Appendix for “Preventive Repression: Two Types of Moral Hazard"

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Before proceeding with the proofs of propositions, notice that, from a technical perspective, our setting is a (non-standard) nonlinear bilevel optimization problem (Colson et al. 2007) in that the ruler maximizes a function of two (continuous) variables that affect the agent's optimal incentives while the agent optimally decides how much to divert from protection to other activities (a continuous variable) and what fraction of the diverted resources to put into politics and corruption (a continuous variable) both of which affect the principal's optimization problem.[1] Moreover, depending on whether politics or corruption is the optimal diversion activity for the agent, we need to consider different objective functions for the principal's maximization problem.

Proof of Lemma 1. For any B and s, given that $d = B - p$, the security agent's maximization problem is

$$\underset{p \in [0,B]}{\text{maximize}} \quad p(B - p + s)(1 - B).$$

The agent's optimal level of p is the solution of the FOC: $s + B - 2p = 0$, which implies that $\hat{p} = \frac{B+s}{2}$ if $s \leq B$ and $\hat{p} = B$ if $s \geq B$ (this is a maximum since the second-order condition is satisfied). As a result, the optimal level of d is $\hat{d} = \frac{B-s}{2}$ if $s \leq B$ and $\hat{d} = 0$ if $s \geq B$. □

Proof of Proposition 1. First, if $s \geq B$, the agent's optimal actions are $\hat{p}(B, s) = B$ and $\hat{d}(B, s) = 0$, and the ruler's optimal allocation of resources is the solution to the following constrained maximization problem:

$$\underset{B,s \in [0,1]}{\text{maximize}} \quad B(1 - s)(1 - B) \quad \text{s.t.} \quad s \geq B.$$

Forming the Lagrangian, $L(B, s, \lambda) = B(1 - s)(1 - B) - \lambda(B - s)$, the first order conditions are $\frac{dL}{dB} = (1 - s)(1 - 2B) - \lambda = 0$; $\frac{dL}{ds} = -(B - B^2) + \lambda = 0$; $\lambda(B - s) = 0$; $\lambda \geq 0$; and

[1] For a review of bilevel optimization problems see Colson, Benoit, Patrice Marcotte, and Gilles Savard. 2007. "An overview of bilevel optimization." *Annals of Operations Research* 153 (1): 235-256.

$s - B \geq 0$. The critical points of this constrained maximization are: 1) $B = s = 1$, $\lambda = 0$; 2) $B = 0$, $s = 1$, and $\lambda = 0$; and 3) $B = s = \frac{1}{3}$, $\lambda = \frac{2}{9}$. Checking the second order conditions, the optimal solutions of this constrained optimization problem are $B^* = \frac{1}{3}$ and $s^* = \frac{1}{3}$.

Second, if $s \leq B$, the agent's optimal actions are $\hat{p}(B,s) = \frac{B+s}{2}$ and $\hat{d}(B,s) = \frac{B-s}{2}$, and the ruler's optimal allocation of resources is the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad \frac{B+s}{2}\big(1 - \frac{B+s}{2}\big)(1-B) \quad \text{s.t.} \quad s \leq B.$$

Forming the Lagrangian, $L(B,s,\lambda) = \frac{B+s}{2}\big(1 - \frac{B+s}{2}\big)(1-B) - \lambda(s-B)$, the first order conditions are $\frac{dL}{dB} = \frac{1}{4}(3B^2 - 2B(3-2s) + 2 - 4s + s^2) + \lambda = 0$; $\frac{dL}{ds} = \frac{1}{2}(1-B)(1-B-s) - \lambda = 0$; $\lambda(s-B) = 0$; $\lambda \geq 0$; and $B - s \geq 0$. The critical points of this constrained maximization are: 1) $B = s = 1$, $\lambda = 0$; and 2) $B = s = \frac{1}{3}$, $\lambda = \frac{1}{9}$. Checking the second order conditions, the optimal solutions of this constrained optimization problem are $B^* = \frac{1}{3}$ and $s^* = \frac{1}{3}$.

