Mass Purges

Supplemental Appendix

We (re)introduce some notation used throughout. Recall that $V_2(\tau) = R + \frac{(v(S,\tau))^2}{2}$ denotes agent i's expected payoff in period 2 as a function of his type. The (ex-ante) average payoffs are denoted by $\overline{v} = \lambda v(S,c) + (1-\lambda)v(S,nc)$ and $\overline{V_2} = \lambda V_2(c) + (1-\lambda)V_2(nc)$. For the autocrat, $W_2(\tau) = v(S,\tau)$ denotes her second-period expected payoff induced by an agent of type $\tau \in \{c, nc\}$. The highest feasible intensity of violence satisfies $\overline{L} = 1 - v(S,c) - V_2(c)$. In all that follows, since second-period actions are subsumed in $V_2(\tau)$ for a subordinate and $W_2(\tau)$ for the autocrat, we ignore the first-period time subscript in all the proofs.

A Proofs of 'Effort and incentive to purge'

Proof of Lemma 1

First, observe that due to our equilibrium refinement, there is no equilibrium in which agents exert zero effort. If there is, a successful project is an out-of-equilibrium event. As it is treated as a mistake, it does not affect the autocrat's purging decision. Hence, congruent types have a profitable deviation to exert effort.

Anticipating the proof of Lemma 2, an agent's effort as a function of his type is $e^i(\tau) = \max\{(1-\kappa_S)v(S,\tau) + (\kappa_F - \kappa_S)(V_2(\tau) + L), 0\}$.

First, proceeding by contradiction, suppose there is an equilibrium in which $\kappa_F, \kappa_S \in (0,1)^2$. The autocrat must then be indifferent between purging from the success and failure pools. One type of agents, however, exerts more effort than the other for all κ_S, κ_F and the autocrat's posterior after success differs from her posterior after observing failure. Hence, the autocrat is never indifferent, a contradiction.

We now show that $\kappa_S > 0$ and $\kappa_F = 0$ cannot be part of an equilibrium strategy proceeding again by contradiction. In this case, both types exert no effort (which contradicts the first paragraph of the proof) or a congruent type exerts more effort than a non-congruent. The autocrat's posterior conditional on success is then higher than conditional on failure and her best response is to first purge from the pool of failure, a contradiction.

Given the reasoning above, $\kappa_S > 0$ only if $\kappa_F = 1$. But note that then success is a positive signal of congruence. Since the proportion of congruent types in the replacement pool is the same as in the pool of existing agents at the beginning of the game and since purging is costly, there cannot be an equilibrium in which $\kappa_S > 0$. □

Proof of Lemma 2

We first determine the level of effort taking κ_F and κ_S as given and assuming that effort is always weakly positive. The maximization problem of a type $\tau \in \{c, nc\}$ agent assumes the following form:

$$\max_{e\in[0,1]} \; R + e\big[(1-\kappa_S)(v(S,\tau)+V_2(\tau))+\kappa_S(-L)\big]+(1-e)\big[(1-\kappa_F)(V_2(\tau))+\kappa_F(-L)\big]-\frac{e^2}{2} \quad \text{(A.1)}$$

If his project is successful (probability e), an agent survives the purge with probability $1-\kappa_S$ and receives a flow payoff $v(S,\tau)$ and as well as his second period expected payoff. If his project fails, he survives the purge with probability $1-\kappa_F$ and only receives his second period expected payoff then. When the agent is purged, he suffers a loss L.

Taking the first-order condition, we obtain:

$$e^i(\tau) = (1-\kappa_S)v(S,\tau)+(\kappa_F-\kappa_S)(V_2(\tau)+L) \quad \text{(A.2)}$$

Using Lemma 1 yields the claim. □

Before proceeding to the rest of the analysis, we briefly discuss why our results hold substantively if non-congruent types get a negative payoff from successful project. More specifically, suppose that $-R < v(S,nc) < 0$. Then there exists $\kappa_F^0(L) = \frac{-v(S,nc)}{R+L} \in (0,1)$ such that for all $\kappa_F \leq \kappa_F^0(L)$, a non-congruent subordinate exerts zero effort in period 1. It can be checked that for all $\kappa_F \in [0,\kappa_F^0(L))$, the autocrat's posterior μ^F is strictly decreasing with κ_F (see the proof of Lemma B.1) and L (see the proof of Lemma B.3) as only congruent types exit the failure pool. As a result, all the comparative statics with respect to violence we establish in the main text (purge incidence, purge breadth, effort, selection) hold for all L such that the equilibrium incidence $\kappa_F^*(L)$ satisfies $\kappa_F^*(L) \in [0,\kappa_F^0(L))$. Hence, the assumption that $v(S,nc) \geq 0$ is meant to simplify the analysis.

B Proofs of 'The consequences of violence'

In all that follows, we slightly abuse notation (given that agents fully anticipate the purge incidence in equilibrium) and denote $e^i(\kappa_F, L; \tau)$ a type $\tau \in \{c, nc\}$ agent's effort. Average effort, in turn, is $\bar{e}(\kappa_F, L)$. Similarly, again slightly abusing notation (given that the autocrat correctly anticipates agents' effort in equilibrium), we denote $\mu^F(\kappa_F, L)$ the autocrat's posterior that an agent is congruent conditional on failure. Before proving the main results, we need some preliminary lemmas to establish some properties of the equilibrium purge incidence.

The next Lemma characterizes some properties of the autocrat's posterior treating the purge incidence κ_F as exogenous.

Lemma B.1. *In a discriminate purge with some failures surviving ($\kappa_F < 1$), the autocrat's posterior after failure is strictly decreasing and concave in κ_F.*

Proof. We prove the lemma using a slightly general reasoning to illustrate that our results do not depend on our functional form assumptions.

By Bayes' Rule, $\mu^F(\kappa_F, L) = \lambda \frac{1-e^i(\kappa_F,L;c)}{1-\bar{e}(\kappa_F,L)}$. The relevant comparative statics is then (omitting superscript):

$$\begin{aligned}\frac{\partial \mu^F(\kappa_F, L)}{\partial \kappa_F} =& \frac{\lambda}{(1-\bar{e}(\kappa_F, L))^2}\Bigg[-\frac{e^i(\kappa_F, L; c)}{\partial \kappa_F}(1 - \lambda e^i(\kappa_F, L; c) - (1-\lambda)e^i(\kappa_F, L; nc)) \\ &+ (1 - e^i(\kappa_F, L; c))\left(\lambda \frac{\partial e^i(\kappa_F, L; c)}{\partial \kappa_F} + (1-\lambda)\frac{\partial e^i(\kappa_F, L; nc)}{\partial \kappa_F}\right)\Bigg] \\ =& \frac{\lambda(1-\lambda)(1-e^i(\kappa_F, L; c))(1-e^i(\kappa_F, L; nc))}{(1-\bar{e}(\kappa_F, L))^2}\left[\frac{\frac{\partial e^i(\kappa_F,L;nc)}{\partial \kappa_F}}{1-e^i(\kappa_F, L; nc)} - \frac{\frac{\partial e^i(\kappa_F,L;c)}{\partial \kappa_F}}{1-e^i(\kappa_F, L; c)}\right]\end{aligned} \tag{B.1}$$

By examination of Equation 5, $e^i(\kappa_F, L; c) > e^i(\kappa_F, L; nc)$ and $\frac{\partial e^i(\kappa_F,L;c)}{\partial \kappa_F} > \frac{\partial e^i(\kappa_F,L;nc)}{\partial \kappa_F}$. This directly implies: $\frac{\frac{\partial e^i(\kappa_F,L;nc)}{\partial \kappa_F}}{1-e^i(\kappa_F,L;nc)} - \frac{\frac{\partial e^i(\kappa_F,L;c)}{\partial \kappa_F}}{1-e^i(\kappa_F,L;c)} < 0$ and $\frac{\partial \mu^F(\kappa_F,L)}{\partial \kappa_F} < 0$ as claimed.

To see that the posterior is strictly concave in κ_F, notice that:

$$\begin{aligned}\frac{\partial^2 \mu^F(\kappa_F, L))}{\partial \kappa_F^2} \propto& \left[\frac{\partial^2 e^i(\kappa_F, L; nc)}{\partial \kappa_F^2}(1 - e^i(\kappa_F, L; c)) - \frac{\partial^2 e^i(\kappa_F, L; c)}{\partial \kappa_F^2}(1 - e^i(\kappa_F, L; nc))\right](1 - \bar{e}(\kappa_F, L)) \\ &+ 2\frac{\partial \bar{e}(\kappa_F, L)}{\partial \kappa_F}\left[\frac{\partial e^i(\kappa_F, L; nc)}{\partial \kappa_F}(1 - e^i(\kappa_F, L; c)) - \frac{\partial e^i(\kappa_F, L; c)}{\partial \kappa_F}(1 - e^i(\kappa_F, L; nc))\right]\end{aligned} \tag{B.2}$$

Equation 5 yields that $\frac{\partial^2 e^i(\kappa_F,L;\tau)}{\partial \kappa_F^2} = 0,\ \tau \in \{c, nc\}$. Further, the term on the second line is negative by Equation B.1 and $\frac{\partial \bar{e}(\kappa_F,L)}{\partial \kappa_F} > 0$. □

For our next preliminary result, denote the proportion of failure when all failed agents are purged $\hat{\alpha}_F(L) = 1 - \overline{v} - \overline{V_2} - L$. Further, define $\mathcal{W}^F(\kappa_F, \kappa_S) \equiv \beta(r - \mu^F(1, L))(W_2(c) - W_2(nc))$ the marginal benefit of purging an agent who failed when the purge incidences after failure and success are κ_F and κ_S, respectively. Our definition is slightly more general than needed (since $\kappa_S^*(L) = 0$ by Lemma 1) so that we can use it when we examine the possibility that the autocrat purges from the success pool in Appendix D.

