

# Online Appendix for: Tactical Extremism

Jon X. Eguia

Francesco Giovannoni

# Appendix A: Supplemental Information

This is the online Appendix to the Letter “Tactical Extremism”, in the *American Political Science Review*. This version of the online Appendix is from September 14th, 2018.

This online Appendix contains the following five sections.

1. An extended literature review.
2. Motivating historical examples.
3. Formal notation and definitions.
4. Formal statement and proofs of the results in the Letter.
5. Extensions.

## 1. Extended Literature Review

Following the Great Recession, concerns about political extremism have made understanding its causes and evolution a salient priority in political research.

Motivated by the rise of previously fringe parties in Europe, a branch of the literature on extremism analyzes the rise of outsider candidates (Buisseret and van Weelden 2017; Karakas 2017) and populist parties (Guiso, Herrera, Morelli and Sono 2017). Our work belongs instead to a literature that studies why mainstream parties sometimes also choose extremist (or at least non-median) policies.

A large strand within this latter literature explains policy divergence as a consequence of parties’ policy preferences. Parties with policy preferences choose non-median policies if there is sufficient uncertainty about voters’ preferences (Wittman 1983; Calvert 1985; Roemer 1994; Matakos, Troumpounis and Xefteris 2016), or if candidates cannot commit to a platform (Osborne and Slivinski 1996; Besley and Coate 1997; Aragonès, Palfrey and Postlewaite 2007; Kalandrakis 2009; Acemoglu, Egorov and Sonin 2013; van Weelden 2013).

One may wonder whether parties that are purely office motivated ever choose extremist policies in their pursuit of office. Office motivated parties may choose non-median positions if they do not know the location of the median voter (Bernhardt, Duggan and Squintani, 2009b). Even if they know the location of the median, they may choose non-median po-

sitions to prevent third-party entry (Palfrey 1984), or to change voters' ideologies (Prato 2018). Parties may also tactically announce extreme platforms if their announcements are discounted by voters (Grofman 1985; Bawn and Somer-Topcu 2012), so that an extreme announcement is needed to signal a moderate policy.

Valence differences across parties help us predict which party is likely to adopt extreme policies. The classical notion of valence is a party specific attribute that makes the party more appealing to all voters, as if it were a quality dimension (Stokes 1992). Theories that focus on office motivated parties and on the role of valence differences tend to find that the stronger party adopts a more mainstream policy than the weaker one (Ansolabehere and Snyder 2000, Groseclose 2001). Assuming such party valences, Aragonès and Palfrey (2002, 2004), and Moon (2004), show that a weak candidate moves away from the center and chooses policy stochastically, to differentiate itself from a stronger rival.

Soubeyran (2009) and Krasa and Polborn (2010) independently introduce the notion of policy-specific party valence. A policy-specific party valence is a quality attribute that a party holds but it is specific for a particular policy: the party is good at something in particular, not good in general. In the context of a theory of public good provision, Soubeyran (2009) and Krasa and Polborn (2010) assume that candidates differ in their competences to provide each one of two public goods, and they compete by choosing quantities of production of each good. They find that candidates specialize, proposing to provide a greater quantity of the good in which they are more competent. Krasa and Polborn (2012) consider a theory of electoral competition with very general voter preferences, which can depend on the fixed attributes of candidates, on policy choices, and on the interaction of attributes and policies. They find conditions on voter preferences that lead to policy convergence, or to policy divergence. Namely, if preferences satisfy a weak separability condition (called “uniform candidate ranking”) between attributes and policies, then in any strict equilibrium, candidates' policies converge. Whereas, complementarities between attributes and policies lead to violations of this separability condition, and to policy divergence.

Krasa and Polborn (2014) apply a fusion of their “differentiated candidates” theories (2010 and 2012) to taxation. A candidate  $R$  has an (exogenous) competence advantage in

providing low levels of taxation, and a candidate  $D$  has a similar (exogenous) advantage in providing high levels of taxation. Candidates choose tax rates. In equilibrium, policies diverge, and Candidate  $D$  proposes a higher level of taxation.

All these valence theories, whether it a global valence difference (Aragonès and Palfrey 2002 and 2004; Moon 2004), or a policy-specific valence difference (Soubeyran 2009; Krasa and Polborn 2010, 2012 and 2014) explain policy divergence, but in every equilibrium of any of these theories, each party chooses the policy that maximizes its probability of winning the election, or its vote share. While there is policy divergence, policy proposals are not too extreme, in the sense that each party chooses policy so as to maximize its appeal to voters.

We are interested in a more puzzling form of extremism: why would a party that can win (with some probability) by choosing a centrist policy, ever choose to lurch to a platform so extreme that it reduces the party's probability of winning? Why would an office-motivated party ever go against the wishes of its electorate, defying its voters with a proposal that voters do not want and most voters will not vote for? In any static theory (such as Aragonès and Palfrey 2002 and 2004; Moon 2004; Soubeyran 2009; and Krasa and Polborn 2010, 2012 and 2014) a party that is exclusively office-motivated must by definition maximize its probability of winning the election, so choosing such an extreme policy that reduces its probability of winning cannot be optimal. Nevertheless, parties sometimes do choose to run an electoral campaign on a platform that is so extreme that it reduces the party's chances of winning the election (see our motivating examples in the next section). To explain these choices, we need a dynamic theory.

We explain why an exclusively office-motivated party sometimes chooses such an extreme policy by introducing dynamic considerations and endogenous valences. In Soubeyran (2009) and Krasa and Polborn (2010, 2012 and 2014), policy-specific competence advantages are exogenous, and candidates' policies diverge because each candidate proposes the policy she's good at. In contrast, we assume that parties endogenously acquire (or lose) policy-specific valences over time. Policies choices that are "extreme" in the sense that they lead to a strictly lower probability of winning in the current election are chosen because they help accumulate policy-specific valence to increase the probability of winning future elections. We call this

choice “tactical extremism.” A party that cares about future elections, may regard losing the current election with an extreme platform as an investment that helps the party build its reputation to win a future election. This intuition is key to our theory.

Endogenous non-policy party valences had been considered by Ashworth and Bueno de Mesquita (2009) and Serra (2010). Hirsch and Shotts (2015) endogenize policy-specific valence: candidates choose a policy position, and also make productive investments in the quality of their policy. While we do not explicitly model these investments, we follow Hirsch and Shotts (2015) in letting policy-specific valence be determined by parties’ actions: we assume that a higher quality (relative to the opponent’s quality) on a given policy comes through the expertise acquired by sticking with that policy repeatedly over time and specifically for more than one election cycle.<sup>6</sup>

A party that first gains a competence advantage on a given policy position owns it, in the sense of Petrocik (1996) and Egan (2013). A party that is weak on a particular policy cannot match the reputation of the strong party by merely matching its policy: if a party mimics the policy of a party that has pre-existing competence advantage, voters prefer the authentic version to the imitation. For instance, “*A Democrat’s promise to attack crime by hiring more police, building more prisons and punishing with longer sentences would too easily be trumped by greater GOP enthusiasm for such solutions.*” (Petrocik 1996, page 826).

We introduce an initial asymmetry so that one party (perhaps due to past history of play outside the model) has an exogenous competence disadvantage on the policy position preferred by a representative voter. We show that the party chooses an alternative position to develop a competence advantage on this alternative position. In Extension 5.2 below, we microfound this initial asymmetry, by introducing parties’ abilities as unobserved types: as in Dewan and Hortala-Vallve (2017), the incumbent’s success or failure in office sends a signal about her ability, and this signal generates an asymmetry on perceived competence over the implemented policy.

---

<sup>6</sup>A party cannot gain expertise suddenly, just by announcing a policy. Rather, crafting a high quality proposal requires time: to first draft a preliminary proposal; to let think-tanks and public policy centers evaluate it; to hire experts to revise and improve it; to explain it to the party so that all members embrace it; to refine how best to communicate it to the public; and to repeat the same message consistently for years so that voters find the party credible on this policy.

We obtain results on the welfare effects of tactical extremism. These results relate to previous studies on whether policy polarization and extremism are detrimental or beneficial to voters. In a multidimensional model, Nunnari and Zapal (2017) differentiate between “antagonism” (the degree of policy differentiation between parties), and “extremism” (the distance between a party’s position and the median voter). They show that antagonism is beneficial but extremism is detrimental to voters.

In a Calvert-Wittman model with policy-motivated parties and uncertainty about the location of the median voter, Bernhardt, Duggan and Squintani (2009) show that the resulting platform divergence is welfare-enhancing for voters as it insures them by giving them more choice. Our theory yields related welfare implications: tactical extremism benefits voters if they are not too confident about the mainstream platform. Voters who currently prefer the mainstream, but are unsure about their future policy preferences are better off if parties diversify their offerings so that one of them invests in developing a high quality alternative that the voter could choose in the future.

## 2. Motivating Historical Examples

We describe the political context and aftermath of three instances in which mainstream parties chose to contest a general election under an extreme leader and platform: the US Republican party under Goldwater in 1964, the UK Labour party under Foot in 1983, and the UK Labour party under Corbyn in 2015.

### 2.1. The US Republican Party under Goldwater in 1964. Tactical Extremism Succeeds

After the United States resorted to expensive government-run programs to overcome the Great Depression and to win World War II, the 1950s represented a rare period of consensus around the ideas of liberal interventionism in the US. “*The onset of the Great Depression during the presidency of Herbert Hoover (1929-1933) discredited conservative Republicanism in the popular mind*” (Grossmann and Hopkins 2016, page 80). A moderate bloc of Republicans, led by Governor Dewey of New York, “*gradually came to accept that the modern*

*economy required government intervention*” (Richardson 2014, page 204). Under President Eisenhower (1953–1960), the GOP stopped fighting against the modern welfare state.

However, this ideological concession put the Republican party at a disadvantage: as long as the political contest was about who is best qualified to lead big government programs, the Democratic party, with its Franklin D. Roosevelt’s New Deal legacy, had the upper hand over a Republican party that had only reluctantly made peace with the very idea of government interventions. With the exception of Eisenhower’s reelection in 1956, Republican electoral defeats accumulated: in midterm elections in 1954 (in which the GOP lost its Senate majority), 1958, and 1962, and in the 1960 presidential elections. By 1963, Democrats controlled the Presidency, had a veto-proof majority of 68 to 32 in the Senate, and a majority of 258 to 176 in the House. As noted by Hofstadter in 1964: “*So long as the Republican moderates are committed to keeping their party in the American mainstream, they have had little to offer but a choice that is only an echo.*”

Dissatisfied conservatives did not want to keep the GOP as a copycat party. Instead, they sought to shatter the liberal consensus and to offer a conservative alternative. If in 1950 Trilling could write that “*in the United States at this time liberalism is not only the dominant but even the sole intellectual tradition*”, in 1960, Barry Goldwater’s *Conscience of a Conservative* sought to pull the Republican party away from this mainstream and to return it to conservatism: “*We cannot win as a dime-store copy of the opposition’s platform*” [...] “*We must be different*” (Perlstein 2001, page 137). Goldwater, with a DW-Nominate score of 0.64, was the second most conservative senator in the 88th Congress (1963-64). In 1964 he became the GOP presidential candidate, and in his convention speech declared: “*extremism in the defense of liberty is no vice.*”

The electorate rejected this extremism and Goldwater lost in a landslide. However, conservative leaders had anticipated this loss, and viewed it as an investment. Back in 1963, Goldwater had spoken favorably about running, even in a losing race, in order to advance the conservative cause (Perlstein 2001, page 200). Campaigning for Goldwater, William F. Buckley argued that the campaign was destined to lose, and that its purpose was to win recruits “*for future Novembers: to infuse the conservative spirit in enough people to entitle*

*us to look about us... not at the ashes of defeat, but at the well planted seeds of hope, which will flower on a great November date in the future.”*<sup>7</sup>

These conservative seeds planted by the Goldwater 1964 campaign flowered indeed, and they bore fruit: “*What appeared to be a defeat for conservatives was actually a dramatic success: [...] Out of the ruins of the 1964 campaign emerged a well-organized, experienced movement.*”<sup>8</sup> This movement went on to win local and state races, House and Senate seats, and governorships in the late 1960s and 70s. Most notably, Ronald Reagan first gained national exposure working in the 1964 campaign, and then defeated a Democratic incumbent in 1966 to become Governor of California, and another in 1980 to become US President. “*Goldwater lost 44 states, but he won the future*” (Edwards 2014).<sup>9</sup> Support for liberalism dropped sharply after 1964-65: the mass of voters who self-identified as liberals shrunk by a quarter, never to recover in the next four decades (Ellis and Stimson 2009). Small government conservatism became part of the American political mainstream, and it has been at the heart of every GOP campaign platform since 1980.

## **2.2. The UK Labour Party under Foot in 1983. Tactical Extremism Fails**

The Labour party had won the 1974 election under Harold Wilson, a moderate member of the so called “soft-left” of Labour. An economic crisis hit, and GDP per capita growth, which had fluctuated between 2% and 6.5% for the previous decade, fell to -2.5% in 1974 and -1.5% in 1975. In March 1976, Wilson resigned, to be replaced by the new Labour leader, James Callaghan. Callaghan’s fight against the Trade Union Congress over anti-inflationary measures led to widespread strikes during the “winter of discontent” of 1978-79. A motion of no-confidence triggered a general election in 1979, which Labour lost to the Conservatives under Margaret Thatcher.

According to Labour MP Golding, the “hard-left” and its leader Tony Benn, hoped that

---

<sup>7</sup>Speech at the Annual Convention of Young Americans for Freedom, September 11th, 1964.

<sup>8</sup>Matthew Dalek. “The Conservative 1960s.” *The Atlantic Monthly*. 276.6: 130-135 (December 1995). For a more thorough analysis of the build-up and legacy of the conservative movement built around the Goldwater campaign, see Brennan (2000).

<sup>9</sup>Paraphrasing George F. Will. “The Cheerful Malcontent.” *Washington Post*. Sunday, May 31, 1998; Page C07.



Thatcher would “*upset the people with her reactionary policies, and provide the scenario whereby he [Benn] would emerge as the great left leader that the people were looking for.*” (Golding and Farrelly 2003, page 20). Following Labour’s 1979 defeat, Michael Foot was elected leader, and Labour’s 1983 general election manifesto lurched to hard-left positions. Labour MP Gerald Kaufman described this party platform as “*the longest suicide note in history.*” It led to Labour’s most decisive defeat since 1945. Yet, following this defeat, Labour’s hard-left still believed in the strategy of persuading voters to vote for an extremist platform against future policy failures by the Thatcher government. Benn wrote: “*...the 1983 Labour manifesto commanded the loyalty of millions of voters and a democratic socialist bridgehead has been established from which further advances in public understanding and support can be made.*”<sup>10</sup>

Following the 1983 defeat, Neil Kinnock replaced Foot as Labour leader. Against a background of strong economic growth, public opinion remained sufficiently supportive of the Conservative government during Thatcher’s second term; Labour’s bet on extremism had failed. After another electoral defeat in 1987, Kinnock began a sharp policy shift to the center, which led to a much more narrow defeat in 1992, and finally to victory in 1997 under Tony Blair.<sup>11</sup>

### **2.3. The UK Labour Party under Jeremy Corbyn in 2015. Tactical Extremism in Progress**

Tony Blair led the Labour party to large victories in the 1997, 2001 and 2005 elections under the slogan “New Labour,” which emphasized the party’s moderation, breaking away from its more socialist tradition. In 2007, Gordon Brown, who had served as Chancellor of the Exchequer from 1997 to 2007, replaced Blair as Prime Minister. Almost immediately, the UK entered its greatest recession in living memory: GDP per capita growth was -1.3% in 2008 and -4.9% in 2009.<sup>12</sup> Labour’s reputation for competence in managing the economy

---

<sup>10</sup>Source for both quotes: “The Spirit of Labour Reborn” by Tony Benn, in *The Guardian*, 20/06/1983. Read more about Labour’s strategy in the early 1980s in Wickham-Jones (1996).

<sup>11</sup>For an analysis of Labour’s transition to New Labour, see Heath et al. (2001, Chapter 6).

<sup>12</sup>Data: World Bank at [data.worldbank.org](http://data.worldbank.org).

suffered, the party lost the 2010 election, earning only 258 seats (68 shy of a majority), and Brown resigned.<sup>13</sup> Ed Miliband, another moderate and former minister in Blair and Brown's cabinets, emerged as new leader. Voters continued to distrust Labour's competence throughout Miliband's tenure as leader,<sup>14</sup> and after Labour contested the 2015 election on platform similar to the one in 2010, it lost again, its group reduced to 232 seats.<sup>15</sup>

Following these two defeats, and with the Conservative advantage in trust in managing the economy unabated, Labour elected its most extreme-left Member of Parliament, Jeremy Corbyn, as new party leader. Corbyn had been the most rebellious MP during Gordon Brown's 2007-2010 Labour government, as measured by dissent votes against the party line (Cowley and Stuart 2014, Table 3), but he earned the strong backing from Unite (the UK's largest trade union) and other unions. The 2nd most rebellious MP, John McDonnell, became Shadow Chancellor. Both Corbyn and McDonnell belonged to the hard-left Socialist Campaign Group at the time Corbyn was first elected party leader in 2015.<sup>16</sup>

Labour soon went on to suffer heavy losses of seats in the Scottish Parliament, the Welsh Assembly and local and council elections in 2016 and 2017.<sup>17</sup> In 2017 its electoral prospects became so poor that the UK's Conservative government called an early election, anticipating a much larger majority.<sup>18</sup> After the Conservatives ran a poor campaign, Labour beat expectations at the election but lost, earning 262 seats, 55 fewer than the Conservatives.

---

<sup>13</sup>Later, Labour politicians openly discussed how the party had lost the voters' trust on the economy during Brown's government. In 2014, Chuka Umunna (Shadow Business Secretary at the time) said that: "*the seeds were sown under the last government and Gordon [Brown]... gave the impression we didn't understand that debt and deficit would have to be dealt with.*" (Source: quoted in Nicholas Brown "Gordon Brown blamed for Labour's lost credibility on economy", *The Guardian*, 1st of September 2014).

<sup>14</sup>Source: Tom Clark. "Voters trust Cameron-Osborne most with the economy, poll finds." *The Guardian*, 6 Oct 2014.

<sup>15</sup>That the 2010 and 2015 manifestos were similar is not surprising, given that Ed Miliband had been assigned by Gordon Brown to draft the 2010 manifesto. (Source: Christopher Hope "General Election 2010: How Labour's campaign manifesto took shape." *The Telegraph*, 12th April 2010).

<sup>16</sup>A YouGov poll estimated that Corbyn was perceived by voters as "twice as left-wing as Ed Miliband." Source: <https://yougov.co.uk/news/2015/09/29/jeremy-corbyn-twice-left-wing-ed-miliband>

<sup>17</sup>It recorded a net loss of 13 out of 37 defending seats, and 30% of its vote share in the Scottish election; 1 out of 30 seats and 16% of its vote share in the Welsh one; and control of 7 out of the 74 councils and 400 out of 2878 council seats in the local elections.

<sup>18</sup>On the date the election was called (April 18th, 2017), according to Britain Elects's average poll tracker, Labour's party polling stood at 26%, the Conservatives at 42%. Betting house William Hill offered odds of 14/1 for Labour to win most seats. During the campaign, the betting markets priced the number of Labour seats between 162 and 210 (source: James Moore, "If you want to know who will win the election, follow the bookies." *The Independent*, June 6th, 2017).

This result was almost identical to Labour’s 2010 defeat, which led Brown to resign as party leader four days after the election. Yet, notwithstanding it all, Corbyn remained party leader, presumably to lead Labour into the 2022 election. Indeed, at the beginning of the campaign, the leader of Unite (Len McCluskey) had predicted a large defeat and pledged to continue supporting Corbyn after such defeat.<sup>19</sup>

While the standard Downsian rationale argues that only moderate platforms lead to victory, “...lefties have the inverse policy strategy: if you are losing, you need a more differentiated, passionate policy vision to win.” And if this passionate policy does not lead to victory today, it will tomorrow: “[Corbyn] regards himself as a soldier in a longer fight. The Bolsheviks were this. It was about being there when the end comes, capitalism unravels, and the envelope opens.”<sup>20</sup>

As of summer 2018, Corbyn is still the leader of the Labour party, the two main parties are close in the polls and Theresa May’s Conservative government faces the exceptional uncertainty surrounding Brexit, making it conceivable that the left’s tactical gamble could prove successful.

We highlight three patterns common to these three examples: First, a party with a weak reputation on mainstream policies chooses extremism after having contested (and lost) the most recent elections on a moderate platform. Second, choosing extremism leads to a landslide loss in the short term. And third, those responsible for choosing extremism have a longer horizon in mind. In the case of the GOP in 1964, extremism paid off in long-term success. In the case of Labour in 1983, it decisively did not. In the case of Labour in 2015, it is too soon to tell.