Given that in both scenarios, the ruler's optimal allocations are $B^* = s^* = \frac{1}{3}$, this implies that the equilibrium resources for protection and distribution of rents are $B^* = 1/3$ and $s^* = 1/3$. Finally, this implies that the agent's equilibrium actions are $p^* = 1/3$ and $d^* = 0$, as claimed. □

Proof of Lemma 2. For any given B and s, given that $d = B - p$, the agent's optimization problem is

$$\underset{p\in[0,B]}{\text{maximize}} \quad p\big[s(1-B) + (B-p)\gamma\big].$$

The optimal p is the solution to the FOC: $s(1-B) + \gamma B - 2\gamma p = 0$ (the solution is a maximum since the second order condition is satisfied). The optimal solution is $\tilde{p} = \frac{\gamma B + s(1-B)}{2\gamma}$ if $s \leq \frac{\gamma B}{1-B}$ and $\tilde{p} = B$ if $s \geq \frac{\gamma B}{1-B}$. This implies that the optimal diversion of resources is $\tilde{d} = \frac{\gamma B - s(1-B)}{2\gamma}$ if $s \leq \frac{\gamma B}{1-B}$ and $\tilde{d} = 0$ if $s \geq \frac{\gamma B}{1-B}$. □

Proposition 2. First, if $s \geq \frac{\gamma B}{1-B}$, the agent's optimal actions are $\tilde{p}(B,s) = B$ and $\tilde{d}(B,s) = 0$.

Thus, the ruler's optimal B and s are the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad B(1-s)(1-B) \quad \text{s.t.} \quad s(1-B) \geq \gamma B.$$

Forming the Lagrangian $L(B,s,\lambda) = B(1-s)(1-B) - \lambda\big(\gamma B - s(1-B)\big)$, the first order conditions are $\frac{dL}{dB} = (1-s)(1-2B) - \lambda(\gamma+s) = 0$; $\frac{dL}{ds} = -(B-B^2) + \lambda(1-B) = 0$; $\lambda\big(\gamma B - s(1-B)\big) = 0$; $\lambda \geq 0$; and $s(1-B) - \gamma B \geq 0$. The critical points of this constrained maximization problem are: 1) $B=0$, $s=1$, $\lambda=0$; and 2) $B = \frac{1}{2(1+\gamma)}$, $s = \frac{\gamma}{1+2\gamma}$, $\lambda = \frac{1}{2(1+\gamma)}$. Checking the second order conditions, the optimal solutions of this constrained optimization problem are $B^* = \frac{1}{2(1+\gamma)}$ and $s^* = \frac{\gamma}{1+2\gamma}$.

Second, if $s \leq \frac{\gamma B}{1-B}$, the agent's optimal actions are $\tilde{p}(B,s) = \frac{\gamma B + s(1-B)}{2\gamma}$ and $\tilde{d}(B,s) = \frac{\gamma B - s(1-B)}{2\gamma}$. This implies that the ruler's optimal B and s are the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad \frac{\gamma B + s(1-B)}{2\gamma}\big(1-s\big)\big(1-B\big) \quad \text{s.t.} \quad s(1-B) \leq \gamma B.$$