Lemma B.2. *The unique equilibrium purge incidence* $\kappa_F^*(L)$ *satisfies:*
(i) $\kappa_F^*(L) < 1$ *if and only if* $C_0 + C_1 \times \hat{\alpha}_F(L) > \mathcal{W}^F(1, 0)$*;*
(ii) $\kappa_F^*(L) = 1$ *if and only if* $C_0 + C_1 \times \hat{\alpha}_F(L) \leq \mathcal{W}^F(1, 0)$*.*

Proof. Point (i). Consider the function

$$K_{PD}(\kappa_F, L) = \beta(r - \mu^F(\kappa_F, L))(W_2(c) - W_2(nc)) - \Big(C_0 + C_1\kappa_F\big(1 - (\overline{v} + \kappa_F(\overline{V_2} + L))\big)\Big). \quad \text{(B.3)}$$

The first term of $K_{PD}(\cdot)$ $(\beta(r-\mu^F(\kappa_F, L))(W_2(c)-W_2(nc)))$ corresponds to the autocrat's marginal benefit of purging an additional subordinate in the failure pool. The second term is the marginal cost of purging an additional subordinate in the failure pool. The function takes into account the effect of a change in κ_F on effort and on the autocrat's posterior. Hence, if it exists, the highest value of $\kappa_F \in (0, 1)$ solving $K_{PD}(\kappa_F, L) = 0$ is an equilibrium point (given our imposed restrictions). Observe that $K_{PD}(\kappa_F, L)$ is strictly convex in κ_F (using Lemma B.1). Under the condition of point (i), note that $K_{PD}(1, L) < 0$. Given the properties of $K_{PD}(\cdot, L)$, $K_{PD}(\kappa_F, L)$ crosses 0 either once (from above) or zero.[1] The equilibrium purge incidence is thus unique and equals $\kappa_F^*(L) = 0$ if $K_{PD}(0, L) < 0$ or the unique solution to $K_{PD}(\kappa_F^*(L), L) = 0$ otherwise.
Point (ii). Under the condition of point (ii), we have three possibilities (a) $K_{PD}(\kappa_F, L) \geq 0$ for all $\kappa_F \in [0, 1]$, (b) there exists a unique solution to $K_{PD}(\kappa_F, L) = 0$ (with $K_{PD}(\kappa_F, L)$ crossing 0 from below), (c) there exists two solutions to $K_{PD}(\kappa_F, L) = 0$. In case (a), the unique equilibrium purge incidence is $\kappa_F^*(L) = 1$. In cases (b) and (c), denote $\kappa_F' \in (0, 1)$ an interior solution (unique or not) and $\kappa_F^c = 1$ the corner solution. Since our equilibrium selection selects the purge with the largest purge incidence, the equilibrium must then satisfy $\kappa_F^*(L) = 1$ as claimed. □

As the proof of Lemma B.2 highlights, our equilibrium criterion plays a role only when the conditions of point (ii) of the Lemma are satisfied. Alternative criterion selection might select a different

[1] If $K_{PD}(\kappa_F, L)$ crosses 0 from below at some κ_F' then it must be that $K_{PD}(\kappa_F, L) > 0$ for $\kappa_F > \kappa_F'$ since the function is strictly convex. This contradicts $K_{PD}(1, L) < 0$.

purge inference. For example, it can be checked that the equilibrium criterion based on the autocrat's welfare-maximizing purge inference would select either the highest interior solution or the corner solution (as in our baseline). All our comparative statics would remain unchanged then (at the cost though of complicating the analysis). Selecting the lowest interior purge inference would change some of our comparative statics, but imposing parameter values such that purge inference is continuous (as we do later) would reestablish them. As such, our results are robust to change in the equilibrium criterion.

We first study the effect of an exogenous change in the intensity of violence on beliefs. Note that this is not an equilibrium analysis. Indeed, as we will see below, the purge inference changes with the intensity of violence.

Lemma B.3. *Fixing the purge breadth, in a discriminate purge, $\mu^F(\kappa_F, L)$ is strictly decreasing in L.*

Proof. A similar reasoning as in Lemma 2 yields:

$$\frac{\partial \mu^F(\kappa_F, L)}{\partial L} = \frac{\lambda(1-\lambda)(1-e^i(\kappa_F, L; c))(1-e^i(\kappa_F, L; nc))}{(1-\overline{e}(\kappa_F, L))^2}\left[\frac{\frac{\partial e^i(\kappa_F, L; nc)}{\partial L}}{1-e^i(\kappa_F, L; nc)} - \frac{\frac{\partial e^i(\kappa_F, L; c)}{\partial L}}{1-e^i(\kappa_F, L; c)}\right] \tag{B.4}$$

Using agents' efforts (Equation 5), $\frac{\partial e^i(\kappa_F, L; c)}{\partial L} = \frac{\partial e^i(\kappa_F, L; nc)}{\partial L}$. Since $e^i(\kappa_F, L; nc) < e^i(\kappa_F, L; c)$, $\frac{\partial \mu^F(\kappa_F, L)}{\partial L} < 0$. □

To facilitate the exposition, we use subscript x to denote the partial derivative of some variable z with respect to x (i.e., $\partial z/\partial x = z_x$) and a similar notation for the second partial derivative. We also ignore superscript and arguments whenever possible.

Proof of Proposition 1

First, consider a partially discriminate purge. Observe that $K_{PD}(\kappa_F, L) = \beta(r - \mu^F(\kappa_F, L))\mathcal{D}_2^{c,nc} - \Big(C_0 + C_1\kappa_F\big(1 - (\overline{v} + \kappa_F(\overline{V_2} + L))\big)\Big)$ is strictly increasing in L using Lemma B.3. Since $\kappa_F^*(L)$ is defined as the solution to $K_{PD}(\kappa_F^*(L), L) = 0$ by the Implicit Function Theorem we must have that $\kappa_F^*(L)$ is continuously and strictly increasing with L (recall that $\partial K_{PD}(\kappa_F^*(L), L)/\partial \kappa_F < 0$ from the proof of Lemma B.2).

We now need to consider two cases: (a) $C_0 + C_1\hat{\alpha}(\overline{L}) < \beta r(W_2(c) - W_2(nc))$ and (b) $C_0 + C_1\hat{\alpha}(\overline{L}) \geq$

$\beta r(W_2(c) - W_2(nc))$.

In case (a), there exists a unique $L^{full} < \overline{L}$ such that $\kappa_F^*(L) = 1$ for all $L \geq L^{full}$. To see this, slightly abusing notation (using equalities, instead of limits), observe that at $L = \overline{L}$ and $\kappa_F = 1$, we have $e^i(1, \overline{L}, c) = 1$ by Equation 5 so $\mu^F(1, \overline{L}) = 0$. Further, $K_{PD}(1, \overline{L}) = \beta r \mathcal{D}^{c,nc} - \Big(C_0 + C_1\big(1 - (\overline{v} + \overline{V_2} + \overline{L})\big)\Big) > 0$ (since $\hat{\alpha}_F(\overline{L}) = 1 - \overline{v} - \overline{V_2} - \overline{L}$). By Lemma B.2, point (ii), the equilibrium purge breadth satisfies $\kappa_F^*(\overline{L}) = 1$. Since $\kappa_F^*(L)$ is strictly increasing with L when interior, here exists $L^{full} < \overline{L}$ such that $\kappa_F^*(L) = 1$ for all $L \geq L^{full}$ and $\kappa_F^*(L) < 1$ otherwise.

In case (b), we must have $\kappa_F^*(\overline{L}) < 1$ which implies $\kappa_F^*(L) < 1$ for all L. In this case, arbitrarily pick L^{full} satisfying $L^{full} > \overline{L}$ such that point (i) of Proposition 1 incorporates the full scope of feasible intensity of violence (i.e., $[0, \overline{L}]$). □

In what follows, to focus on the most interesting cases, we impose the following two restrictions on the cost of purging parameters.

Assumption B.1. *The cost of purging parameters satisfy:*
(1) $C_0 < \beta(r - \lambda \frac{1-(v(S,c)+V_2(c))}{1-(\overline{v}+\overline{V_2})})(W(c) - W(nc))$ *and*
(2) $C_0 + C_1\big(1 - (\overline{v} + \overline{V_2} + \overline{L})\big) < \beta r(W(c) - W(nc))$.

Point (1) of Assumption B.1 is equivalent to $K_{PD}(0, 0) > 0$. That is, it guarantees that $\kappa_F^*(0) > 0$. Point (2) of Assumption B.1 is discussed in the proof of Proposition 1. It guarantees that $L^{full} < \overline{L}$ so $\kappa_F^*(L) = 1$ for L sufficiently large. From the onset, let us stress that these restrictions are simply meant to limit the number of cases to be considered. The reasoning can easily be extended to incorporate cases when this assumption is relaxed.[2]

Proof of Proposition 2

Suppose some failures survive ($\kappa_F^*(L) \in (0, 1)$), the purge breadth is $\kappa^*(L) = \alpha_F^*(L)\kappa_F^*(L)$ with $\alpha_F^*(L) = 1 - (\overline{v} + \kappa_F^*(L)(\overline{V_2} + L))$. It is defined by the following equation

$$C_0 + C_1\kappa^*(L) = \beta(r - \mu^F(\kappa_F^*(L), L))(W_2(c) - W_2(nc))$$

The total derivative of the posterior with respect to L is $\frac{d\mu^F(\kappa_F^*(L),L)}{dL} = \frac{\partial \kappa_F^*(L)}{\partial L}\mu^F_{\kappa_F}(\kappa_F^*(L), L) + \mu^F_L(\kappa_F^*(L), L)$, with $\frac{\partial \kappa_F^*(L)}{\partial L} > 0$ (proof of Proposition 1), $\mu^F_{\kappa_F} < 0$ (Lemma B.1), and $\mu^F_L < 0$

[2]For example, if point (1) of Assumption B.1 does not hold, then there exists $L^0 > 0$ such that $\kappa_F^*(L) > 0$ if and only if $L > L^0$ and the comparative statics we characterize holds above say threshold L^0.

(Lemma B.3). So $\frac{d\mu^F(\kappa_F^*(L),L)}{dL} < 0$ and by the Implicit Function Theorem, $d\kappa^*(L)/dL > 0$.
When all failures are purged ($\kappa_F^*(L) = 1$), the purge breadth is simply $\kappa^*(L) = 1 - \overline{e}(1, L) = 1 - (\overline{v} + \overline{V_2} + L)$ strictly decreasing with L. □

Before proving the remaining two propositions of this section, we prove the following property of the purge inference when interior.

Lemma B.4. *Suppose $\kappa_F^*(L) \in (0, 1)$. Then the purge inference is convex in L.*

Proof. Recall that $\kappa_F^*(L)$ is the solution to $K_{PD}(\kappa_F, L) = 0$ with $K_{PD}(\kappa_F, L)$ defined in Equation B.3. Simple algebra yields (using $\mu^F_{\kappa_F\kappa_F} < 0$, $\mu^F_{LL} < 0$ and $\mu^F_{\kappa_F L} < 0$ from $\mu^F = \frac{1-(v(S,c)+\kappa_F(V_2(c)+L))}{1-(\overline{v}+\kappa_F(\overline{V_2}+L))}$, see also the proof of Lemma C.1 below): $\partial^2 K_{PD}(\kappa_F, L)/\partial\kappa_F^2 > 0$, $\partial^2 K_{PD}(\kappa_F, L)/\partial L^2 > 0$, and $\partial^2 K_{PD}(\kappa_F, L)/\partial\kappa_F\partial L > 0$. Totally differentiating at $\kappa_F = \kappa_F^*(L)$, we obtain (ignoring arguments):

$$\frac{\partial^2\kappa_F}{\partial L^2}\frac{\partial K_{PD}}{\partial\kappa_F} + \frac{\partial\kappa_F}{\partial L}\left(\frac{\partial^2 K_{PD}}{\partial\kappa_F^2}\frac{\partial\kappa_F}{\partial L} + 2\frac{\partial^2 K_{PD}}{\partial\kappa_F\partial L}\right) + \frac{\partial^2 K_{PD}}{\partial L^2} = 0$$

Since $\partial K_{PD}(\kappa_F^*(L), L)/\partial\kappa_F < 0$ (Lemma B.2), $\frac{\partial^2\kappa_F}{\partial L^2} > 0$. □

Proof of Proposition 3

When some failures survive the purge ($\kappa_F^*(L) \in (0, 1)$), the total derivative of average effort with respect to violence is (using Equation 5):

$$\frac{d\overline{e}(\kappa_F^*(L), L)}{dL} = \frac{\partial\kappa_F^*(L)}{\partial L}(\overline{V_2} + L) + \kappa_F^*(L) \tag{B.5}$$

