Online Appendix: Proofs

Proof of Lemma 1: First, observe that in the limit when the noise goes to zero, we have (Morris and Shin 2003):

$$Pr(x_i < \hat{x}|\theta = \hat{\theta}) = 1 - Pr(\theta < \hat{\theta}|x_i = \hat{x}), \quad \text{for all } \hat{x} \text{ and } \hat{\theta}. \tag{21}$$

To show this, we mirror the steps in Morris and Shin (2003):

$$\begin{aligned}
Pr(\theta < \hat{\theta}|x_i = \hat{x}) &= \int_{\theta=-\infty}^{\hat{\theta}} pdf(\theta|\hat{x})d\theta \\
&= \int_{\theta=-\infty}^{\hat{\theta}} \frac{pdf(\hat{x}|\theta)\ g(\theta)}{\int_{-\infty}^{\infty} pdf(\hat{x}|\theta)\ g(\theta)\ d\theta}\ d\theta \\
&= \int_{\theta=-\infty}^{\hat{\theta}} \frac{f_\epsilon(\frac{\hat{x}-\theta}{\sigma_w})\ g(\theta)}{\int_{-\infty}^{\infty} f_\epsilon(\frac{\hat{x}-\theta}{\sigma_w})\ g(\theta)\ d\theta}\ d\theta \\
&= \int_{z(\hat{\theta})}^{z=\infty} \frac{f_\epsilon(z)\ g(\hat{x} - \sigma_w\ z)}{\int_{-\infty}^{\infty} f_\epsilon(z) g(\hat{x} - \sigma_w\ z)\ dz}\ dz \\
&= 1 - F_\epsilon(z(\hat{\theta})) \ \ \text{(in the limit when } \sigma_w \to 0) \\
&= 1 - F_\epsilon\left(\frac{\hat{x} - \hat{\theta}}{\sigma_w}\right) \\
&= 1 - Pr(x_i < \hat{x}|\theta = \hat{\theta}).
\end{aligned}$$

We now use equation (21) to prove the lemma:

$$\begin{aligned}
H(p|\theta = \hat{\theta}) &= Pr(Pr(\theta < \hat{\theta}|x_i = \hat{x}) < p|\theta = \hat{\theta}) && \text{(definition of } H) \\
&= Pr(1 - Pr(x_i < \hat{x}|\theta = \hat{\theta}) < p|\theta = \hat{\theta}) && \text{(from (21))} \\
&= Pr(\ 1 - F_\epsilon((\hat{x} - \hat{\theta})/\sigma_w) < p\ |\theta = \hat{\theta}\ ) \\
&= Pr(\ \hat{\theta} + \sigma_w\ F^{-1}(1-p) < \hat{x}\ |\theta = \hat{\theta}\ ) \\
&= 1 - F_\epsilon(F_\epsilon^{-1}(1-p)) \\
&= p.
\end{aligned}$$

□

Proof of Lemma 2: Let $\Delta(x_i; x^*)$ be worker i's net expected payoff from revolting versus not revolting. We show that as x_i traverses the real line from $-\infty$ to ∞, $\Delta(x_i; x^*)$ changes sign at a unique point.

$$
\begin{aligned}
\Delta(x_i;x^*) &= Pr(\theta<\theta^{**}|x_i,\tilde{r}_f,K)\times s-(1-\alpha)\ E\left[\left(\frac{\underline{K}+K}{\underline{L}+Pr(x_j\geq x^*|\theta)(1-\underline{L})}\right)^{\alpha}\middle| x_i,\tilde{r}_f,K\right] \\
&= \int_{\theta=-\infty}^{\infty}\left(\mathbf{1}_{\{\theta<\theta^{**}\}}\ s-(1-\alpha)\ \left(\frac{\underline{K}+K}{\underline{L}+Pr(x_j\geq x^*|\theta)(1-\underline{L})}\right)^{\alpha}\right) f(\theta|x_i,\tilde{r}_f,K)\ d\theta \\
&= \int_{\theta=-\infty}^{\infty}\pi(\theta)\ f(\theta|x_i,\tilde{r}_f,K)\ d\theta,
\end{aligned}
$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function, and $\pi(\theta)\equiv\mathbf{1}_{\{\theta<\theta^{**}\}}\ s-(1-\alpha)\ \left(\frac{\underline{K}+K}{\underline{L}+Pr(x_j\geq x^*|\theta)(1-\underline{L})}\right)^{\alpha}$. Observe that

$$
\begin{aligned}
\lim_{\theta\to-\infty}\pi(\theta) &= s-(1-\alpha)\ \left(\frac{\underline{K}+K}{\underline{L}}\right)^{\alpha} > s-(1-\alpha)\ \left(\frac{\overline{K}}{\underline{L}}\right)^{\alpha} > 0.\ \text{(Assumption 1)} \\
\lim_{\theta\to\infty}\pi(\theta) &= -(1-\alpha)\ \left(\frac{\underline{K}+K}{1}\right)^{\alpha} < 0.
\end{aligned}
$$

Moreover, inspection of $\pi(\theta)$ reveals that $\pi(\theta)$ changes sign from positive to negative at a unique point $\theta=\theta^{**}$.