Forming the Lagrangian $L(B,s,\lambda) = \frac{\gamma B + s(1-B)}{2\gamma}\big(1-s\big)\big(1-B\big) - \lambda\big(s(1-B) - \gamma B\big)$, the first order conditions are $\frac{dL}{dB} = \frac{1}{2\gamma}(1-s)[\gamma(1-2B) - 2s(1-B)] + \lambda(s+\gamma) = 0$; $\frac{dL}{ds} = \frac{1}{2\gamma}(1-B)[-\gamma B + (1-B)(1-2s)] - \lambda(1-B) = 0$; $\lambda\big(s(1-B) - \gamma B\big) = 0$; $\lambda \geq 0$; and $s(1-B) - \gamma B \leq 0$. The critical points of this constrained maximization problem are: 1) $B=1$, $s=1$, $\lambda=0$; and 2) $B = \frac{1}{2(1+\gamma)}$, $s = \frac{\gamma}{1+2\gamma}$, $\lambda = \frac{1-\gamma}{4\gamma(1+\gamma)}$. Checking the second order conditions, the optimal solutions of this constrained optimization problem are $B^* = \frac{1}{2(1+\gamma)}$ and $s^* = \frac{\gamma}{1+2\gamma}$.

Given that in both scenarios, the ruler's optimal allocations are $B^* = \frac{1}{2(1+\gamma)}$ and $s^* = \frac{\gamma}{1+2\gamma}$, this implies that the equilibrium distribution of rents is $s^* = \frac{\gamma}{1+2\gamma}$ and the equilibrium allocation of resources to protection is $B^* = \frac{1}{2(1+\gamma)}$. Finally, this implies that the agent's equilibrium actions are $p^* = \frac{1}{2(1+\gamma)}$ and $d^* = 0$, as claimed. $\square$

Proof of Proposition 3. In text. $\square$

Proof of Proposition 4. First, if $1-B \geq \gamma$ and $s \geq B$, the agent's optimal actions are $\sigma^*(B,s)=1$, $\hat{p}(B,s)=B$, and $\hat{d}(B,s)=0$. Thus the ruler's optimal allocation of resources is the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad B(1-s)(1-B) \quad \text{s.t.} \quad s \geq B \quad \text{and} \quad 1-B \geq \gamma.$$

Forming the Lagrangian, $L(B,s,\lambda_1,\lambda_2)=B(1-s)(1-B)-\lambda_1(B-s)-\lambda_2(\gamma+B-1)$, the first order conditions are $\frac{dL}{dB}=(1-s)(1-2B)-\lambda_1-\lambda_2=0$; $\frac{dL}{ds}=-(B-B^2)+\lambda_1=0$; $\lambda_1(B-s)=0$; $\lambda_2(\gamma+B-1)=0$; $\lambda_1 \geq 0$; $\lambda_2 \geq 0$; $s-B \geq 0$; and $1-B-\gamma \geq 0$. If $\gamma \leq \frac{2}{3}$, the critical points of this constrained maximization are: 1) $B=0$, $s=1$, $\lambda_1=\lambda_2=0$; 2) $B=s=\frac{1}{3}$, $\lambda_1=\frac{2}{9}$, $\lambda_2=0$. If $\frac{2}{3} \leq \gamma < 1$, the critical points are: 1) $B=0$, $s=1$, $\lambda_1=\lambda_2=0$; and 2) $B=s=1-\gamma$, $\lambda_1=\gamma(1-\gamma)$, $\lambda_2=\gamma(3\gamma-2)$. And if $\gamma=1$, the critical points are: $B=0$, $s=\alpha$, $\lambda_1=0$, $\lambda_2=1-\alpha$ for any $\alpha \in [0,1]$. Checking the second order conditions, the optimal solutions of this constrained optimization problem are $B^*=s^*=\frac{1}{3}$ for $\gamma \leq \frac{2}{3}$ and $B^*=s^*=1-\gamma$ for $\gamma \geq \frac{2}{3}$. The ruler's payoff is $U_R^*=\frac{4}{27}$ if $\gamma \leq \frac{2}{3}$ and $U_R^*=\gamma^2(1-\gamma)$ if $\gamma \geq \frac{2}{3}$.