From the proof of Proposition 1, $\frac{\partial\kappa_F^*(L)}{\partial L} > 0$ so $\frac{d\overline{e}(\kappa_F^*(L),L)}{dL} > 0$. Further given the convexity of $\kappa_F^*(L)$ (Lemma B.4), effort is also convex in the intensity of violence: $\frac{d^2\overline{e}(\kappa_F^*(L),L)}{dL^2} > 0$.
Suppose that there exists a unique solution to $K_{PD}(\kappa_F, L) = 0$ (we provide a precise condition for this assumption to hold below, see Equation C.11). As $L \to L^{full}$, $\kappa_F^*(L) \to 1$. Since $\frac{\partial\kappa_F^*(L)}{\partial L}$ is continuous and increasing in L, so is $\frac{d\overline{e}(\kappa_F^*(L),L)}{dL}$ and, given the convexity of effort in L, there exists a unique $L^{eff} \in [0, L^{full})$ such that for all $L > L^{eff}$, $\frac{d\overline{e}(\kappa_F^*(L),L)}{dL} > 1$. When there are multiple solutions to $K_{PD}(\kappa_F, L) = 0$, there exists a discontinuity in the equilibrium purge inference using Lemma B.2. Then, $\lim_{L\uparrow L^{full}} \kappa_F^*(L) < 1$. If $\lim_{L\uparrow L^{full}} \frac{d\overline{e}(\kappa_F^*(L),L)}{dL} > 1$, then there exists a unique $L^{eff} < L^{full}$ such that the claim holds (by convexity of average effort). If not, denote $L^{eff} = L^{full}$ which is uniquely defined under Assumption B.1.
When all failures are purged ($\kappa_F^*(L) = 1$), average effort is simply: $\overline{e}(1, L) = \overline{v} + \overline{V_2} + L$ so

$\frac{d\overline{e}(\kappa_F^*(L),L)}{dL} = 1.$ □

To ease the exposition, we ignore the equilibrium '$*$' superscript and arguments whenever possible in what follows.

Proof of Proposition 4

Point (i). For $L < L^{full}$, the proportion of ideologues in the pool of survivors is:

$$\begin{aligned}\mathcal{S}(L) =& \frac{(1-\overline{e})(1-\kappa_F)\mu^F + \overline{e}\mu^S}{1-(1-\overline{e})\kappa_F} \\ =& \frac{\lambda - (1-\overline{e})\mu^F\kappa_F}{1-(1-\overline{e})\kappa_F}\end{aligned} \tag{B.6}$$

Consider the function $F(x) = \frac{1-x\mu^F}{1-x}$, its derivative is $F'(x) = \frac{(\lambda-\mu^F)}{(1-x)^2} > 0$.

We obtain

$$\frac{d\mathcal{S}(L)}{dL} = \lambda\frac{d(1-\overline{e}(\kappa_F(L),L))\kappa_F(L)}{dL}F'((1-\overline{e}(\kappa_F(L),L))\kappa_F(L)) - \frac{d\mu^F(\kappa_F(L),L)}{dL}\frac{(1-\overline{e})\kappa_F}{1-(1-\overline{e})\kappa_F} \tag{B.7}$$

From the proof of Proposition 2, we know that $(1-\overline{e}(\kappa_F(L),L))\kappa_F(L) = \kappa(L)$ and $d\kappa(L)/dL > 0$, and $\frac{d\mu^F(\kappa_F(L),L)}{dL} < 0$. Hence $\frac{d\mathcal{S}(L)}{dL} > 0$

For $L \geq L^{full}$, the proportion of ideologues in the pool of survivors is

$$\mathcal{S}(L) = \mu^S \tag{B.8}$$

Since $\mu^S = \frac{\lambda e(\kappa_F,L;\tau)}{\overline{e}(\kappa_F,L)}$, we obtain $\mu^S_L < 0$ and $\frac{d\mathcal{S}(L)}{dL} < 0$.

Point (ii). For $L < L^{full}$, the proportion of congruent subordinates in the second period is:

$$\begin{aligned}\mathcal{P}(L) =& (1-\overline{e})\kappa_F r + (1-\overline{e})(1-\kappa_F)\mu^F + \overline{e}\mu^S \\ =& (1-\overline{e})\kappa_F(r-\mu^F) + \lambda\end{aligned} \tag{B.9}$$

Given $r = \lambda > \mu^F$, $d(1-\overline{e})\kappa_F/dL > 0$, and $d\mu^F/dL < 0$ so $\frac{d\mathcal{P}(L)}{dL} > 0$.

For $L \in [L^{full}, \overline{L})$, the proportion of congruent subordinates in the second period is:

$$\mathcal{P}(L) = (1-\overline{e})r + \overline{e}\mu^S \tag{B.10}$$

Using $\mu^S = \lambda\frac{v(S,c)+V_2(c)+L}{\overline{v}+\overline{V_2}+L}$ and $\overline{e} = \overline{v} + \overline{V_2} + L$, we obtain: $\mathcal{P}(L) = (1-(\overline{v}+\overline{V_2}+L))r + \lambda(v(S,c) + V_2(c) + L)$ and

$$\frac{d\mathcal{P}(L)}{dL} = \lambda - r \tag{B.11}$$

Under our assumption that $\lambda = r$, $\mathcal{P}_L(L) = 0$ as claimed. □

C Proofs of 'Intensity of violence'

Denote $B(L)$ the autocrat's expected benefit from violence. The next Lemmas characterize the properties of $B(L)$ ignoring superscript and arguments whenever possible. In what follows, we still assume that Assumption B.1 holds so $L^{full} < \overline{L}$.

Lemma C.1. *The expected benefit of violence is $\mathcal{C}^{\infty}$, strictly increasing, and convex in L for $L \in [0, L^{full})$.*

Proof. For $L \leq L^{full}$, the expected benefit of violence is:

$$\begin{aligned}
B(L) =& \overline{e} + \beta(1-\overline{e})\Big(\kappa_F(rW_2(c) + (1-r)W_2(nc)) + (1-\kappa_F)(\mu^F W_2(c) + (1-\mu^F)W_2(nc))\Big) \\
&+ \beta\overline{e}\Big(\mu^S W_2(c) + (1-\mu^S)W_2(nc)\Big) - C((1-\overline{e})\kappa_F) \\
B(L) =& \overline{e} + \beta\Big(\lambda W_2(c) + (1-\lambda)W_2(nc)\Big) + \beta(1-\overline{e})\kappa_F(r-\mu^F)(W_2(c) - W_2(nc)) - C((1-\overline{e})\kappa_F)
\end{aligned} \tag{C.1}$$

Since in the interval $[0, L^{full}]$ all functions are $\mathcal{C}^{\infty}$, so is $B(L)$.

Making use of the Envelop Theorem as κ_F^* is interior, we obtain:

$$\frac{dB(L)}{dL} = \frac{d\overline{e}(\kappa_F^*(L), L)}{dL} - \beta(1-\overline{e})\kappa_F \frac{d\mu^F(\kappa_F^*(L), L)}{dL}(W_2(c) - W_2(nc)) \tag{C.2}$$

Since $\frac{d\overline{e}(\kappa_F^*(L),L)}{dL} > 0$ (Proposition 3) and $\frac{d\mu^F(\kappa_F^*(L),L)}{dL} < 0$ (proof of Proposition 2), $dB(L)/dL > 0$.

To prove that the expected benefit of violence is strictly convex, we proceed in three steps. First, we compute the second (total) derivative of the marginal benefit of violence. Second, we look at the second (partial) derivatives of effort and autocrat's posterior with respect to L and κ_F. The last step proves the claim.

Step 1. Using Equation C.2, we obtain:

$$\begin{aligned}
\frac{d^2B(L)}{dL^2} =& \frac{d^2\overline{e}(\kappa_F^*(L), L)}{dL^2} - \beta(1-\overline{e})\kappa_F \frac{d^2\mu^F(\kappa_F^*(L), L)}{dL^2}(W_2(c) - W_2(nc)) \\
&- \beta\frac{d(1-\overline{e})\kappa_F}{dL}\frac{d\mu^F(\kappa_F^*(L), L)}{dL}(W_2(c) - W_2(nc)),
\end{aligned} \tag{C.3}$$

with

$$\frac{d^2\overline{e}(\kappa_F^*(L), L)}{dL^2} = 2\frac{\partial\kappa_F^*(L)}{\partial L} + \frac{\partial^2\kappa_F^*(L)}{\partial L^2}(\overline{V_2} + L) \tag{C.4}$$

$$\frac{d^2\mu^F(\kappa_F^*(L), L)}{dL^2} = \mu^F_{LL} + 2\frac{\partial\kappa_F^*(L)}{\partial L}\mu^F_{\kappa_F L} + \left(\frac{\partial\kappa_F^*(L)}{\partial L}\right)^2\mu^F_{\kappa_F\kappa_F} + \frac{\partial^2\kappa_F^*(L)}{\partial L^2}\mu^F_{\kappa_F} \tag{C.5}$$

Step 2. Using $\mu^F = \lambda\frac{1-e(i)}{1-\overline{e}}$, we obtain for $j \in \{\kappa, L\}$:

$$\mu^F_{jj} = \frac{\lambda(1-\lambda)(1-e(c))(1-e(nc))}{(1-\overline{e})^3} \times \left[\left(\frac{e_{jj}(nc)}{(1-e(nc))} - \frac{e_{jj}(c)}{(1-e(c))}\right)(1-\overline{e}) + 2\overline{e}_j\left(\frac{e_j(nc)}{1-e(nc)} - \frac{e_j(c)}{1-e(c)}\right)\right] \tag{C.6}$$

$$\begin{aligned}
\mu^F_{\kappa_F L} =& \frac{\lambda(1-\lambda)(1-e(c))(1-e(nc))}{(1-\overline{e})^3} \times \left[\left(\frac{e_{\kappa_F L}(nc)}{(1-e(nc))} - \frac{e_{\kappa_F L}(c)}{(1-e(c))}\right)(1-\overline{e}) + \overline{e}_L\left(\frac{e_{\kappa_F}(nc)}{1-e(nc)} - \frac{e_{\kappa_F}(c)}{1-e(c)}\right)\right] \\
&+ \frac{\lambda(1-\lambda)}{(1-\overline{e})^3}\left[\overline{e}_L(e_{\kappa_F}(nc)(1-e(c)) - e_{\kappa_F}(c)(1-e(nc))) + (1-\overline{e})(e_{\kappa_F}(c)e_L(nc) - e_L(c)e_{\kappa_F}(nc))\right] \\
=& \frac{\lambda(1-\lambda)(1-e(c))(1-e(nc))}{(1-\overline{e})^3} \times \left[\left(\frac{e_{\kappa_F L}(nc)}{(1-e(nc))} - \frac{e_{\kappa_F L}(c)}{(1-e(c))}\right)(1-\overline{e}) + \overline{e}_L\left(\frac{e_{\kappa_F}(nc)}{1-e(nc)} - \frac{e_{\kappa_F}(c)}{1-e(c)}\right)\right] \\
&+ \frac{\lambda(1-\lambda)}{(1-\overline{e})^3}\left[e_{\kappa_F}(nc)\big(\overline{e}_L(1-e(c)) - e_L(c)(1-\overline{e})\big) + e_{\kappa_F}(c)\big(e_L(nc)(1-\overline{e}) - \overline{e}_L(1-e(nc))\big)\right] \\
\mu^F_{\kappa_F L} =& \frac{\lambda(1-\lambda)(1-e(c))(1-e(nc))}{(1-\overline{e})^3} \times \left[\left(\frac{e_{\kappa_F L}(nc)}{(1-e(nc))} - \frac{e_{\kappa_F L}(c)}{(1-e(c))}\right)(1-\overline{e}) + \overline{e}_L\left(\frac{e_{\kappa_F}(nc)}{1-e(nc)} - \frac{e_{\kappa_F}(c)}{1-e(c)}\right)\right] \\
&+ \frac{\lambda(1-\lambda)(1-e(c))(1-e(nc))}{(1-\overline{e})^3}\left[((1-\lambda)e_{\kappa_F}(nc) + \lambda e_{\kappa_F}(c))\left(\frac{e_L(nc)}{(1-e(nc))} - \frac{e_L(c)}{(1-e(c))}\right)\right]
\end{aligned} \tag{C.7}$$