Next, because $f(\theta|x_i,\tilde{r}_f,K)$ is TP_2 (i.e., has MLRP between θ and x_i), by Karlin's theorem (Karlin 1968, Ch. 1, Theorem 3.1), $\Delta(x_i;x^*)$ has, at most one sign change. Finally, the inspection of $\Delta(x_i;x^*)$ reveals that $\lim_{x_i\to-\infty}\Delta(x_i;x^*)>0>\lim_{x_i\to\infty}\Delta(x_i;x^*)$. Thus, $\Delta(x_i;x^*)$, indeed, has one sign change from positive to negative. □

Proof of Lemma 3: Recalling that

$$
L(\theta)=Pr(x_i\geq x^*|\theta)\ (1-\underline{L})=\left(1-F_\epsilon\left(\frac{x^*-\theta}{\sigma_w}\right)\right)\ (1-\underline{L}), \tag{22}
$$

we have:

$$
\begin{aligned}
Pr(L(\theta)/(1-\underline{L})<A|x_i=x^*) &= Pr(1-F_\epsilon((x^*-\theta)/\sigma_w)<A|x_i=x^*) \qquad \text{(from (22))} \\
&= Pr(\theta<x^*-\sigma_w F_\epsilon^{-1}(1-A)|x_i=x^*) \\
&= 1-Pr(x_i<x^*|\theta=x^*-\sigma_w F_\epsilon^{-1}(1-A)) \quad \text{(from (21))} \\
&= 1-F_\epsilon\left(\frac{x^*-x^*+\sigma_w F_\epsilon^{-1}(1-A)}{\sigma_w}\right) \\
&= 1-F_\epsilon(F_\epsilon^{-1}(1-A)) \\
&= A.
\end{aligned}
$$

Hence, the marginal worker with signal $x_i = x^*$ believes that $Pr(x_i \geq x^*|\theta)$ is distributed uniformly on $[0,1]$, and hence $L(\theta)|x_i = x^* \sim U[0, 1-\underline{L}]$. □

Proof of Proposition 3: Given a level of aggregate domestic capital $\underline{K}+K$, the equilibrium is characterized by a pair (x^*, θ^*) such that:

$$Pr(\theta < \theta^{**}|x_i = x^*, \tilde{r}_f, K) \times s = E[w(\theta)|x_i = x^*, \tilde{r}_f, K]. \tag{23}$$

$$w(\theta) = (1-\alpha)\left(\frac{\underline{K}+K}{\underline{L} + Pr(x_i \geq x^*|\theta)(1-\underline{L})}\right)^{\alpha}. \tag{24}$$

$$Pr(x_i < x^*|\theta^{**}, \tilde{r}_f, K)\ (1-\underline{L}) = \theta^{**}. \tag{25}$$

First, observe that in the limit where the noise in the workers' private signals approaches zero, $pdf(\theta|x_i, \tilde{r}_f, K)$ approaches $pdf(\theta|x_i)$.[17] Now,

$$\begin{aligned}
E[w(\theta)|x_i = x^*] &= (1-\alpha)\ (\underline{K}+K)^{\alpha} \int_{-\infty}^{\infty} \frac{1}{[\underline{L} + (1 - Pr(x_i < x^*|\theta))(1-\underline{L})]^{\alpha}}\ pdf(\theta|x_i = x^*)\ d\theta \\
&= (1-\alpha)\ (\underline{K}+K)^{\alpha} \int_0^1 \frac{dz}{(\underline{L} + z(1-\underline{L}))^{\alpha}} \quad \text{(from Lemma 3)} \\
&= (1-\alpha)\ \frac{(\underline{K}+K)^{\alpha}}{1-\underline{L}} \left[\frac{(\underline{L}+z(1-\underline{L}))^{1-\alpha}}{1-\alpha}\right]_0^1 \\
&= (\underline{K}+K)^{\alpha}\ \frac{1-\underline{L}^{1-\alpha}}{1-\underline{L}}.
\end{aligned}$$

Thus, in the limit, equations (23) and (25) simplify to:

$$Pr(\theta < \theta^{**}|x_i = x^*) \times s = (\underline{K}+K)^{\alpha}\ \frac{1-\underline{L}^{1-\alpha}}{1-\underline{L}}. \tag{26}$$

$$Pr(x_i < x^*|\theta^{**})\ (1-\underline{L}) = \theta^{**}. \tag{27}$$

Because $Pr(\theta < \theta^{**}|x^*) = 1 - Pr(x_i < x^*|\theta^{**})$ in the limit, the result for $\theta^{**}(K)$ follows. Given this $\theta^{**}(K)$, equation (27) implies a unique x^*.

