t} = c'(v_{n,t})$$

And so:

$$\frac{\partial^2 v_t}{\partial c_t \partial \beta} = c'(v_{n,t}) \frac{\partial v_{n,t}}{\partial \beta} > 0$$

This implies that $\frac{\partial^2 p(v_{n,t})}{\partial c_t \partial \beta} > 0$, i.e., when electoral violence becomes cheaper (or available at all)

Figure 4: The effect of allowing violence to be moderately more effective in the pre-election period.



violence becomes more effective in the sense that increasing the cost paid (when choosing the optimal allocation between electoral and non-electoral violence) has a larger marginal impact on the chance of taking over the incumbency. So, introducing a new technology has the exact effect we assume of increasing the marginal effect of violence on the probability of taking over office.

Smoother Cycles of Violence

The dip in violence leading up to elections results from the fact that when violence is only more effective in the electoral period itself, the incentives to commit violence are weakest right before the election. However, pre-electoral violence can begin weeks or months before the actual election date, leading to a smoother cycle as shown in the SCAD data. One way to generate such a pattern with the model is to assume that violence is also somewhat more effective in the period prior to elections.

Figure 4 illustrates this claim, with the solid curve in the left panel showing how the parameterization in figure 3 in the main text changes if the effectiveness of violence is equal to $k_t = 0.6$ in pre-election periods (it is equal to 0.3 in other non-electoral periods and 1 in electoral periods). Compared to the case where the pre-election period is the same as other non-electoral periods (the dashed curve), there is more violence in the pre-election period but less violence in other non-electoral periods. The right panel shows that making violence more effective in the pre-election period increases the average level of violence until it becomes nearly as effective as in electoral periods, but the average is still below that which it would be with no elections (the grey curve).

Simulated Comparative Statics

To test the robustness of the conclusions from the numerical examples in the main text and generate suggesting results about when elections lead to more or less violence, we conduct a simulation exercise for a range of (randomly generated) parameterizations of the model. In these simulations, we vary the exogenous parameters ψ, δ, k_t, k_n, Δ^*_{T+1}, and the frequency of elections. We also consider more general cost functions of the form

$$c(v_t) = v_t^{\alpha}$$

where $\alpha > 1$ affects the convexity of the cost function, i.e., how strongly the marginal cost increases. Finally, we also vary the probability of taking over office by considering functions of the form:

$$p(v_t; k_t, v_0) = k_t \frac{v_t}{v_0 + v_t}$$

When $v_0 > 0$ is low, it is generally "easier" to take over office, as the returns to violence are very high for low levels. When v_0 is higher low levels of violence have a lower marginal return and it is generally hard to take over office.

By proposition 2, since c is convex with $c'(0) = 0$ and p is concave, there is a unique solution to each period's equilibrium condition, which is strictly positive if $\Delta_t^* > 0$.

In particular, the equilibrium condition for an interior solution is:

$$\alpha v_t^{\alpha-1} = k_t \frac{v_0}{(v_0 + v_t)^2} \delta(\pi_I^*(T+1) - \pi_O^*(T+1))$$

We now explore how varying the exogenous parameters of the model affects average violence levels for these classes of cost and p functions. For each parameter, we first plot the change in the average violence level in a 100 period model when keeping the other parameters fixed at the following values: $k_e = 0.9, k_n = 0.3, \delta = 0.9, \psi = 1, v_0 = 1, \alpha = 2$, elections occur every 4 periods, and Δ_{T+1}^* is set such that the level of violence is stationary without elections.

