Online Appendix for Partisan Affect and Elite Polarization

Omitted Proofs

To simplify notation, we will often drop the subscript i when referring to a generic voter. Also, let κ_b denote the measure of b voters.

PROOF OF PROPOSITION 1

Suppose the incumbent is A (the analysis when party B is the incumbent is similar). For a fixed π, and for any $p > p'$ and $\delta < \delta'$, Ingroup Responsiveness implies that

$$\alpha(p, \delta, \pi) - \alpha(p, \delta', \pi) > \alpha(p', \delta, \pi) - \alpha(p', \delta', \pi). \tag{1}$$

In other words, the difference $\alpha(p, \delta, \pi) - \alpha(p, \delta', \pi)$ is strictly increasing in p. Also, affective polarization implies that G_b FOSD G_{-b} for any $b < 0$ and $-b$. Therefore, the property of first order stochastic dominance implies that

$$\mathbb{E}_{G_b}\left[\alpha(p, \delta, \pi) - \alpha(p, \delta', \pi)\right] > \mathbb{E}_{G_{-b}}\left[\alpha(p, \delta, \pi) - \alpha(p, \delta', \pi)\right], \tag{2}$$

where expectation is taken with respect to p (the subscript denotes the distribution of p). Rearrange (2), and letting $\alpha_b = \mathbb{E}_{G_b}\left[\alpha(p, \delta, \pi)|\pi\right]$ denote the conditional expectation of α given π, we have,

$$\underbrace{\mathbb{E}\left[\alpha_b(p, \delta, \pi) + \alpha_{-b}(p, \delta', \pi)\right]}_{\equiv W} > \underbrace{\mathbb{E}\left[\alpha_b(p, \delta', \pi) + \alpha_{-b}(p, \delta, \pi)\right]}_{\equiv V}, \tag{3}$$

where expectation is taken over π. Now, for a policy $\theta < 0$, define $\delta' = |\theta + b|$ and $\delta = |\theta - b|$. Thus, for $b < 0$, W is the expected vote share among b and $-b$ voters given policy θ, and V is the expected vote share among b and $-b$ voters given policy $-\theta$.

Inequality (3) implies that party A gains more votes from b and $-b$ voters by choosing θ than by choosing θ'. Also, θ and $-\theta$ induce the same number of votes from centrist ($b = 0$) voters since they are equidistant from 0. Because (3) holds for any $b < 0$ and the distribution of voters is symmetric around 0, it follows that the expected vote share for A under $\theta < 0$ is weakly greater than under $-\theta$.

$\square$

PROOF OF COROLLARY 1

Suppose party A is the incumbent (the argument is the similar when B is the incumbent). We would like to show that A has a strict incentive to deviate from the median policy when the measure of centrist voters is low (i.e., κ_0 is below a threshold). Formally, the expected vote share for A under policy θ is:

$$\underbrace{\sum_{b<0} \kappa_b \mathbb{E}\left[\alpha(p, \delta_b(\theta), \pi)\right]}_{\equiv C} + \underbrace{\kappa_0 \mathbb{E}\left[\alpha(p, \delta_0(\theta), \pi)\right]}_{\equiv D} + \underbrace{\sum_{b>0} \kappa_b \mathbb{E}\left[\alpha(p, \delta_b(\theta), \pi)\right]}_{\equiv E}$$

where $\delta_b(\theta) = |\theta - b|$ is expressed explicitly as a function of θ. Note that $\frac{\partial E}{\partial \theta}|_{\theta=0} > 0$, $\frac{\partial D}{\partial \theta}|_{\theta=0} < 0$, $\frac{\partial C}{\partial \theta}|_{\theta=0} < 0$. Therefore, a deviation of the policy to the left of the median results in a gain of votes from $b < 0$ voters, and a loss of votes from $b \geq 0$ voters. Note that ingroup responsiveness implies that $\frac{\partial C}{\partial \theta}|_{\theta=0} + \frac{\partial E}{\partial \theta}|_{\theta=0} < 0$. Now, party A would have a strict incentive to deviate to the left if

$$\left| \frac{\partial C}{\partial \theta}|_{\theta=0} + \frac{\partial E}{\partial \theta}|_{\theta=0} \right| > \left| \frac{\partial D}{\partial \theta}|_{\theta=0} \right|.$$

It is straightforward to see that the inequality holds when κ_0 is sufficiently small. $\square$

PROOF OF PROPOSITION 2

Let $\theta^I(G_b)$ be the optimal policy for party $I \in \{A, B\}$ as a function of G_b, holding $G_{b' \neq b}$

fixed. We seek to prove the following three statements:

- Given G_l' FOSD G_l,

$$\theta^A\left(G_l'\right) \leq \theta^A\left(G_l\right) \text{ and } \theta^B\left(G_l\right) \leq \theta^B\left(G_l'\right).$$

- Given G_r FOSD G_r',

$$\theta^A\left(G_r'\right) \leq \theta^A\left(G_r\right) \text{ and } \theta^B\left(G_r\right) \leq \theta^B\left(G_r'\right).$$

- Given G_m' FOSD G_m,

$$\theta^A\left(G_m'\right) \geq \theta^A\left(G_m\right) \text{ and } \theta^B\left(G_m\right) \leq \theta^B\left(G_m'\right).$$

A given inequality holds strictly if $\theta^I\left(G_b\right) \in (-1,0) \cup (0,1)$.