Second, if $1-B \geq \gamma$ and $s \leq B$, the agent's optimal actions are $\sigma^*(B,s)=1$, $\hat{p}(B,s)=\frac{B+s}{2}$, and $\hat{d}(B,s)=\frac{B-s}{2}$. The ruler's optimal allocation of resources is the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad \frac{B+s}{2}\Big(1-\frac{B+s}{2}\Big)(1-B) \quad \text{s.t.} \quad s \leq B \quad \text{and} \quad 1-B \geq \gamma.$$

Forming the Lagrangian, $L(B,s,\lambda_1,\lambda_2)=\frac{B+s}{2}\big(1-\frac{B+s}{2}\big)(1-B)-\lambda_1(s-B)-\lambda_2(\gamma+B-1)$, the first order conditions are $\frac{dL}{dB}=\frac{1}{4}(3B^2-2B(3-2s)+2-4s+s^2)+\lambda_1-\lambda_2=0$; $\frac{dL}{ds}=\frac{1}{2}(1-B)(1-B-s)-\lambda_1=0$; $\lambda_1(s-B)=0$; $\lambda_2(\gamma+B-1)=0$; $\lambda_1 \geq 0$; $\lambda_2 \geq 0$; $1-B-\gamma \geq 0$; and $B-s \geq 0$. If $\gamma \leq \frac{2}{3}$, the critical points of this constrained maximization are: $B=S=\frac{1}{3}$, $\lambda_1=\frac{1}{9}$, $\lambda_2=0$. And if $\gamma \geq \frac{2}{3}$, the critical points are: $B=s=1-\gamma$, $\lambda_1=\frac{1}{2}\gamma(2\gamma-1)$, $\lambda_2=\gamma(3\gamma-2)$. Checking the second order conditions, the optimal solution of this constrained optimization problem is $B^* = s^* = \frac{1}{3}$ for $\gamma \leq \frac{2}{3}$ and $B^* = s^* = 1-\gamma$ for $\gamma \geq \frac{2}{3}$. The ruler's payoff is $U_R^* = \frac{4}{27}$ if $\gamma \leq \frac{2}{3}$ and $U_R^* = \gamma^2(1-\gamma)$ if $\gamma \geq \frac{2}{3}$.

Third, if $1-B \leq \gamma$ and $s \geq \frac{\gamma B}{1-B}$, the agent's optimal actions are $\sigma^*(B,s) = 0$, $\tilde{p}(B,s) = B$, and $\tilde{d}(B,s) = 0$. Thus, the ruler's optimal B and s are the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad B(1-s)(1-B) \quad \text{s.t.} \quad s(1-B) \geq \gamma B \quad \text{and} \quad 1-B \leq \gamma.$$

Forming the Lagrangian $L(B,s,\lambda_1,\lambda_2) = B(1-s)(1-B) - \lambda_1\big(\gamma B - s(1-B)\big) - \lambda_2(1-B-\gamma)$, the first order conditions are $\frac{dL}{dB} = (1-s)(1-2B) - \lambda_1(\gamma+s) + \lambda_2 = 0$; $\frac{dL}{ds} = -(B-B^2) + \lambda_1(1-B) = 0$; $\lambda_1\big(\gamma B - s(1-B)\big) = 0$; $\lambda_2(1-B-\gamma) = 0$; $\lambda_1 \geq 0$; $\lambda_2 \geq 0$; $s(1-B) - \gamma B \geq 0$; and $1-B-\gamma \leq 0$. If $\gamma \leq \sqrt{\frac{1}{2}}$, the critical points of this constrained maximization problem are: $B = s = 1-\gamma$, $\lambda_1 = 1-\gamma$, $\lambda_2 = 1-2\gamma^2$. If $\sqrt{\frac{1}{2}} \leq \gamma < 1$, the critical points are: $B = \frac{1}{2(1+\gamma)}$, $s = \frac{\gamma}{1+2\gamma}$, $\lambda_1 = \frac{1}{2(1+\gamma)}$, $\lambda_2 = 0$. And if $\gamma = 1$, the critical points are: 1) $B = \frac{1}{4}$, $s = \frac{1}{3}$, $\lambda_1 = \frac{1}{4}$, $\lambda_2 = 0$; and 2) $B = 0$, $s = 1$, $\lambda_1 = \lambda_2 = 0$. Checking the second order conditions, the optimal solution of this constrained optimization problem is $B^* = \frac{1}{2(1+\gamma)}$, $s^* = \frac{\gamma}{1+2\gamma}$ if $\gamma \geq \sqrt{\frac{1}{2}}$ and $B^* = s^* = 1-\gamma$ if $\gamma \leq \sqrt{\frac{1}{2}}$. The ruler's payoff is $U_R^* = \frac{1}{4\gamma+4}$ if $\gamma \geq \sqrt{\frac{1}{2}}$ and $U_R^* = \gamma^2(1-\gamma)$ if $\gamma \leq \sqrt{\frac{1}{2}}$.