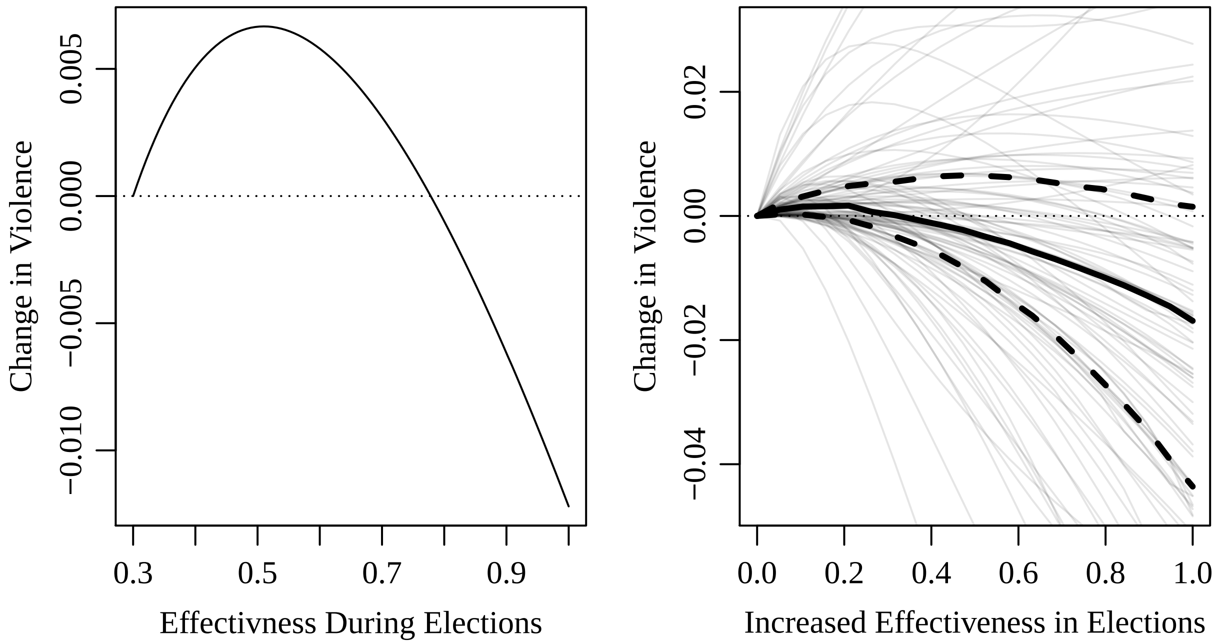
Next, we run plot the effect of these variables for 100 simulations where each of the "control" parameters is randomly drawn from a distribution generally centered around the values of the main parameterization:

- k_e is uniform on $[.1, .6]$ and k_n is uniform on $[.6, 1]$. When looking at the effect of changing k_e, we first drawn k_n and then plot over the range $[0, k_n]$. When looking at the effect of changing k_e, we first draw k_n and then plot the range $[k_n, 1]$,
- δ is drawn from a beta distribution with shape parameters $a = 9$ and $b = 1$,[1]
- v_0 is drawn from an exponential distribution with mean 1,
- α is equal to 1.1 plus an exponential distribution with mean 0.9.[2],
- ψ is equal to 0.5 plus an exponential random variable with mean 0.5,
- the periodicity of elections is uniform between every 1 and every 10 periods, and

[1] I.e., the density is proportional to $\delta^{a-1}(1-\delta)^{b-1}$.

[2] When α is very close to 1 some of the optimization procedures fail for some draws of the other parameters, so putting a lower bound at 1.1 avoids this issue.

Figure 5: Simulations for the effect of k_e.



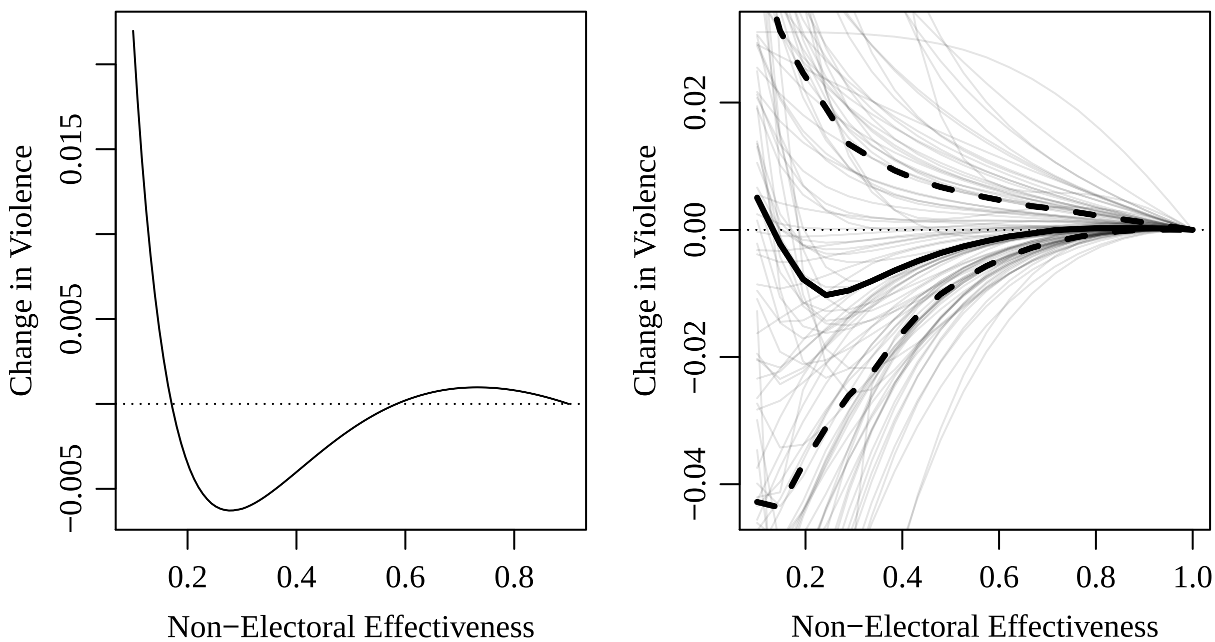
- Δ^*_{T+1} is set so the level if violence is stationary without elections.