In all three of these examples, actors have policy motivations, which parties don’t have in our model. Introducing such policy concerns would only make it easier to explain extremism. We show that extremism can arise as tactic even in the hardest case in which policy

---

<sup>19</sup>Source: Jessica Elgot. “Success for Labour party would be 200 seats, says Len McCluskey” *The Guardian*. 16th May, 2017.

<sup>20</sup>The first quote is excerpted from Pete Davis, “How to Heal the Left-Liberal Divide”, *Current Affairs*, October 4th, 2017. The second quote is attributed to a Miliband adviser, excerpted from Sam Knight, “Enter Left”, *The New Yorker*, May 23rd, 2016.

considerations are absent. All three examples show that short-term and long-term electoral considerations were indeed quite prominent in the debate about the merits of extremism. In order for a party as a whole to embrace the extreme faction's policies, pivotal actors within the party must believe that such policies can, in the long run, lead to electoral success.

### 3. Formal Notation and Definitions

Let  $\{j, -j\} = \{A, D\}$  so for any party  $j \in \{A, D\}$ , we use notation  $-j$  to refer to the other party. Recall the policy space is  $\{e, m\}$ .<sup>21</sup>

Given the party platforms pair  $(x_t^A, x_t^D)$  in period  $t$ , let  $x_t \equiv (x_t^A, x_t^D)$ , and let  $x^j \equiv (x_1^j, x_2^j)$  and  $x \equiv (x_1; x_2)$ . Let the policy implemented in period  $t$  be denoted by  $x_t^w \in \{e, m\}$  (note that  $x_t^w = x_t^{W_t}$ ). For each  $j \in \{A, D\}$  and for each  $t \in \{1, 2\}$ ,  $x_t^w = x_t^j$  if the voter votes  $j$  in period  $t$ , and if the voter abstains, then  $x_t^w$  is  $x_t^A$  or  $x_t^D$  with equal probability.

We denote by  $o_t \in \{0, 1\}$  the *realized* economic outcome in period  $t$ . The probability that the economic outcome is good ( $o_t = 1$ ) depends on the state of Nature and the implemented policy as follows:

$$\Pr[o_t = 1 | x_t^w, \theta] = \begin{cases} \pi_l & \text{if } x_t^w \neq \theta, \\ \pi_h & \text{if } x_t^w = \theta. \end{cases}$$

With the remaining probability,  $o_t = 0$ . Outcome  $o_t$  is a random variable. A belief about  $\theta$  is a subjective probability  $\Pr[\theta = m]$ . For any such belief  $\hat{\mu} \in [0, 1]$ , and for any implemented policy  $x_t^w \in \{e, m\}$ , we denote by  $E[o_t | (x_t^w, \hat{\mu})]$  the expectation over  $o_t$ , given belief  $\hat{\mu}$  and policy  $x_t^w$ .

Given our assumption that party  $A$ 's platform in the first period is  $m$ , a pure strategy  $s^A : X \times [-\frac{1}{4}, \frac{1}{4}] \times \{A, D\} \times \{0, 1\} \rightarrow X$  for party  $A$  is a platform in period 2 as a function of  $x_1^D$ ,  $\varepsilon_1$ ,  $W_1$ , and  $o_1$ . For party  $D$ , a strategy  $s^D$  is a pair  $(s_1^D, s_2^D)$ , where  $s_1^D \in X$  is an unconditional choice of platform, and  $s_2^D : X \times [-\frac{1}{4}, \frac{1}{4}] \times \{A, D\} \times \{0, 1\} \rightarrow X$  is a platform in period 2 as a function of  $x_1^D$ ,  $\varepsilon_1$ ,  $W_1$  and  $o_1$ .

For the voter, a strategy  $s^v$  is a pair  $(s_1^v, s_2^v)$ , where  $s_1^v : X \times [-\frac{1}{4}, \frac{1}{4}] \rightarrow \{A, D, \emptyset\}$

---

<sup>21</sup>An alternative formulation of our model in which candidates can choose any policy over the real line, is available from the authors upon request. Since a reduced policy space  $\{e, m\}$  suffices to convey our results, we choose the simplest formulation.

is a party choice in period 1 as a function of  $x_1^D$  and the non-policy valence  $\varepsilon_1$ , and  $s_2^v : X \times \{A, D\} \times \{0, 1\} \times X^2 \times [-\frac{1}{4}, \frac{1}{4}]^2 \longrightarrow \{A, D, \emptyset\}$  is a party choice in period 2 as a function of  $x_1^D$ ,  $W_1$ ,  $o_1$ ,  $x_2$ ,  $\varepsilon_1$  and  $\varepsilon_2$ . Let  $S^A$ ,  $S^D$ , and  $S^v$  denote the strategy sets of each party and of the voter, respectively.

For the voter,  $s_t^v = \emptyset$  denotes abstention in period  $t$ . Because turnout is costless, the voter only abstains in equilibrium if she is indifferent. Given the random popularity shock  $\varepsilon_t$ , the probability that the voter is indifferent at any given election is zero, so in equilibrium, the voter votes for  $A$  or  $D$  with probability one. Hence, abstention plays no relevant role in our model.<sup>22</sup>

Party  $j$ 's optimization problem in period  $t = 1$  is:

$$\max_{s^j \in S^j} \{ \Pr[W_1 = j | (s^{-j}, s^v)] + \Pr[W_2 = j | (s^{-j}, s^v)] \},$$

and in period 2 it reduces to

$$\max_{x_2^j \in \{e, m\}} \Pr[W_2 = j | (s^{-j}, s^v)].$$

The voter's preferences over the two candidates in period  $t$ , given that the voter has belief  $\mu_t$  over  $\theta$  and that candidates propose  $x_t$ , are representable by the following net expected utility function:

$$EU^v[A | (x_t, \mu_t)] - EU^v[D | (x_t, \mu_t)] = E[o_t | (x_t^A, \mu_t)] + c_t^A + \varepsilon_t - E[o_t | (x_t^D, \mu_t)] - c_t^D, \quad (2)$$

where the first term on the right hand side is the expected economic outcome if party  $A$  is elected, the second and third represent the competence and (relative) charisma of party  $A$ , the fourth is the expected economic outcome if party  $D$  is elected and the fifth is the competence of party  $D$ . The voter prefers  $A$  to  $D$  in period  $t \in \{1, 2\}$  if Expression (2) is strictly positive, prefers  $D$  to  $A$  if it is strictly negative, and is indifferent if the expression

---

<sup>22</sup>Results would hold unchanged if we rule out abstention, and we let the voter choose either  $A$  or  $D$  arbitrarily in case of indifference.

is equal to zero.

For any implemented policy  $x_1^w \in \{m, e\}$  and any economic outcome  $o_1 \in \{0, 1\}$ , let  $\mu^*(x_1^w, o_1) \equiv \Pr[\theta = m | (x_1^w, o_1)]$  be the posterior probability that the state is  $m$ , conditional on observing  $x_1^w$  and  $o_1$ , and given an unconditional prior probability  $\Pr[\theta = m] = \mu$ . By Bayes' rule,

$$\mu^*(m, 0) = \frac{\mu(1 - \pi_h)}{\mu(1 - \pi_h) + (1 - \mu)(1 - \pi_l)} = \frac{\mu(1 - \pi_h)}{(1 - \mu)(\pi_h - \pi_l) + (1 - \pi_h)}, \quad (3)$$

$$\text{and } \mu^*(m, 1) = \frac{\mu\pi_h}{\mu(\pi_h - \pi_l) + \pi_l}, \mu^*(e, 0) = \frac{\mu(1 - \pi_l)}{1 - \pi_h + \mu(\pi_h - \pi_l)}, \mu^*(e, 1) = \frac{\mu\pi_l}{(1 - \mu)(\pi_h - \pi_l) + \pi_l}.$$

**Definition 1** *Agents' beliefs satisfy consistency if they follow Bayes' rules wherever applicable, and:*

- i) period 2 beliefs about  $\theta$  are equal to  $\mu^*(x_1^w, o_1)$ ,*
- ii) beliefs about any unobserved  $\varepsilon_t$  for each  $t \in \{1, 2\}$  are that  $\varepsilon_t$  is distributed uniformly in  $[-\frac{1}{4}, \frac{1}{4}]$ .*

This consistency requirement means that even off-path, after observing an unexpected action by another player, players stick to their correct beliefs about Nature.<sup>23</sup>

**Definition 2** *A strategy profile  $(s^A, s^D, s^v)$  and a system of consistent beliefs are an equilibrium if:*

- i) Each party  $j \in \{A, D\}$  is sequential rational, and, if indifferent at any period  $t$  between  $e$  or  $m$ , it chooses  $x_t^j = m$ ;*
- ii) In period 1, the voter votes for  $A$  if*

$$E[o_1 | (m, \mu)] + c + \varepsilon_1 > E[o_1 | (x_1^D, \mu)] \quad (4)$$

*and for  $D$  if the inequality is reversed;*

- iii) In period 2, for any pair of platforms in each period  $(x_1, x_2)$ , the voter votes for  $A$  if*

$$E[o_2 | (x_2^A, \mu^*(x_1^w, o_1))] + c_2(x_2^A | x_1^A) + \varepsilon_2 > E[o_2 | (x_2^D, \mu^*(x_1^w, o_1))] + c(x_2^D | x_1^D)$$

---

<sup>23</sup> A Sequential equilibrium satisfies this consistency requirement. A Weak Perfect Bayesian equilibrium need not.

and for  $D$  if the inequality is reversed.

## 4. Formal Results and Proofs

We begin by showing that under the assumption that  $\mu \geq \bar{\mu}$  the mainstream policy always wins in the first period election.

**Lemma 1** *If  $x_1 = (m, e)$ , then  $s_1^v = A$  so  $x_1^w = m$  and the probability that party  $A$  wins the election is 1. If  $x_1 = (m, m)$ , then  $x_1^w = m$  and the probability that party  $A$  wins the election is  $\frac{1}{2} + 2c$ .*

**Proof.** Recall that the voter is myopic which means that she will maximize only her expected utility from the current period in each period. Then if  $x_1 = (m, e)$  the voter's expected period payoff from voting  $A$  is:

$$E[o_1|(m, \mu)] + c + \varepsilon_1 = \pi_l + (\pi_h - \pi_l)\mu + c + \varepsilon_1,$$

and the expected period payoff from voting  $D$  is

$$E[o_1(e, \mu)] = \pi_l + (\pi_h - \pi_l)(1 - \mu),$$

so the net of the two is

$$\begin{aligned} & (\pi_h - \pi_l)\mu + c + \varepsilon_1 - (\pi_h - \pi_l)(1 - \mu) = 2(\pi_h - \pi_l)\mu - (\pi_h - \pi_l) + c + \varepsilon_1 \\ & \geq 2(\pi_h - \pi_l) \left( \frac{1}{2} + \frac{1 - 4c}{8(\pi_h - \pi_l)} \right) - (\pi_h - \pi_l) + c - \frac{1}{4} = 0. \end{aligned}$$

since  $\varepsilon_1 \geq -\frac{1}{4}$ . This implies that party  $A$  wins with certainty.

Alternatively, if  $x_1 = (m, m)$ , then  $E[o_1(x_1^D, \mu)] = \pi_l + (\pi_h - \pi_l)\mu$ , and party  $A$  wins if  $\varepsilon_1 > -c$ , which occurs with probability  $\frac{1}{2} + 2c$ . ■

We next show that in the second period, the platform profile  $x_2 = (e, m)$  cannot occur.

**Lemma 2**  $x_2 = (x_2^A, x_2^D) = (e, m)$  is not part of any pure strategy equilibrium.

**Proof.** Suppose that  $x_2 = (e, m)$  occurred in any pure strategy equilibrium. If the probability of victory for  $A$  in this equilibrium is not strictly greater than  $\frac{1}{2}$ ,  $A$  deviates to  $x_2^A = m$ ; if it is strictly greater, then  $D$  deviates to  $x_2^D = e$ . ■

The following Lemma shows that if the economic outcome is positive in the first period (and given Lemma 1, this is due to the mainstream policy) then in the second period both parties will choose the mainstream policy.

**Lemma 3** *Let  $(s^A, s^D, s^v)$  be an equilibrium strategy profile. If  $o_1 = 1$ , then*

$$(s_2^A(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 1), s_2^D(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 1))) = (m, m).$$

**Proof.** By Lemma 1,  $x_1^w = m$ . Given  $x_1^w = m$  and  $o_1 = 1$ ,  $\mu^*(m, 1) = \frac{\mu\pi_h}{\mu(\pi_h - \pi_l) + \pi_l} > \mu$  and thus in period 2, if  $x_2^j = m$  and  $x_2^{-j} = e$ , then the voter prefers party  $j$ , for any valence realization. Hence, both parties strictly prefer to propose  $m$ . ■

We are now ready to study the case where the economic outcome is bad at the end of the first period. We have to consider two cases: that with TE in the first period and that without. We begin with the former. For notational convenience, define the following three cutoffs.

$$\begin{aligned} \mu_1 &\equiv \frac{(\pi_h - \pi_l - c)(1 - \pi_l)}{(\pi_h - \pi_l)(2 - \pi_h - \pi_l - c)}, \\ \mu_2 &\equiv \frac{1 - \pi_l}{2 - \pi_h - \pi_l}, \\ \mu_3 &\equiv \frac{(\pi_h - \pi_l + c)(1 - \pi_l)}{(\pi_h - \pi_l)(2 - \pi_h - \pi_l + c)}. \end{aligned} \tag{5}$$

**Lemma 4** *Let  $(s^A, s^D, s^v)$  be an equilibrium strategy profile. Then  $(s_2^A(e, \varepsilon_1, A, 0), s_2^D(e, \varepsilon_1, A, 0))$  is equal to*

$$\begin{aligned} (e, e) &\quad \text{if } \mu \in [0, \mu_1), \\ (m, e) &\quad \text{if } \mu \in [\mu_1, \mu_3), \\ (m, m) &\quad \text{if } \mu \in [\mu_3, 1], \end{aligned}$$



with second period expected utility for party A

$$\begin{aligned} & \frac{1}{2} - 2c && \text{if } \mu \in [0, \mu_1), \\ \frac{1}{2} - 2(\pi_h - \pi_l) \left( 1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} \right) && \text{if } \mu \in [\mu_1, \mu_3), \\ & \frac{1}{2} + 2c && \text{if } \mu \in [\mu_3, 1]. \end{aligned}$$

**Proof.** Note that  $s_1^v(e, \varepsilon_1) = A$  implies  $x_1^w = m$ , and  $x_1^w = m$  and  $o_1 = 0$  imply that

$$\mu^*(m, 0) = \frac{\mu(1 - \pi_h)}{(1 - \mu)(\pi_h - \pi_l) + (1 - \pi_h)}, \quad (6)$$

which is greater than  $\frac{1}{2}$  iff  $\mu > \frac{1 - \pi_l}{2 - \pi_l - \pi_h}$ . Let  $EU_2^v[j|(x_1^D, x_2, \mu^*)]$  denote the expected utility for the voter from electing party  $j$ , given  $x_1^D$  and  $x_2$  and posterior  $\mu^*$  on the state of Nature.

Now since

$$\begin{aligned} EU_2^v[D|(e, (e, e), \mu^*)] &= \pi_l + (\pi_h - \pi_l)(1 - \mu^*) + c \text{ and} \\ EU_2^v[A|(e, (e, e), \mu^*)] &= \pi_l + (\pi_h - \pi_l)(1 - \mu^*) + \varepsilon_2, \end{aligned}$$

then

$$\Pr[A \text{ wins} | (e, (e, e), \mu^*)] = \Pr \left[ \varepsilon_2 \in \left( c, \frac{1}{4} \right) \right] = \frac{1}{2} - 2c \in \left[ 0, \frac{1}{2} \right].$$

Whereas,

$$\begin{aligned} EU_2^v[D|(e, (m, e), \mu^*)] &= \pi_l + (\pi_h - \pi_l)(1 - \mu^*) + c \text{ and} \\ EU_2^v[A|(e, (m, e), \mu^*)] &= \pi_l + (\pi_h - \pi_l)\mu^* + c + \varepsilon_2, \end{aligned}$$

thus

$$\Pr[A \text{ wins} | (e, (m, e), \mu^*(m, 0))] = \Pr \left[ \varepsilon_2 \in \left( (\pi_h - \pi_l)(1 - 2\mu^*(m, 0)), \frac{1}{4} \right) \right], \quad (7)$$

which is equal to

$$\begin{aligned}
& 0 \text{ if } \mu \in \left( 0, \frac{(1 - \pi_l)(4(\pi_h - \pi_l) - 1)}{(\pi_h - \pi_l)(7 - 8\pi_l - 4(\pi_h - \pi_l))} \right); \\
& \quad \frac{1}{2} - 2(\pi_h - \pi_l) \left( 1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} \right) \\
& \text{if } \mu \in \left[ \frac{(1 - \pi_l)(4(\pi_h - \pi_l) - 1)}{(\pi_h - \pi_l)(7 - 8\pi_l - 4(\pi_h - \pi_l))}, \frac{(1 - \pi_l)(1 + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(9 - 8\pi_l - 4(\pi_h - \pi_l))} \right); \text{ and} \\
& 1 \text{ if } \mu \in \left[ \frac{(1 + 4(\pi_h - \pi_l))(1 - \pi_l)}{(\pi_h - \pi_l)(9 - 8\pi_l - 4(\pi_h - \pi_l))}, 1 \right),
\end{aligned}$$

where the cutoffs are obtained by substituting (6) for  $\mu^*(m, 0)$  in (7) and solving:

$$(\pi_h - \pi_l) \left( 1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} \right) = \mp \frac{1}{4}.$$

Similarly,

$$\begin{aligned}
EU_2^v[D|(e, (m, m), \mu^*)] &= \pi_l + (\pi_h - \pi_l) \mu^* \text{ and} \\
EU_2^v[A|(e, (m, m), \mu^*)] &= \pi_l + (\pi_h - \pi_l) \mu^* + c + \varepsilon_2,
\end{aligned}$$

so

$$\Pr[A \text{ wins} | (e, (m, m), \mu^*(m, 0))] = \Pr[\varepsilon_2 \in \left( -c, \frac{1}{4} \right] = \frac{1}{2} + 2c \in \left( \frac{1}{2}, 1 \right].$$

Therefore

$$\begin{aligned}
\Pr[A \text{ wins} | (e, (e, e), \mu^*(m, 0))] &> \Pr[A \text{ wins} | (e, (m, e), \mu^*(m, 0))] \Leftrightarrow c < (\pi_h - \pi_l)(1 - 2\mu^*(m, 0)) \\
&\Leftrightarrow \mu^*(m, 0) < \frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}, \text{ and}
\end{aligned}$$

$$\begin{aligned}
\Pr[A \text{ wins} | (e, (m, e), \mu^*)] &> \Pr[A \text{ wins} | (e, (m, m), \mu^*(m, 0))] \\
&\Leftrightarrow (\pi_h - \pi_l)(1 - 2\mu^*(m, 0)) < -c \\
&\Leftrightarrow \mu^*(m, 0) > \frac{1}{2} + \frac{c}{2(\pi_h - \pi_l)}.
\end{aligned}$$

Since  $A$  has greater incentives to deviate from  $(e, e)$  than  $D$ , it follows that

$$(s_2^A(e, \varepsilon_1, A, 0), s_2^D(e, \varepsilon_1, A, 0)) = (e, e)$$

is a mutual best response for the second period given  $x_1^D = e$ ,  $s_1^v(e, \varepsilon_1) = A$  and  $o_1 = 0$  if and only if  $\mu^*(m, 0) < \frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}$ . Similarly,  $D$  has greater incentives to deviate from  $(m, m)$  than  $A$ , so  $(s_2^A(e, \varepsilon_1, A, 0), s_2^D(e, \varepsilon_1, A, 0)) = (m, m)$  is a mutual best response for the second period given  $x_1^D = e$ ,  $s_1^v(e, \varepsilon_1) = A$  and  $o_1 = 0$  if and only if  $\mu^*(m, 0) > \frac{1}{2} + \frac{c}{2(\pi_h - \pi_l)}$ . If  $\mu^* \in \left(\frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}, \frac{1}{2} + \frac{c}{2(\pi_h - \pi_l)}\right)$ , then the mutual best response is  $(s_2^A(e, \varepsilon_1, A, 0), s_2^D(e, \varepsilon_1, A, 0)) = (m, e)$ . The intervals for  $\mu$  follow by simple substitution:

$$\begin{aligned} \mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} = \frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)} &\Leftrightarrow \mu = \frac{(\pi_h - \pi_l - c)(1 - \pi_l)}{(\pi_h - \pi_l)(2 - \pi_h - \pi_l - c)} \text{ and} \\ \mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} = \frac{1}{2} + \frac{c}{2(\pi_h - \pi_l)} &\Leftrightarrow \mu = \frac{(\pi_h - \pi_l + c)(1 - \pi_l)}{(\pi_h - \pi_l)(2 - \pi_h - \pi_l + c)}. \end{aligned}$$

■

Next we consider the branch of the tree without TE in the first period.