Using $e(\tau) = v(\tau) + \kappa_F(V_2(\tau) + L)$, we obtain:

$$\begin{aligned}
e_{\kappa_F\kappa_F}(\tau) =& 0 \\
e_{LL}(\tau) =& 0 \\
e_{\kappa_F L}(\tau) =& 1
\end{aligned}$$

This implies that $\mu^F_{LL} < 0$ ($\mu^F_{\kappa_F\kappa_F} < 0$ by Lemma B.1) and $\mu^F_{L\kappa_F} < 0$ (since $e(c) > e(nc)$ and $e_j(c) \geq e_j(nc),\ j \in \{\kappa_F, L\}$).

Step 3. Plugging all partial derivatives into Equation C.4 and Equation C.5 and given that $\kappa^*_F(L)$ is convex in L (Lemma B.4), we obtain: $\frac{d^2\overline{e}(\kappa^*_F(L),L)}{dL^2} > 0$ and $\frac{d^2\mu^F(\kappa^*_F(L),L)}{dL^2} < 0$. Given that $\frac{d(1-\overline{e})\kappa_F}{dL} > 0$ and $\frac{d(1-\overline{e})\kappa_F}{dL}\frac{d\mu^F(\kappa^*_F(L),L)}{dL} < 0$ (see Proposition 2), Equation C.3 yields $\frac{d^2B(L)}{dL^2} > 0$. □

Lemma C.2. *The expected benefit of violence is $\mathcal{C}^\infty$, strictly increasing, and strictly concave in L for $L \in [L^{full}, \overline{L}]$.*

Proof. For $L \in [L^{full}, \overline{L}]$, the expected benefit of violence is

$$B(L) = \overline{e} + \beta(1-\overline{e})\Big(rW_2(c) + (1-r)W_2(nc)\Big) + \beta\overline{e}\Big(\mu^S W_2(c) + (1-\mu^S)W_2(nc)\Big) - C(1-\overline{e}), \tag{C.8}$$

with $e(\tau) = v(\tau) + V(\tau) + L,\ \tau \in \{c, nc\}$. Taking the derivative, we obtain:

$$\frac{dB(L)}{dL} = 1 + \beta(\lambda - r)(W_2(c) - W_2(nc)) + C'(1 - \bar{e}) \tag{C.9}$$

Since $v(S,\tau) < 1,\ \tau \in \{c, nc\}$ and $\mathcal{D}^{c,nc} = v(S,c) - \max\{0, v(S,nc)\} < 1$, we obtain: $\frac{dB(L)}{dL} > 0$. Further, from Equation C.9,

$$\frac{d^2B(L)}{dL^2} = -C''(1 - \bar{e}) < 0 \tag{C.10}$$

□

From Proposition 1, L^{full} is the unique solution to $K_{PD}(1, L) = 0 \Leftrightarrow C_0 + C_1(1 - (\overline{v} + \overline{V_2} + L)) = \beta(r - \mu^F(1, L))(W_2(c) - W_2(nc))$. The next Lemma establishes a necessary and sufficient condition such that $B(L)$ is continuous in L.

Lemma C.3. *The expected benefit of violence $B(L)$ is continuous if and only if*

$$\frac{\partial K_{PD}(1, L^{full})}{\partial \kappa_F} \leq 0 \tag{C.11}$$

Proof. If the condition stated in (C.11) does not hold, it must be that at $L = L^{full}$, there exists two solutions to $K_{PD}(\kappa_F, L^{full}) = 0$: $\kappa'_F(L) \in (0, 1)$ and $\kappa''_F(L) = 1$. Using the proof of Lemma B.2, we then have that $\lim_{L \uparrow L^{full}} \kappa^*_F(L) < 1 = \lim_{L \downarrow L^{full}} \kappa^*_F(L)$ (recall we select the highest purge inference). Rearranging Equation C.1, we obtain for $L < L^{full}$:

$$B(L) = \bar{e} + \beta W_2(nc) + \beta\lambda(W_2(c) - W_2(nc)) + \beta(1 - \bar{e})\kappa_F(r - \mu^F)(W_2(c) - W_2(nc)) - C((1 - \bar{e})\kappa_F)$$

Rearranging Equation C.8, we obtain for $L \geq L^{full}$ (using $\mu^F = \lambda\frac{1-e(c)}{1-\bar{e}}$ and $\mu^S = \lambda\frac{e(c)}{\bar{e}}$):

$$\begin{aligned}
B(L) =& \bar{e} + \beta(1 - \bar{e})W_2(nc) + \beta(1 - \bar{e})r(W_2(c) - W_2(nc)) + \beta\bar{e}W_2(nc) + \beta\bar{e}\mu^S(W_2(c) - W_2(nc)) - C(1 - \bar{e}) \\
=& \bar{e} + \beta W_2(nc) + \beta(1 - \bar{e})r(W_2(c) - W_2(nc)) + \beta\lambda e(c)(W_2(c) - W_2(nc)) - C(1 - \bar{e}) \\
=& \bar{e} + \beta W_2(nc) + \beta(1 - \bar{e})r(W_2(c) - W_2(nc)) + \beta\lambda(W_2(c) - W_2(nc)) \\
& - \beta\lambda(1 - e(c))(W_2(c) - W_2(nc)) - C(1 - \bar{e}) \\
=& \bar{e} + \beta W_2(nc) + \beta\lambda(W_2(c) - W_2(nc)) + \beta(1 - \bar{e})(r - \mu^F)(W_2(c) - W_2(nc)) - C((1 - \bar{e}))
\end{aligned}$$

For any interior solution for the purge inference $\kappa^*_F(L)$, using the quadratic cost of purging, we obtain $B(L) = \bar{e} + \beta W_2(nc) + \beta\lambda(W_2(c) - W_2(nc)) + \frac{C_1}{2}((1 - \bar{e})\kappa^*_F(L))^2$. Since at $L = L^{full}$, both $\kappa'_F(L) \in (0, 1)$ and $\kappa''_F(L) = 1$ are solution of $K_{PD}(\kappa_F, L) = 0$ and $\bar{e}(\kappa'_F(L^{full}), L^{full}) <$

$\overline{e}(\kappa''_F(L^{full}), L^{full})$, we obtain (slightly abusing notation) that $B(L)|_{\kappa_F=\kappa'_F(L)} < B(L)|_{\kappa_F=\kappa''_F(L)}$. Consequently, $\lim_{L\uparrow L^{full}} B(L) < \lim_{L\downarrow L^{full}} B(L)$ and $B(L)$ is not continuous.

In turn, suppose the condition stated in (C.11) holds. Then $\kappa^*_F(L^{full}) = 1$ is the unique solution to $K_{PD}(\kappa_F, L^{full}) = 0$ and $\kappa^*_F(L)$ is continuous in L for all L. As a result, after rearranging Equation C.1 and Equation C.8 as well as using $\overline{e}(1, L^{full})\mu^S(1, L^{full}) = \lambda e(1, L^{full}; c)$ and $(1 - \overline{e}(1, L^{full}))\mu^F(1, L^{full}) = \lambda(1 - e(1, L^{full}; c))$, we obtain $\lim_{L\uparrow L^{full}} B(L) = \overline{e}(1, L^{full}) + \beta W_2(nc) + \beta\big((1 - \overline{e}(1, L^{full}))r + \lambda e(1, L^{full}; c)\big)(W_2(c) - W_2(nc)) - C(1 - \overline{e}(1, L^{full})) = \lim_{L\downarrow L^{full}} B(L)$. Hence $B(L)$ is continuous for all L. □

In what follows, we assume that the condition stated in (C.11) holds. Observe that since the properties of $B(L)$ (Lemmas C.1 and C.2) do not depend on the continuity of $B(L)$, the analysis below remains valid. To find the equilibrium intensity of violence when Equation C.11 does not hold, in addition to the analysis below, it is necessary to consider cases when the marginal cost interacts the marginal benefit before and after the discontinuity at $L = L^{full}$. Consequently, assuming that $B(L)$ is continuous simply limits the number of cases to be analyzed.[3] We nonetheless establishes existence and (generically) uniqueness of an equilibrium when the condition stated in (C.11) does not hold and $B(L)$ is not continuous in Remark C.1 below.

Lemma C.4. *The marginal benefit of violence satisfies:*

$$\lim_{L\uparrow L^{full}} \frac{dB(L)}{dL} > \frac{dB(L_1)}{dL} \text{ for all } L_1 \in [L^{full}, \overline{L}]$$

Proof. The proof of a discontinuity in the marginal benefit at $L = L^{full}$ proceeds in three steps. First, we show that $-\kappa(L)\frac{d\mu^F}{dL} > \lambda - \mu^F$ as $L \uparrow L^{full}$. Second, we show that $\frac{dB(L)}{dL} = 1 + \beta(\lambda - \mu^F)(W_2(c) - W_2(nc))$ as $L \downarrow L^{full}$, Finally, we prove the claim.

Step 1. Using the definition of μ^F, $-(1-\overline{e})\kappa_F(L)\frac{d\mu^F}{dL} > -\kappa_F(L)(1-\overline{e})\mu^F_L$ for all $L \in [0, L^{full}]$, we obtain

$$\begin{aligned} -(1-\overline{e})\kappa_F(L)\frac{d\mu^F}{dL} >& \kappa_F(L)\left((1-\overline{e})\lambda\frac{e_L(c)(1-\overline{e}) - \overline{e}_L(1-e(c))}{(1-\overline{e})^2}\right) \\ =& \kappa_F(L)(\lambda e_L(c) - \mu^F\overline{e}_L) \end{aligned}$$

As $e_L(\tau) = \kappa_F(L)$ (Equation 5) and $\kappa_F(L) \xrightarrow{L\uparrow L^{full}} 1$, we obtain that $\kappa_F(L)(\lambda e_L(c) - \mu^F\overline{e}_L) \xrightarrow{L\uparrow L^{full}} \lambda - \mu^F$.

[3] Further, when $B(L)$ is not continuous, there exists additional conditions such that small changes in parameter values can lead to discontinuous changes in the equilibrium intensity of violence, purge breadth, and effort. It would reinforce the result described in Remark 1.