Figure 5 shows the effect of changing k_e. For this and later parameters, left panel plots the effect of changing k_e on the absolute change in average violence when going from no elections to elections. The right panel plots the change in average violence when introducing elections for 100 simulations in translucent lines, with the median effect in a solid black curve and the 25th and 75th percentile effects in dashed black curves.

The results for k_e largely reinforce the conclusion in the main text. As $k_e \to k_n$, introducing elections has no effect on violence and electoral periods are no different than non-electoral periods. When k_e is slightly higher than k_n there is generally more violence, but as violence gets much more effective during electoral periods there is usually less violence on average with elections.

Figure 6 looks at varying the effectiveness of violence in *non-electoral* periods. The left panel indicates that for the main parameterization, when k_n is very small introducing elections leads to

Figure 6: Simulations for the effect of k_n.



a substantial increase in violence. This is not too surprising: if society is very peaceful without elections, the indirect effect of introducing elections will be small since there is little violence to prevent in the non-electoral baseline. So, the direct effect of increasing violence in elections dominates. On the other extreme, the effect of introducing elections when society is already very violent (k_n close to k_e) is close to zero, as there is little difference in electoral and non-electoral periods. However, when the effectiveness of violence is intermediate, so the indirect effect is meaningful and there is a sufficient difference between non-electoral and electoral periods, introducing elections decreases the average violence levels.

The right panel shows that this pattern holds generally for a wide variety of simulations, though for some (e.g., following the 75th percentile curve) there is always more violence with elections. Even when this is true, though increasing k_n generally makes introducing elections less bad (i.e., the change is less positive).

Figure 7: Simulations for the effect of ψ.