**Lemma 5** *Let  $(s^A, s^D, s^v)$  be an equilibrium strategy profile. Then for each  $j \in \{A, D\}$ ,  $(s_2^A(m, \varepsilon_1, j, 0), s_2^D(m, \varepsilon_1, j, 0))$  is equal to*

$$\begin{aligned} (e, e) &\text{ if } \mu \in [0, \mu_1), \\ (m, e) &\text{ if } \mu \in [\mu_1, \mu_2), \\ (m, m) &\text{ if } \mu \in [\mu_2, 1], \end{aligned}$$

with second period utility for party  $A$

$$\begin{aligned} \frac{1}{2} &\text{ if } \mu \in [0, \mu_1), \\ \frac{1}{2} + 2c - 2(\pi_h - \pi_l) \left(1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)}\right) &\text{ if } \mu \in [\mu_1, \mu_2), \\ \frac{1}{2} + 2c &\text{ if } \mu \in [\mu_2, 1]. \end{aligned}$$

**Proof.** Since

$$EU_2^v[D|(m, (e, e), \mu^*)] = \pi_l + (\pi_h - \pi_l)(1 - \mu^*) \text{ and}$$

$$EU_2^v[A|(m, (e, e), \mu^*)] = \pi_l + (\pi_h - \pi_l)(1 - \mu^*) + \varepsilon_2,$$

then

$$\Pr[A \text{ wins}|(m, (e, e), \mu^*)] = \Pr \left[ \varepsilon_2 \in \left( 0, \frac{1}{4} \right] \right] = \frac{1}{2}.$$

Whereas,

$$EU_2^v[D|(m, (m, e), \mu^*)] = \pi_l + (\pi_h - \pi_l)(1 - \mu^*) \text{ and}$$

$$EU_2^v[A|(m, (m, e), \mu^*)] = \pi_l + (\pi_h - \pi_l)\mu^* + c + \varepsilon_2,$$

thus

$$\begin{aligned} \Pr[A \text{ wins}|(e, (m, e), \mu^*(m, 0))] &= \Pr [(\pi_h - \pi_l)\mu^*(m, 0) + c + \varepsilon_2 > (\pi_h - \pi_l)(1 - \mu^*(m, 0))] \\ &= \Pr \left[ \varepsilon_2 \in \left( (\pi_h - \pi_l)(1 - 2\mu^*(m, 0)) - c, \frac{1}{4} \right] \right], \end{aligned}$$

which is equal to:

$$\begin{aligned} &0 \text{ if } \mu \in \left[ 0, \frac{(1 - \pi_l)(-1 - 4c + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(7 - 8\pi_l - 4(\pi_h - \pi_l) - 4c)} \right); \\ &\quad \frac{1}{2} + 2c - 2(\pi_h - \pi_l) \left( 1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} \right) \\ &\text{if } \mu \in \left[ \frac{(1 - \pi_l)(-1 - 4c + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(7 - 8\pi_l - 4(\pi_h - \pi_l) - 4c)}, \frac{(1 - \pi_l)(1 - 4c + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(9 - 8\pi_l - 4(\pi_h - \pi_l) - 4c)} \right); \text{ and} \\ &1 \text{ if } \mu \in \left[ \frac{(1 - \pi_l)(1 - 4c + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(9 - 8\pi_l - 4(\pi_h - \pi_l) - 4c)}, 1 \right), \end{aligned}$$

where the cutoffs are obtained from

$$(\pi_h - \pi_l) \left( 1 - 2\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} \right) - c = \mp \frac{1}{4}.$$

Similarly,

$$EU_2^v[D|(m, (m, m), \mu^*)] = \pi_l + (\pi_h - \pi_l) \mu^* \text{ and}$$

$$EU_2^v[A|(m, (m, m), \mu^*)] = \pi_l + (\pi_h - \pi_l) \mu^* + c + \varepsilon_2,$$

so

$$\Pr[A \text{ wins}|(m, (m, m), \mu^*(m, 0))] = \Pr[\varepsilon_2 \in \left(-c, \frac{1}{4}\right] = \frac{1}{2} + 2c \in \left(\frac{1}{2}, 1\right].$$

Therefore

$$\begin{aligned} \Pr[A \text{ wins}|(m, (e, e), \mu^*(m, 0))] &> \Pr[A \text{ wins}|(m, (m, e), \mu^*(m, 0))] \\ &\Leftrightarrow 0 < (\pi_h - \pi_l) (1 - 2\mu^*(m, 0)) - c \\ &\Leftrightarrow \mu^*(m, 0) < \frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}, \text{ and} \\ \Pr[A \text{ wins}|(m, (m, e), \mu^*(m, 0))] &\geq \Pr[A \text{ wins}|(m, (m, m), \mu^*(m, 0))] \\ &\Leftrightarrow (\pi_h - \pi_l) (1 - 2\mu^*(m, 0)) - c \leq -c \\ &\Leftrightarrow \mu^*(m, 0) \geq \frac{1}{2}. \end{aligned}$$

Since  $A$  has greater incentives to deviate from  $(e, e)$  than  $D$ , it follows that for each  $j \in \{A, D\}$ ,  $(s_2^A(m, \varepsilon_1, j, 0), s_2^D(m, \varepsilon_1, j, 0)) = (e, e)$  is a mutual best response for the second period given  $x_1^D = m$ ,  $s_1^v(m, \varepsilon_1) = j$  and  $o_1 = 0$  if and only if  $\mu_2 < \frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}$ . Similarly,  $D$  has greater incentives to deviate from  $(m, m)$  than  $A$ , so  $(s_2^A(m, \varepsilon_1, j, 0), s_2^D(m, \varepsilon_1, j, 0)) = (m, m)$  is a mutual best response for the second period given  $x_1^D = m$ ,  $s_1^v(m, \varepsilon_1) = j$  and  $o_1 = 0$  if and only if  $\mu_2 \geq \frac{1}{2}$ . If  $\mu_2 \in \left(\frac{1}{2} - \frac{c}{2(\pi_h - \pi_l)}, \frac{1}{2}\right)$ , then the mutual best response is  $(s_2^A(m, \varepsilon_1, j, 0), s_2^D(m, \varepsilon_1, j, 0)) = (m, e)$ . The intervals for  $\mu$  follow by simple substitution as before where the new term is

$$\mu \frac{1 - \pi_h}{1 - \pi_l - \mu(\pi_h - \pi_l)} = \frac{1}{2} \Leftrightarrow \mu = \frac{1 - \pi_l}{2 - \pi_h - \pi_l}.$$

■

For completeness, we should also consider the actions in the second period, after  $x_1^D = e$  and the voter deviates to vote  $D$ . Since voters are myopic, this would never occur in equilibrium and so we omit this analysis which is available upon request. We now move to characterizing the expected probability of winning for party  $A$  in the both periods as a function of party  $D$ 's decision to pursue TE or not. We then have the following.

**Lemma 6** *The total expected utility for party  $A$  over the two periods given  $x_1^D = m$  is*

$$\begin{aligned} & \frac{1}{2} + 2c + \frac{1}{2} + 2c(\pi_l + \mu(\pi_h - \pi_l)) \text{ if } \mu \in (\bar{\mu}, \mu_1); \\ & \frac{1}{2} + 2c + \frac{1}{2} + 2c - 2(\pi_h - \pi_l)(1 - \pi_l) + 2\mu(\pi_h - \pi_l)(2 - \pi_h - \pi_l) \text{ if } \mu \in [\mu_1, \mu_2]; \quad (8) \\ & \text{and } \frac{1}{2} + 2c + \frac{1}{2} + 2c \text{ if } \mu \in [\mu_2, 1). \end{aligned}$$

while the total expected utility for party  $A$  over the two periods given  $x_1^D = e$  is

$$\begin{aligned} & 1 + \frac{1}{2} - 2c(1 - 2\pi_l - 2\mu(\pi_h - \pi_l)) && \text{if } \mu \in (\bar{\mu}, \mu_1); \\ & 1 + \frac{1}{2} + 2c(\pi_l + \mu(\pi_h - \pi_l)) - 2(\pi_h - \pi_l)(1 - \pi_l) + 2\mu(\pi_h - \pi_l)(2 - \pi_h - \pi_l) && \text{if } \mu \in [\mu_1, \mu_3); \\ & 1 + \frac{1}{2} + 2c && \text{if } \mu \in [\mu_3, 1). \end{aligned} \quad (9)$$

**Proof.** Let  $E[P_2(x_1^D)]$  denote the probability that  $A$  wins the second period election, as a function of  $x_1^D$ , evaluated before the realization of  $o_1$ . This can be calculated by noting that in the second period both parties will choose platform profile  $(m, m)$  if either  $o_1 = 1$  (which happens with probability  $\pi_l + (\pi_h - \pi_l)\mu$ ) or if  $o_1 = 0$  (which happens with probability  $1 - \pi_h + (\pi_h - \pi_l)(1 - \mu)$ ) and  $\mu$  is large enough ( $\mu \geq \mu_2$  without TE and  $\mu \geq \mu_3$  with TE). In the remaining cases, where  $o_1 = 0$  and  $\mu$  is not that large, the probability that  $A$  wins follows from substitution from Lemmas 4 and 5. Putting these together with the probabilities of winning in the first period (1 under TE and  $\frac{1}{2} + c$  if not) given by 1 provides us the result. ■

We are now ready to describe our main result which builds on the preceding Lemmas. Define  $\bar{\mu}_3$  be the value of  $\mu_3$  in Expression 5 evaluated at  $c = \frac{1}{4}$ , namely,

$$\bar{\mu}_3 \equiv \frac{(1 - \pi_l)(1 + 4(\pi_h - \pi_l))}{(\pi_h - \pi_l)(9 - 4\pi_l - 4\pi_h)}.$$

**Proposition 1** *Define the function  $\gamma$  by*

$$\gamma(\mu, \pi_h, \pi_l) = \begin{cases} \frac{1}{4(2 - \pi_l - \mu(\pi_h - \pi_l))} & \text{if } \mu \in (\bar{\mu}, \mu_2) \\ \frac{1 + 4(\pi_h - \pi_l)(2\mu + \pi_l(1 - \mu) - \mu\pi_h - 1)}{4(2 - \pi_l - \mu(\pi_h - \pi_l))} & \text{if } \mu \in (\mu_2, \bar{\mu}_3]. \end{cases}$$

*Then, if  $\mu \in (\bar{\mu}, \bar{\mu}_3]$  in equilibrium TE occurs if and only if  $c \geq \gamma$ . Otherwise, there is no TE in equilibrium.*

**Proof.** The probability that party  $D$  wins an election is the reciprocal of the probability that party  $A$  wins an election. Therefore, an equilibrium in which party  $D$  chooses  $x_1^D = e$  exists if and only if, for the given  $\mu$ , the utility value in Expression 9 is strictly lower than the value in Expression 8.

For  $\mu \in (\bar{\mu}, \mu_1)$ , the condition is

$$\begin{aligned} \frac{1}{2} + 2c + \frac{1}{2} + 2c(\pi_l + \mu(\pi_h - \pi_l)) &> 1 + \frac{1}{2} - 2c(1 - 2\pi_l - 2\mu(\pi_h - \pi_l)) \\ \Leftrightarrow c &> \frac{1}{4(2 - \pi_l - \mu(\pi_h - \pi_l))}. \end{aligned}$$

For  $\mu \in (\mu_1, \mu_2)$ , the condition is

$$\begin{aligned} \frac{1}{2} + 2c + \frac{1}{2} + 2c - 2(\pi_h - \pi_l)(1 - \pi_l) + 2\mu(\pi_h - \pi_l)(2 - \pi_h - \pi_l) &> \\ 1 + \frac{1}{2} + 2c(\pi_l + \mu(\pi_h - \pi_l)) - 2(\pi_h - \pi_l)(1 - \pi_l) + 2\mu(\pi_h - \pi_l)(2 - \pi_h - \pi_l) & \\ \Leftrightarrow c &> \frac{1}{4(2 - \pi_l - \mu(\pi_h - \pi_l))}. \end{aligned}$$

which the same condition as in the first case.

For  $\mu \in (\mu_2, \mu_3)$ , the condition is

$$\begin{aligned} & \frac{1}{2} + 2c + \frac{1}{2} + 2c \\ & > 1 + \frac{1}{2} + 2c(\pi_l + \mu(\pi_h - \pi_l)) - 2(\pi_h - \pi_l)(1 - \pi_l) + 2\mu(\pi_h - \pi_l)(2 - \pi_h - \pi_l) \\ & \Leftrightarrow c > \frac{1 + 4(\pi_h - \pi_l)(2\mu + \pi_l(1 - \mu) - \mu\pi_h - 1)}{4(2 - \pi_l - \mu(\pi_h - \pi_l))}. \end{aligned}$$

The function  $\gamma$  represents these lower bounds for different values of  $\mu$  and it is easy to show that  $\gamma$  is continuous. Finally, since

$$\frac{\partial \mu_3}{\partial c} = \frac{2}{(\pi_h - \pi_l)} (\pi_l - 1) \frac{\pi_h - 1}{(c - \pi_h - \pi_l + 2)^2} > 0,$$

the largest value of  $\mu$  for which this case applies is the case with  $\bar{\mu}_3$ . ■

We next look at comparative statics, beginning with  $\mu$ :

**Proposition 2** *For any  $\mu \in (\bar{\mu}, \bar{\mu}_3)$ ,  $\gamma$  is a strictly increasing function of  $\mu$ .*

**Proof.** For any  $\mu \in (\bar{\mu}, \mu_2)$ ,

$$\frac{\partial}{\partial \mu} \gamma = \frac{1}{4} \frac{\pi_h - \pi_l}{(2 - \pi_l - \mu(\pi_h - \pi_l))^2} > 0.$$

For any  $\mu \in (\mu_2, \bar{\mu}_3)$ ,

$$\begin{aligned} \frac{\partial}{\partial \mu} \gamma &= \frac{\partial}{\partial \mu} \left( \frac{1 + 4(\pi_h - \pi_l)(2\mu + \pi_l(1 - \mu) - \mu\pi_h - 1)}{4(2 - \pi_l - \mu(\pi_h - \pi_l))} \right) \\ &= \frac{1}{4} (\pi_h - \pi_l) \frac{-24\pi_l - 12(\pi_h - \pi_l) + 8\pi_l^2 + 8\pi_l(\pi_h - \pi_l) + 17}{(\pi_l + \mu(\pi_h - \pi_l) - 2)^2}, \end{aligned}$$

which is strictly positive if and only if

$$-24\pi_l - 12(\pi_h - \pi_l) + 8\pi_l^2 + 8\pi_l(\pi_h - \pi_l) + 17 > 0,$$

which holds. Finally,  $\gamma$  is not differentiable at  $\mu = \mu_2$  but since  $\frac{\partial}{\partial \mu} \gamma \Big|_{\mu \rightarrow (\mu_2)^+}$  and  $\frac{\partial}{\partial \mu} \gamma \Big|_{\mu \rightarrow (\mu_2)^-}$  are strictly positive, the result still holds. ■



Proposition 3 below looks at the comparative statics when we change  $\pi_l$  holding  $\pi_h - \pi_l$  constant. Define

$$\tilde{\pi}_l = \left( \frac{1}{4} \sqrt{2} \right) \frac{2(\pi_h - \pi_l) + 2\sqrt{2} \sqrt{2(\pi_h - \pi_l)^2 + 1} + 2(\pi_h - \pi_l)^2 - \sqrt{2}(\pi_h - \pi_l) \sqrt{2(\pi_h - \pi_l)^2 + 1} - 4}{\sqrt{2(\pi_h - \pi_l)^2 + 1} - \sqrt{2}} \quad \text{and}$$

$$\tilde{\mu} = \frac{1}{2} + \frac{1}{2(\pi_h - \pi_l)} - \frac{\sqrt{4(\pi_h - \pi_l)^2 + 2}}{4(\pi_h - \pi_l)}.$$

**Proposition 3** *If  $\pi_l \leq \tilde{\pi}_l$ , then  $\gamma$  is a strictly increasing function of  $\pi_l$  for  $\mu < \mu_2$  and strictly decreasing for  $\mu \in (\mu_2, \bar{\mu}_3)$ . If  $\pi_l > \tilde{\pi}_l$  then  $\tilde{\mu} > \mu_2$  and  $\gamma$  is a strictly increasing function of  $\pi_l$  for  $\mu < \tilde{\mu}$  and strictly decreasing for  $\mu > \tilde{\mu}$ .*

**Proof.** For any  $\mu \in (\bar{\mu}, \mu_2)$ ,

$$\left. \frac{\partial}{\partial \pi_l} \gamma \right|_{\pi_h - \pi_l = \text{const.}} = \frac{1}{4(2 - \pi_l - \mu(\pi_h - \pi_l))^2} > 0.$$

For any  $\mu \in (\mu_2, \bar{\mu}_3)$ ,

$$\left. \frac{\partial}{\partial \pi_l} \gamma \right|_{\pi_h - \pi_l = \text{const.}} = \frac{1 + 4(\pi_h - \pi_l)(-2\mu + 2\mu^2(\pi_h - \pi_l) + 1 - 2\mu(\pi_h - \pi_l))}{4(2 - \pi_l - \mu(\pi_h - \pi_l))^2},$$

which is strictly positive if

$$1 + 4(\pi_h - \pi_l)(-2\mu + 2\mu^2(\pi_h - \pi_l) + 1 - 2\mu(\pi_h - \pi_l)) > 0$$

$$\Leftrightarrow \mu < \tilde{\mu},$$

and strictly negative if vice-versa.

Note that for  $\pi_l \leq 1 - (\pi_h - \pi_l)$  (true by definition) and  $\pi_h - \pi_l \geq \frac{1}{4}$  (true by assumption),  $\pi_l \leq \tilde{\pi}_l$  implies  $\tilde{\mu} \leq \mu_2$ , and then  $\mu \in (\mu_2, \bar{\mu}_3)$  implies  $\mu > \tilde{\mu}$ , so if  $\pi_l \leq \tilde{\pi}_l$ ,  $\gamma$  is a strictly increasing function of  $\pi_l$  for  $\mu < \mu_2$  and strictly decreasing for  $\mu \in (\mu_2, \bar{\mu}_3)$ . Whereas, if  $\pi_l > \tilde{\pi}_l$ , then  $\tilde{\mu} > \mu_2$  and the derivative of interest is strictly positive for  $\mu < \tilde{\mu}$  and strictly negative for  $\mu > \tilde{\mu}$ . ■

Remark 2 follows as a corollary: if  $\mu$  is low (below a threshold that takes the value  $\mu_2$  or  $\tilde{\mu}$  in different cases), then  $\gamma$  is strictly increasing  $\pi_l$ , which means that if the environment deteriorates,  $\gamma$  takes a lower value, hence TE holds for a larger set of values of parameter  $c$ . Whereas, if  $\mu$  is above the relevant threshold, then  $\gamma$  is strictly decreasing in  $\pi_l$ , which means that if the environment deteriorates,  $\gamma$  takes a higher value, and hence TE holds for a lower set of values of  $c$ .

Next we consider a decrease in the relevance of luck, keeping the ratio of bad luck  $(1 - \pi_h)$  over total luck  $(\pi_l + 1 - \pi_h)$  constant. Define the ratio  $\rho \equiv \frac{1 - \pi_h}{\pi_l + 1 - \pi_h}$  constant. In order to simplify the analysis we use the notation  $\omega = 1 - \pi_h$ .

**Proposition 4** For each  $\rho \in (0, \infty)$ , define

$$\omega^* = \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho) \frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} \text{ and}$$

$$\mu_\rho = \frac{-11\rho + 4\rho^2 + 8}{-24\rho + 8\rho^2 + 17}.$$

If  $\pi_h \leq 1 - \omega^*$  and  $\mu > \mu_\rho$ , then  $\gamma$  is a strictly increasing function of  $\pi_h - \pi_l$ ; whereas, if  $\pi_h > 1 - \omega^*$  or  $\mu \leq \mu_\rho$ , then  $\gamma$  is a strictly decreasing function of  $\pi_h - \pi_l$ .