Step 2. As $L \downarrow L^{full}$, $C'(1-\overline{e}) = \beta(r-\mu^F)(W_2(c) - W_2(nc))$. Hence, we can rewrite Equation C.9 as $L \downarrow L^{full}$ as (slightly abusing notation by using equalities):

$$\begin{aligned}\frac{dB(L)}{dL} =& 1 + \beta(\lambda - r(W_2(c) - W_2(nc)) + \beta(r-\mu^F)(W_2(c) - W_2(nc)) \\ =& 1 + \beta(\lambda - \mu^F)(W_2(c) - W_2(nc))\end{aligned}$$

Step 3. From Proposition 3, $\lim\limits_{L\uparrow L^{full}} \frac{d\overline{e}}{dL} > 1$. Using Equation C.2 and step 2, as $L \uparrow L^{full}$

$$\begin{aligned}\frac{dB(L)}{dL} =& \frac{d\overline{e}}{dL} - \beta(1-\overline{e})\frac{d\mu^F}{dL}\mathcal{D}^{c,nc} \\ & > 1 + \beta(r-\mu^F)\mathcal{D}^{c,nc},\end{aligned}$$

which proves $\lim_{L\uparrow L^{full}} \frac{dB(L)}{dL} > \lim_{L\downarrow L^{full}} \frac{dB(L)}{dL}$.
Since $B(L)$ is strictly concave for $L \in (L^{full}, L^{ind}]$, this directly implies that $\lim_{L\uparrow L^{full}} \frac{dB(L)}{dL} > \frac{dB(L_1)}{dL}$ for all $L_1 \in [L^{full}, \overline{L}]$. □

The next Lemma establishes existence of an equilibrium intensity of violence when the condition stated in C.11) holds.

Lemma C.5. *There exists an equilibrium intensity of violence.*

Proof. Existence follows from the fact that $B(L)$ is continuous (Lemma C.3) and the maximization problem is over a compact set $[0, \overline{L}]$. □

We know look at some properties of the equilibrium intensity of violence (Remarks 1 and 2). We first state a more formal version of Remark 1 in the following Lemma.

Lemma C.6. *There exists a non-measure zero set of parameter values $\mathcal{P}^d$ such that if $(\lambda, r, v(S,c), v(S,nc), C_0, \zeta_0) \in \mathcal{P}^d$, there exists C_1^d and ζ_1^d satisfying $\lim\limits_{C_1\uparrow C_1^d} L^* < \lim\limits_{C_1\downarrow C_1^d} L^*$ and $\lim\limits_{\zeta_1\uparrow \zeta_1^d} L^* < \lim\limits_{\zeta_1\downarrow \zeta_1^d} L^*$.*

Proof. The procedure is as such. Step 1: Pick $(\lambda', r', v(S,c)', v(S,nc)', C_0', \zeta_0') \in [0,1]^3 \times [0, v(S,c)] \times \mathbb{R}_+^2$. Step 2: Check whether there exists C_1^d satisfying point (2) of Assumption B.1 and $\zeta_1^d \in \mathbb{R}_+$ such that (i) there exists a local maximum of $B(L) - \zeta(L)$ in $[0, L^{full}]$, which we denote L^1 and (ii) L^1 satisfies $B(L^1) = B(L^{full})$ (notice that C_1^d and ζ_1^d are unique if they exist). Step 3: If conditions (i) and (ii) hold then $(\lambda', r', v(S,c)', v(S,nc)', C_0', \zeta_0') \in \mathcal{P}^d$, if not $(\lambda', r', v(S,c)', v(S,nc)', C_0', \zeta_0') \notin \mathcal{P}^d$. Repeat the steps for all possible $(\lambda, r, v(S,c), v(S,nc), C_0, \zeta_0)$. $\mathcal{P}^d$ is non-empty as we can always pick C_1 such that a discriminate purge with all failures purged is possible and ζ_0 and ζ_1 such that conditions (i) and (ii) hold by convexity of the marginal benefit (Lemma C.1). $\mathcal{P}^d$ is not measure

0 as we can always perturb the parameters slightly and adjust ζ_0 and ζ_1. Due to the convexity of the marginal benefit of violence and conditions (i) and (ii), the claim holds directly.[4] $\square$

Proof of Remark 1

Follows directly from Lemma C.6. $\square$

For our next result, we characterize a condition such that the equilibrium intensity of violence is (weakly) lower than the threshold L^{full}.

Lemma C.7. *If at* $L = L^{full}$, $\zeta_0 + \zeta_1 L > \frac{d\bar{e}(\kappa_F^*(L),L)}{dL} - \beta(1-\bar{e})\kappa_F \frac{d\mu^F(\kappa_F^*(L),L)}{dL}(W_2(c) - W_2(nc))$, *then* $L^* < L^{full}$.

Proof. Suppose the claim holds. Recall that $\lim_{L \uparrow L^{full}} B(L) = \frac{d\bar{e}(\kappa_F^*(L),L)}{dL} - \beta(1-\bar{e})\kappa_F \frac{d\mu^F(\kappa_F^*(L),L)}{dL}(W_2(c) - W_2(nc))\Big|_{L=L^{full}}$ (proof of Lemma C.4). By Lemma C.4, it must then be that $\zeta_0 + \zeta_1 L > B(L)$ for all $L \geq L^{full}$. Hence, $L^* < L^{full}$. Notice that the inequality is well-defined since L^{full} is the solution of $K_{PD}(1, L) = 0$ and does not depend on ζ_0 and ζ_1, see Equation B.3. $\square$

Proof of Remark 2

The claim follows directly from Lemma C.7 which establishes an upper bound on the possible value of ζ_0 and ζ_1 such that $L^* \geq L^{full}$ and the purge is discriminate.

Note that the claim only establishes a necessary condition, but not a sufficient one. To this suppose that $\lim_{L \downarrow L^{full}} \frac{dB(L)}{dL} < \zeta_0 + \zeta_1 L^{full} \leq \lim_{L \uparrow L^{full}} \frac{dB(L)}{dL}$, Given that $B(L)$ is convex over the interval $[0, L^{full})$, L^{full} is not necessarily the unique argmax of the autocrat's problem. If the equation $\zeta_0 + \zeta_1 L = \frac{dB(L)}{dL}$ has no solution or a single solution (which must be a local minimum) in the interval $[0, L^{full})$, then the optimal choice satisfies $L^* = L^{full}$. However, if $\zeta_0 + \zeta_1 L = \frac{dB(L)}{dL}$ admits two solutions, then the smallest solution L^1 is a local maximum, whereas the highest solution L^2 is a local minimum. The autocrat then compares $B(L^1)$ and $B(L^{full})$ and she may choose $L^* = L^1$. $\square$

For completeness, we discuss the equilibrium intensity of violence when the benefit of violence is not continuous.

[4] It is important to observe that for all $(\lambda, r, v(S,c), v(S,nc), C_0, \zeta_0) \in \mathcal{P}^d$, the condition described in the text of the proposition is knife-edge. However, the properties of $\mathcal{P}^d$ indicate that this knife edge condition can arise for a non-trivial set of parameter values.

Remark C.1. *Suppose the condition stated in C.11 does not hold and $B(L)$ exhibits a discontinuity at $L = L^{full}$. There exists a generically unique equilibrium intensity of violence.*

Proof. We amend the proof of Remark 2 to take into account the discontinuity at $L = L^{full}$ (Lemma C.3). First, note that $B(L)$ is always bounded so a maximum exists. Suppose there exists L' such that $\zeta_0 + \zeta_1 L' = \frac{dB(L')}{dL}$ and $L' < L^{full}$. If there exists $L'' > L^{full}$ such that $\zeta_0 + \zeta_1 L'' = \frac{dB(L'')}{dL}$, then the equilibrium intensity of violence satisfies $L^* = \arg \max_{L \in \{L', L''\}} B(L)$ and is generically unique. If there is no such L'', then the equilibrium intensity of violence satisfies $L^* = \arg \max_{L \in \{L', L^{full}\}} B(L)$ and is generically unique. □

D Proofs of 'Another form of purges'

In this Appendix, we consider semi-indiscriminate purge which (recall) we define as mass purges with some successful agents purged ($\kappa_S^*(L) > 0$). Using the proof of Lemma 1, $\kappa_S > 0$ only if $\kappa_F = 1$. This yields Equation 7 from Equation 5. The next preliminary lemmas discuss some properties of the equation determining the autocrat's marginal benefit of purging a greater proportion of successful agent: $\mathcal{W}^S = [r - \mu^S](W_2(c) - W_2(nc))$ (Equation 8). As before, this is equivalent to considering the comparative statics of the posterior with respect to the purge inference κ_S and the intensity of violence L.

To do so, we denote $e^i(\kappa_S, L; \tau) = (1 - \kappa_S)(v(S, \tau) + V_2(\tau) + L)$ a type-τ agent's effort in a semi-indiscriminate purge. Average effort is $\overline{e}(\kappa_S, L) = (1 - \kappa_S)(\overline{v} + \overline{V_2} + L)$. The autocrat's posterior (slightly abusing notation) is denoted $\mu^S(\kappa_S, L)$.

Lemma D.1. *In a semi-indiscriminate purge, the autocrat's posterior after success is:*

(i) constant in κ_S;

(ii) strictly decreasing in L.

Proof. Point (i). By Bayes' rule, the autocrat's posterior after success is: $\mu^S(\kappa_S, L) = \lambda \frac{e^i(\kappa_S, L; c)}{\overline{e}(\kappa_S, L)}$. By a similar reasoning as in the proof of Lemma B.1, we obtain:

$$\begin{aligned}\frac{\partial \mu^S(\kappa_S, L)}{\partial \kappa_S} =& \frac{\lambda}{\overline{e}(\kappa_S, L)^2}\Bigg[\frac{e^i(\kappa_S, L; c)}{\partial \kappa_S}(\lambda e^i(\kappa_S, L; c) + (1-\lambda) e^i(\kappa_S, L; nc)) \\ & - e^i(\kappa_S, L; c)\left(\lambda \frac{\partial e^i(\kappa_S, L; c)}{\partial \kappa_S} + (1-\lambda)\frac{\partial e^i(\kappa_S, L; nc)}{\partial \kappa_S}\right)\Bigg] \\ =& \frac{\lambda(1-\lambda) e^i(\kappa_S, L; c) e^i(\kappa_S, L; nc)}{\overline{e}(\kappa_S, L)^2}\left[\frac{\frac{\partial e^i(\kappa_S, L; c)}{\partial \kappa_S}}{e^i(\kappa_S, L; c)} - \frac{\frac{\partial e^i(\kappa_S, L; nc)}{\partial \kappa_S}}{e^i(\kappa_S, L; nc)}\right]\end{aligned} \quad \text{(D.1)}$$

Using Equation 5, we obtain: $\frac{\frac{\partial e^i(\kappa_S,L;\tau)}{\partial \kappa_S}}{e^i(\kappa_S,L;\tau)} = \frac{1}{1-\kappa_S}$, $\tau \in \{c, nc\}$. So $\frac{\partial \mu^S(\kappa_S,L)}{\partial \kappa_S} = 0$ as claimed.