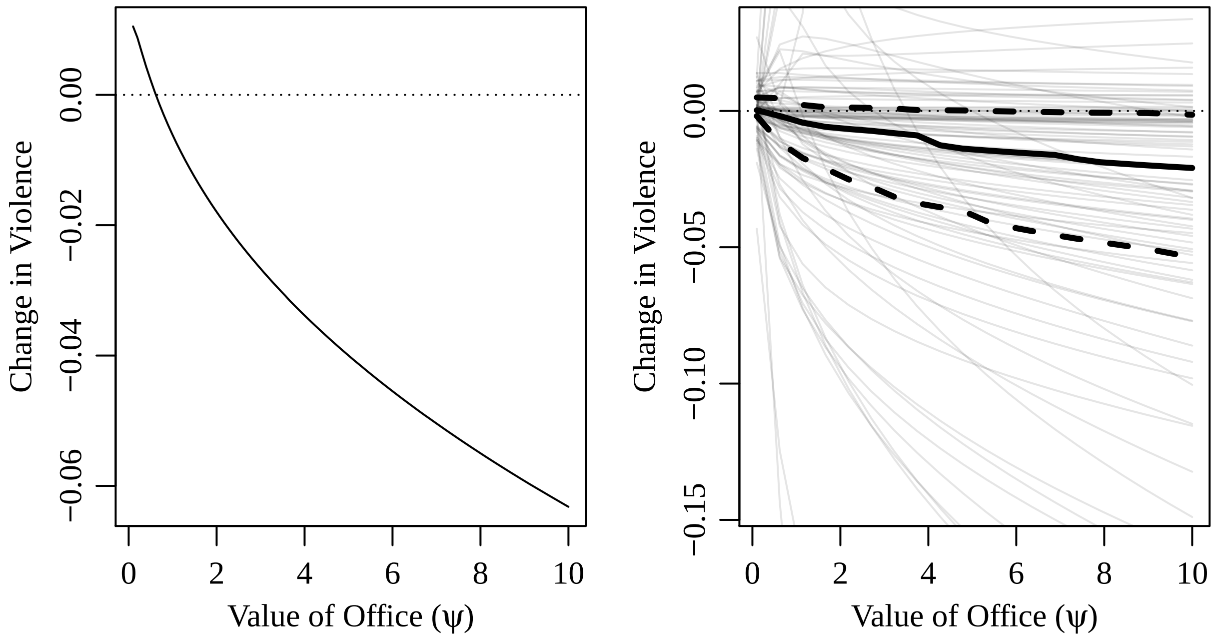


Figure 7 examines the effect of increasing the stakes of office ψ. As discussed in the main text, for the the main parameterization increasing ψ *always* leads to elections having a more (if always modest) pacifying effect. This also holds for most of the simulations: the 75th percentile line hugs zero and is slightly decreasing, while the median and 25th percentile are negative for all but tiny values of ψ and decreasing. So, it seems elections have a pacifying effect not only when elections are consequential, but when officeholding is consequential.

Figure 8 shows similar results when changing the value of holding office at the end of the game Δ^*_{T+1}. This likely follows from a similar logic, as increasing the value of holding office at the end of the game increase the value of holding office earlier, though this effect tends to be smaller compared to increasing the value of holding office in every period. This also demonstrates that setting this parameter to the stationary value is unlikely to have large consequences on the conclusions reached.

Figure 8: Simulations for the effect of Δ^*_{T+1}.

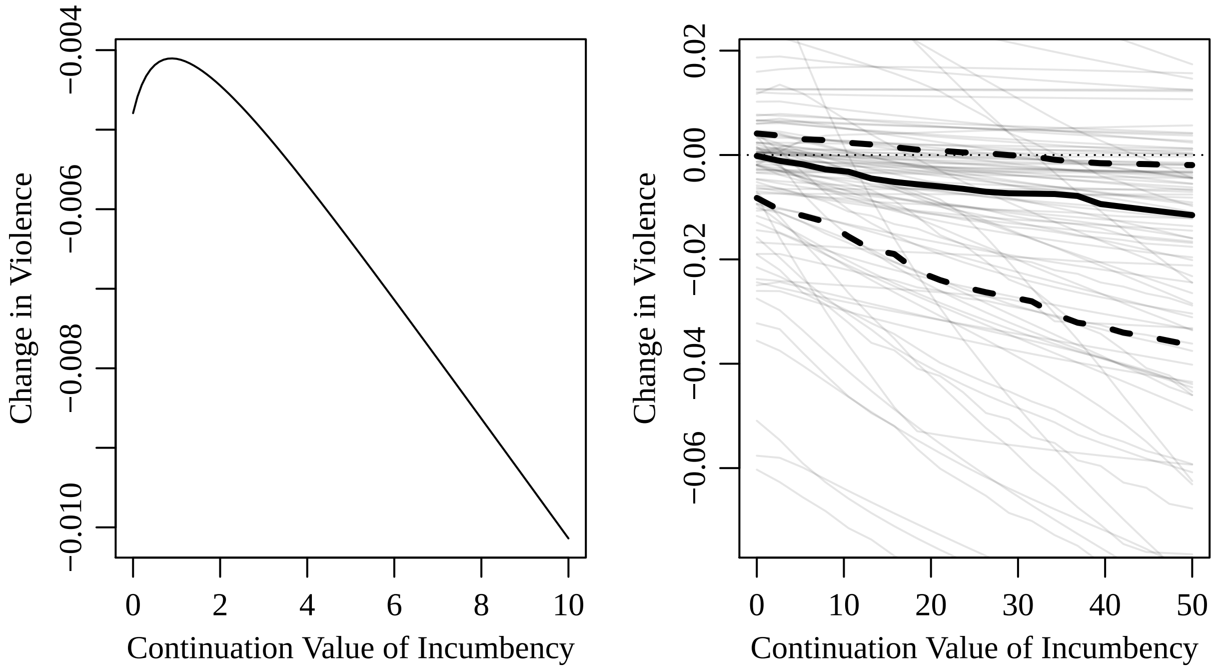


Figure 9: Simulations for the effect of δ.