**Proof.** We have two cases.

1. For any  $\mu \in \left(\bar{\mu}, \frac{1-\pi_l}{2-\pi_h-\pi_l}\right) = (\bar{\mu}, \mu_2)$ ,

$$\gamma = \frac{1 - \rho}{4(-\mu - 2\rho + \mu\rho + \mu\omega - \rho\omega + 2)},$$

so that the derivative of this with respect to  $\omega$ , given the constraint, is equivalent to an increase in  $\pi_l + \omega$  and so a decrease in  $\pi_h - \pi_l$ . Thus,

$$\frac{\partial\gamma}{\partial\omega} > 0 \Leftrightarrow \frac{\partial\gamma}{\partial(\pi_h - \pi_l)} < 0.$$

So

$$\frac{\partial\gamma}{\partial\omega} = \frac{1}{4}(1-\rho) \frac{\rho - \mu}{(-\mu - 2\rho + \omega\mu - \omega\rho + \mu\rho + 2)^2}.$$

This implies

$$\frac{\partial \gamma}{\partial (\pi_h - \pi_l)} > 0 \Leftrightarrow \mu > \rho.$$

2.  $\mu \in \left( \mu_2, \frac{(1-\pi_l)(1+4(\pi_h-\pi_l))}{(\pi_h-\pi_l)(9-8\pi_l-4(\pi_h-\pi_l))} \right)$ . Now

$$\gamma = \frac{1}{4(1-\rho)} \frac{4\mu+6\rho+4\omega-3\rho^2-8\mu\rho+4\mu\rho^2-4\mu\omega^2-4\rho\omega^2-4\rho^2\omega+8\mu\rho\omega^2+8\mu\rho^2\omega-8\mu\rho\omega-3}{-\mu-2\rho+\mu\rho+\mu\omega-\rho\omega+2}.$$

We get

$$\frac{\partial \gamma}{\partial \omega} = \frac{1}{4} \frac{4(\rho-\mu)(\mu+\rho-2\mu\rho)\omega^2-8(1-\rho)(2-\mu)(\mu+\rho-2\mu\rho)\omega+(1-\rho)^2(-\mu+5\rho-4\mu^2-16\mu\rho+8\mu^2\rho+8)}{(1-\rho)(-\mu-2\rho+\mu\omega-\rho\omega+2)^2}.$$

The sign depend on the sign of the numerator which is a quadratic function of  $\omega$  with extremum at

$$\omega^* = \frac{(1-\rho)(2-\mu)}{\rho-\mu},$$

and two roots

$$\begin{aligned} \omega^A &= \frac{(1-\rho)(2-\mu)}{\rho-\mu} + \frac{1}{2}(1-\rho) \frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)}, \\ \omega^B &= \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho) \frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)}. \end{aligned}$$

For our parameter values both roots are real valued. This means that if  $\rho > \mu$  then we have a strictly convex quadratic function with global minimum at  $\omega^*$  which has roots at  $\omega^A > \omega^*$  and  $\omega^B < \omega^*$ . So this is negative in the interval  $(\omega^B, \omega^A)$ . If  $\rho < \mu$  then we have a strictly concave function with global maximum at  $\omega^*$  which has roots at  $\omega^A < \omega^*$  and  $\omega^B > \omega^*$ . So this is positive in the interval  $(\omega^A, \omega^B)$ . Further, since

$$\pi_l + \omega = \frac{1}{1-\rho}\omega = 1 - \pi_h + \pi_l \Rightarrow \frac{1}{1-\rho}\omega \leq 1 \Leftrightarrow \omega \leq 1 - \rho$$

it easy to see that if  $\rho > \mu$  then  $\omega^* > 1 - \rho$  while if  $\rho < \mu$  then  $\omega^* < 0$ . So:

- If  $\rho > \mu$  then we have a strictly convex function with constrained global minimum

at  $1 - \rho$ . So this is negative in the interval  $(\min(\omega^B, 1 - \rho), 1 - \rho)$ .

- If  $\rho < \mu$  then we have a strictly concave function with constrained global maximum at 0. So this is positive in the interval  $(0, \max(\omega^B, 0))$ .

From now on, let  $\omega^* = \omega^B$ . We can now compare  $\omega^*$  with 0 and  $1 - \rho$ . We can show that

$$\begin{aligned} \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho) \frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} &> 0 \\ \text{iff } \mu < \frac{1}{16\rho-8} \left( 16\rho - \sqrt{3}\sqrt{-48\rho + 32\rho^2 + 43} + 1 \right) \\ \text{or } \mu > \frac{1}{16\rho-8} \left( 16\rho + \sqrt{3}\sqrt{-48\rho + 32\rho^2 + 43} + 1 \right), \end{aligned}$$

and since both expressions are greater than one this always holds. So  $\omega^* > 0$ . Now

$$\begin{aligned} \frac{(1-\rho)(2-\mu)}{\rho-\mu} - \frac{1}{2}(1-\rho) \frac{\sqrt{(\mu+\rho-2\mu\rho)(24\mu+8\rho-17\mu^2-5\rho^2-8\mu^2\rho^2-42\mu\rho+16\mu\rho^2+24\mu^2\rho)}}{(\rho-\mu)(\mu+\rho-2\mu\rho)} < 1 - \rho \\ \text{iff } \mu > \frac{-11\rho+4\rho^2+8}{-24\rho+8\rho^2+17} = \mu_\rho. \end{aligned}$$

Noting that  $\mu_\rho > \rho$  if and only  $\rho < \frac{1}{2} < \mu$ , we can summarize the previous results as follows:

- If  $\mu \leq \mu_\rho < \rho$ , then  $\frac{\partial\gamma}{\partial\omega} > 0$  is positive.
- If  $\rho > \mu > \mu_\rho$ , then  $\frac{\partial\gamma}{\partial\omega}$  is positive in the interval  $(0, \omega^*)$  and negative otherwise.
- If  $\mu > \mu_\rho > \rho$  then  $\frac{\partial\gamma}{\partial\omega}$  is positive in the interval  $(0, \omega^*)$  and negative otherwise.

Therefore,

$$\frac{\partial\gamma}{\partial(\pi_h - \pi_l)} > 0 \Leftrightarrow \mu > \mu_\rho \text{ and } \omega \geq \omega^*.$$

■

In addition,  $\mu_\rho$  is increasing in  $\rho$ . This means that for a given  $\pi_h - \pi_l$ , the condition  $\frac{\partial\gamma}{\partial(\pi_h - \pi_l)} > 0$  holds for a larger range of values of  $\mu$  if  $1 - \pi_h$  is high relative to  $\pi_l$ .

We now turn to welfare analysis where the comparison is between the equilibrium outcome in Proposition 1 and the platform choices that a social planner would choose if this planner is trying to maximize the voter's utility over two periods. We obtain the result below which implies that in equilibrium there is too little TE from the planner's perspective when  $\mu$  is relatively low and too much when  $\mu$  is relatively high.

**Proposition 5** *There exists  $\gamma^O \in [0, \frac{1}{4}]$  such that the social planner prefers tactical extremism iff  $c > \gamma^O$ . Further, there exists  $\hat{\mu} \in (\mu_2, \mu_3)$  such that  $\gamma^O < \gamma$  if  $\mu < \hat{\mu}$ , whereas  $\gamma^O > \gamma$  if  $\mu > \hat{\mu}$ .*

**Proof.** We consider three cases where  $\bar{\mu} < \mu < \mu_1$ ,  $\mu_1 \leq \mu < \mu_2$  and  $\mu_2 \leq \mu < \mu_3$  and for each we determine conditions on  $c$  such that the planner would prefer TE and then compare with the equilibrium  $\gamma$ . Note first that in the first election, if both parties choose platform  $m$  then the voter has a choice of voting for  $A$  which gives her non-policy utility  $c + \varepsilon_1$  or for party  $D$  which gives her non-policy utility 0. In case of TE, given our parametric assumptions, then  $A$  will win for sure so that the voter gets non-policy utility  $c + \varepsilon_1$ . This means that TE lowers the voter's expected utility whenever  $c + \varepsilon_1 < 0$  or, more precisely, the expected loss from TE is

$$E[c + \varepsilon_1 | c + \varepsilon_1 < 0] \Pr[c + \varepsilon_1 < 0] = \int_{-\frac{1}{4}}^{-c} 2(c + t) dt = -c^2 + \frac{1}{2}c - \frac{1}{16}.$$

1. If  $\mu \in (\bar{\mu}, \mu_1)$ , then we have that conditional on a bad outcome, TE has an advantage since  $\max(\varepsilon_2, c) > \max(\varepsilon_2, 0)$ . In particular, this matters when  $\varepsilon_2 < c$  and so the expected gain from TE is

$$\begin{aligned} & E[c - \varepsilon_2 | \varepsilon_2 \in (0, c)] \Pr[\varepsilon_2 \in (0, c)] + c \Pr[\varepsilon_2 < 0] \\ &= \int_0^c 2(c - t) dt + c \int_{-\frac{1}{4}}^0 2 dt = \frac{1}{2}c(2c + 1). \end{aligned}$$

The voter prefers TE if

$$\begin{aligned}
& -c^2 + \frac{1}{2}c - \frac{1}{16} + (1 - \pi_h + (1 - \mu)(\pi_h - \pi_l)) \frac{1}{2}c(2c + 1) > 0 \\
\Leftrightarrow & -(\pi_l + \mu(\pi_h - \pi_l))c^2 + \frac{1}{2}(2 - \pi_l - \mu(\pi_h - \pi_l))c - \frac{1}{16} > 0 \\
& \Leftrightarrow -\Lambda c^2 + \frac{1}{2}(2 - \Lambda)c - \frac{1}{16} > 0,
\end{aligned}$$

where we have used the substitution  $\Lambda \equiv \pi_l + \mu(\pi_h - \pi_l)$ . Then the two roots are

$$\frac{2 + \sqrt{-5\Lambda + \Lambda^2 + 4}}{4\Lambda} - \frac{1}{4} \text{ and } \frac{2 - \sqrt{-5\Lambda + \Lambda^2 + 4}}{4\Lambda} - \frac{1}{4},$$

The term under square root is positive and so the two roots are well defined. Also, defining  $f \equiv \frac{\sqrt{-5\Lambda + \Lambda^2 + 4}}{4\Lambda} > 0$ , we can rewrite the two roots as

$$\frac{1}{2\Lambda} - \frac{1}{4} + f \text{ and } \frac{1}{2\Lambda} - \frac{1}{4} - f,$$

and since  $\Lambda < 1$ , the first root is greater than  $\frac{1}{4}$ , which means it is outside of the admissible range. So, the correct root is  $\gamma^O = \frac{1}{2\Lambda} - \frac{1}{4} - f$ . Comparing  $\gamma^O$  to the equilibrium boundary

$$\gamma = \frac{1}{4(2 - \pi_l - \mu(\pi_h - \pi_l))} = \frac{1}{4(2 - \Lambda)}$$

yields

$$\frac{1}{2\Lambda} - \frac{1}{4} - f < \frac{1}{4(2 - \Lambda)},$$

so for  $\mu < \mu_1$ ,  $\gamma^O < \gamma$ .

2. If  $\mu \in (\mu_1, \mu_2)$  then we have that conditional on a bad outcome, TE yields a second period benefit, since

$$\max\{\mu^*(m, 0) + c + \varepsilon_2, 1 - \mu^*(m, 0) + c\} > \max\{\mu^*(m, 0) + c + \varepsilon_2, 1 - \mu^*(m, 0)\}.$$

In particular, there is no benefit from TE if

$$\mu^*(m, 0) + c + \varepsilon_2 > 1 - \mu^*(m, 0) + c,$$

or equivalently, if  $\varepsilon_2 > 1 - 2\mu^*(m, 0)$ ; whereas if

$$1 - \mu^*(m, 0) + c > \mu^*(m, 0) + c + \varepsilon_2 > 1 - \mu^*(m, 0),$$

or equivalently, if  $\varepsilon_2 \in (1 - 2\mu^*(m, 0) - c, 1 - 2\mu^*(m, 0))$ , then the expected gain from TE (times the probability for this case) is

$$\int_{1-2\mu^*(m,0)-c}^{1-2\mu^*(m,0)} 2(1 - \mu^*(m, 0) + c - \mu^*(m, 0) - c - t) dt = c^2.$$

Finally, if

$$1 - \mu^*(m, 0) + c > 1 - \mu^*(m, 0) > \mu^*(m, 0) + c + \varepsilon_2,$$

or equivalently,  $\varepsilon_2 < 1 - 2\mu^*(m, 0) - c$ , then the expected gain from TE (times the probability for this case) is

$$\int_{-\frac{1}{4}}^{1-2\mu^*(m,0)-c} 2c dt = \frac{1}{2}c(5 - 4c - 8\mu^*(m, 0)).$$

This means that the voter prefers TE whenever

$$-c^2 + \frac{1}{2}c - \frac{1}{16} + (1 - \pi_h + (1 - \mu)(\pi_h - \pi_l)) \left( \frac{1}{2}c(5 - 4c - 8\mu^*(m, 0)) + c^2 \right) > 0. \quad (10)$$

Substituting in the value of  $\mu^*(m, 0)$  (Expression 3), Inequality 10 becomes

$$\begin{aligned}
& - (2 - \pi_l - \mu (\pi_h - \pi_l)) c^2 \\
& + \frac{1}{2} \left( 1 + (1 - \pi_h + (1 - \mu) (\pi_h - \pi_l)) \left( 5 - 8 \frac{\mu (1 - \pi_h)}{(1 - \mu) (\pi_h - \pi_l) + 1 - \pi_h} \right) \right) c - \frac{1}{16} > 0 \\
\Leftrightarrow & - (2 - \pi_l - \mu (\pi_h - \pi_l)) c^2 + \frac{1}{2} (-8\mu - 5\pi_l + 8\mu\pi_l + 3\mu (\pi_h - \pi_l) + 6) c - \frac{1}{16} > 0 \\
\Leftrightarrow & -\Lambda c^2 + \frac{1}{2} (2 - \Lambda) c - \frac{1}{16} + 2 [(\Lambda - 1) c^2 + ((2\mu - 1) \Lambda + 1 + 2\mu ((1 - \mu) (\pi_h - \pi_l) - 1)) c] > 0.
\end{aligned}$$

The term  $-\Lambda c^2 + \frac{1}{2} (2 - \Lambda) c - \frac{1}{16}$  is the same as in the previous case. Consider the term in square brackets. Its derivative with respect to  $\mu$  is negative:

$$\frac{d \left( [(\Lambda - 1) c^2 + ((2\mu - 1) (\pi_l - 1) + \mu (\pi_h - \pi_l))]_{\Lambda = \pi_l + \mu (\pi_h - \pi_l)} \right)}{d\mu} = \pi_l + \pi_h + c^2 (\pi_h - \pi_l) - 2,$$

which is strictly negative for any  $c$ . The derivative of the term outside square brackets is also negative

$$\frac{d \left( [-\Lambda c^2 + \frac{1}{2} (2 - \Lambda) c - \frac{1}{16}]_{\Lambda = \pi_l + \mu (\pi_h - \pi_l)} \right)}{d\mu} = -\frac{1}{2} c (\pi_h - \pi_l) (2c + 1) < 0$$

for any  $c$ . Define

$$F(c, \mu) \equiv -\Lambda c^2 + \frac{1}{2} (2 - \Lambda) c - \frac{1}{16} + 2 [(\Lambda - 1) c^2 + ((2\mu - 1) (\pi_l - 1) + \mu (\pi_h - \pi_l)) c].$$

It follows that the expression  $F(c, \mu)$  as a function of  $\mu$ , for any  $c$ , is minimized in our range  $(\mu_1, \mu_2)$  for  $\mu = \mu_2$ , and maximized for  $\mu = \mu_1$ . Since  $F(c, \mu)$  is quadratic and concave in  $c$  and negative for  $c = 0$ , for  $\mu = \mu_2$ ,  $F(c, \mu)|_{\mu = \mu_2}$  considered as a function only of  $c$  is the lowest value of  $F(c, \mu)$ , and so the roots of  $c$  that solve

$$F(c, \mu_2) = 0$$

are going to be the closest to each other, and in particular, the lower root of this



equation, is larger than for any other value of  $\mu$ . If the upper root is above  $c = \frac{1}{4}$  then only the lower root matters and this will therefore be the highest possible value of the root in our range - the worst case scenario. We now study these roots. So

$$F(c, \mu_2) = \frac{1}{16} (16c + 16c^2\Lambda - 8c\Lambda - 32c^2 - 1),$$

which has roots

$$\frac{1}{4} \frac{2 - \Lambda + \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda} \text{ and } \frac{1}{4} \frac{2 - \Lambda - \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda}.$$

The term in square roots is always positive, so the first root above is clearly greater than  $\frac{1}{4}$ . So the relevant root is

$$\frac{1}{4} \frac{2 - \Lambda - \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda} \tag{11}$$

and  $\frac{1}{4} \frac{2 - \Lambda - \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda} < \gamma$ . This is the worst-case scenario cut-off so, although this is not  $\gamma^O$ , still we must have  $\gamma^O \leq \frac{1}{4} \frac{2 - \Lambda - \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda} < \gamma$ .

3. If  $\mu_2 \leq \mu < \mu_3$  then we have that conditional on a bad outcome, TE does not have an obvious advantage. We have

$$\begin{aligned} 1 - \mu^* + c > \mu^* &\Leftrightarrow \mu^* < \frac{1 + c}{2} \\ \Leftrightarrow \mu < \frac{(1 - \pi_l)(1 + c)}{2 - 2\pi_l - (\pi_h - \pi_l)(1 - c)}, \end{aligned}$$

but

$$\begin{aligned}
\mu_3 - \frac{(1 - \pi_l)(1 + c)}{2 - 2\pi_l - (\pi_h - \pi_l)(1 - c)} \\
&= 2c(1 - (\pi_h - \pi_l))(1 - \pi_l) \frac{1 - \pi_h}{(\pi_h - \pi_l)(2 - \pi_h - \pi_l + c)(2 - 2\pi_l - (\pi_h - \pi_l)(1 - c))} > 0; \\
\frac{(1 - \pi_l)(1 + c)}{2 - 2\pi_l - (\pi_h - \pi_l)(1 - c)} - \mu_2 \\
&= 2c(1 - \pi_l) \frac{1 - \pi_h}{(2 - \pi_h - \pi_l)(2 - 2\pi_l - (\pi_h - \pi_l)(1 - c))} > 0,
\end{aligned}$$

so that this condition discriminates between the two cases. This means that if

$$\frac{(1 - \pi_l)(1 + c)}{2 - 2\pi_l - (\pi_h - \pi_l)(1 - c)} < \mu < \mu_3,$$

then there is either no advantage of TE or a disadvantage. We therefore, from now on, assume

$$\mu_2 < \mu < \frac{(1 - \pi_l)(1 + c)}{2 - 2\pi_l - (\pi_h - \pi_l)(1 - c)}.$$

In that case, if

$$\mu^* + c + \varepsilon_2 > 1 - \mu^* + c > \mu_2 \Leftrightarrow \varepsilon_2 > 1 - 2\mu^*,$$

then there is no advantage to TE. If

$$1 - \mu^* + c > \mu^* + c + \varepsilon_2 > \mu^* \Leftrightarrow -c < \varepsilon_2 < 1 - 2\mu^*,$$

then the expected gain from TE (times the probability for this case) is

$$\int_{-c}^{1-2\mu^*} 2(1 - \mu^* + c - \mu^* - c - t) dt = (c - 2\mu^* + 1)^2.$$

If

$$1 - \mu^* + c > \mu^* > \mu^* + c + \varepsilon_2 \Leftrightarrow -c > \varepsilon_2,$$

then the expected gain from TE (times the probability for this case) is

$$\int_{-\frac{1}{4}}^{-c} 2(1 - \mu^* + c - \mu^*) dt = \frac{1}{2}(1 - 4c)(c - 2\mu^* + 1).$$

This means that the voter prefers TE whenever

$$\begin{aligned} -c^2 + \frac{1}{2}c - \frac{1}{16} + (1 - \pi_h + (1 - \mu)(\pi_h - \pi_l)) \left( (c - 2\mu^* + 1)^2 + \frac{1}{2}(1 - 4c)(c - 2\mu^* + 1) \right) &> 0 \\ \Leftrightarrow -(2 - \pi_l - \mu(\pi_h - \pi_l))c^2 + \frac{1}{2}(2 - \pi_l - \mu(\pi_h - \pi_l))c - \frac{1}{16}\Sigma &> 0 \\ \Leftrightarrow -(2 - \Lambda)c^2 + \frac{1}{2}(2 - \Lambda)c - \frac{1}{16}\Sigma &> 0, \end{aligned} \quad (12)$$

where

$$\Sigma = \frac{80\mu + 47\pi_l - 64\mu^2 - 24\pi_l^2 - 8\mu^2\pi_h^2 - 24\mu^2\pi_l^2 - 33\mu\pi_h - 127\mu\pi_l + 48\mu^2\pi_h + 48\mu\pi_l^2 + 80\mu^2\pi_l - 32\mu^2\pi_h\pi_l + 32\mu\pi_h\pi_l - 23}{1 - \Lambda}.$$