Moreover, $\theta^{**}(K)$ is decreasing in K and clearly $\theta^{**}(K) < 1$. To see that $\theta^{**}(K) > 0$, note that $\frac{1-\underline{L}^{1-\alpha}}{1-\underline{L}} < \frac{1-\alpha}{\underline{L}^{\alpha}}$, and hence $\frac{(\underline{K}+K)^{\alpha}\ (1-\underline{L})^{1-\alpha}}{1-\underline{L}} < (1-\alpha)\left(\frac{\overline{K}}{\underline{L}}\right)^{\alpha} < s$, where the last inequality follows from Assumption 1. □

Proof of Lemma 4: Let $\Gamma(y_i; \rho)$ be a capitalist's net expected payoff from investing one unit of capital in the country versus abroad, given his private signal y_i and given the strategies

[17] As we discussed in footnote 10, $\tilde{r}_f$ and K constitute a noisy public signal of θ, which becomes irrelevant for calculating the posterior when the noise in private signals becomes sufficiently accurate.

of other capitalists (ρ) and workers (x^*). We show that $\Gamma(y_i;\rho)$ has single-crossing property.

$$
\begin{aligned}
\Gamma(y_i;\rho) &= Pr(\theta \geq \theta^*|y_i)\ E[r_d(\theta)|\theta \geq \theta^*, y_i] - r_f \\
&= \int_{-\infty}^{\infty} \left[\mathbf{1}_{\{\theta\geq\theta^*\}}\ \alpha \left(\frac{\underline{L} + Pr(x_k \geq x^*|\theta)\ (1-\underline{L})}{\underline{K} + K(\theta)} \right)^{1-\alpha} - r_f \right] pdf(\theta|y_i)\ d\theta \\
&= \int_{-\infty}^{\infty} \Pi(\theta)\ pdf(\theta|y_i) d\theta, \qquad (28)
\end{aligned}
$$

where $\Pi(\theta) \equiv \mathbf{1}_{\{\theta\geq\theta^*\}}\ \alpha \left(\frac{\underline{L}+Pr(x_k\geq x^*|\theta)\ (1-\underline{L})}{\underline{K}+K(\theta)} \right)^{1-\alpha} - r_f$. Observe that:

$$
\lim_{\theta\to-\infty} \Pi(\theta) = -r_f < 0 \text{ and } \lim_{\theta\to\infty} \Pi(\theta) \geq \alpha \left(\frac{1}{\overline{K}} \right)^{1-\alpha} - r_f > 0, \qquad (29)
$$

where the last inequality follows from Assumption 2 that $\overline{f} < \alpha(1/\overline{K})^{1-\alpha}$, where we recall that $r_f \in [0, \overline{f}]$. From (28) and (29), $\lim_{y_i\to-\infty} \Gamma(y_i;\rho) < 0 < \lim_{y_i\to\infty} \Gamma(y_i;\rho)$. Thus, $\Gamma(y_i;\rho)$ has at least one sign change.

We will show that there exists a $\overline{\sigma} > 0$ such that if $\sigma_w < \overline{\sigma}$, then $\Pi(\theta;\sigma_w)$ has exactly one sign change as θ traverses the real line from $-\infty$ to ∞, where we have made the dependence of Π on σ_w explicit.[18] Then, because $pdf(\theta|y_i)$ is TP_2 (i.e., has MLRP between θ and y_i), by Karlin's theorem, $\Gamma(y_i;\rho,\sigma_w)$ has at most one sign change for $\sigma_w < \overline{\sigma}$.

Now, we show that there exists a $\overline{\sigma} > 0$ such that if $\sigma_w < \overline{\sigma}$, then $\Pi(\theta;\sigma_w)$ has exactly one sign change as a function of θ. Clearly, $\Pi(\theta;\sigma_w) = -r_f < 0$ for $\theta < \theta^*$. Let $\widehat{\Pi}(\theta;\sigma_w)$ be the restriction of $\Pi(\theta;\sigma_w)$ to $[\theta^*,\infty)$, so that

$$
\widehat{\Pi}(\theta;\sigma_w) \equiv \alpha \left(\frac{1 - F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L})}{\overline{K} - F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K} \right)^{1-\alpha} - r_f, \quad \text{for} \quad \theta \in [\theta^*,\infty),
$$

where $\Delta K \equiv \overline{K} - \underline{K} < \overline{K}$, and we used the cdf of the noise in the signals of workers (F_ϵ) and capitalists (F_η). By continuity, for every $\gamma > 0$, there exists a $\delta > 0$ such that if $\theta \in [\theta^*, \theta^*+\delta]$, then $\overline{K} - F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K \in [\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K, \overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K + \gamma]$. Moreover, because $\lim_{\sigma_w\to 0} x^*(\sigma_w) = \theta^*$, for sufficiently small σ_w, we have $x^*(\sigma_w) - (\theta^*+\delta) < 0$. Thus, for every