Fourth, if $1-B \leq \gamma$ and $s \leq B\frac{\gamma}{1-B}$, the agent's optimal actions are $\sigma^*(B,s) = 0$, $\tilde{p} = \frac{\gamma B + s(1-B)}{2\gamma}$, and $\tilde{d} = \frac{\gamma B - s(1-B)}{2\gamma}$. This implies that the ruler's optimal B and s are the solution to the following constrained maximization problem:

$$\underset{B,s\in[0,1]}{\text{maximize}} \quad \frac{\gamma B + s(1-B)}{2\gamma}\big(1-s\big)\big(1-B\big) \quad \text{s.t.} \quad s(1-B) \leq \gamma B \quad \text{and} \quad 1-B \leq \gamma.$$

Forming the Lagrangian $L(B,s,\lambda_1,\lambda_2) = \frac{\gamma B + s(1-B)}{2\gamma}\big(1-s\big)\big(1-B\big) - \lambda_1\big(s(1-B) - \gamma B\big) - \lambda_2(1-B-\gamma)$, the first order conditions are $\frac{dL}{dB} = \frac{1}{2\gamma}(1-s)[\gamma(1-2B) - 2s(1-B)] + \lambda_1(\gamma + s) + \lambda_2 = 0$; $\frac{dL}{ds} = \frac{1}{2\gamma}(1-B)[-\gamma B + (1-B)(1-2s)] - \lambda_1(1-B) = 0$; $\lambda_1\big(s(1-B) - \gamma B\big) = 0$; $\lambda_2(1-B-\gamma) = 0$; $\lambda_1 \geq 0$; $\lambda_2 \geq 0$; $s(1-B) - \gamma B \leq 0$; and $1-B-\gamma \leq 0$. If $\gamma \leq \frac{2}{3}$, the critical points of this constrained maximization problem are: 1) $B = 1-\gamma$, $s = \frac{\gamma}{2}$, $\lambda_1 = 0$, $\lambda_2 = \frac{1}{4}(2-\gamma)(1-\gamma)$; and 2) $B = 1$, $s = 1$, $\lambda_1 = \lambda_2 = 0$. If $\frac{2}{3} \leq \gamma \leq \sqrt{\frac{1}{2}}$, the critical points are: 1) $B = 1-\gamma$, $s = 1-\gamma$, $\lambda_1 = \frac{3\gamma-2}{2}$, $\lambda_2 = 1-2\gamma^2$; and 2) $B = 1$, $s = 1$, $\lambda_1 = \lambda_2 = 0$. And if $\gamma \geq \sqrt{\frac{1}{2}}$, the critical points are: 1) $B = \frac{1}{2(1+\gamma)}$, $s = \frac{\gamma}{1+2\gamma}$, $\lambda_1 = \frac{1-\gamma}{4\gamma(1+\gamma)}$, $\lambda_2 = 0$; and 2) $B = 1$, $s = 1$, $\lambda_1 = \lambda_2 = 0$. Checking the second order conditions, the optimal solution of this constrained optimization problem is $B^* = \frac{1}{2(1+\gamma)}$ and $s^* = \frac{\gamma}{1+2\gamma}$ if $\gamma \geq \sqrt{\frac{1}{2}}$; $B^* = 1-\gamma$ and $s^* = 1-\gamma$ if $\frac{2}{3} \leq \gamma \leq \sqrt{\frac{1}{2}}$; and $B = 1-\gamma$, $s = \frac{\gamma}{2}$ if $\gamma \leq \frac{2}{3}$. The ruler's payoff is $U_R^* = \frac{1}{4(\gamma+1)}$ if $\gamma \geq \sqrt{\frac{1}{2}}$; $U_R^* = \gamma^2(1-\gamma)$ if $\frac{2}{3} \leq \gamma \leq \sqrt{\frac{1}{2}}$; and $U_R^* = \frac{1}{8}\gamma(2-\gamma)^2$ if $\gamma \leq \frac{2}{3}$.