Denote the left hand side of Inequality 12 as  $G(c, \mu)$ . Function  $G(c, \mu)$  has roots

$$\frac{1}{4} \frac{2 - \Lambda - \sqrt{(2 - \Lambda)(2 - \Sigma - \Lambda)}}{2 - \Lambda} \text{ and } \frac{1}{4} \frac{2 - \Lambda + \sqrt{(2 - \Lambda)(2 - \Sigma - \Lambda)}}{2 - \Lambda},$$

where the second root for the usual arguments does not apply. In the first root the term in square root is positive for  $\mu = \mu_2$  because

$$\left[ \frac{1}{4} \frac{2 - \Lambda - \sqrt{(2 - \Lambda)(2 - \Sigma - \Lambda)}}{2 - \Lambda} \right]_{\mu=\mu_2} = \left[ \frac{1}{4} \frac{2 - \Lambda - \sqrt{-3\Lambda + \Lambda^2 + 2}}{2 - \Lambda} \right]_{\mu=\mu_2},$$

where the term on the right hand side is the root in Expression 11 above. We know this root is well-defined and implies

$$G(c, \mu_2) = F(c, \mu_2).$$

So let

$$\gamma^O = \frac{1}{4} \frac{2 - \Lambda - \sqrt{(2 - \Lambda)(2 - \Sigma - \Lambda)}}{2 - \Lambda}.$$

Now note also that

$$G\left(c, \frac{(1-\pi_l)(1+c)}{2-2\pi_l-(\pi_h-\pi_l)(1-c)}\right) = -\frac{1}{16}(4c-1)^2 < 0.$$

All of this implies that if  $\gamma^O$  is a strictly increasing function of  $\mu$  (clearly it is continuous), there must exist a  $\hat{\mu} < \frac{(1-\pi_l)(1+c)}{2-2\pi_l-(\pi_h-\pi_l)(1-c)}$  such that

$$\gamma^O(\hat{\mu}) = \gamma(\hat{\mu}) = \frac{1+4(\pi_h-\pi_l)(\hat{\mu}(2-\pi_h-\pi_l)+\pi_l-1)}{4(2-\pi_l-\hat{\mu}(\pi_h-\pi_l))}$$

and  $\gamma^O < \gamma$  for  $\mu < \hat{\mu}$  and vice-versa for  $\mu > \hat{\mu}$  proving our result. So now we study

$$\begin{aligned} \frac{\partial \gamma^O}{\partial \mu} &= \frac{1}{8(\Lambda-1)} \frac{\sqrt{\frac{2-\Lambda}{1-\Lambda}(-8\mu-5\pi_l+8\mu\pi_l+3\mu(\pi_h-\pi_l)+5)^2}}{(-8\mu-5\pi_l+8\mu\pi_l+3\mu(\pi_h-\pi_l)+5)(\Lambda-2)^2} \\ &\quad \left( \begin{aligned} &27\pi_h+53\pi_l-21\pi_l^2+24\mu\pi_h-24\mu\pi_l-43\pi_h\pi_l-19\mu\pi_h^2 \\ &+21\mu\pi_l^2+16\pi_h\pi_l^2-16\mu\pi_h\pi_l^2+16\mu\pi_h^2\pi_l-2\mu\pi_h\pi_l-32 \end{aligned} \right) \end{aligned} \quad (13)$$

The first term that needs to be signed is

$$(-8\mu - 5\pi_l + 8\mu\pi_l + 3\mu(\pi_h - \pi_l) + 5),$$

which is decreasing in  $\mu$ . But

$$\begin{aligned} &[(-8\mu - 5\pi_l + 8\mu\pi_l + 3\mu(\pi_h - \pi_l) + 5)]_{\mu=\frac{(1-\pi_l)(1+c)}{2-2\pi_l-(\pi_h-\pi_l)(1-c)}} \\ &= 2(1-\pi_l)(1-4c) \frac{1-\pi_h}{2-2\pi_l-(\pi_h-\pi_l)(1-c)} > 0, \end{aligned}$$

and so this is positive over our interval of interest. Now to economize on notation, let

$$\pi_h - \pi_l = \chi.$$

$$\begin{aligned} &\left( \begin{aligned} &27\pi_h+53\pi_l-21\pi_l^2+24\mu\pi_h-24\mu\pi_l-43\pi_h\pi_l-19\mu\pi_h^2 \\ &+21\mu\pi_l^2+16\pi_h\pi_l^2-16\mu\pi_h\pi_l^2+16\mu\pi_h^2\pi_l-2\mu\pi_h\pi_l-32 \end{aligned} \right) = \\ &\chi(-40\pi_l - 19\pi_h - \pi_l + 16\pi_l^2 + 16\pi_l\chi + 24)\mu - (1-\pi_l)(-48\pi_l - 27\chi + 16\pi_l^2 + 16\pi_l\chi + 32). \end{aligned}$$

Define

$$\begin{aligned}\mathcal{B} &\equiv (-40\pi_l - 19\chi + 16\pi_l^2 + 16\pi_l\chi + 24) \text{ and} \\ \mathcal{C} &\equiv (-48\pi_l - 27\chi + 16\pi_l^2 + 16\pi_l\chi + 32).\end{aligned}$$

$\mathcal{B}$  is decreasing in  $\pi_l$  so setting  $\pi_l = 1 - \chi$  (as large as possible) makes it as small as possible and we get

$$[\mathcal{B}]_{\pi_l=1-\chi} = [(-40\pi_l - 19\chi + 16\pi_l^2 + 16\pi_l\chi + 24)]_{\pi_l=1-\chi} = 5\chi.$$

So  $\chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}$  is increasing in  $\mu$ . That is,

$$\chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}$$

is maximized over  $\mu$  in the interval  $(\mu_2, \mu_3)$  at  $\mu = \mu_3$  and we get

$$\begin{aligned}& [\chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}]_{\mu=\frac{(1-\pi_l)(1+c)}{2-2\pi_l-\chi(1-c)}} \\ &= 2(1 - \pi_l) (48\pi_l + 23\chi - 4c\chi - 16\pi_l^2 - 16\pi_l\chi - 32) \frac{1 - \pi_h}{2 - \pi_h - \pi_l + c(\pi_h - \pi_l)},\end{aligned}$$

where every component is positive except for

$$48\pi_l + 23\chi - 4c\chi - 16\pi_l^2 - 16\pi_l\chi - 32,$$

which is increasing in  $\pi_l$ . So setting  $\pi_l = 1 - \chi$  (as large as possible) makes it as large as possible and we get

$$[48\pi_l + 23\chi - 4c\chi - 16\pi_l^2 - 16\pi_l\chi - 32]_{\pi_l=1-\chi} = -\chi(4c + 9) < 0.$$

This proves that

$$0 > [\chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}]_{\mu=\mu=\frac{(1-\pi_l)(1+c)}{2-2\pi_l-\chi(1-c)}} > \chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}.$$

But, then going back to  $\frac{\partial\gamma^O}{\partial\mu}$  (Expression 13), all component are positive except for  $\Lambda - 1$  at the denominator which is negative and the expression  $\chi\mathcal{B}\mu - (1 - \pi_l)\mathcal{C}$  which we just studied. Hence  $\frac{\partial\gamma^O}{\partial\mu} > 0$ .

■

## 5. Extensions

We next provide three extensions to the theory:

1. Forward-looking voters, who are fully rational and sophisticated, and vote today taking into account how the current period outcome indirectly affects their expected utility in future periods.
2. Uncertainty about party-specific competence on each policy, instead of about which policy is correct.
3. An infinite horizon of elections, with impatient parties.

Qualitative results are robust across these extensions: with sufficient uncertainty, if a party faces a disadvantage over the mainstream policy and can build up a better reputation for competence on an alternative policy, then if the party is sufficiently patient, it faces incentives to choose tactical extremism.

An alternative robustness check, in which we assume that parties know the state and can signal it to the voter through their platform choices, is available from the authors upon request. We show that if there is sufficient uncertainty about the mainstream policy and competence matters enough, then an equilibrium with Tactical Extremism exists in this environment as well.

### 5.1. Forward-looking voters

In this extension, we consider a fully rational voter who maximizes her expected utility summed across both periods, under the simplifying assumption that  $\pi_h = \pi$  and  $\pi_l = 1 - \pi$  for some  $\pi \in (\frac{1}{2}, 1)$ .

In the second period, the expected utility for the voter from voting to party  $A$ , and the expected utility of voting for party  $D$  are, respectively,

$$E[o_2|(x_2^A, \mu^*(x_1^w, o_1))] + c_2^A(x_2^A) + \varepsilon_2, \text{ and}$$

$$E[o_2|(x_2^D, \mu^*(x_1^w, o_1))] + c_2^A(x_2^D|x_1^D).$$

The voter optimizes by choosing the party that maximizes her expected utility.

For any party strategies  $s^A \in S^A$  and  $s^D \in S^D$ , and for any voter strategy  $s_1^v \in \{A, D\}$ , let  $EU_2(s^A, s^D, s_1^v)$  be the expected utility of the voter in the second period -evaluated before the voter observes  $o_1$ ), given that parties played the strategies  $s^A$  and  $s^D$  and the voter played  $s_1^v$  in the first period, and given that the voter will choose optimally in the second period. The voter's expected utility over the whole game, subject to voting to party  $A$  in the first period, and subject to voting to party  $D$  is, respectively,

$$E[o_1|(m, \mu)] + c + \varepsilon_1 + EU_2^v(s^A, s^D, A), \text{ and}$$

$$E[o_1|(s_1^D, \mu)] + EU_2^v(s^A, s^D, D).$$

In the first period the voter optimizes by choosing the party that maximizes this aggregate expected utility.

#### Results.

Note that

$$\mu^*(m, 0) = \mu^*(e, 1) = \frac{\mu(1 - \pi)}{\mu(1 - \pi) + (1 - \mu)\pi}, \text{ and}$$

$$\mu^*(m, 1) = \mu^*(e, 0) = \frac{\mu\pi}{\mu\pi + (1 - \mu)(1 - \pi)}.$$

If  $x_1^w = m$ , the second period will be solved exactly as in the case with myopic voters. Therefore, Lemma 2 applies. A modified Lemma 3 applies as well.

**Lemma 3b.** *For any equilibrium strategy profile  $(s^A, s^D, s^v)$  such that  $x_1^w = m$ ,*

$$(s_2^A(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 1), s_2^D(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 1)) = (m, m),$$

and for any equilibrium strategy profile  $(s^A, s^D, s^v)$  such that  $x_1^w = e$ ,

$$(s_2^A(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 0), s_2^D(x_1^D, \varepsilon_1, s_1^v(x_1^D, \varepsilon_1), 0)) = (m, m).$$

The intuition for the proof is the same: a good outcome given  $x_1^w = m$ , or a bad outcome given  $x_1^w = e$  both induce a posterior  $\mu^* > \mu$ , so that the voter will only vote for mainstream policies in the second period.

Similarly, a modified Lemma 4 applies.

**Lemma 4b** *Let  $(s^A, s^D, s^v)$  be an equilibrium strategy profile. Then  $(s_2^A(e, \varepsilon_1, A, 0), s_2^D(e, \varepsilon_1, A, 0))$  and  $(s_2^A(e, \varepsilon_1, D, 1), s_2^D(e, \varepsilon_1, D, 1))$  are equal to*

$$\begin{aligned} (e, e) & \text{ if } \mu \in [0, \mu_1), \\ (m, e) & \text{ if } \mu \in [\mu_1, \mu_3), \\ (m, m) & \text{ if } \mu \in [\mu_3, 1], \end{aligned}$$

with second period expected utility for party A

$$\begin{aligned} \frac{1}{2} - 2c & \text{ if } \mu \in [0, \mu_1), \\ \frac{1}{2} - 2(2\pi - 1) \left( \frac{\pi - \mu}{\pi - \mu(2\pi - 1)} \right) & \text{ if } \mu \in [\mu_1, \mu_3), \\ \frac{1}{2} + 2c & \text{ if } \mu \in [\mu_3, 1]. \end{aligned}$$

The proof is the proof of Lemma 4, now applied as well to the case in which the voter updates negatively on the mainstream policy after voting  $D$  with  $x_1^D = e$  and obtaining a good economic outcome in the first period. Lemma 5 holds as stated, with its proof.

We conclude with the key insight, regarding voting behavior in period 1, subject to



Tactical Extremism. By Lemma 3b and Lemma 4b, the equilibrium actions in the second period do not depend on  $s_1^v$ . Hence, the expected voter's second period payoff is unaffected by the voter's first period play and a fully rational, forward-looking voter optimizes over the two periods by optimizing her vote over each period myopically.

Therefore, under the assumption that  $\pi_h = 1 - \pi_l$ , Propositions 1, 2 and 3 and Remark 1 are robust, whether voters are fully rational (forward looking, sophisticated), or myopic.<sup>24</sup>

## 5.2. Uncertainty about party-specific policy competence

Consider an extension in which the economic outcome in a given period reveals information about the incumbent's competence at implementing the chosen policy, as in Butt (2006) or Dewan and Hortala-Vallve (2017), but it does not reveal any information about the opposition.

That is, assume that the uncertainty is not just about the policy, but rather, it is about the policy-party pair. Specifically, suppose that the state of the world  $\theta \in \{0, 1\}^4$  has four components

$$\theta \equiv (\theta_m^A, \theta_e^A, \theta_m^D, \theta_e^D),$$

where for each party  $j \in \{A, B\}$  and each policy  $p \in \{m, e\}$ ,  $\theta_p^j \in \{0, 1\}$  denotes the intrinsic ability of party  $j$  on policy  $p$ . We interpret  $\theta_p^j = 0$  to mean that party  $j$  has no ability on  $p$ , and  $\theta_p^j = 1$  to mean that party  $j$  has high intrinsic ability on  $p$ .

To micro-found the asymmetry in beliefs about the ability of the two parties, we now consider a model with four periods  $\{0, 1, 2, 3\}$ . periods 0 and 1 occurred before the strategic environment we analyze, and they constitute the history that leads to the asymmetry, starting from a symmetric environment at period 0.

In this model, we dispense with the additive parameter  $c > 0$ , and we instead let the competence of party  $j \in \{A, D\}$  on a given policy  $p \in \{m, e\}$  affect the probability that the economic outcome is good if party  $j$  implements policy  $p$ .

For each period  $t \in \{0, 1, 2, 3\}$ , for each party  $j \in \{A, D\}$ , and for each policy  $p \in (m, e)$ , let  $c_t^j(p) \in \{0, 1\}$  denote the competence of party  $j$  on policy  $p$  in period  $t$ . To capture that

---

<sup>24</sup>Proposition 3 and Remark 2 involve a comparative static that violates the assumption that  $\pi_h = 1 - \pi_l$ .

acquiring competence requires both preparation and ability, we assume that the first time that a party chooses a policy, it is not competent on this policy, but if it chooses the same policy again a second consecutive time, and the party has intrinsic ability on this policy, then this second time the party is competent on the policy in question.

Formally, for any period  $t \in \{0, 1, 2, 3\}$ , for any party  $j \in \{A, D\}$ , and for any policy  $p \in \{m, e\}$ ,  $c_t^j(p) = \theta_t^j$  if  $x_t^j = x_{t-1}^j$  and  $c_t^j(p) = 0$  if  $x_t^j \neq x_{t-1}^j$ .

In this model, the advantage of competence on a given policy is that it makes a good economic outcome more likely, so we do not need to incorporate an additive parameter  $c$  (equivalently, we can assume  $c = 0$ ). Instead, we assume that a policy executed with competence delivers a good economic outcome with probability  $\pi > \frac{1}{2}$ , whereas executed incompetently, it delivers a good economic outcome with probability  $1 - \pi$ . Formally,

$$\Pr[o_t = 1 | W_t = j] = \begin{cases} \pi & \text{if } c_t^j(x_t^j) = 1 \\ 1 - \pi & \text{if } c_t^j(x_t^j) = 0. \end{cases}$$

Suppose there is a common independent prior at period 0 that  $\Pr[\theta_e^D = 1] = \Pr[\theta_e^A = 1] = \frac{1}{2}$  and  $\Pr[\theta_m^D = 1] = \Pr[\theta_m^A = 1] = \mu \in (\frac{1}{2}, \pi)$ , so that both parties are more likely to be good at the mainstream policy, than at the extreme policy, which is another way of saying that ex-ante, economic outcomes are more likely to be good choosing the mainstream policy than the extreme one.<sup>25</sup> For each party  $j \in \{A, D\}$ , each policy  $p \in \{m, e\}$  and each period  $t \in \{1, 2, 3\}$ , let  $\mu_t^j(p)$  denote the posterior on  $\Pr[\theta_p^j = 1]$ .

We consider the strategic scenario at the beginning of period 2, given that both parties had proposed the mainstream policy  $m$  in periods 0 and 1. In period 0, neither party had experience, so they were both incompetent, and the probability of a good outcome is  $1 - \pi$ , regardless of ability, so nothing can be learned about the state of the world from the economic outcome. In period 1, on the other hand, both parties had previous experience on policy  $m$ , so their competence is equal to their ability.

Without loss of generality, suppose that  $D$  is the party that won the election in period

---

<sup>25</sup>The assumption that the priors are independent is a simplification, so that if Party  $j$  delivers a bad economic outcome implementing policy  $p$ , we do not learn anything about Party  $j$ 's ability to implement the other policy, nor about the other party's ability to implement  $p$ .

1, and implemented policy  $m$ . Conditioning on  $\theta_m^D = 1$ ,  $c_1^D(m) = 1$  and the probability of a good economic outcome is  $\pi$ ; while conditioning on  $\theta_m^D = 0$ ,  $c_1^D(m) = 0$  and the probability of a good economic outcome is  $1 - \pi$ . Suppose the realized economic outcome was bad,  $o_1 = 0$ . The economic outcome  $o_1$  is now a signal about  $\theta_m^D$ , and agents can make an inference about  $\Pr[\theta_m^D = 1]$ . Define

$$\mu^F = \frac{\mu(1 - \pi)}{\mu(1 - \pi) + (1 - \mu)\pi} < \frac{1}{2}.$$

Note that  $\mu^F$  is the posterior about the party-specific ability on the mainstream policy, after one economic failure realized at a time the party had expertise on the policy.

This history of play in periods 0 and 1 constitutes the starting point of our model of the strategic environment in period 2 and period 3, where parties face the asymmetry that the posterior on party  $D$  is  $\mu_2^D(m) = \mu^F < \frac{1}{2}$ , while  $\mu_2^A(m) = \mu > \frac{1}{2}$ . Intuitively, economic failures have dented the reputation of the previously incumbent party, which now faces a disadvantage. This fits our intuition about the status of Labour in 2010-2015.

We now model the agents' decisions in periods 2 and 3.

For party  $A$  in period 2, the policy that is more likely to deliver a good outcome if  $A$  wins the election is  $m$ . It is thus intuitive that party  $A$  proposes policy  $m$ . We prove below that this is indeed the equilibrium choice, but for the time being merely assume  $x_2^A = m$ . If party  $A$  is elected, the outcome is good with probability  $\pi$  if  $\theta_m^A = 1$  and with probability  $1 - \pi$  if  $\theta_m^A = 0$ . Since  $\mu_2^A(m) = \mu$ , the expected economic outcome in period 2 voting for party  $A$  is

$$\mu\pi + (1 - \mu)(1 - \pi). \tag{14}$$

If party  $D$  proposes policy  $m$ , the expected economic outcome voting for party  $D$  is only  $\mu^F\pi + (1 - \mu^F)(1 - \pi)$ , because the beliefs about party  $D$ 's ability and competence on policy  $m$ , and hence its likelihood of delivering a good economic outcome out of policy  $m$  in period 2 have been damaged by party  $D$ 's failure to deliver a good economic outcome out of policy  $m$  in period 1.

If party  $D$  proposes policy  $e$ , the expected economic outcome in period 2 voting for party

$D$  is only  $(1 - \pi)$ , because this is a new policy in which the party has no expertise, and hence no competence.

Thus, subtracting  $(1 - \pi)$  from Expression 14,

$$\mu\pi + (1 - \mu)(1 - \pi) - (1 - \pi) = \mu(2\pi - 1),$$

and if  $\mu(2\pi - 1)$  is more than the maximum value of  $\varepsilon_t$ , then by choosing  $x_2^D = e$ , party  $D$  foregoes any chance of winning the period 2 election.