[18] A stronger assumption, $\overline{f} < \alpha(\underline{L}/\overline{K})^{1-\alpha}$, immediately implies that, for any $\sigma_w > 0$, $\Pi(\theta)$ switches sign from negative to positive at the unique point θ^*. Then, because $pdf(\theta|y_i)$ is TP_2 (i.e., has MLRP between θ and y_i), by Karlin's theorem, $\Gamma(y_i;y^*)$ has at most one sign change.

$\beta > 0$, there exists a $\sigma_\beta > 0$ such that if $\sigma_w < \sigma_\beta$, then $1 - F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L}) > 1-\beta$, for all $\theta \geq \theta^* + \delta$. Now, choose a $\hat{\beta} > 0$ such that $\alpha\ \left(\frac{1-\hat{\beta}}{\overline{K}}\right)^{1-\alpha} > r_f$ (by Assumption 2, such a $\hat{\beta}$ exists). Thus, there exists a $\sigma_{\hat{\beta}} > 0$ such that if $\sigma_w < \sigma_{\hat{\beta}}$, then for $\theta \geq \theta^* + \delta$, we have:

$$\begin{aligned}
\widehat{\Pi}(\theta;\sigma_w) &> \alpha\ \left(\frac{1 - F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L})}{\overline{K}}\right)^{1-\alpha} - r_f \\
&\geq \alpha\ \left(\frac{1 - F_\epsilon\left(\frac{x^*(\sigma_w)-(\theta^*+\delta)}{\sigma_w}\right)(1-\underline{L})}{\overline{K}}\right)^{1-\alpha} - r_f \\
&> \alpha\ \left(\frac{1-\hat{\beta}}{\overline{K}}\right)^{1-\alpha} - r_f \\
&> 0 \qquad\qquad (\text{for } \theta \geq \theta^* + \delta). \qquad (30)
\end{aligned}$$

Next, we show that there is at most one sign change in $\theta \in [\theta^*, \theta^* + \delta]$. By continuity, at any $\theta_0 \in (\theta^*, \theta^* + \delta]$ at which there is a sign change, we must have $\widehat{\Pi}(\theta = \theta_0; \sigma_w) = 0$:

$$1 - F_\epsilon\left(\frac{x^*(\sigma_w)-\theta_0}{\sigma_w}\right)(1-\underline{L}) = \left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta_0}{\sigma_c}\right)\Delta K\right). \qquad (31)$$

By Assumption 2, $\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K\right) < 1$. If $\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K\right) < \underline{L}$, then choose γ (and the corresponding δ) such that, for all $\theta \in [\theta^*, \theta^* + \delta]$,

$$\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K\right) \leq \left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K + \gamma\right) < \underline{L}.$$

Thus, $\widehat{\Pi}(\theta;\sigma_w) > 0$ for all $\theta \in [\theta^*, \theta^* + \delta]$.

Next, consider the case where $\underline{L} < \left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K\right) < 1$. Then, choose γ (and the corresponding δ) such that $\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K + \gamma\right) < 1$. This implies that, for all $\theta \in [\theta^*, \theta^* + \delta]$, $\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\ \left(\overline{K} - F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K\right) \in [I_1, I_2] \subset (\underline{L}, 1)$, for some $I_1 < I_2$. Thus, from equation (31), at any $\theta_0 \in (\theta^*, \theta^* + \delta]$ at which there is a crossing, we must have $\frac{y^*-\theta_0}{\sigma_c} \in [k_1, k_2]$ and $\frac{x^*(\sigma_w)-\theta_0}{\sigma_w} \in [l_1, l_2]$, for some $k_1 < k_2$ and $l_1 < l_2$. Define

$$f_\eta^M \equiv \max_{x\in[k_1,k_2]} f_\eta(x) \ \text{ and } \ f_\epsilon^m \equiv \min_{x\in[l_1,l_2]} f_\epsilon(x) > 0. \qquad (32)$$