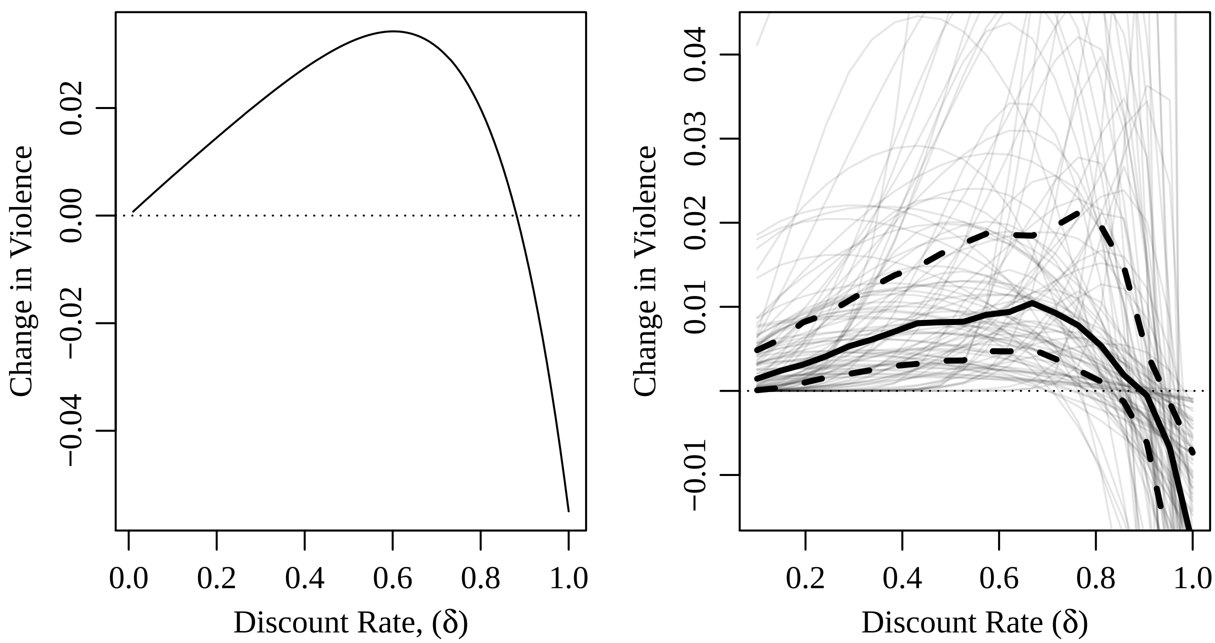


Figure 10: Simulations for the effect of α.