At the beginning of period 2, the difference in posteriors  $\mu_2^A(m) - \mu_2^D(m)$  is  $\mu - \mu^F$ . So the difference in expected payoff from electing  $A$  versus  $D$  if both propose  $m$  is

$$(\mu - \mu^F)(2\pi - 1) = \left( \frac{\mu(\mu(1 - \pi) + (1 - \mu)\pi) - \mu(1 - \pi)}{\mu(1 - \pi) + (1 - \mu)\pi} \right) (2\pi - 1) = \frac{\mu(1 - \mu)(2\pi - 1)^2}{\mu(1 - \pi) + (1 - \mu)\pi}. \quad (15)$$

If this value is less than the maximum value of  $\varepsilon_t$ , then by choosing  $x_2^D = m$ , party  $D$  has a chance to win the period 2 election. So let us keep the assumption that the maximum value of  $\varepsilon_t$  is  $\frac{1}{4}$ , and then assume

$$\frac{\mu(1 - \mu)(2\pi - 1)^2}{\mu(1 - \pi) + (1 - \mu)\pi} \leq \frac{1}{4} \leq \mu(2\pi - 1). \quad (16)$$

For the first inequality to hold, we need

$$\mu(1 - \pi) + (1 - \mu)\pi - 4\mu(1 - \mu)(2\pi - 1)^2 \geq 0.$$

Solving for  $\pi$ , we need

$$\pi \in \left( \mu, \frac{1}{32\mu - 32\mu^2} \left( 14\mu + \sqrt{28\mu - 28\mu^2 + 1} - 16\mu^2 + 1 \right) \right).$$

Equivalently, solving for  $\mu$ , we need  $\mu < \pi$  and

$$\mu \notin \left( -\frac{1}{16\pi - 8} \left( -8\pi + \sqrt{-64\pi + 64\pi^2 + 9} + 3 \right), \frac{1}{16\pi - 8} \left( 8\pi + \sqrt{-64\pi + 64\pi^2 + 9} - 3 \right) \right).$$

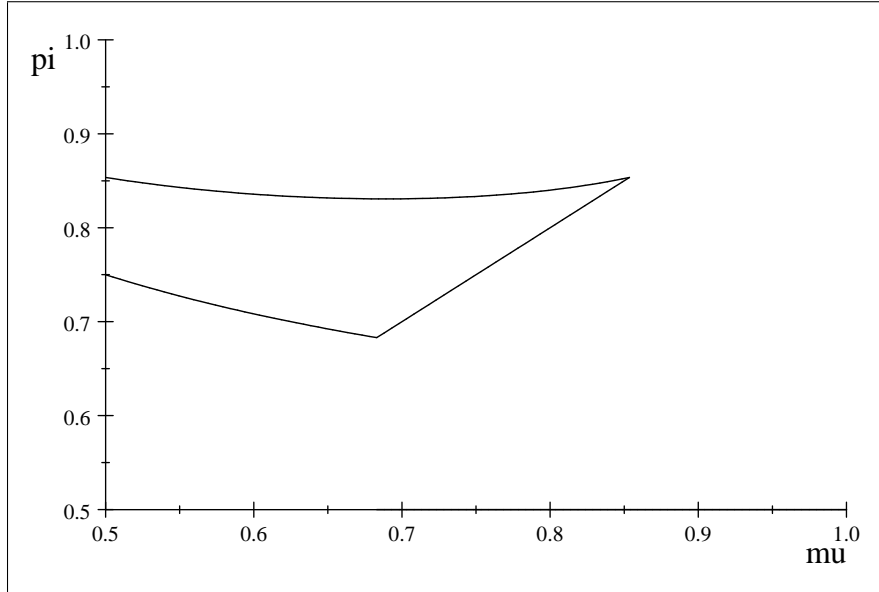
For the second inequality in Expression 16, we need

$$\pi \geq \frac{1}{2} + \frac{1}{8\mu}.$$

Summarizing, both inequalities in Expression 16 together with our initial assumptions are satisfied if and only if  $(\mu, \pi)$  is in the set:

$$\left\{ (x, y) \in \left( \frac{1}{2}, 1 \right)^2 : y \in \left( \max \left\{ x, \frac{1}{2} + \frac{1}{8x} \right\}, \frac{1}{32x - 32x^2} \left( 14x + \sqrt{28x - 28x^2 + 1} - 16x^2 + 1 \right) \right) \right\}, \quad (17)$$

graphically depicted in the following figure.



The vertices of this area are  $\left\{ \left( \frac{1+\sqrt{3}}{4}, \frac{1+\sqrt{3}}{4} \right), \left( \frac{1}{2}, \frac{3}{4} \right), \left( \frac{1}{2}, \frac{1}{2} + \frac{\sqrt{2}}{4} \right), \left( \frac{1}{2} + \frac{\sqrt{2}}{4}, \frac{1}{2} + \frac{\sqrt{2}}{4} \right) \right\}$ .

**Period 3 election.** Period 3 is the last in our model, so each party maximizes its chance of winning the current election. If party  $A$  has played  $x_2^A = m$  in period 2 (as assumed), then regardless of the electoral and economic outcome in period 2, now in period 3, if  $x_3^A = m$ ,

the expected outcome if  $A$  wins is  $\mu_3^A(m)\pi + (1 - \mu_3^A(m))(1 - \pi)$  whereas if  $x_3^A = e$ , the economic outcome is good with probability only  $1 - \pi$ . Since, for any  $\mu_3^A(m) > 0$ ,

$$\mu_3^A(m)\pi + (1 - \mu_3^A(m))(1 - \pi) > 1 - \pi,$$

the best response for party  $A$  is to play  $x_3^A = x_2^A = m$ .

By a similar logic, party  $D$  also chooses  $x_3^D = x_2^D$ . Specifically:

1. If  $x_2^D = m$ , then exactly as in the case of party  $A$ , if  $D$  switches to  $x_3^D = e$  it has no expertise, and the expected economic outcome if it wins is only  $1 - \pi$ , whereas sticking to  $x_3^D = x_2^D = m$ , the expected economic outcome if it wins is  $\mu_3^D(m)\pi + (1 - \mu_3^D(m))(1 - \pi) > 1 - \pi$ . So if  $x_2^D = m$ , then  $x_3^D = m$ .
2. If  $x_2^D = e$ , then the expected economic outcome if  $D$  wins given  $x_3^D = e$ , now that  $D$  has accumulated expertise in policy  $e$ , is  $\mu_3^D(e)\pi + (1 - \mu_3^D(e))(1 - \pi) = \frac{1}{2}$ , whereas if it returns to  $m$  it is  $\mu_3^D(m)\pi + (1 - \mu_3^D(m))(1 - \pi) = \mu^F\pi + (1 - \mu^F)(1 - \pi) < \frac{1}{2}$ . Hence if  $x_2^D = e$ , then  $x_3^D = e$ .

So, if in periods 2 and 3, party  $D$  chooses  $(x_2^D, x_3^D) = (e, e)$ , then subject to party  $D$  winning the period 3 election, the expected economic outcome is  $\frac{1}{2}$ . On the other hand, if party  $D$  chooses  $(x_2^D, x_3^D) = (m, m)$  and wins the period 3 election, then the expected economic outcome depends on  $\mu_3^D(m)$ , which itself depends on the realization of the election and the economy in period 2. In particular,

1. If  $x_2^D = m$  and  $W_2 = A$ , then  $\mu_3^D(m) = \mu_2^D(m) = \mu^F$ , because period 2 did not reveal any further information about  $\theta_m^D$ .
2. If  $x_2^D = m$ ,  $W_2 = D$ , and the economic outcome is bad, the posterior is  $\mu_3^D(m) = \mu^{FF}$ , where the posterior following two failures  $\mu^{FF}$  is defined by

$$\mu^{FF} \equiv \frac{\mu^F(1 - \pi)}{\mu^F(1 - \pi) + (1 - \mu^F)\pi} = \frac{\mu(1 - \pi)^2}{\mu(1 - \pi)^2 + (1 - \mu)\pi^2}$$

3. If  $x_2^D = m$ ,  $W_2 = D$ , and the economic outcome is good, the posterior is back to  $\mu_3^D(m) = \mu$ .

**Period 2 election.**

If  $x_2^D = m$ , from Expression 15, the probability that  $D$  wins in period 2 is

$$\Pr \left[ \varepsilon_t < -\frac{(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi) + (1-\mu)\pi} \right] = \frac{1}{2} - \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi) + (1-\mu)\pi}, \quad (18)$$

which is positive by assumption.

Subject to  $x_2^D = m$  and  $W_2 = D$ , the probability that the outcome is good ( $o_2 = 1$ ) is  $\mu^F\pi + (1-\mu^F)(1-\pi)$ , or equivalently, in primitives,

$$\frac{\mu(1-\pi)}{\mu(1-\pi) + (1-\mu)\pi}\pi + \left(1 - \frac{\mu(1-\pi)}{\mu(1-\pi) + (1-\mu)\pi}\right)(1-\pi) = \frac{\pi(1-\pi)}{\mu(1-\pi) + (1-\mu)\pi}, \quad (19)$$

and that the outcome is bad is

$$1 - \frac{\pi(1-\pi)}{\mu(1-\pi) + (1-\mu)\pi} = \frac{\pi^2 - \mu(2\pi-1)}{\mu(1-\pi) + (1-\mu)\pi}. \quad (20)$$

So at the beginning of period 2, if party  $D$  chooses  $x_2^D = m$ , from Expressions 18, 19 and 20, party  $D$  can anticipate that

$$\begin{aligned} \Pr[\mu_3^D(m) = \mu^{FF}] &= \left( \frac{1}{2} - \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi) + (1-\mu)\pi} \right) \left( \frac{\pi^2 - \mu(2\pi-1)}{\mu(1-\pi) + (1-\mu)\pi} \right), \\ \Pr[\mu_3^D(m) = \mu^F] &= \frac{1}{2} + \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi) + (1-\mu)\pi}, \text{ and} \\ \Pr[\mu_3^D(m) = \mu] &= \left( \frac{1}{2} - \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi) + (1-\mu)\pi} \right) \frac{\pi(1-\pi)}{\mu(1-\pi) + (1-\mu)\pi}. \end{aligned}$$

Subject to  $x_2^D = m$  and  $W_2 = A$ ,  $\Pr[\mu_3^A(m) = \mu^F] = \mu(1-\pi) + (1-\mu)\pi$  and  $\Pr[\mu_3^A(m) = \mu^G] = \mu\pi + (1-\mu)(1-\pi)$ , where  $\mu^G$  is the posterior that follows one good outcome, defined by

$$\mu^G \equiv \frac{\mu\pi}{\mu\pi + (1-\mu)(1-\pi)} > \mu.$$

**Back to period 3 election.**

Note

$$\begin{aligned}\mu^G - \mu^F &= \mu \left( \frac{\pi}{\mu\pi + (1-\mu)(1-\pi)} - \frac{1-\pi}{\mu(1-\pi) + (1-\mu)\pi} \right) \\ &= \mu \frac{2\pi - 2\mu\pi - 1 + \mu}{(\mu - 2\mu\pi + \pi)(2\mu\pi + 1 - \mu - \pi)}\end{aligned}$$

So subject to  $x_2^D = x_2^A = x_3^D = x_3^A = m$ , the posteriors  $\mu_3^D(m)$  and  $\mu_3^A(m)$ , and their net difference are

$\mu_3^D(m)$	$\mu_3^A(m)$	$\mu_3^A(m) - \mu_3^D(m)$	with probability
$\mu^{FF}$	$\mu$	$\frac{\mu(1-\mu)(2\pi-1)}{\mu-2\pi\mu+\pi^2}$	$2 \left( \frac{1}{4} - \frac{(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi)+(1-\mu)\pi} \right) \left( \frac{\pi^2 - \mu(2\pi-1)}{\mu(1-\pi)+(1-\mu)\pi} \right) \equiv P_1$
$\mu$	$\mu$	$0$	$2 \left( \frac{1}{4} - \frac{(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi)+(1-\mu)\pi} \right) \frac{\pi(1-\pi)}{\mu(1-\pi)+(1-\mu)\pi} \equiv P_2$
$\mu^F$	$\mu^F$	$0$	$\left( \frac{1}{2} + \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi)+(1-\mu)\pi} \right) (\mu(1-\pi) + (1-\mu)\pi) \equiv P_3$
$\mu^F$	$\mu^G$	$\frac{\mu(1-\mu)(2\pi-1)}{(\mu-2\mu\pi+\pi)(2\mu\pi+1-\mu-\pi)}$	$\left( \frac{1}{2} + \frac{2(1-\mu)\mu(2\pi-1)^2}{\mu(1-\pi)+(1-\mu)\pi} \right) (\mu\pi + (1-\mu)(1-\pi)) \equiv P_4$

where the first row corresponds to the event in which  $W_2 = D$  and  $o_2 = 0$ , the second row to  $W_2 = D$  and  $o_2 = 1$ , the third to  $W_2 = A$  and  $o_2 = 0$  and the fourth to  $W_2 = A$  and  $o_2 = 1$ .

Now, the difference in expected outcomes in voting for  $A$  over  $D$  is

$$(\mu_3^A(m) - \mu_3^D(m)) (2\pi - 1).$$

So the probability that party  $D$  wins in period 3, subject to  $x_2^D = m$ , is as follows

$$\begin{aligned}P_1 \Pr \left[ \varepsilon_t < -\frac{\mu(1-\mu)(2\pi-1)^2}{\mu-2\pi\mu+\pi^2} \right] &+ (P_2 + P_3) \frac{1}{2} \\ + P_4 \Pr \left[ \varepsilon_t < -\frac{\mu(1-\mu)(2\pi-1)}{(\mu-2\mu\pi+\pi)(2\mu\pi+1-\mu-\pi)} \right].\end{aligned}\tag{21}$$

If instead  $x_2^D = x_3^D = e$ , then  $\mu_3^D(e) = \frac{1}{2}$ . If  $x_2^D = e$ , party  $A$  wins the period 2 election with certainty (by assumption on the parameter range), and the posterior about party  $A$  is  $\mu_3^A(m) = \mu^F$  with probability  $\mu(1-\pi) + (1-\mu)\pi$  and  $\mu^G > \mu$ , with probability  $\mu\pi + (1-\mu)(1-\pi)$ .



So, with probability  $\mu(1 - \pi) + (1 - \mu)\pi$ ,  $\mu_3^A(m) - \mu_3^D(e)$  is equal to

$$\mu^F - \frac{1}{2} = \frac{1}{2} - \frac{\mu(1 - \pi)}{\mu(1 - \pi) + (1 - \mu)\pi} = \frac{1}{2} \frac{\mu - \pi}{\mu(1 - \pi) + (1 - \mu)\pi} < 0$$

which translates into a difference in expected economic outcome of

$$\frac{1}{2} \frac{(\mu - \pi)(2\pi - 1)}{\mu(1 - \pi) + (1 - \mu)\pi} < 0.$$

Whereas, with probability  $\mu\pi + (1 - \mu)(1 - \pi)$ ,  $\mu_3^A(m) - \mu_3^D(e)$  is equal to

$$\mu^G - \frac{1}{2} = \frac{1}{2} \frac{\pi + \mu - 1}{\mu\pi + (1 - \mu)(1 - \pi)}$$

which translates into a difference in expected economic outcome equal to

$$\frac{1}{2} \frac{(\pi + \mu - 1)(2\pi - 1)}{\mu\pi + (1 - \mu)(1 - \pi)},$$

So, subject to  $x_2^D = x_3^D = e$  and  $x_2^A = x_3^A = m$ , the probability that  $D$  wins in period 3, is

$$(\mu\pi + (1 - \mu)(1 - \pi)) \Pr \left[ \varepsilon_t < -\frac{(\mu + \pi - 1)(2\pi - 1)}{2(2\mu\pi + 1 - \mu - \pi)} \right] + (\mu(1 - \pi) + (1 - \mu)\pi) \Pr \left[ \varepsilon_t < \frac{(\pi - \mu)(2\pi - 1)}{2\pi - 2\mu(2\pi - 1)} \right]. \quad (22)$$

So the period 3 gain for  $D$  from Tactical Extremism is Expression 22 minus Expression 21. This is equal to

$$(2\pi - 1) \frac{\pi - \mu}{\pi + \mu - 2\pi\mu}. \quad (23)$$

### Parameters for which TE holds.

Comparing the gain  $(2\pi - 1) \frac{\pi - \mu}{\pi + \mu - 2\pi\mu}$  in period 3 (Expression 23), with the loss in probability of election in period 2,  $\frac{1}{2} - \frac{2(1 - \mu)\mu(2\pi - 1)^2}{\mu(1 - \pi) + (1 - \mu)\pi}$  (Expression 18), and always under the parameter range given by Expression 12 and Expression 11, we obtain that

$$(2\pi - 1) \frac{\pi - \mu}{\pi + \mu - 2\pi\mu} - \left( \frac{1}{2} - \frac{2(1 - \mu)\mu(2\pi - 1)^2}{\mu(1 - \pi) + (1 - \mu)\pi} \right) \geq 0$$

if and only if

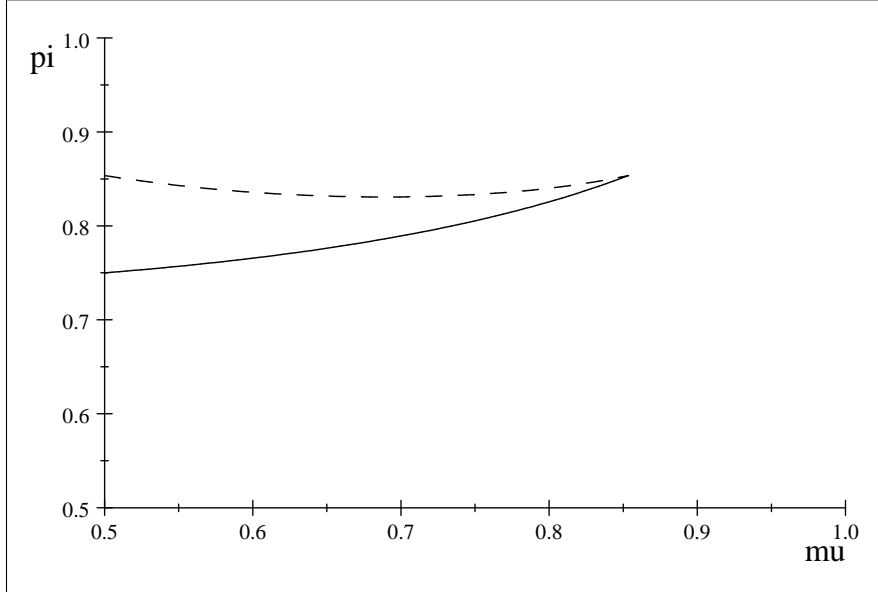
$$\pi \geq \frac{1}{32\mu - 32\mu^2 + 8} \left( 18\mu + \sqrt{28\mu - 28\mu^2 + 9} - 16\mu^2 + 3 \right). \quad (24)$$

From the characterization of the set of parameters in Expression 17, and from Inequality 24, the following result holds.

**Claim 3** *The set of parameters  $(\mu, \pi)$  for which in equilibrium we observe Tactical Extremism is*

$$TE \equiv \left\{ (\mu, \pi) \in \left( \frac{1}{2}, 1 \right)^2 : \pi \in \left( \frac{1}{32\mu - 32\mu^2 + 8} \left( 18\mu + \sqrt{28\mu - 28\mu^2 + 9} - 16\mu^2 + 3 \right), \frac{1}{32\mu - 32\mu^2} \left( 14\mu + \sqrt{28\mu - 28\mu^2 + 1} - 16\mu^2 + 1 \right) \right) \right\}.$$

The set  $TE \subset \left( \frac{1}{2}, 1 \right)^2$  is depicted in the following graph.



Its vertices are  $\left\{ \left( \frac{1}{2}, \frac{3}{4} \right), (1, 1), \left( \frac{1}{2}, \frac{1}{2} + \frac{\sqrt{2}}{4}, \frac{1}{2} + \frac{\sqrt{2}}{4} \right), \left( \frac{1}{2} + \frac{\sqrt{2}}{4}, \frac{1}{2} + \frac{\sqrt{2}}{4} \right) \right\}$ .

Recall that the upper constraint on  $\pi$ ,  $\pi \leq \frac{1}{32\mu - 32\mu^2} \left( 14\mu + \sqrt{28\mu - 28\mu^2 + 1} - 16\mu^2 + 1 \right)$ , is the restriction that guarantees that party  $D$  has some probability of winning the period 2 election by choosing  $x_2^D = m$ . The parameter range of substantive interest is such that this constraint is satisfied; if it is not satisfied (for  $(\mu, \pi)$  above the line), in equilibrium we

also observe TE, but due to more trivial reasons: in such region, since party  $D$  has zero probability of winning the period 2 election anyway, TE has no cost, and it follows that party  $D$  chooses extremism to best prepare for period 3.

Therefore, within the parameter range of interest, the qualitative result we have obtained across all models holds here as well: TE occurs in equilibrium if and only if there is sufficient uncertainty about the moderate policy ( $\mu$  is sufficiently low, given  $\pi$ ), and if competence matters enough ( $\pi$  sufficiently high, given  $\mu$ ).

### 5.3. An Infinite Horizon

We present an infinite horizon extension, with an election at every period  $t \in \mathbb{N}$ . We find that if parties are sufficiently patient, the main result of our theory is robust in this extension: TE obtains if competence is sufficiently important, and beliefs that the mainstream policy is correct are relatively low.