Differentiating $\widehat{\Pi}(\theta;\sigma_w)$ with respect to θ yields:

$$\frac{d\widehat{\Pi}(\theta;\sigma_w)}{d\theta} > 0 \ \Leftrightarrow\ \frac{\frac{1}{\sigma_w} f_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L})}{1 - F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L})} > \frac{\frac{1}{\sigma_c} f_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K}{\overline{K} - F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K}. \qquad (33)$$

Thus, at any θ_0 at which $\widehat{\Pi}(\theta=\theta_0;\sigma_w)=0$, we have:

$$\left.\frac{d\widehat{\Pi}(\theta;\sigma)}{d\theta}\right|_{\theta=\theta_0}>0 \quad\Leftrightarrow\quad \frac{1}{\sigma_w}f_\epsilon\left(\frac{x^*(\sigma_w)-\theta_0}{\sigma_w}\right)>\frac{1}{\sigma_c}f_\eta\left(\frac{y^*-\theta_0}{\sigma_c}\right)\frac{\Delta K}{1-\underline{L}}\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{34}$$

where we used equation (31). Moreover, from (32),

$$\frac{1}{\sigma_w}f_\epsilon\left(\frac{x^*(\sigma_w)-\theta_0}{\sigma_w}\right)\geq\frac{1}{\sigma_w}f_\epsilon^m \text{ and } \frac{1}{\sigma_c}\frac{\Delta K}{1-\underline{L}}\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}} f_\eta^M\geq\frac{1}{\sigma_c}\frac{\Delta K}{1-\underline{L}}\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}} f_\eta\left(\frac{y^*-\theta_0}{\sigma_c}\right). \tag{35}$$

Thus, from (34) and (35), if $\sigma_w<\frac{f_\epsilon^m}{f_\eta^M}\left(\frac{1}{\sigma_c}\frac{\Delta K}{1-\underline{L}}\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\right)^{-1}\in(0,\infty)$, then $\frac{d\widehat{\Pi}(\theta;\sigma_w)}{d\theta}\big|_{\theta=\theta_0}>0$. That is, for sufficiently small σ_w, at any point $\theta_0\in(\theta^*,\theta^*+\delta]$ at which $\widehat{\Pi}(\theta)$ crosses 0, the derivative is strictly positively. Thus, there is at most one such crossing. In particular, either $\Pi(\theta)$ switches sign only at θ^*, or it switches sign only at some $\theta_0\in(\theta^*,\theta^*+\delta)$.

Finally, consider the special case, where $\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\left(\overline{K}-F_\eta\left(\frac{y^*-\theta^*}{\sigma_c}\right)\Delta K\right)=\underline{L}$. Then, $\widehat{\Pi}(\theta^*;\sigma_w)>0$ for all $\sigma_w>0$. If $x^*(\sigma_w)$ goes to θ^* from below, for sufficiently small σ_w, there exists a $\delta>0$ such that $\widehat{\Pi}(\theta;\sigma_w)>0$ for all $\theta\in[\theta^*,\theta^*+\delta)$. To see this, observe that

$$1-F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L})\geq\underline{L}+(1-F_\epsilon(0))(1-\underline{L})>\underline{L},\ \forall\theta\geq\theta^*.$$

Thus, we can pick a $\gamma>0$ (with the corresponding δ) small enough, so that for $\theta\in[\theta^*,\theta^*+\delta)$ we have:

$$\left(\frac{r_f}{\alpha}\right)^{\frac{1}{1-\alpha}}\left(\overline{K}-F_\eta\left(\frac{y^*-\theta}{\sigma_c}\right)\Delta K\right)<\underline{L}+(1-F_\epsilon(0))(1-\underline{L})\leq 1-F_\epsilon\left(\frac{x^*(\sigma_w)-\theta}{\sigma_w}\right)(1-\underline{L}),$$