Point (ii). Regarding the comparative statics with respect to L, a similar reasoning as above yields:

$$\frac{\partial \mu^S(\kappa_S, L)}{\partial L} = \frac{\lambda(1-\lambda)e^i(\kappa_S, L; c)e^i(\kappa_S, L; nc)}{\overline{e}(\kappa_S, L)^2}\left[\frac{\frac{\partial e^i(\kappa_S,L;c)}{\partial L}}{e^i(\kappa_S, L; c)} - \frac{\frac{\partial e^i(\kappa_S,L;nc)}{\partial L}}{e^i(\kappa_S, L; nc)}\right] \tag{D.2}$$

Using agents' efforts (Equation 5), $\frac{\partial e^i(\kappa_S,L;c)}{\partial L} = \frac{\partial e^i(\kappa_S,L;nc)}{\partial L}$. Since $e^i(\kappa_S, L; nc) < e^i(\kappa_S, L; c)$, $\frac{\partial \mu^F(\kappa_F,L)}{\partial L} < 0$. □

We now establish some conditions on the cost of purging so that the purge is semi-indiscriminate. Recall that $\hat{\alpha}(L) = 1 - \overline{v} - \overline{V_2} - L$ corresponds to the proportion of failures when all failures are purged.

Lemma D.2. *The purge is semi-indiscriminate if and only if* $C_0+C_1\times\hat{\alpha}_F(L) < \beta(r-\mu^S(0,L))(W_2(c)-W_2(nc))$.

Proof. Consider the function:

$$K_{SD}(\kappa_S, L) = \beta(r - \mu^S(\kappa_S, L))(W_2(c) - W_2(nc)) - \Big(C_0 + C_1\big(1 - (1-\kappa_S)^2(\overline{v} + \overline{V_2} + L)\big)\Big). \tag{D.3}$$

As in the proof of Lemma B.2, the first term corresponds to the marginal benefit of purging an additional agent in the success pool and the second term to the associated marginal benefit. Hence, the highest value of κ_S such that $K_{SD}(\kappa_S, L) = 0$ (if it exists) is an equilibrium point.

Using Lemma D.1, $K_{SD}(\kappa_S, L)$ is decreasing and convex in κ_S. Under the condition of the lemma, $K_{SD}(0, L) > 0$. Further, given our assumption that $\beta r(W_2(c) - W_2(nc)) < C_0 + C_1$, $\lim_{\kappa_S \to 1} K_{SD}(\kappa_S, L) < 0$. This implies that there exists a unique interior solution to $K_{SD}(\kappa_S, L) = 0$ and thus a unique equilibrium purge incidence $\kappa_S^*(L) > 0$. □

Proof of Proposition 5

Point (i). It can be checked that as in the proof of Proposition 1 highlights, if $C_0 + C_1\hat{\alpha}(\overline{L}) \geq \beta r(W_2(c) - W_2(nc)$, then $\kappa_F^*(L) < 1$ for all L. In this case, denote $L^{ind} = \overline{L}$. Suppose in what follows that $C_0 + C_1\hat{\alpha}(\overline{L}) < \beta r(W_2(c) - W_2(nc)$ so that there exists $L^{full} < \overline{L}$ such that for $L > L^{full}$, the purge is discriminate or semi-indiscriminate.

Recall that $K_{SD}(\kappa_S, L) = \beta(r-\mu^S(\kappa_S, L))(W_2(c)-W_2(nc)) - \Big(C_0+C_1\big(1-(1-\kappa_S)^2(\overline{v}+\overline{V_2}+L)\big)\Big)$ and $K_{PD}(\kappa_F, L) = \beta(r-\mu^F(\kappa_F, L))(W_2(c)-W_2(nc)) - \Big(C_0+C_1\kappa_F\big(1-(\overline{v}+\kappa_F(\overline{V_2}+L))\big)\Big)$. Observe that at $L = L^{full}$, $K_{SD}(0, L^{full}) < 0$ (since $\mu^F(\cdot) < \mu^S(\cdot)$ and $K_{PD}(1, L^{full}) = 0 > K_{SD}(0, L^{full})$.

We need to consider two cases.

Case (a) $K_{SD}(0,\overline{L}) \leq 0$. In this case, it is never profitable for the autocrat to purge from the success pool. In this case, pick an arbitrary L^{ind} satisfying $L^{ind} > \overline{L}$.

Case (b) $K_{SD}(0,\overline{L}) > 0$. By Lemma D.2, it must then be that $\kappa_S^*(\overline{L}) > 0$. By Lemma D.1, $\mu^S(\kappa_S, L)$ is strictly increasing with L so by the Implicit Function Theorem (recall that $\partial K_{SD}(\kappa_S^*(L), L)/\partial \kappa_S < 0$ from the proof of Lemma D.2), any interior equilibrium incidence $\kappa_S^*(L)$ is continuously and strictly increasing in L. In case (b), we thus obtain that there exists a unique $L^{ind} \in (L^{full}, \overline{L})$ such that $\kappa_S^*(L) > 0$ for all $L > L^{ind}$ and $\kappa_S^*(L) = 0$ otherwise.

Point (ii). Suppose $K_{SD}(0,\overline{L}) > 0$ (otherwise, the purge is never semi-indiscriminate). The purge breadth is then $\kappa^*(L) = 1 \times \alpha_F^*(L) + \kappa_S^*(L) \times \alpha_S^*(L)$ characterized by

$$C_0 + C_1\kappa^*(L) = \beta(r - \mu^S(\kappa_S^*(L), L))\mathcal{D}^{c,nc}$$

The total derivative of the posterior with respect to L is $\frac{d\mu^S(\kappa_S^*(L),L)}{dL} = \frac{\partial \kappa_S^*(L)}{\partial L}\mu^S_{\kappa_S}(\kappa_S^*(L), L) + \mu^S_L(\kappa_S^*(L), L)$, with $\frac{\partial \kappa_S^*(L)}{\partial L} > 0$ (see point (i)), $\mu^S_{\kappa_S} = 0$ (Lemma D.1, point (i)), and $\mu^S_L < 0$ (Lemma D.2, point (ii)). So $\frac{d\mu^S(\kappa_S^*(L),L)}{dL} < 0$ and by the Implicit Function Theorem, $d\kappa^*(L)/dL > 0$. □

Proof of Proposition 6

We first prove the result regarding effort. Assume that $L^{ind} < \overline{L}$ and $L > L^{ind}$ so that $\kappa_S^*(L) \in (0,1)$. The total derivative of average effort with respect to violence is (using Equation 5):

$$\frac{d\overline{e}(\kappa_S^*(L), L)}{dL} = \frac{\partial(1-\kappa_S^*(L))}{\partial L}(\overline{v} + \overline{V_2} + L) + (1 - \kappa_S^*(L)) \tag{D.4}$$

From the proof of Proposition 1, $\frac{\partial \kappa_S^*(L)}{\partial L} > 0$ so $\frac{d\overline{e}(\kappa_S^*(L),L)}{dL} < 1$. It remains to show that $\frac{d\overline{e}(\kappa_S^*(L),L)}{dL} > 0$. To see this, recall that $\kappa_S^*(L)$ is the solution to

$$C_0 + C_1 - C_1(1-\kappa_S)^2(\overline{v} + \overline{V_2} + L) = \beta\left(r - \lambda\frac{v(S,c) + V_2(c) + L}{\overline{v} + \overline{V_2} + L}\right)(W_2(c) - W_2(nc)) \tag{D.5}$$

We thus obtain:

$$\frac{\partial(1-\kappa_S^*(L))}{\partial L} = -\frac{\beta\lambda(1-\lambda)}{2C_1(1-\kappa_S^*(L))}\frac{(v(S,c) - v(S,nc)) + (V_2(c) - V_2(nc))}{(\overline{v} + \overline{V_2} + L)^3}(W_2(c) - W_2(nc)) - \frac{(1-\kappa_S)}{2(\overline{v} + \overline{V_2} + L)}$$

Plugging this into Equation D.4, we obtain

$$\frac{d\overline{e}(\kappa_S^*(L), L)}{dL} = -\frac{\beta\lambda(1-\lambda)}{2C_1(1-\kappa_S^*(L))}\frac{(v(S,c) - v(S,nc)) + (V_2(c) - V_2(nc))}{(\overline{v} + \overline{V_2} + L)^2}(W_2(c) - W_2(nc)) + \frac{1-\kappa_S^*(L)}{2}$$

So after rearranging,

$$\frac{d\overline{e}(\kappa_S^*(L),L)}{dL} \propto C_1(1-\kappa_S^*(L))^2(\overline{v}+\overline{V_2}+L) - \beta\lambda(1-\lambda)\frac{(v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))}{(\overline{v}+\overline{V_2}+L)}(W_2(c)-W_2(nc))$$

Denote $\mathcal{H} := C_1(1-\kappa_S^*(L))^2(\overline{v}+\overline{V_2}+L) - \beta\lambda(1-\lambda)\frac{(v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))}{(\overline{v}+\overline{V_2}+L)}(W_2(c)-W_2(nc))$.
Using Equation D.5 and our assumption that $C_0+C_1 > \beta r(W_2(c)-W_2(nc))$, we obtain:

$$\begin{aligned}
\mathcal{H} =& C_0+C_1-\beta\left(r-\lambda\frac{v(S,c)+V_2(c)+L}{\overline{v}+\overline{V_2}+L}\right)(W_2(c)-W_2(nc)) \\
&-\beta\lambda(1-\lambda)\frac{(v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))}{(\overline{v}+\overline{V_2}+L)}(W_2(c)-W_2(nc)) \\
>& \beta r(W_2(c)-W_2(nc))-\beta\left(r-\lambda\frac{v(S,c)+V_2(c)+L}{\overline{v}+\overline{V_2}+L}\right)(W_2(c)-W_2(nc)) \\
&-\beta\lambda(1-\lambda)\frac{(v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))}{(\overline{v}+\overline{V_2}+L)}(W_2(c)-W_2(nc)) \\
=& \frac{\beta\lambda}{\overline{v}+\overline{V_2}+L}(W_2(c)-W_2(nc))\big(v(S,c)+V_2(c)+L-(1-\lambda)\big((v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))\big)\big) \\
=& \beta\lambda(W_2(c)-W_2(nc)) > 0
\end{aligned}$$

We now turn to the selection of subordinates. The proof for surviving agents is exactly the same as in the baseline model (see the proof of Proposition 4 and Equation B.8). We thus focus on the proportion of congruent subordinates in the second period: $\mathcal{P}(L)$.
First notice that using the proof of Proposition 4, $d\mathcal{P}(L)/dL > 0$ for all $L < L^{full}$. Further, using Equation B.11, for all $L \in [L^{full}, L^{ind})$, $d\mathcal{P}(L)/dL < 0$ since $r > \lambda$. If $L^{ind} > \overline{L}$, then we have proved the claim. Suppose therefore that $L^{ind} < \overline{L}$ and $L \geq L^{ind}$, the proportion of congruent types in the second period is:

$$\begin{aligned}
\mathcal{P}(L) =& ((1-\overline{e})+\overline{e}\kappa_S)r+\overline{e}(1-\kappa_S)\mu^S \\
=& r-\overline{e}(1-\kappa_S)(r-\mu^S) \qquad \text{(D.6)}
\end{aligned}$$