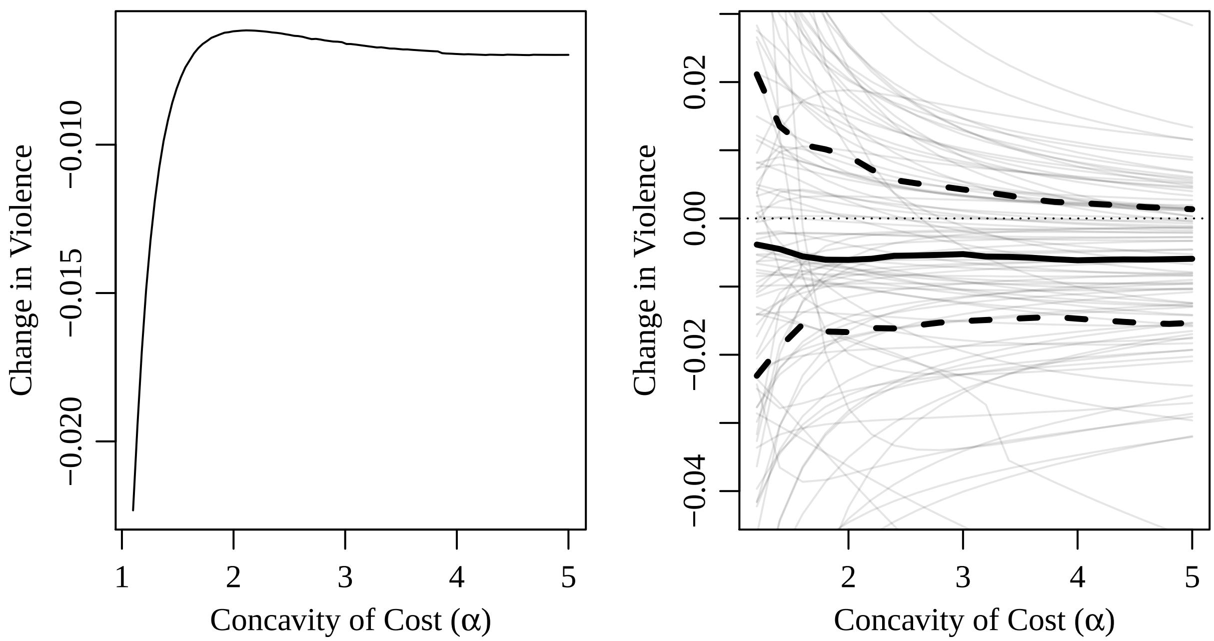
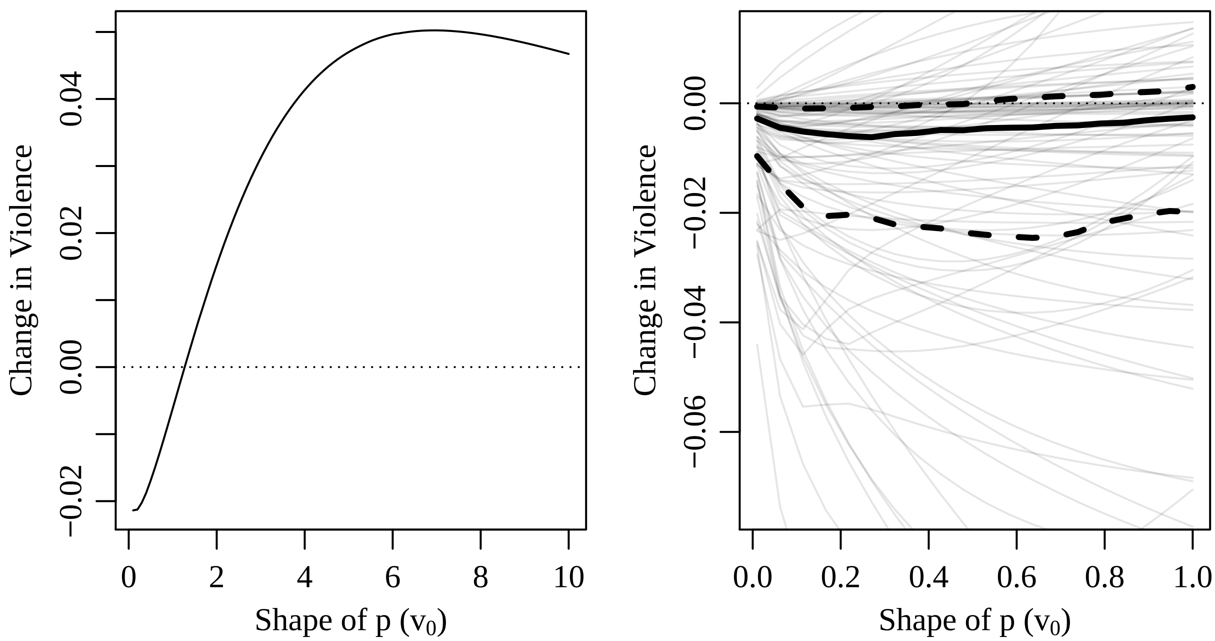


Figure 9 examines the effect of increasing the discount rate. As discussed in the main text in the main parameterization Changing δ has a a non-monotone effect, and elections lead to less violence when the actors are sufficiently patenit. The shape of this effect is similar for the vast majority of the simulations.

Next we plot the effect of making the cost function more convex (increasing α). For most of the range of α (particularly greater than 2) the effect of changing α is modest on average, though as the individual simulations show the effect of elections can become very positive or negative as $\alpha \to 1$ depending on the other parameters. On average for the distributions used here, elections always have a modest but negative impact on violence for any α.

Figure 11 looks at how changing the shape of the function capturing the probability of taking office changes the effect of holding elections. In general, when v_0 is small, the effect of committing violence is larger for small values of v_t but smaller for large values of v_t, i.e., when v_0 is bigger

Figure 11: Simulations for the effect of v_0.



the returns to violence are lower at first but diminish more slowly. For the fixed parameter values, elections tend to have a pacifying effect when v_0 is low but lead to more violence when v_0 is high. This could be related to the fact that when v_0 gets very high there will tend to be less violence in equilibrium, so by a similar logic discussed when considering changes in ψ it may be the case that the direct effect matters more when violence tends to be low while the indirect effect is stronger when violence is already high.

However, it also seems that the effect of elections is always increasing or always decreasing in v_0 for some simulations, so drawing strong conclusions on this parameter is likely unwarranted.

Finally, we examine the effect of making elections more or less frequent (figure 12). The general pattern where elections lead to more violence when very frequent and generally lead to a reduction in violence for moderate frequency holds for the majority of the simulations.

Figure 12: Simulations for the effect of the frequency of elections.