and hence $\widehat{\Pi}(\theta;\sigma_w)>0$ for all $\theta\in[\theta^*,\theta^*+\delta)$. Next, suppose $x^*(\sigma_w)$ approaches θ^* from above. From (33), for sufficiently small σ_w, $\frac{d\widehat{\Pi}(\theta;\sigma_w)}{d\theta}|_{\theta=x^*(\sigma_w)}>0$, and it can be made arbitrarily large. Further, by log-concavity, the left hand side of the second inequality in (33) is decreasing in θ. Thus, for sufficiently small σ_w, when $\theta\in(\theta^*,x^*(\sigma_w)]$, we have $\frac{d\widehat{\Pi}(\theta;\sigma_w)}{d\theta}>0$, and hence $\widehat{\Pi}(\theta;\sigma_w)>0$ for all $\theta\in[\theta^*,x^*(\sigma_w)]$. Thus, any crossing must happen at some $\theta_0(\sigma_w)>x^*(\sigma_w)$, and hence $\frac{x^*(\sigma_w)-\theta_0(\sigma_w)}{\sigma_w}<0$, where we made explicit the possible dependence of θ_0 on σ_w. Now, if $\frac{x^*(\sigma_w)-\theta_0(\sigma_w)}{\sigma_w}$ is finite for all sufficiently small σ_w, then the logic of equations (32)-(35) goes through because we can find some $f_\epsilon^m>0$. That is, there exists a $\overline{\sigma}_a>0$ such that if $\sigma_w<\overline{\sigma}_a$, then $\Pi(\theta;\sigma)$ has one sign change as a function of θ. Otherwise, $\frac{x^*(\sigma_w)-\theta_0(\sigma_w)}{\sigma_w}$ must become unboundedly negative. But then $F\left(\frac{x^*(\sigma_w)-\theta_0(\sigma_w)}{\sigma_w}\right)$ approaches 0, and the logic of (30) applies. That is, there exists a $\overline{\sigma}_b > 0$ such that if $\sigma_w < \overline{\sigma}_b$, then $\Pi(\theta;\sigma)$ has one sign change as a function of θ. If $x^*(\sigma_w)$ approaches θ^* from both above and below, then set $\overline{\sigma} = \min\{\overline{\sigma}_b, \overline{\sigma}_a\}$. □

Proof of Proposition 4: First, we calculate the expected payoff from domestic investment for a capitalist whose signal is at the equilibrium threshold $y_j = y^*$. The left hand side of equation (8) is:

$$
\begin{aligned}
& Pr(\theta \geq \theta^* | y_j = y^*)\ E[r_d(\theta) | \theta \geq \theta^*, y_j = y^*] \\
= \;& Pr(\theta \geq \theta^* | y_j = y^*)\ \alpha \int_{-\infty}^{\infty} \left(\frac{\underline{L} + L(\theta)}{\underline{K} + K(\theta)} \right)^{1-\alpha} pdf(\theta | \theta \geq \theta^*, y_j = y^*)\ d\theta \\
= \;& Pr(\theta \geq \theta^* | y_j = y^*)\ \alpha \int_{\theta^*}^{\infty} \left(\frac{\underline{L} + L(\theta)}{\underline{K} + K(\theta)} \right)^{1-\alpha} \frac{pdf(\theta | y_j = y^*)}{Pr(\theta \geq \theta^* | y_j = y^*)}\ d\theta \\
= \;& \alpha \int_{\theta^*}^{\infty} \frac{[\underline{L} + Pr(x_i \geq x^* | \theta)\ (1 - \underline{L})]^{1-\alpha}}{[\underline{K} + Pr(y_l \geq y^* | \theta)\ (\overline{K} - \underline{K})]^{1-\alpha}}\ pdf(\theta | y_j = y^*)\ d\theta \\
= \;& \alpha \int_{\theta^*}^{\infty} \frac{1}{[\underline{K} + Pr(y_l \geq y^* | \theta)\ (\overline{K} - \underline{K})]^{1-\alpha}}\ pdf(\theta | y_j = y^*)\ d\theta, \quad \text{(because } \lim_{\sigma_w \to 0} Pr(x_i \geq x^* | \theta > \theta^*) = 1) \\
= \;& \alpha \int_{z(\theta^*)}^{1} \frac{1}{[\underline{K} + (\overline{K} - \underline{K})\ z]^{1-\alpha}}\ dz, \quad \text{(change of variable from } \theta \text{ to } z = Pr(y_l \geq y^* | \theta)) \qquad (36) \\
= \;& \alpha\ \frac{1}{\overline{K} - \underline{K}} \left[\frac{[\underline{K} + (\overline{K} - \underline{K})\ z]^{\alpha}}{\alpha} \right]_{z = z(\theta^*)}^{1} \\
= \;& \frac{1}{\overline{K} - \underline{K}} \{ \overline{K}^{\alpha} - [\underline{K} + (\overline{K} - \underline{K})\ z(\theta^*)]^{\alpha} \} \\
= \;& \frac{\overline{K}^{\alpha} - [\underline{K} + K(\theta^*)]^{\alpha}}{\overline{K} - \underline{K}}. \qquad (37)
\end{aligned}
$$

Substituting from equation (37) into equation (8) yields:

$$
[\underline{K} + K(\theta^*)]^{\alpha} = \overline{K}^{\alpha} - (\overline{K} - \underline{K})\ r_f. \qquad (38)
$$