Let party A be the incumbent (the argument is similar when party B is the incumbent). Consider first the effect of changing G_l, holding G_r and G_m fixed. We substitute θ in for δ in the function α (i.e., take $\alpha_b(p,\theta,\pi) \equiv \alpha\left(p,\delta_b(\theta),\pi\right)$), and define $V(\theta|G_l) = \sum_b \mathbb{E}\left[\alpha_b(p,\theta,\pi)\right]$ to be the expected vote share given policy θ given G_l. We want to show that, for any $\theta' < \theta$ and G_l' FOSD G_l

$$V(\theta'|G_l') - V(\theta|G_l') > V(\theta'|G_l) - V(\theta|G_l). \tag{4}$$

First, notice that the inequality depends only on the change in the vote share of l voters under the different distributions of affinities. Now, Ingroup Responsiveness implies that the difference $\alpha_l(p,\theta',\pi) - \alpha_l(p,\theta,\pi)$ is increasing in p. As G_l' FOSD G_l, the following inequality holds

$$\mathbb{E}_{G_l'}\left[\alpha_l(p,\theta',\pi)-\alpha_l(p,\theta,\pi)|\pi\right] > \mathbb{E}_{G_l}\left[\alpha_l(p,\theta',\pi)-\alpha_l(p,\theta,\pi)|\pi\right],$$

where the expectation is taken with respect to p for a fixed π. Since this holds for all π, Inequality 4 follows immediately. Now, Inequality 4 implies that V satisfies decreasing differences in θ and G_l. Therefore, by results in monotone comparative statics, it is the case that $\theta^A\left(G_l'\right) \leq \theta^A\left(G_l\right)$ with strict inequality if $\theta^A\left(G_l\right) \in (0,1)$.

Similar arguments apply to the case of G_r FOSD G_r'. Here, V exhibits increasing differences in θ and G_r i.e., for $\theta' > \theta$

$$V(\theta'|G_r) - V(\theta|G_r) > V(\theta'|G_r') - V(\theta|G_r').$$

It follows that that $\theta^A\left(G_r'\right) \leq \theta^A\left(G_r\right)$, with strict inequality if $\theta^A\left(G_r\right) \in (0,1)$. The case of G_m FOSD-increases follows a similar logic, and the detail is omitted for brevity. □

Generalization of Proposition 2

We proved Proposition 2 under the assumption that the set of bliss points has three elements. The result below generalizes Proposition 2 to the case where the set of bliss points is finite. As before, let $\theta^A(G_b)$ and $\theta^B(G_b)$ stand for the optimal policy for party A and B as a function of G_b, holding G_b' constant for $b' \neq b$.

Proposition. *If G_b FOSD-increases for $b < \theta^A(G_b)$ or G_b FOSD-decreases for $b > \theta^B(G_b)$, then $\theta^A(G_b)$ weakly decreases and $\theta^B(G_b)$ weakly increases. If G_b FOSD-increases for $\theta^A(G_b) \leq b \leq \theta^B(G_b)$, then $\theta^A(G_b)$ weakly increases and $\theta^B(G_b)$ weakly increases. "Weakly" can be replaced with "strictly" whenever $\theta^A(G_b), \theta^B(G_b) \notin \{-1,0,1\}$.*

The proof is omitted as it is a straightforward generalization of the arguments for Proposition 2. As extreme voters (those with bliss points outside of the optimal policies of the two parties) become more partisan, the parties respond by adopting more extreme policies. If the incumbent finds more support from voters with moderate bliss points, then it adopts a more moderate policy.

Maximizing the Probability of Winning

In the benchmark model, we assumed that the incumbent's objective is to maximize the expected vote share. The main result goes through if the incumbent maximizes the probability of winning instead. However, we need to impose the following assumption.

Assumption 2. *α is increasing in $\pi_i = \hat{\pi}_c + \hat{\pi}_i$, where $\hat{\pi}_c$ is a variable common to all voters, and $\hat{\pi}_i$ is iid across voters.*

The monotonicity assumption simplifies the argument. The more substantial assumption is that the shock can be decomposed into two components: a common shock $\hat{\pi}_c$ which captures events that have an effect on the entire electorate (e.g., global oil price) and a private shock $\hat{\pi}_i$ that affects only voter i.