**Setup.** Consider a model of electoral competition with an infinite horizon, two purely office-motivated parties and one strategic representative voter. In each period  $t \in \{1, 2, \dots\}$ , parties  $A$  and  $D$  compete in an election. The policy space is  $X = \{e, m\}$ . Parties seek to maximize the discounted sum of the probabilities of being elected over the whole horizon.

In each period  $t$ , before the election, each party  $j \in \{A, D\}$  simultaneously announces a platform  $x_t^j \in X$ , which is the policy that the party will implement in period  $t$  if it wins office. Let  $x_t \equiv (x_t^A, x_t^D)$  and  $\mathbf{x}_t^j = (x_1^j, x_2^j, \dots, x_t^j)$  and let  $\mathbf{x}_t = (x_1, x_2, \dots, x_t)$ . The voter observes  $x_t$  and votes for either  $A$  or  $D$  or abstains. The winning party  $W_t \in \{A, D\}$  implements  $x_t^w$ , which is equal to its announced policy  $x_t^{W_t}$ . The set of states of Nature is  $\Theta = \{e, m\}$ , and  $\theta_t \in \Theta$  is the state of Nature in period  $t$ . For each  $t \in \mathbb{N}$ , at the beginning of period  $t$  the state  $\theta_t$  is unknown. However, all agents know that the state is more likely to be  $m$ , and has some inertia, specifically, they know that

$$\eta_t \equiv \Pr[\theta_t = m | \theta_{t-1}] = \begin{cases} \eta_H & \text{if } \theta_{t-1} = m \\ \eta_L & \text{if } \theta_{t-1} = e \end{cases} \quad (25)$$

with  $\eta_H > \eta_L > \frac{1}{2}$ .<sup>26</sup> We refer to  $m$  as the *mainstream* platform because all agents agree that in any period,  $m$  is the policy most likely to be correct. We assume  $\eta_1 \in \{\eta_L, \eta_H\}$ .<sup>27</sup>

Let  $o_t \in \{0, 1\}$  denote the economic outcome in period  $t$ . The probability of a good outcome is 1 if the implemented policy  $x_t$  matches the state  $\theta_t$ , and it is 0 otherwise.

We model *policy-specific* valence (or *competence*) by assuming that whether the government implements its chosen policy competently affects the utility of the voter. Competence is policy specific and it is a function of current and previous platforms, because acquiring competence on a given policy requires time to build the necessary expertise. As in our two-period model, we introduce an ex-ante asymmetry by assuming that party  $A$  enjoys a competency advantage in period 1 on the mainstream platform, implicitly due to policy choices made before the game starts in period 1. In all subsequent periods, we assume that party  $j$  has competence on any given platform if it has proposed this platform for the last two periods, and for at least as many consecutive periods as the other party.

For any policy pair  $(y, z) \in \{e, m\}^2$ , let  $c_t^j((y, z))$  denote the competence of party  $j$  in period  $t$ , conditional on platform pair  $(x_t^A, x_t^D) = (y, z)$ . Note that the value of  $c_t^j((y, z))$  also depends on the past platforms  $\mathbf{x}_{t-1}$ , as follows. Introduce the notation  $x_0^A = x_0^D = x_{-1}^A = m$  and  $x_{-1}^D = \emptyset$ . Then, with this notation, for each period  $t \in \mathbb{N}$ , for each policy pair  $(y, z) \in \{e, m\}^2$  and for each party  $j \in \{A, D\}$ ,  $c_t^j((y, z)) = c$  if  $x_{t-1}^j = x_t^j$  and there does not exist  $\tau \in \{-1, 0, 1, 2, \dots, t\}$  such that  $x_\tau^j \neq x_t^j$  and  $x_k^{-j} = x_t^j$  for any  $k \in \{\tau, \tau + 1, \dots, t\}$ . Otherwise,  $c_t^j((y, z)) = 0$ .

Notice that under this formulation of the policy-specific valence, a party  $j$  that has gained an advantage on any given policy  $x \in \{e, m\}$  in period  $t$ , relinquishes such advantage by flip-flopping to  $x_{t+1}^j = x' \neq x$ . Even if party  $j$  returns to  $x_{t+2}^j = x$ , the flip-flop would have caused  $j$  to lose its advantage, and party  $j$  would have no policy-specific valence on  $x$  in period  $t + 2$ . To regain a policy-specific valence advantage on  $x$ , party  $j$  would need to

---

<sup>26</sup>Assuming that  $\eta_L > \frac{1}{2}$  makes it harder for TE to obtain as it implies that policy  $m$  is always more likely to generate a good outcome than policy  $e$ .

<sup>27</sup>We can consider instead a model with a constant state  $\theta$ , in which voters hold the following (non-Bayesian) beliefs: they believe that  $\Pr[\theta = m] = \eta_H$  after any period in which the mainstream policy delivers a good outcome (or the extreme policy delivers a bad one) and  $\Pr[\theta = m] = \eta_L$  after the mainstream policy delivers a bad outcome (or the extreme policy delivers a good one). The two models yield the exact same results.

again choose  $x_{t+3} = x$  and to persevere on choosing policy  $x$  for at least as many consecutive periods as party  $-j$ .

For each  $t \in \mathbb{N}$ , let  $k_t \in \{A, D, \emptyset\}$  denote the party that enjoys a net competence advantage in period  $t$ , given  $x_t$ , (and given  $\mathbf{x}_{t-1}$ ). That is,  $k_t = j$  if  $c_t^j(x_t) - c_t^{-j}(x_t) = c$ .

We also model *non-policy* valence (or *charisma*) by assuming that  $\varepsilon_t$  represents the voter's idiosyncratic preference for party  $A$  in period  $t$ . This shock captures non-policy attributes that may affect the voter's preferences. For each period  $t \in \{1, 2, \dots\}$ ,  $\varepsilon_t$  is drawn independently from a uniform distribution over  $[-\frac{1}{4}, \frac{1}{4}]$ . Its draw is the voter's private information.

**Timing.** At the beginning of period  $t \in \{1, 2, \dots\}$ ,  $\theta_t$  and  $\varepsilon_t$  are unknown to all players. Parties choose platforms  $x_t^A \in \{e, m\}$  and  $x_t^D \in \{e, m\}$  simultaneously. Then all players observe  $x_t = (x_t^A, x_t^D)$  and  $\varepsilon_t$  and after this observation, the voter chooses a vote in  $\{A, D, \emptyset\}$ . If the voter chooses a party  $j \in \{A, B\}$ , then this party wins, while if the voter abstains ( $\emptyset$ ), the winning party is randomly chosen with equal probability. The winning party  $j$  implements its policy so that  $x_t^w = x_t^j$ . The economic outcome  $o_t$  is realized and observed by all players. The rules of the game, and parameters  $(\eta_H, \eta_L, c)$  are common knowledge.

At the beginning of each period  $t$  there is a state of the game

$$\lambda_t \equiv (x_{t-1}, k_{t-1}, \theta_{t-1}),$$

which describes parties' platform choices and net competence advantage, and the state of Nature, at the end of the previous period. Note that  $x_{t-1}$  and  $k_{t-1}$  are directly observed by all agents; and at the end of period  $t-1$ ,  $\theta_{t-1}$  can be inferred by Bayesian updating from the implemented policy  $x_{t-1}^w$  and the economic outcome  $o_{t-1}$ : namely,  $o_{t-1} = 1 \implies \theta_{t-1} = x_{t-1}^w$  and  $o_{t-1} = 0 \implies \theta_{t-1} = \{e, m\} \setminus \{x_{t-1}^w\}$ . Hence, at the beginning of period  $t$ , all agents share the same degenerate (and correct) belief about the state (of the game)  $\lambda_t$ .

Let  $\Lambda \equiv \{e, m\}^2 \times \{A, D, \emptyset\} \times \{e, m\}$  denote the set of states of the game, and let  $\lambda \in \Lambda$  denote an arbitrary state of the game. The state  $\lambda_t \in \Lambda$  determines updated beliefs about  $\theta_t$  (from  $\theta_{t-1}$ ), and it also determines the competencies  $(c_t^A(x_t), c_t^D(x_t))$ , as a function of  $x_{t-1}$

and  $k_{t-1}$ , as indicated in the following Table 26:

$(x_{t-1}^A, x_{t-1}^D, k_{t-1})$	$(x_t^A, x_t^D)$			
	$(e, e)$	$(e, m)$	$(m, e)$	$(m, m)$
$(e, e, A)$	$(c, 0)$	$(c, 0)$	$(0, c)$	$(0, 0)$
$(e, e, \emptyset)$	$(c, c)$	$(c, 0)$	$(0, c)$	$(0, 0)$
$(e, e, D)$	$(0, c)$	$(c, 0)$	$(0, c)$	$(0, 0)$
$(e, m, \cdot)$	$(c, 0)$	$(c, c)$	$(0, 0)$	$(0, c)$
$(m, e, \cdot)$	$(0, c)$	$(0, 0)$	$(c, c)$	$(c, 0)$
$(m, m, A)$	$(0, 0)$	$(0, c)$	$(c, 0)$	$(c, 0)$
$(m, m, \emptyset)$	$(0, 0)$	$(0, c)$	$(c, 0)$	$(c, c)$
$(m, m, D)$	$(0, 0)$	$(0, c)$	$(c, 0)$	$(0, c)$

(26)

One feature emphasized by Table 26 is that if  $x_{t-1}^A \neq x_{t-1}^D$  then  $k_{t-1}$  is not needed to determine  $(c_t^A(x_t), c_t^D(x_t))$ .

**Utilities.** Parties are purely office motivated. They maximize the discounted sum of the probabilities of being elected over the whole horizon, with  $\delta \in (0, 1)$  the discount factor. The voter optimizes period by period, myopically. In each period  $t$ , and for each party  $j$ , the voter calculates the expected utility that it would attain if she elects party  $j$ . This expected utility is computed as the sum of three terms: the expected economic performance under party  $j$  (given the voter's beliefs), the policy-specific valence of party  $j$ , and the non-policy valence of party  $j$ . The voter then optimizes for the period by voting for the party with the highest expected utility.

**Solution concept.** We assume that parties are strategic and sequentially rational while in each period the voter chooses party  $A$  if the net expected utility function for that period, conditional on her beliefs, is non-negative and party  $D$  otherwise. Beliefs about the state follow the rules described above. We further focus on stationary equilibria, in the sense that we only consider equilibria where for each player, strategies in period  $t$  are solely a function of the state and not of the whole history. This means that if in two periods  $t$  and  $t'$  we have  $\lambda_t = \lambda_{t'}$  then all players play the same strategies in both periods, even if the histories

between the two periods differ.

For each party  $j \in \{A, D\}$ , let  $x^j : \Lambda \rightarrow \{e, m\}$  denote a stationary strategy.

We will say that there is *Tactical Extremism (TE)* in an equilibrium, if there is at least one state  $\lambda \in \Lambda$  along the equilibrium path, in which party  $j$  chooses the platform that is less likely to be correct, given beliefs at such state.

We begin the analysis by considering the voter's decision in any period  $t$  as a function of the platforms  $(x_t^A, x_t^D)$  chosen by the parties, given  $c_t$ , the realization of  $\varepsilon_t$ , and the voter's belief about  $\Pr[\theta_t = m]$ .

For each party  $j \in \{A, D\}$ , for each  $x_t^j \in \{e, m\}$  and for each state  $\lambda_t$ , let  $E_{\theta_t} [o_t | (x_t^j, \lambda_t)]$  denote the expectation over the economic outcome  $o_t$  given that  $x_t^w = x_t^j$  and given state  $\lambda_t$ , where the source of uncertainty is the state  $\theta_t$ . The voter votes for  $D$  in period  $t$  if

$$E_{\theta_t} [o_t | (x_t^A, \lambda_t)] + c_t^A(x_t) + \varepsilon_t < E_{\theta_t} [o_t | (x_t^D, \lambda_t)] + c_t^D(x_t)$$

Given that the voter votes for  $D$  in period  $t$  if

$$\varepsilon_t < E_{\theta_t} [o_t | (x_t^D, \lambda_t)] - E_{\theta_t} [o_t | (x_t^A, \lambda_t)] + c_t^D(x_t) - c_t^A(x_t), \quad (27)$$

then the probability of winning for party  $D$  as a function of platform  $x_t$  and the state  $\lambda_t$  is

$$\begin{aligned} & \frac{E_{\theta_t} [o_t | (x_t^D, \lambda_t)] - E_{\theta_t} [o_t | (x_t^A, \lambda_t)] + c_t^D(x_t) - c_t^A(x_t) + \frac{1}{4}}{\frac{1}{2}} \\ &= \frac{1}{2} + 2(c_t^D(x_t) - c_t^A(x_t)) + 2E_{\theta_t} [o_t | (x_t^D, \lambda_t)] - E_{\theta_t} [o_t | (x_t^A, \lambda_t)]. \end{aligned}$$

Let  $(x^{A*}, x^{D*})$  denote the parties' strategies in an stationary equilibrium. The infinitely repeated 2-player game played by the two parties, taken the voter strategy as given, is a constant-sum stochastic game and so that the strategy pursued by a given party will be a maxmin strategy.

We obtain the first preliminary result: party  $A$  always plays  $m$ .

**Lemma 7** *In any stationary equilibrium of the game, party A plays  $x_t^A = m$  along the equilibrium path in every period.*

**Proof.** In any period  $t$ , the probability of winning for party A is

$$\frac{1}{2} + 2(c_t^A(x_t) - c_t^D(x_t)) + 2E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - 2E_{\theta_t}[o_t | (x_t^D, \lambda_t)],$$

subject to bounds at zero and one.

Since  $\frac{1}{2}$  is a constant, without loss of generality we consider the game without it, and normalize payoffs so that the period payoff for party A is

$$c_t^A(x_t) - c_t^D(x_t) + E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - E_{\theta_t}[o_t | (x_t^D, \lambda_t)]. \quad (28)$$

Consider the lowest possible payoff that party A may obtain, if it plays  $x_t^A = m$  for every period. Playing  $x_t^A = m$  for every period implies  $c_t^A(x_t) = c$  and  $c_t^D((m, m)) = 0$  for every  $t \in \mathbb{N}$ . So, for any period such that  $x_t^D = m$ ,  $c_t^A(x_t) - c_t^D(x_t) = c$  and  $E_{\theta_t}[o_t | (x_t^A, \lambda_t)] = E_{\theta_t}[o_t | (x_t^D, \lambda_t)]$ , so Payoff 28 is strictly positive. Further, for any period such that  $x_t^D = e$ ,  $c_t^A(x_t) - c_t^D(x_t) \in \{0, c\}$  and since for any  $t \geq \tau$ ,

$$E_{\theta_t}[o_t | (m, \lambda_t)] - E_{\theta_t}[o_t | (e, \lambda_t)] = 2(2\eta_t - 1) > 0,$$

it follows that  $E_{\theta_t}[o_t | (x_t^A, \lambda_t)] > E_{\theta_t}[o_t | (x_t^D, \lambda_t)]$ . Hence, Payoff 28 is strictly positive as well.

Therefore, strategy  $x^A(\lambda) = m$  yields A a strictly positive payoff in every period.

Consider a putative equilibrium where at some time  $\tau \geq 1$ , for the first time, party A chooses platform  $e$ .<sup>28</sup>

Case 1: Assume  $x_\tau^D = x_{\tau-1}^D = e$ .

Suppose party D plays  $x_t^D = m$  for any  $t > \tau$ . Then  $c_\tau^A(x_\tau) - c_\tau^D(x_\tau) = c_\tau^A((e, m)) -$

---

<sup>28</sup>Recall that we use the convention that  $x_0^A = x_0^D = x_{-1}^A = m$  and  $x_{-1}^D = \emptyset$ , so that  $k_0 = A$ .



$c_\tau^D((e, m)) = -c$  and  $c_t^A(x_t) - c_t^D(x_t) \in \{-c, 0\}$  for any  $t > \tau$ . Further, since for any  $t \geq \tau$ ,

$$E_{\theta_t}[o_t | (e, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] = 2(1 - 2\eta_t) < 0,$$

it follows

$$E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] \leq 0,$$

so the Payoff 28 is zero in period  $\tau$ , and weakly negative for any period after  $\tau$ , for any strategy played by  $A$ . Since the game is constant sum, the equilibrium payoff for  $A$  must be at most the payoff obtained if party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then deviating to  $x^A(\lambda) = m$  is profitable.

Case 2: Assume  $x_{\tau-1}^D = m, x_\tau^D = e$ .

Suppose party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then  $c_\tau^A(x_\tau) - c_\tau^D(x_\tau) = c_\tau^A((e, e)) - c_\tau^D((e, e)) = 0$  and  $c_t^A(x_t) - c_t^D(x_t) \in \{-c, 0\}$  for any  $t > \tau + 1$ . Further, since for any  $t \geq \tau$ ,

$$E_{\theta_t}[o_t | (e, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] = 2(1 - 2\eta_t) < 0,$$

it follows

$$E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] \leq 0,$$

so the Payoff 28 is zero in period  $\tau$ ,  $c - 2(2\eta_t - 1)$  in period  $\tau + 1$ , and weakly negative for any period after  $\tau$ , for any strategy played by  $A$ . Since the game is constant sum, the equilibrium payoff for  $A$  must be at most the payoff obtained if party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ .

Deviating to  $x^A(\lambda) = m$ , in period  $\tau$  party  $A$  obtains  $c + 2(2\eta_t - 1)$  and a strictly positive payoff in every period after  $\tau$ . Since  $c + 2(2\eta_t - 1) > \delta(c - 2(2\eta_t - 1))$ , deviating to  $x^A(\lambda) = m$  is profitable.

Case 3: Assume  $x_{\tau-1}^D = e, x_\tau^D = m$ .

Suppose party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then  $c_\tau^A(x_\tau) - c_\tau^D(x_\tau) = 0$  and  $c_t^A(x_t) -$

$c_t^D(x_t) \in \{-c, 0\}$  for any  $t > \tau$ . Further, since for any  $t \geq \tau$ ,

$$E_{\theta_t}[o_t | (e, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] = 2(1 - 2\eta_t) < 0,$$

it follows

$$E_{\theta_\tau}[o_\tau | (e, \lambda_\tau)] - E_{\theta_\tau}[o_\tau | (m, \lambda_\tau)] = 2(1 - 2\eta_\tau) < 0,$$

and

$$E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] \leq 0,$$

so the Payoff 28 is strictly negative in period  $\tau$ , and weakly negative for any period after  $\tau$ , for any strategy played by  $A$ . Since the game is constant sum, the equilibrium payoff for  $A$  must be at most the payoff obtained if party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then deviating to  $x^A(\lambda) = m$  is profitable.

Case 4: Assume  $x_{\tau-1}^D = x_\tau^D = m$ .

Suppose party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then  $c_\tau^A((e, m)) - c_\tau^D((e, m)) = -c$  and  $c_t^A(x_t) - c_t^D(x_t) \in \{-c, 0\}$  for any  $t > \tau$ . Further, since for any  $t \geq \tau$ ,

$$E_{\theta_t}[o_t | (e, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] = 2(1 - 2\eta_t) < 0,$$

it follows

$$E_{\theta_\tau}[o_\tau | (e, \lambda_\tau)] - E_{\theta_\tau}[o_\tau | (m, \lambda_\tau)] = 2(1 - 2\eta_\tau) < 0,$$

and

$$E_{\theta_t}[o_t | (x_t^A, \lambda_t)] - E_{\theta_t}[o_t | (m, \lambda_t)] \leq 0 \text{ for any } t > \tau,$$

so the Payoff 28 is strictly negative in period  $\tau$ , and weakly negative for any period after  $\tau$ , for any strategy played by  $A$ . Since the game is constant sum, the equilibrium payoff for  $A$  must be at most the payoff obtained if party  $D$  plays  $x_t^D = m$  for any  $t > \tau$ . Then deviating to  $x^A(\lambda) = m$  is profitable. ■

For any period  $t \in \mathbb{N}$ , for any sequence of policy platforms and economic outcomes  $((x_s, o_s))_{s=t}^\infty$ , and for any period  $s \in \{t, t+1, \dots\}$ , define  $p_s(((x_s, o_s))_{s=t}^\infty)$  as the probability

that party  $j$  wins in period  $s$ , given  $((x_s, o_s))_{s=t}^\infty$ , and given the voting behavior specified by (27). Then let

$$V_t^j(((x_s, o_s))_{s=t}^\infty) \equiv \sum_{s=t}^{\infty} \delta^{s-t} p_s$$

denote the present value evaluated at period  $t$ , of the infinite stream of expected period utilities for party  $j$  in that sequence from time  $t$  onwards. Then

$$V_t^{-j}(((x_s, o_s))_{s=t}^\infty) = \sum_{s=t}^{\infty} \delta^{s-t} (1 - p_s) = \frac{1}{1 + \delta} - V_t^j(((x_s, o_s))_{s=t}^\infty),$$

and for any  $t$ ,  $V_t^D(((x_s, o_s))_{s=t}^\infty)$  is equal to

$$\sum_{s=t}^{\infty} \delta^{s-t} \left( 2(c_s^D(x_s) - c_s^A(x_s)) + 2E_{\theta_s} [o_s | (x_s^D, \lambda_s)] - 2E_{\theta_s} [o_s | (x_s^A, \lambda_s)] + \frac{1}{2} \right),$$

and  $V_t^A(((x_s, o_s))_{s=t}^\infty)$  is equal to

$$\frac{1}{1 + \delta} - \sum_{s=t}^{\infty} \delta^{s-t} \left( 2(c_s^D(x_s) - c_s^A(x_s)) + 2E_{\theta_s} [o_s | (x_s^D, \lambda_s)] - 2E_{\theta_s} [o_s | (x_s^A, \lambda_s)] + \frac{1}{2} \right).$$

For any pair of stationary strategies  $x^A$  and  $x^D$ , for any  $t \in \mathbb{N}$ , for any policy  $x_t^D \in \{e, m\}$ , and for any state  $\lambda_t \in \Lambda$ , slightly abusing notation, let  $V(x^A, (x_t^D; x^D), \lambda_t)$  denote the expected value of  $V_t^D(((x_s, o_s))_{s=t}^\infty)$  given that party  $A$  plays strategy  $x^A$ , party  $D$  plays  $x_t^D$  in period  $t$  and strategy  $x^D$  thereafter, the voter votes according to (27), and the state is  $\lambda_t$ , and let  $V(x^A, x^D, \lambda_t) \equiv V^D(x^A, (x^D(\lambda_t), x^D), \lambda_t)$  denote the special case in which  $D$  also plays  $x^D$  in period  $t$ . Note that the expectation is over the realization of  $o_t$  in each period, and the period subscript on the  $V$  disappears because  $x^A$  and  $x^D$  are stationary strategies (we drop the party superscript because we do not use the analogous notation for party  $A$ ).