Recall that κ_S^* is the solution to $C_0 + C_1(1-\overline{e}(1-\kappa_S)) = \beta(r-\mu^S)(W_2(c) - W_2(nc))$ so $\frac{d\overline{e}(1-\kappa_S)}{dL} = \frac{\beta}{C_1}\frac{d\mu^S}{dL}(W_2(c) - W_2(nc))$. Hence, we obtain:

$$\begin{aligned}\frac{d\mathcal{P}(L)}{dL} &= -\frac{d\overline{e}(1-\kappa_S)}{dL}(r-\mu^S) + \frac{d\mu^S}{dL}\overline{e}(1-\kappa_S) \\ &= -\frac{d\mu^S}{dL}\frac{\beta}{C_1}(r-\mu^S)(W_2(c) - W_2(nc)) + \frac{d\mu^S}{dL}\overline{e}(1-\kappa_S) \\ &= \frac{d\mu^S}{dL}\frac{1}{C_1}\left(C_1\overline{e}(1-\kappa_S) - \beta(r-\mu^S)(W_2(c) - W_2(nc))\right) \\ &= \frac{d\mu^S}{dL}\left(C_0 + C_1 - 2\beta(r-\mu^S)(W_2(c) - W_2(nc))\right)\end{aligned}$$

Recall that by assumption $C_0 + C_1 > \beta r(W_2(c) - W_2(nc))$ so $C_0 + C_1 - 2\beta(r-\mu^S)(W_2(c) - W_2(nc)) > \beta(2\mu^S - r)(W_2(c) - W_2(nc))$. Since $\lambda < \mu^S$, $C_0 + C_1 - 2\beta(r-\mu^S)(W_2(c) - W_2(nc)) > 0$ for all $r \in (\lambda, 2\lambda]$. Given $\frac{d\mu^S}{dL} < 0$ (Lemma B.3), we obtain $\frac{d\mathcal{P}(L)}{dL} < 0$ as claimed. $\square$

To prove the last result of the section, we assume that $L^{ind} < \overline{L}$. Further, a similar reasoning as above yields that when $L \geq L^{ind}$, the expected benefit of violence is:

$$\begin{aligned}B(L) =& \overline{e} + \beta(1-\overline{e})\Big(rW_2(c) + (1-r)W_2(nc)\Big) \\ &+ \beta\overline{e}\Big(\kappa_S(rW_2(c) + (1-r)W_2(nc)) + (1-\kappa_S)(\mu^S W_2(c) + (1-\mu^S)W_2(nc))\Big) - C((1-\overline{e}) + \kappa_S\overline{e}) \\ B(L) =& \overline{e} + W_2(nc) + \beta\mu^S(W_2(c) - W_2(nc)) + \beta((1-\overline{e}) + \kappa_S\overline{e}))(r-\mu^S)(W_2(c) - W_2(nc)) - C((1-\overline{e}) + \overline{e}\kappa_S)\end{aligned} \tag{D.7}$$

Taking the derivative and using the Envelop Theorem, we obtain:

$$\frac{dB(L)}{dL} = \frac{d\overline{e}}{dL} + \beta\frac{d\mu^S}{dL}\overline{e}(1-\kappa_S)(W_2(c) - W_2(nc)) \tag{D.8}$$

Lemma D.3. *The marginal benefit of violence satisfies:*

$$\frac{dB(L_1)}{dL} > \frac{dB(L_2)}{dL} \text{ for all } L_1 \in (L^{full}, L^{ind}] \text{ and } L_2 \in (L^{ind}, \overline{L}]$$

Proof. By Equation C.9, $\frac{dB(L_1)}{dL} > 1$ for all $\in (L^{full}, L^{ind}]$. By Equation D.8, $\frac{dB(L_2)}{dL} < 1$ for all $L_2 > L^{ind}$ since $d\overline{e}(L)/dL < 1$ (Proposition 3) and the other term is negative. $\square$

Lemma D.4. *Assume $L^{ind} < \overline{L}$. If $C_1 \leq \frac{1}{2(1-\lambda)(v(S,c) - v(S,nc) + V_2(c) - V_2(nc))}$ the marginal benefit of violence is strictly positive for $L > L^{ind}$.*

Proof. From the proof of Proposition 3, recall that

$$\frac{d\overline{e}}{dL} > \frac{\beta\lambda(W_2(c) - W_2(nc))}{2C_1\overline{e}}$$

Using Equation D.8 and $\mu^S_{\kappa_F} = 0$, this implies that

$$\begin{aligned}\frac{dB(L)}{dL} >&\frac{\beta\lambda(W_2(c) - W_2(nc))}{2C_1\overline{e}} + \beta\mu^S_L(W_2(c) - W_2(nc))\overline{e}(1 - \kappa_S)\\ =&\frac{\beta\lambda(W_2(c) - W_2(nc))}{C_1\overline{e}}\Big(\frac{1}{2} - C_1(1-\lambda)(v(S,c) - v(S,nc) + V_2(c) - V_2(nc))(1-\kappa_S)^3\Big)\end{aligned}$$

The second line uses: $\mu^S_L = -\lambda(1-\lambda)\frac{(v(S,c)-v(S,nc))+(V_2(c)-V_2(nc))}{(\overline{v}+\overline{V_2}+L)^2}$ and $\overline{e} = (1-\kappa_S)(\overline{v}+\overline{V_2}+L)$. Since $\kappa^*_S(L) \geq 0$ for all $L \geq L^{ind}$, if $C_1 \leq \frac{1}{2(1-\lambda)(v(S,c)-v(S,nc)+V_2(c)-V_2(nc))}$, then $dB(L)/dL > 0$. □

Lemma D.5. *If at* $L = L^{ind}$, $\zeta_0+\zeta_1 L < 1-\frac{\partial\kappa^*_S(L)}{\partial L}(\overline{v}+\overline{V_2}+L)+\frac{\partial\mu^S(0,L)}{\partial L}\beta(\overline{v}+\overline{V_2}+L)(W_2(c)-W_2(nc))$, *then* $L^* > L^{ind}$.

Proof. Suppose at $L = L^{ind}$, $\zeta_0 + \zeta_1 L < \max_{L\in(L^{ind},\overline{L}]} \frac{dB(L)}{dL}$. The equilibrium intensity is $L = L^{ind}$ if the equation $\zeta_0 + \zeta_1 L = \frac{dB(L)}{dL}$, with $\frac{dB(L)}{dL}$ defined by Equation D.8, has no solution. The equilibrium intensity is the solution to $\zeta_0 + \zeta_1 L = \frac{dB(L)}{dL}$ if it is unique. It is either the smallest solution to $\zeta_0 + \zeta_1 L = \frac{dB(L)}{dL}$ or $L = \overline{L}$ if there are multiple solutions.

To complete the proof, recall that $\kappa^*_S(L^{ind}) = 0$, $\overline{e}(1, L^{ind}) = \overline{v} + \overline{V_2} + L$ (Equation 5), and $\frac{d\overline{e}}{dL} = (1-\kappa^*_S(L)) - \frac{\partial\kappa^*_S(L)}{\partial L}(\overline{v}+\overline{V_2}+L)$. Hence, $\lim_{L\downarrow L^{ind}} \frac{dB(L)}{dL} = 1 - \frac{\partial\kappa^*_S(L)}{\partial L}(\overline{v}+\overline{V_2}+L) + \beta\frac{d\mu^S}{dL}(\overline{v}+\overline{V_2}+L)\mathcal{D}^{c,nc}$ so the condition in the Lemma is a special case of the more general condition in the previous paragraph and $L^* > L^{ind}$. □

Proof of Remark 3

We provide sufficient (and some necessary) conditions for a purge to be semi-discriminate.

Denote $\underline{r} := \mu^S(0,\overline{L})$. Recall that $\mu^S(\cdot)$ is decreasing with L (Lemma B.3)and constant in κ_S. If $r \leq \underline{r}$, then the marginal benefit of purging a successful agent is negative. Using the proof of Proposition 5, $L^{ind} > \overline{L}$ for all C_0, C_1 so a purge is never semi-indiscriminate. So $r > \underline{r}$ is a necessary condition. This is condition 1.

Supposing condition 1. holds, define $\overline{C_0}(r) = \beta[r - \mu^S(0,\overline{L})](W_2(c) - W_2(nc))$. If $C_0 \geq \overline{C_0}(r)$ then for all $C_1 > 0$, the purge cannot be semi-indiscriminate as the marginal cost is always greater than the marginal benefit. When $C_0 < \overline{C_0}(r)$, define $\check{C}_1(r, C_0)$ such that at $L = \overline{L}$, $K_{SD}(0, L) = \beta[r - \mu^S(0,L)](W_2(c) - W_2(nc)) - \big(C_0 + C_1(1-\overline{e}(0,L))\big) = 0$. Similarly, if $C_1 \geq \check{C}_1(r, C_0)$, a purge can never be semi-indiscriminate. If $dB(L)/dL > 0$ for all $L \leq L^{ind}$ at $C_1 = \check{C}_1(r, C_0)$, then denote $\overline{C_1}(r, C_0) := \check{C}_1(r_i, c_0)$. If not, denote $\overline{C_1}(r, C_0)$, the smallest C_1 such that for all $C_1 < \overline{C_1}(r, C_0)$, the marginal benefit satisfies $dB(L)/dL > 0$ for all $L \geq L^{ind}$ (such $\overline{C_1}$ exists by Lemma D.4). This is condition 2.[5]

Finally define $\overline{\zeta_0}(r) := \frac{dB(L^{ind})}{dL}$ (condition 2 ensures $\frac{dB(L)}{dL} > 0$ over this range). And for all $\zeta_0 \leq \overline{\zeta_0}(r)$, denote $\overline{\zeta_1}(r, \zeta_0) := \frac{\frac{dB(L)}{dL} - \zeta_0}{L}$ at $L = L^{ind}$. This guarantees that for all $\zeta_0 < \overline{\zeta_0}(r)$ and $\zeta_1 < \overline{\zeta_1}(r, \zeta_0)$, the condition described in the text of Lemma D.5 holds and the purge is semi-discriminate. This is condition 3. □

E Proofs of 'Extensions'

E.1 Endogenous reward

As explained in the text, the autocrat can supplement agents' second-period benefit with R_2 at marginal cost $\chi'(R_2) = \chi_0 + \chi_1 R_2$ with $\chi_0 = \zeta_0$ to simplify the analysis. In all this subsection, we denote equilibrium value by $\widehat{\cdot}$. The previous analysis corresponds to the case when R_2 is constrained to be 0. We also amend the notation of the baseline model and use $L^*(0)$ to denote the equilibrium intensity of violence characterized in Remarks 2 and 3 as well as Lemmas C.7 and D.5. We do not make restrictions on r in what follows (i.e., r can be strictly greater than λ as in Section).

In the setting with endogenous reward R_2, agents' efforts become:

$$e_1^i(\tau) = \begin{cases} v(S,\tau) + \kappa_F(V_2(\tau) + L + R_2) & \text{if } \kappa_S = 0 \\ (1-\kappa_S)(v(S,\tau) + V_2(\tau) + L + R_2) & \text{if } \kappa_S > 0 \end{cases} \tag{E.1}$$

It is useful to denote $T = R_2 + L$ and $\overline{T} = 1 - v(S,c) - V_2(c)$. We can rewrite effort as:

(i) $e_1^i(\kappa_F, T; \tau) = v(S,\tau) + \kappa_F(V_2(\tau) + T)$ in a discriminate purge;

(ii) $e^i(\kappa_S, T; \tau) = (1-\kappa_S)(v(S,\tau) + V_2(\tau) + T)$ in a semi-indiscriminate purge.