Substituting from equation (38) into equation (10) yields the unique θ^* in equation (11). Finally, given a unique θ^*, we show that a unique y^* solves equation (38), and hence y^* exists and is unique. Recall that $K(\theta^*) = Pr(y_j \geq y^* | \theta^*)\ (\overline{K} - \underline{K})$. From equation (38), for a given θ^*, as y^* traverses the real line from $-\infty$ to ∞, the left hand side (strictly) falls from $\overline{K}^\alpha$ to $\underline{K}^\alpha$. Clearly, $\overline{K}^\alpha > \overline{K}^\alpha - (\overline{K} - \underline{K})\ r_f$. Next, we show $\underline{K}^\alpha < \overline{K}^\alpha - (\overline{K} - \underline{K})\ r_f$, i.e., $\frac{\overline{K}^\alpha - \underline{K}^\alpha}{\overline{K} - \underline{K}} > r_f$. Observe that from (36) and (37) we have:

$$
\begin{aligned}
\frac{\overline{K}^\alpha - \underline{K}^\alpha}{\overline{K} - \underline{K}} &= \lim_{y^* \to \infty} \frac{\overline{K}^\alpha - [\underline{K} + K(\theta^*)]^\alpha}{\overline{K} - \underline{K}} \\
&= \lim_{y^* \to \infty} \alpha \int_{z(\theta^*)}^{1} \frac{1}{[\underline{K} + (\overline{K} - \underline{K})\ z]^{1-\alpha}}\ dz \\
&= \alpha \int_{0}^{1} \frac{1}{[\underline{K} + (\overline{K} - \underline{K})\ z]^{1-\alpha}}\ dz \geq \alpha\ \frac{1}{\overline{K}^{1-\alpha}} > \overline{f} \geq r_f,
\end{aligned}
$$

where second to last inequality is true by Assumption 2. Thus, there is a unique y^* that satisfies equation (38) and hence equation (8). □

Proof of Corollary 1: From Proposition 4,

$$
\frac{\partial \theta^*}{\partial \underline{L}} = -1 + \frac{1}{s}\ (1-\alpha)\ \underline{L}^{-\alpha}\ [\overline{K}^\alpha - (\overline{K} - \underline{K})\ r_f] \leq -1 + \frac{1}{s}\ (1-\alpha)\ \left(\frac{\overline{K}}{\underline{L}}\right)^\alpha < 0,
$$

where the last inequality follows from Assumption 1. $\frac{\partial \theta^*}{\partial \overline{K}} > 0$ follows from Assumption 2. Other results are immediate. □

Proof of Proposition 5: With capital control, a capitalist's expected payoff is:

$$
U_1 = (1 - G(\theta_1^*))\ \alpha\ \overline{K}^\alpha,
$$

where we used $\lim_{\sigma_w \to 0} Pr(x_i \geq x^* | \theta \geq \theta_1^*) = 1$. Without capital control, a capitalist's expected payoff is:

$$
\begin{aligned}
U_0 &= Pr(\theta \geq \theta_0^*, y_i \geq y^*)\ \alpha\ E\left[\left(\frac{1}{\underline{K} + Pr(y_j \geq y^* | \theta)\ (\overline{K} - \underline{K})}\right)^{1-\alpha} \middle| \theta \geq \theta_0^*, y_i \geq y^*\right] \overline{K} \\
&\quad + Pr(y_i < y^*) r_f\ \Delta K \\
&= Pr(\theta \geq \theta_0^*, y_i \geq y^*)\ \alpha\ \left(\frac{1}{\overline{K}}\right)^{1-\alpha} \overline{K} + Pr(y_i < y^*)\ r_f\ \Delta K \\
&= (1 - G(\theta_0^*))\ \alpha\ \overline{K}^\alpha + G(\theta_0^*)\ r_f\ \Delta K, \qquad (39)
\end{aligned}
$$

where we used the facts that $\lim_{\sigma_c \to 0} y^* = \theta_0^*$, and the distribution of y_j approaches that of θ.

Lemma 5 *Fix $\overline{K}$, and suppose $\sigma_c \to 0$ and $g(\theta)$ is log-concave. For $R_f \in \left[0, \alpha\ \overline{K}^\alpha\right]$, either $U_0(R_f)$ is monotone, or it has a unique extremum, which is minimum.*

Proof of Lemma 5: Differentiating $U_0(r_f)$ from (39) with respect to r_f yields:[19]

$$\frac{dU_0(R_f)}{dR_f} = G(\theta_0^*) \ - \frac{\partial \theta_0^*}{\partial R_f} \ g(\theta_0^*) \left(\alpha \ \overline{K}^{\alpha} - R_f \right). \tag{40}$$

Moreover, from equation (14),

$$\frac{\partial \theta_0^*}{\partial R_f} = \frac{1 - \underline{L}^{1-\alpha}}{s}. \tag{41}$$