Comparing the ruler's payoff for different values of γ in the above four scenarios, the ruler's highest payoff is $\frac{4}{27}$ for $\gamma \leq \frac{2}{3}$, $\gamma^2(1-\gamma)$ for $\gamma \in [\frac{2}{3}, \sqrt{\frac{1}{2}}]$, and $\frac{1}{4\gamma+4}$ for $\gamma \geq \sqrt{\frac{1}{2}}$. As a result, the ruler's equilibrium choices are

$$B^* = \begin{cases} \frac{1}{3} & \text{if } \gamma \leq \overline{\gamma}_1 \\ 1-\gamma & \text{if } \gamma \in [\overline{\gamma}_1, \overline{\gamma}_2] \\ \frac{1}{2(\gamma+1)} & \text{if } \gamma \geq \overline{\gamma}_2 \end{cases} \quad \text{and} \quad s^* = \begin{cases} \frac{1}{3} & \text{if } \gamma \leq \overline{\gamma}_1 \\ 1-\gamma & \text{if } \gamma \in [\overline{\gamma}_1, \overline{\gamma}_2], \\ \frac{\gamma}{1+2\gamma} & \text{if } \gamma \geq \overline{\gamma}_2 \end{cases}$$

where $\overline{\gamma}_1 = \frac{2}{3}$ and $\overline{\gamma}_2 = \sqrt{\frac{1}{2}}$. □

Proof of Proposition 5. In the game in which the agent can choose between diverting to politics and diverting to corruption, the ruler's equilibrium payoff is the following:

$$U_R^* = \begin{cases} \frac{4}{27} & \text{if } \gamma \leq \overline{\gamma}_1 \\ \gamma^2(1-\gamma) & \text{if } \gamma \in [\overline{\gamma}_1, \overline{\gamma}_2]. \\ \frac{1}{4(\gamma+1)} & \text{if } \gamma \geq \overline{\gamma}_2 \end{cases}$$

where $\overline{\gamma}_1 = \frac{2}{3}$ and $\overline{\gamma}_2 = \sqrt{\frac{1}{2}}$. Notice that the ruler's equilibrium payoff is continuous in γ and is constant in γ for $\gamma \leq \overline{\gamma}_1$ and is decreasing in γ for $\gamma > \overline{\gamma}_1$. To see this, notice that at $\gamma = \gamma_1$, we have $(\gamma_1)^2(1-\gamma_1) = 4/27$ and at $\gamma = \gamma_2$, we have $(\gamma_2)^2(1-\gamma_2) = \frac{1}{4(\gamma_2+1)}$. Notice also that the expression $\frac{d}{d\gamma}\big(\gamma^2(1-\gamma)\big) = \gamma(2-3\gamma) \leq 0$ for any $\gamma \in [\overline{\gamma}_1, \overline{\gamma}_2]$ and that the expression $\frac{d}{d\gamma}\left(\frac{1}{4(\gamma+1)}\right) = -\frac{1}{4(\gamma+1)^2} < 0$ for any $\gamma \geq \overline{\gamma}_2$. On the other hand, in the politics game, the ruler's equilibrium payoff is $U_R = \frac{4}{27}$. Taken together, these arguments imply that the ruler's equilibrium payoff is (weakly) higher in the politics game than in the game in which the agent can choose between diverting to politics and diverting to corruption.