Proposition. *Suppose the electorate is affectively polarized and Assumption 2 is satisfied, then the incumbent who maximizes win-probability weakly biases its policy toward its partisans. The bias is strict if the measure of centrist voters is sufficiently low.*

Proof. Without loss of generality, suppose party A is the incumbent. Let

$$V_\theta(\hat{\pi}_c) = \sum_b \kappa_b \mathbb{E}\left[\alpha\left(p, \theta, \pi\right) | \hat{\pi}_c\right] \tag{5}$$

be A's vote share under policy θ conditional on the realization of the common shock $\hat{\pi}_c$. In other words, the expectation is taken with respect to p and $\hat{\pi}_i$. Note that as a function of $\hat{\pi}_c$, V_θ is a random variable. We want to show that $\Pr(V_\theta > \frac{1}{2})$ is maximized for some policy $\theta \leq 0$. For this, we shall prove that for any $\theta < 0$, V_θ FOSD $V_{-\theta}$, which implies that any policy to the right of the median is dominated by a mirroring policy to the left of the median.

Let $\alpha_b(\theta, \pi) \equiv \mathbb{E}\left[\alpha(p, \theta, \pi) | \theta, \pi\right]$ be the vote share conditional on θ and π (i.e., expectation is taken with respect to p). Let $\theta < 0$ and applying ingroup responsiveness, we obtain the following inequality for any $b < 0$,

$$\underbrace{\mathbb{E}\left[\alpha_b(\theta, \pi) + \alpha_{-b}(-\theta, \pi) | \hat{\pi}_c\right]}_{v_\theta^b(\hat{\pi}_c)} > \underbrace{\mathbb{E}\left[\alpha_b(-\theta, \pi) + \alpha_{-b}(\theta, \pi) | \hat{\pi}_c\right]}_{v_{-\theta}^b(\hat{\pi}_c)}, \tag{6}$$

where the expectation is taken with respect to $\hat{\pi}_i$. Define $v_\theta^0(\hat{\pi}_c) \equiv \mathbb{E}\left[\alpha_0(\theta, \pi) | \hat{\pi}_c\right]$, we have that

$$V_\theta(\hat{\pi}_c) = \sum_{b \leq 0} \kappa_b v_\theta^b(\hat{\pi}_c).$$

Now, for any realizations of the common shock $\hat{\pi}_c$ and $\theta < 0$, Inequality (6) implies that

$$V_\theta(\hat{\pi}_c) > V_{-\theta}(\hat{\pi}_c). \tag{7}$$

It remains to show that V_θ FOSD $V_{-\theta}$ for any $\theta < 0$. Take $\mathbb{I}$ to be the indicator function, the CDF of V_θ and $V_{-\theta}$ can be expressed as

$$\Pr(V_\theta < x) = \int_{\hat{\pi}_c} \mathbb{I}\left[V_\theta(\hat{\pi}_c) < x\right] d\hat{\pi}_c$$
$$\Pr(V_{-\theta} < x) = \int_{\hat{\pi}_c} \mathbb{I}\left[V_{-\theta}(\hat{\pi}_c) < x\right] d\hat{\pi}_c.$$

Given Assumption 2 and Inequality (7), we have that $\Pr(V_\theta < x) < \Pr(V_{-\theta} < x)$ for all x. Therefore, V_θ FOSD $V_{-\theta}$, and thus $-\theta$ is dominated by θ.

When the measure of the centrist voters is sufficiently low, the incumbent has as strict incentive to deviate from the median policy. The argument for this is similar to the proof of Corollary 1, and the details are omitted for brevity. $\square$

Statements of Results for the Case of Multiple Optimal Policies

In this section, we consider the versions of the main results that account for the possibility of multiple optimal policies. The proofs for the more general versions of the results follow immediately from their counterparts in the body text and are therefore omitted.

Define Θ^A and Θ^B to be the set of optimal policies for party A and B, respectively. The generalization of Proposition 1 is:

Proposition. *Suppose the electorate is affectively polarized, then every element of Θ^A is biased to the left (i.e., $\theta^A \leq 0$ for all $\theta^A \in \Theta^A$), and every element of $\Theta^B$is biased to the right (i.e., $\theta^B \geq 0$ for all $\theta^B \in \Theta^B$).*

The generalization of Corollary 1 is:

Corollary. *If the measure of centrist voters is sufficiently small, then the incumbent chooses a non-median policy (i.e., $\theta^A < 0$ for all $\theta^A \in \Theta^A$ and $\theta^B > 0$ for all $\theta^B \in \Theta^B$).*

The generalization of Proposition 2 requires a definition of monotonicity of sets that is based on the idea of the strong set order.

Definition. A set S increases (to S') if $\forall s \in S$, $\exists s' \in S'$ such that $s' \geq s$, and strictly increases if $s' > s$. A set S decreases (to S') if S' increases to S.

Proposition. *Let $\mathcal{B} = \{l, m, r\}$. If G_l FOSD-increases or G_r FOSD-decreases, then Θ^A decreases, and Θ^B increases. If G_m FOSD-increases, then both Θ^A and Θ^B increase. The change for Θ^A is strict if $\Theta^A \setminus \{-1, 0\} \neq \emptyset$; the change for Θ^B is strict if $\Theta^B \setminus \{0, 1\} \neq \emptyset$.*