Substituting from (41) into (40) yields:

$$\frac{dU_0(R_f)}{dR_f} = G(\theta_0^*) - g(\theta_0^*) \ \frac{1 - \underline{L}^{1-\alpha}}{s} \ \left(\alpha \ \overline{K}^{\alpha} - R_f \right).$$

Thus,

$$\frac{dU_0(R_f)}{dR_f} > 0 \quad \Leftrightarrow \quad \frac{g(\theta_0^*)}{G(\theta_0^*)} < \left[\ \frac{1 - \underline{L}^{1-\alpha}}{s} \ \left(\alpha \ \overline{K}^{\alpha} - R_f \right) \ \right]^{-1}. \tag{42}$$

As R_f increases from 0 to $\alpha \overline{K}^{\alpha}$, (i) the right hand side rises, and (ii), from equation (41), θ_0^* increases, and hence the left hand side falls by log-concavity of $g(\theta)$. Thus, $U_0(r_f)$ is either monotone, or it has a unique extremum, which is a minimum. □

From (42),

$$\left. \frac{dU_0(R_f)}{dR_f} \right|_{R_f = 0} < 0 \quad \Leftrightarrow \quad \frac{g(\theta_{0,m}^*)}{G(\theta_{0,m}^*)} > \left[\ \frac{1 - \underline{L}^{1-\alpha}}{s} \ \alpha \ \overline{K}^{\alpha} \ \right]^{-1} = \frac{1}{\alpha[(1 - \underline{L}) - \theta_{0,m}^*]}.$$

The result follows because $U_0(R_f = 0) = U_1$ and $U_0(R_f = \alpha \overline{K}^{\alpha}) > U_1$. □

[19] Results are the same if one differentiates first, and then takes the limits.

Online Appendix: Karlin's Theorem

For completeness we state Karlin's Theorem. We first provide the definitions of the objects used in the theorem. All the material is quoted from Chapter 1 of Karlin's (1968) book, *Total Positivity, Vol. I.*

Definition 1 *A real function (frequently called kernel) $K(x, y)$ of two variables ranging over linearly ordered sets X and Y, respectively, is said to be totally positive of order r (abbreviated TP_r) if for all*

$$x_1 < x_2 < \cdots < x_m, y_1 < y_2 < \cdots < y_m \qquad x_i \in X, y_j \in Y; 1 \leq m \leq r \tag{43}$$

we have the inequalities

$$K\begin{pmatrix} x_1, x_2, \cdots, x_m \\ y_1, y_2, \cdots, y_m \end{pmatrix} = \begin{vmatrix} K(x_1, y_1) & K(x_1, y_2) & \cdots & K(x_1, y_m) \\ K(x_2, y_1) & K(x_2, y_2) & \cdots & K(x_2, y_m) \\ \vdots & \vdots & & \vdots \\ K(x_m, y_1) & K(x_m, y_2) & \cdots & K(x_m, y_m) \end{vmatrix} \geq 0$$

A concept more general than total positivity is that of sign regularity.

Definition 2 *A function $K(x, y)$ is sign-regular of order r (abbreviated SR_r) if there exists a sequence of numbers ϵ_m each either $+1$ or -1 such that where conditions (43) apply, we have*

$$\epsilon_m \, K\begin{pmatrix} x_1, x_2, \cdots, x_m \\ y_1, y_2, \cdots, y_m \end{pmatrix} \geq 0$$

Definition 3 *Let $f(t)$ be defined in I, where I is an ordered set of the real line. Let*

$$S^-(f) = S^-[f(t)] = sup \; S^-[f(t_1), f(t_2), \cdots, f(t_m)]$$

where the supremum is extended over all sets $t_1 < t_2 < \cdots < t_m$ $(t_i \in I)$, m is arbitrary but finite, and $S^-(x_1, x_2, \cdots, x_m)$ is the number of sign changes of the indicated sequence, zero terms being discarded.

Let $K(x, y)$ defined on $X \times Y$ be Borel-measurable, and assume for simplicity that the integral $\int_Y K(x, y) d\mu(y)$ exists for every x in X. Here μ represents a fixed sigma-finite regular measure defined on Y such that $\mu(U) > 0$ for each open set U for which $U \cap Y$ is nonempty. Let f be bounded and Borel-measurable on Y, and consider the transformation

$$g(x) = (Tf)(x) = \int_Y K(x, y) f(y) d\mu(y)$$

Theorem 1 *If K is SR_r and satisfies the integrability requirements stated above, then*

$$S^-(g) = S^-(Tf) \leq S^-(f) \quad \text{provided } S^-(f) \leq r - 1$$

In the case in which K is TP_r and f is piecewise-continuous, if $S^-(f) = S^-(g) \leq r - 1$, we further assert that the values of the functions f and g exhibit the same sequence of signs when their respective arguments traverse the domain of definition from left to right.