In the game in which the agent can choose between politics and corruption, the agent's equilibrium payoff is

$$U_A^* = \begin{cases} \frac{2}{27} & \text{if } \gamma \leq \overline{\gamma}_1 \\ \gamma(1-\gamma)^2 & \text{if } \gamma \in [\overline{\gamma}_1, \overline{\gamma}_2], \\ \frac{\gamma}{4(\gamma+1)^2} & \text{if } \gamma \geq \overline{\gamma}_2 \end{cases}$$

where $\overline{\gamma}_1 = \frac{2}{3}$ and $\overline{\gamma}_2 = \sqrt{\frac{1}{2}}$. Notice that the agent's equilibrium payoff is continuous in γ; the agent's equilibrium payoff is constant in γ for $\gamma \leq \overline{\gamma}_1$, is decreasing in γ for $\gamma \in [\overline{\gamma}_1, \overline{\gamma}_2]$ and is less than $2/27$ for $\gamma \in [\overline{\gamma}_2, 1]$. To see this, notice that at $\gamma = \overline{\gamma}_1$, we have $\gamma(1-\gamma)^2 = \frac{2}{27}$ and at $\gamma = \overline{\gamma}_2$, we have $\gamma(1-\gamma)^2 = \frac{\gamma}{4(\gamma+1)^2}$. Also notice that $\frac{d}{d\gamma}\left(\gamma(1-\gamma)^2\right) = 1 - 4\gamma - \gamma^2 < 0$ for any $\gamma \in [\overline{\gamma}_1, \overline{\gamma}_2]$ and that $\frac{\gamma}{4(\gamma+1)^2} < \frac{2}{27}$ for any $\gamma \in [\overline{\gamma}_2, 1]$ since $8\gamma^2 - 11\gamma + 8 > 0$ for all $\gamma \in [\overline{\gamma}_2, 1]$. On the other hand, in the politics game, the agent's equilibrium payoff is $U_A = \frac{2}{27}$. Taken together, these arguments imply that the agent's equilibrium payoff is (weakly) higher in the politics game than in the game in which the agent can choose between diverting to politics and diverting to corruption. □

Proof of Proposition 6. In the politics game, the equilibrium probability of regime survival is $p^* = 1/3$. On the other hand, in the game in which the agent can choose between politics and corruption, the equilibrium probability of regime survival is

$$p^* = \begin{cases} \frac{1}{3} & \text{if } \gamma \leq \overline{\gamma}_1 \\ 1-\gamma & \text{if } \gamma \in [\overline{\gamma}_1, \overline{\gamma}_2], \\ \frac{1}{2(\gamma+1)} & \text{if } \gamma \geq \overline{\gamma}_2 \end{cases}$$

where $\overline{\gamma}_1 = \frac{2}{3}$ and $\overline{\gamma}_2 = \sqrt{\frac{1}{2}}$. Notice that the equilibrium probability of regime survival is continuous in γ since at $\gamma = \gamma_1$, we have $\frac{1}{3} = 1 - \gamma$ and at $\gamma = \gamma_2$, we have $1 - \gamma = \frac{1}{2(\gamma+1)}$. Notice also that this equilibrium probability is constant in γ for $\gamma \leq \overline{\gamma}_1$ and is decreasing in γ for $\gamma \geq \overline{\gamma}_1$ since both $1-\gamma$ and $\frac{1}{2(\gamma+1)}$ are decreasing in γ. Take together, these arguments imply that the equilibrium probability of regime survival is (weakly) higher in the politics game than in the game in which the agent can choose between diverting to politics and diverting to corruption. $\square$