The next proposition defines conditions for an equilibrium where TE exists, in the sense that party  $D$  chooses  $x^{D*}(\lambda) = e$  for any  $\lambda_t$  with  $\theta_{t-1} = \theta_L$ , even though  $E_{\theta_t} [o_t | (m, \lambda_t)] > E_{\theta_t} [o_t | (e, \lambda_t)]$ .

**Proposition 6** *Assume*

$$\eta_L \leq \frac{1 + \delta c}{2 + c} \text{ and } \eta_H \geq \frac{1 + (1 + \delta)c}{2 + \delta c}$$

then there exists a (stationary) equilibrium where  $A$  always chooses policy  $m$  along the equilibrium path, whereas  $D$  choose platform  $e$  in any period  $t$  such that  $\theta_{t-1} = \theta_L$  and platform  $m$  in any period  $t$  such that  $\theta_{t-1} = \theta_H$ .

**Proof.** We know, by Lemma 7, that the equilibrium strategy  $x^{A*}$  is such that  $x_t^A = m$  in every period. Let  $\Lambda' \subset \Lambda$  be the set of states that can arise if  $x_t^A = m$  in every period. In particular, we have the following possible states:

$\Lambda' = \{(m, A, m), (m, A, e), (e, A, m), (e, A, e), (e, \emptyset, m), (e, \emptyset, e)\}$ , where each state is defined simply as  $(x_{t-1}^D, k_{t-1}, \theta_{t-1})$ . Notice further that states  $(e, A, m)$  and  $(e, \emptyset, m)$ , and states  $(e, A, e)$  and  $(e, \emptyset, e)$  are payoff equivalent, so we can restrict the set of states further to  $\Lambda'' \equiv \{(m, A, m), (m, A, e), (e, \cdot, m), (e, \cdot, e)\}$ .

We consider each of the four states in turn. We want to sustain an equilibrium in which  $x_t^D(\lambda) = m$  for  $\lambda \in \{(m, A, m), (e, \cdot, m)\}$  and  $x_t^D(\lambda) = e$  for  $\lambda \in \{(m, A, e), (e, \cdot, e)\}$ .

Case 1. State  $\lambda_t = (m, A, m)$ . Choosing  $x_t^D = x_t^{D*}(\lambda_t) = m$ , party  $D$  obtains utility  $V(x^{A*}, x^{D*}, (m, A, m))$  equal to

$$\frac{1}{2} - 2c + \delta[\eta_H V(x^{A*}, x^{D*}, (m, A, m)) + (1 - \eta_H) V(x^{A*}, x^{D*}, (m, A, e))]. \quad (29)$$

Deviating to  $x_t^D = e$ , party  $D$  obtains utility  $V(x^{A*}, (e, x^{D*}), (m, A, m))$  equal to

$$\frac{1}{2} - 2c + 2(1 - 2\eta_H) + \delta[\eta_H V(x^{A*}, x^{D*}, (e, \cdot, m)) + (1 - \eta_H) V(x^{A*}, x^{D*}, (e, \cdot, e))].$$

To sustain our equilibrium, we need that

$$V(x^{A*}, x^{D*}, (m, A, m)) \geq V(x^{A*}, (e, x^{D*}), (m, A, m)). \quad (30)$$

Case 2. State  $\lambda_t = (m, A, e)$ . Deviating to  $x_t^D = m$ , party  $D$  obtains  $V(x^{A*}, (m, x^{D*}), (m, A, e))$

equal to

$$\frac{1}{2} - 2c + \delta[\eta_L V(x^{A^*}, x^{D^*}, (m, A, m)) + (1 - \eta_L) V(x^{A^*}, x^{D^*}, (m, A, e))]. \quad (31)$$

Choosing  $x_t^D = x_t^{D^*}(\lambda_t) = e$ , party  $D$  obtains utility  $V(x^{A^*}, x^{D^*}, (m, A, e))$  equal to

$$\frac{1}{2} - 2c + 2(1 - 2\eta_L) + \delta[\eta_L V(x^{A^*}, x^{D^*}, (e, \cdot, m)) + (1 - \eta_L) V(x^{A^*}, x^{D^*}, (e, \cdot, e))].$$

To sustain our equilibrium, we need that

$$V(x^{A^*}, (m, x^{D^*}), (m, A, e)) \leq V(x^{A^*}, x^{D^*}, (m, A, e)). \quad (32)$$

Case 3. State  $\lambda_t = (e, \cdot, m)$ . Choosing  $x_t^D = x_t^{D^*}(\lambda_t) = m$ , party  $D$  obtains  $V(x^{A^*}, x^{D^*}, (e, \cdot, m))$  equal to

$$\frac{1}{2} - 2c + \delta[\eta_H V(x^{A^*}, x^{D^*}, (m, A, m)) + (1 - \eta_H) V(x^{A^*}, x^{D^*}, (m, A, e))].$$

Deviating to  $x_t^D = e$ , party  $D$  obtains utility  $V(x^{A^*}, (e, x^{D^*}), (e, \cdot, m))$  equal to

$$\frac{1}{2} + 2(1 - 2\eta_H) + \delta[\eta_H V(x^{A^*}, x^{D^*}, (e, \cdot, m)) + (1 - \eta_H) V(x^{A^*}, x^{D^*}, (e, \cdot, e))]. \quad (33)$$

To sustain our equilibrium, we need that

$$V(x^{A^*}, x^{D^*}, (e, \cdot, m)) \geq V(x^{A^*}, (e, x^{D^*}), (e, \cdot, m)). \quad (34)$$

Case 4. State  $\lambda_t = (e, \cdot, e)$ . Deviating to  $x_t^D = m$ , party  $D$  obtains  $V(x^{A^*}, (m, x^{D^*}), (e, \cdot, e))$  equal to

$$\frac{1}{2} - 2c + \delta[\eta_L V(x^{A^*}, x^{D^*}, (m, A, m)) + (1 - \eta_L) V(x^{A^*}, x^{D^*}, (m, A, e))].$$

Choosing  $x_t^D = x_t^{D*}(\lambda_t) = e$ , party  $D$  obtains utility  $V(x^{A*}, x^{D*}, (e, \cdot, e))$  equal to

$$\frac{1}{2} + 2(1 - 2\eta_L) + \delta[\eta_L V(x^{A*}, x^{D*}, (e, \cdot, m)) + (1 - \eta_L) V(x^{A*}, x^{D*}, (e, \cdot, e))]. \quad (35)$$

To sustain our equilibrium, we need that

$$V(x^{A*}, (m, x^{D*}), (e, \cdot, e)) \leq V(x^{A*}, x^{D*}, (e, \cdot, e)). \quad (36)$$

Notice first that given that  $x_t^{D*}(\lambda) = m$  for  $\lambda \in \{(m, A, m), (e, \cdot, m)\}$  and  $x_t^{D*}(\lambda) = e$  for  $\lambda \in \{(m, A, e), (e, \cdot, e)\}$ ,

$$V(x^{A*}, x^{D*}, (m, A, m)) = V(x^{A*}, x^{D*}, (e, \cdot, m)) \quad (37)$$

$$V(x^{A*}, x^{D*}, (m, A, e)) = V(x^{A*}, x^{D*}, (e, \cdot, e)) - 2c \quad (38)$$

$$V(x^{A*}, (e, x^{D*}), (m, A, m)) = V(x^{A*}, (e, x^{D*}), (e, \cdot, m)) - 2c \quad (39)$$

$$V(x^{A*}, (m, x^{D*}), (m, A, e)) = V(x^{A*}, (m, x^{D*}), (e, \cdot, e)). \quad (40)$$

We use Equality 39 to restate Condition 30 as

$$V(x^{A*}, x^{D*}, (m, A, m)) \geq V(x^{A*}, (e, x^{D*}), (e, \cdot, m)) - 2c. \quad (41)$$

We use Equality 38 to restate Condition 32 as

$$V(x^{A*}, (m, x^{D*}), (m, A, e)) \leq V(x^{A*}, x^{D*}, (e, \cdot, e)) - 2c. \quad (42)$$

We use Equality 37 to restate Condition 34 as

$$V(x^{A*}, x^{D*}, (m, A, m)) \geq V(x^{A*}, (e, x^{D*}), (e, \cdot, m)). \quad (43)$$

We use Equality 40 to restate Condition 36 as

$$V(x^{A*}, (m, x^{D*}), (m, A, e)) \leq V(x^{A*}, x^{D*}, (e, \cdot, e)), \quad (44)$$

Notice that Condition 43 implies Condition 41, and that Condition 42 implies Condition 44. Hence conditions 42 and 43 are necessary, and jointly sufficient, to sustain a TE equilibrium. Expanding Condition 42 using Expression 31 and Expression 35, we obtain

$$\begin{aligned} \delta(1 - \eta_L) V(x^{A^*}, x^{D^*}, (m, A, e)) &\leq 2(1 - 2\eta_L) + \delta(1 - \eta_L) V(x^{A^*}, x^{D^*}, (e, \cdot, e)) \\ \Leftrightarrow 0 &\leq 2(1 - 2\eta_L) + \delta(1 - \eta_L) 2c \\ \Leftrightarrow c &\geq \frac{2\eta_L - 1}{\delta(1 - \eta_L)}. \end{aligned}$$

Expanding Condition 43 using Expression 29 and Expression 33, we obtain

$$\begin{aligned} -2c + \delta(1 - \eta_H) V(x^{A^*}, x^{D^*}, (m, A, e)) &\geq 2(1 - 2\eta_H) + \delta(1 - \eta_H) V(x^{A^*}, x^{D^*}, (e, \cdot, e)) \\ \Leftrightarrow 2(2\eta_H - 1) &\geq 2c + \delta(1 - \eta_H) 2c = 2c(1 + \delta(1 - \eta_H)) \\ \Leftrightarrow c &\leq \frac{2\eta_H - 1}{1 + \delta(1 - \eta_H)} \end{aligned}$$

Therefore, a TE equilibrium exists if and only if

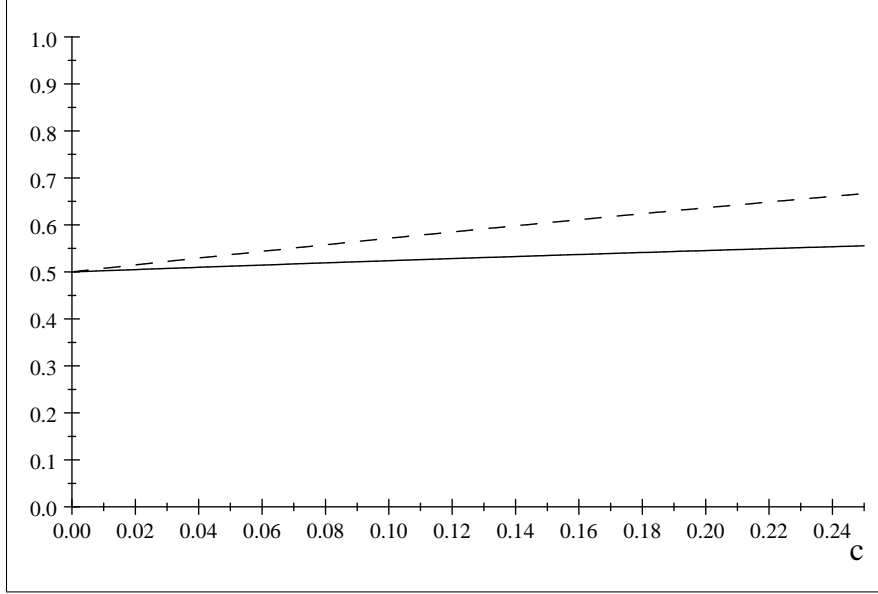
$$c \in \left[ \frac{2\eta_L - 1}{\delta(1 - \eta_L)}, \frac{2\eta_H - 1}{1 + \delta(1 - \eta_H)} \right].$$

Equivalent conditions are

$$\eta_L \leq \frac{1 + \delta c}{2 + c} \text{ and } \eta_H \geq \frac{1 + (1 + \delta)c}{2 + \delta c}.$$

■

The following figure plots the maximum value of  $\eta_L$  (solid line) and the minimum one of  $\eta_H$  (dashed) in order for this equilibrium to hold if  $\delta \simeq 1$  :



Intuitively, if  $\eta_L$  is too high (above  $\frac{1+\delta c}{2+c}$ ), then both parties always play  $m$ , consistent with our robust qualitative result that a TE equilibrium requires that there be sufficient uncertainty that the correct policy is  $m$ : parameter  $\eta_L$  plays a similar role as  $\mu$  in our two period model, namely, they have to be not too high in order for TE to hold. Equivalently, stating the result in terms of the competence parameter  $c$ , we again obtain the same intuition as in the two-period model: the importance of competence has to be sufficiently high, specifically  $c \geq \frac{2\eta_L - 1}{\delta(1-\eta_L)}$ , in order for TE to hold.

If  $\eta_H$  is not sufficiently high (if it not above  $\frac{1+(1+\delta)c}{2+\delta c}$ ), or, equivalently, if  $c$  is too high, then party  $D$  always plays  $e$ . In this case, party  $D$  sacrifices a probability of winning the first period election, for the sake of higher probabilities of victory in future elections, conforming to our definition of a Tactical Extremism equilibrium.

Computations for equilibria for parameter ranges outside those of Proposition 6 are available from the authors.

Notice that Tactical Extremism requires parties to be patient: a party that chooses TE incurs a short term cost in terms of foregone probability of winning the immediate election, for a greater probability of victory in future elections; only a sufficiently patient party is willing to take such a trade-off. Formally, the condition that  $\eta_L \in \left(\frac{1}{2}, \frac{1+\delta c}{2+c}\right]$  can only be satisfied if  $\frac{1}{2} \leq \delta$ .



Below is the list of additional references we cite in the Appendix and not in the main Letter.

## References

- [1] Ansolabehere, Stephen, and James M. Snyder. 2000. “Valence politics and equilibrium in spatial election models.” *Public Choice* 103 (3-4): 327–336.
- [2] Aragonès, Enriqueta, and Thomas R. Palfrey. 2002. “Mixed equilibrium in a downsonian model with a favored candidate.” *Journal of Economic Theory* 103 (1): 131–161.
- [3] Aragonès, Enriqueta, Thomas R. Palfrey, and Andrew Postlewaite. 2007. “Political reputations and campaign promises.” *Journal of the European Economic Association* 5 (4): 846–884.
- [4] Ashworth, Scott, and Ethan Bueno de Mesquita. 2009. “Elections with platform and valence competition.” *Games and Economic Behavior* 67 (1) 191–216.
- [5] Bernhardt, Dan, John Duggan, and Francesco Squintani. 2009b. “Private polling in elections and voter welfare.” *Journal of Economic Theory* 144 (5): 2021–2056.
- [6] Besley, Timothy, and Stephen Coate. 1997. “An economic model of representative democracy.” *Quarterly Journal of Economics* 112 (1): 85–114.
- [7] Brennan, Mary C.. 2000. *Turning Right in the Sixties: The Conservative Capture of the GOP*. University of North Carolina Press.
- [8] Butt, Sarah. 2006. “How voters evaluate economic competence: a comparison between parties in and out of power.” *Political Studies* 54 (4): 743–766.
- [9] Cowley, Philip, and Mark Stuart. 2014. “In the Brown stuff?: Labour backbench dissent under Gordon Brown, 2007–10.” *Contemporary British History* 28 (1): 1–23.
- [10] Edwards, Lee. 2014. “Barry M. Goldwater: The most consequential loser in American politics.” *Makers of American Political Thought* No. 11. Heritage Foundation.
- [11] Egan, Patrick J.. 2013. *Partisan priorities: How issue ownership drives and distorts American politics*. Cambridge University Press.
- [12] Ellis, Christopher, and James A. Stimson. 2009. “Symbolic ideology in the American electorate.” *Electoral Studies* 28 (3): 388–402.
- [13] Golding, John and Paul Farrelly (Ed). 2003. *Hammer of the Left*. Politico’s Publishing.

- [14] Grofman, Bernard. 1985. "The neglected role of the status quo in models of issue voting." *Journal of Politics* 47 (1): 230–237.
- [15] Groseclose, Tim. 2001. "A model of candidate location when one candidate has a valence advantage." *American Journal of Political Science* 45 (4): 862–886.
- [16] Grossmann, Matt, and David A. Hopkins. 2016. *Asymmetric Politics*. Oxford University Press.
- [17] Heath, Anthony F., Roger M. Jowell and John K. Curtice. 2001. *The Rise of New Labour*. Oxford University Press.
- [18] Hofstadter, Richard. 1964. "A long view: Goldwater in history." *New York Review of Books*, October 6th, 1964 Issue.
- [19] Kalandrakis, Tasos. 2009. "A reputational theory of two-party competition." *Quarterly Journal of Political Science* 4: 343–378.
- [20] Moon, Woojin. 2004. "Party activists, campaign resources and candidate position taking: theory, tests and applications." *British Journal of Political Science* 34 (4): 611–33.
- [21] Nunnari, Salvatore, and Jan Zápál. 2017. "Dynamic elections and ideological polarization." *Political Analysis* 25: 505–534.
- [22] Osborne, Martin J., and Al Slivinski. 1996. "A model of political competition with citizen-candidates." *Quarterly Journal of Economics* 111 (1): 65–96.
- [23] Palfrey, Thomas R.. 1984. "Spatial equilibrium with entry." *Review of Economic Studies* 51 (1): 139–156.
- [24] Perlstein, Rick. 2001. *Before the Storm: Barry Goldwater and the Unmaking of the American Consensus*. New York, Hill and Wang.
- [25] Richardson, Heather C.. 2014. *To Make Men Free: A History of the Republican Party*. New York, Basic Books.
- [26] Roemer, John E.. 1994. "A theory of policy differentiation in single issue electoral politics." *Social Choice and Welfare* 11 (4): 355–380.
- [27] Serra, Gilles. 2010. "Polarization of what? A model of elections with endogenous valence." *Journal of Politics* 72 (2): 426–437.
- [28] Soubeyran, Raphaël. 2009. "Does a disadvantaged candidate choose an extremist position?" *Annals of Economics and Statistics* 93/94: 327–348.
- [29] Stokes, Donald E.. 1992. "Valence politics." In D. Kavanagh (Ed.), *Electoral Politics*. Oxford: Clarendon Press.

- [30] Trilling, Lionel. 2008 [1950]. *The Liberal Imagination: Essays on Literature and Society*. New York Review of Books.
- [31] van Weelden, Richard. 2013. “Candidates, credibility, and re-election incentives.” *Review of Economic Studies* 80 (4): 1622–51.
- [32] Wickham-Jones, Mark. 1996. *Economic Strategy and the Labour Party*. MacMillan.
- [33] Wittman, Donald. 1983. “Candidate motivation — a synthesis of alternative theories.” *American Political Science Review* 77 (1): 142–157.