On the agents' side, the problems in the constrained ($R_2 = 0$) and unconstrained (R_2 endogenous) cases are isomorphic. The only difference is that L is replaced by $T = R_2 + L$. Hence, all the comparative statics above hold in this setting replacing L by T. In particular, we recover the following results.

(i) There exist $T^{full} = L^{full}$ and $T^{ind} = L^{ind}$, unique if the threshold is strictly below $\overline{T}$, such that some failures survive the purge if and only if $T \leq T^{full}$, all failures are purged if and only if $T \in (T^{full}, T^{ind}]$, and some successful agents are purged otherwise.

(ii) The expected benefit of T—denoted $B(T)$—is strictly increasing in T for $T < T^{ind}$, strictly

[5] Notice that we assume that $C_0 + C_1 > \beta r \mathcal{D}^{c,nc}$, the conditions stated in Assumption B.1 (especially (2)) and (C.11) hold for all $C_1 \leq \overline{C_1}(\cdot)$. Otherwise, the condition can be appropriately rearranged.

convex for $T \leq T^{full}$ and strictly concave for $T \in [T^{full}, T^{ind})$.

We further assume that the conditions stated in Assumption B.1 hold as well as a modified version (replacing L^{full} by T^{full}) of Equation C.11. Thus, $T^{full} < \overline{T}$ and $B(T)$ is continuous (as before this last assumption only simplifies the analysis). Further, an appropriately modified Lemma C.4 (with T replacing L) holds.

The autocrat's problem can then be conceived into steps: 1) for all T, find $\widehat{L}(T)$ and $\widehat{R_2}(T)$ which minimizes the cost of producing T and 2) Find the optimal T given step 1. Regarding step 1, the autocrat's cost of producing T is thus:

$$\min_{L,R \in \mathbb{R}^2_+} \zeta(L) + \chi(T) \text{ such that } L + R = T$$

Ignoring the non-negativity constraint, the solution to the minimization problem is:

$$\widehat{L}(T) = \frac{\chi_0 - \zeta_0 + \chi_1}{\chi_1 + \zeta_1} T = \frac{\chi_1}{\chi_1 + \zeta_1} T$$
$$\widehat{R_2}(T) = \frac{\zeta_0 - \chi_0 + \zeta_1}{\chi_1 + \zeta_1} T = \frac{\zeta_1}{\chi_1 + \zeta_1} T$$

Hence, under our assumption that $\zeta_0 = \chi_0$, the non-negativity constraint does not bind.

Denote now $\mathcal{T}(T) := \zeta(\widehat{L}(T)) + \chi(\widehat{R_2}(T))$. Observe that $\mathcal{T}(T)$ is strictly increasing and convex, $\mathcal{T}(T) < \zeta(T)$ for all T. Further $\mathcal{T}(T)$ satisfies:

$$\mathcal{T}'(T) = \zeta_0 + \zeta_1 \frac{\chi_1}{\chi_1 + \zeta_1} T = \zeta'(\widehat{L}(T)) \tag{E.2}$$

We can now prove Proposition 7.

Proof of Proposition 7

As a preliminary, we establish that $\widehat{T} \geq L^*(0)$.

Suppose first that $L^*(0) \notin \{L^{full}, L^{ind}, 1 - v(S;c) - V_2(c)\}$. Then it must be that $\zeta'(L^*(0)) = B'(L^*(0))$. Given $\mathcal{T}'(L^*(0)) = \zeta'(\widehat{L}(L^*(0)) < \zeta'(L^*(0))$, we necessarily have $\mathcal{T}'(L^*(0)) < B'(L^*(0))$. Given $L^*(0) \notin \{L^{full}, L^{ind}, 1 - v(S;c) - V_2(c)\}$, there exists $\eta > 0$ such that $\mathcal{T}'(L^*(0) + \eta) < B'(L^*(0) + \eta)$ which implies $\widehat{T} > L^*(0)$.

If $L^*(0) = L^{full}$, then there are two cases to consider: (a) $\lim_{T \downarrow T^{full}} B'(T) \leq \zeta'(\widehat{L}(T^{full}))$ then $\widehat{T} = T^{full} = L^{full}$ and (b) $\lim_{T \downarrow T^{full}} B'(T) > \zeta'(\widehat{L}(T^{full}))$ then $\widehat{T} > L^*(0)$. A similar reasoning holds for $L^*(0) = L^{ind}$. If $L^*(0) = 1 - v(S;c) - V_2(c)$, given the lower marginal cost of producing $\widehat{T}$, we necessarily have $\widehat{T} = L^*(0)$ then.

Using this result, we can now prove points (i)-(iii).

Point (i). Purge inference is weakly increasing in L (proof of Proposition 1) in the baseline model. Hence, it is weakly increasing in T. Noting that since the agents' problem is isomorphic, Lemma B.2 holds in this extension so the purge inference is the same in both settings whenever $T = L$ (i.e., $\widehat{\kappa}_\omega(L) = \kappa^*_\omega(L)$, $\omega \in \{F, S\}$). Since $\widehat{T} \geq L^*(0)$ we have: $\widehat{\kappa}_F(\widehat{T}) \geq \kappa^*_F(L^*(0))$ and $\widehat{\kappa}_S(\widehat{T}) \geq \kappa^*_S(L^*(0))$ with equality only if $L^*(0), \widehat{T} \in \{L^{full}, L^{ind}, 1 - v(S;c) - V_2(c)\}^2$.

Point (ii). By a similar reasoning as above, whenever $T = L$, the purge breadth is the same in the extension as in the baseline model (i.e., $\widehat{\kappa}(L) = \kappa^*(L)$). The purge breadth is strictly increasing for $L \in [0, L^{full}]$ and strictly decreasing for $L \in [L^{full}, L^{ind}]$. Suppose $L^*(0), \widehat{T} \in (0, L^{full})^2$.[6] Since $\widehat{T} > L^*(0)$ by the reasoning above, $\kappa^*(L^*(0)) < \widehat{\kappa}(\widehat{T})$ then. Suppose $L^*(0), \widehat{T} \in (L^{full}, L^{ind})^2$. Since $\widehat{T} > L^*(0)$ by the reasoning above, $\kappa^*(L^*(0)) > \widehat{\kappa}(\widehat{T})$ then.

Point (iii). Suppose $\widehat{T} < T^{full}$ and so is $L^*(0)$. We then have: $B'(\widehat{T}) = \mathcal{T}'(\widehat{T}) = \zeta'(\widehat{L}(\widehat{T}))$ and $B'(L^*(0)) = \zeta'(L^*(0))$. Hence, $\widehat{L}(\widehat{T}) = (\zeta')^{-1}(B'(\widehat{T}))$ and $L^*(0) = (\zeta')^{-1}(B'(L^*(0)))$. Since $B'(\cdot)$ is increasing (Lemma C.1) and $\widehat{T} > L^*(0)$ in that range (see above), $\widehat{L}(\widehat{T}) > L^*(0)$. Suppose $\widehat{T} = L^*(0) = L^{full}$. Then $\widehat{L}(\widehat{T}) = \frac{\xi_1}{\xi_1 + \zeta_1}\widehat{T} < L^*(0)$ (the result holds for other parameter values). Hence, the equilibrium intensity of violence can be greater or lower. □

E.2 Declining replacement pool

Recall that the purge breadth is $\kappa = \alpha_F \kappa_F + \alpha_S \kappa_S$. Suppose that the replacement pool is linearly decreasing in the purge breadth: $r(\kappa) = \overline{r} - r_1 \kappa$. In a partially discriminate purge, the autocrat then maximizes with respect to κ_F:

$$\Big(\int_0^{\kappa_F(1-\overline{e}_1)} r(z)dz - \mu^F \kappa_F(1-\overline{e}_1)\Big)(W_2(c) - W_2(nc))$$

Rearranging, this is equivalent to

$$\kappa_F(1-\overline{e}_1)(\overline{r} - \mu^F)(W_2(c) - W_2(nc)) - r_1(W_2(c) - W_2(nc))\frac{((1-\overline{e}_1)\kappa_F)^2}{2}$$

The autocrat's problem is then as in the baseline model with $r = \overline{r}$, $C_0 = 0$ and $C_1 = r_1(W_2(c) - W_2(nc))$. A similar mapping exists for a semi-indiscriminate purge. Hence, we can apply the same reasoning as in Appendices A-D and show that all our results hold in this setting.

[6] Observe that if $L^*(0) \in (0, L^{full})$, we can always choose χ_1 large enough so that $\widehat{T} \in (0, L^{full})$.

E.3 Autocrat's survival

In this extension, we suppose that the autocrat cares about staying in power and gets a payoff of 1 if so. The probability the autocrat survives is:

$$P(survives) = \gamma\overline{e}_1 + (1-\gamma)\big((1-\epsilon) + \epsilon(1-\overline{e}_1)\big) \times \beta\big(1-\mathcal{P}(L)\big), \tag{E.3}$$

with $\mathcal{P}(L)$ is the proportion of congruent types among second period subordinate (see the proof of Proposition 4). Observe that ϵ measures the complementarity between first-period performance and the proportion of congruent surbordinates. Throughout, we do not make assumptions on the quality of the replacement pool r (i.e., $r \leq \lambda$).

Our first Lemma reproduces Lemmas B.2 and D.2 in this framework. To do so, define $\widehat{\mathcal{P}}(L) := \hat{\alpha}_F(L)r + \lambda(v(S,c) + V_2(c) + L)$ (recall $\hat{\alpha}_F(L) = 1 - \overline{v} - \overline{V_2} - L$) and $\widehat{\mu^F}(L) = \lambda\frac{1-v(S,c)-V_2(c)-L}{\hat{\alpha}_F(L)}$. Recall that $\mu^S(\kappa_S, L) := \lambda\frac{v(S,c)+V_2(c)+L}{\overline{v}+\overline{V_2}+L}$, we obtain:

Lemma E.1. *Further, there exists $\overline{\epsilon}^\kappa(L) > 0$ such that if $\epsilon \leq \overline{\epsilon}^\kappa(L)$, such that the equilibrium purge incidences $\kappa_F^*(L)$, $\kappa_S^*(L)$ satisfy (i) $\kappa_F^*(L) \in [0,1)$ if and only if $C_0 + C_1 \times \hat{\alpha}_F(L) > -(1-\gamma)\big((1-\epsilon) + \hat{\alpha}_F(L)\epsilon\big)(r - \widehat{\mu^F}(L))\beta'\big(1-\widehat{\mathcal{P}}(L)\big)$;*
(ii) $\kappa_S^*(L) \in (0,1)$ if and only if $C_0 + C_1 \times \hat{\alpha}_F(L) < -(1-\gamma)\big((1-\epsilon) + \hat{\alpha}_F(L)\epsilon\big)(r - \mu^S(\kappa_S, L))\beta'\big(1-\widehat{\mathcal{P}}(L)\big)$;
(iii) $\kappa_F^*(L) = 1$ and $\kappa_S^*(L) = 0$ otherwise

Proof. From Equation B.9, if $\kappa_F \in (0,1)$, $\mathcal{P}(L) = (1-\overline{e}_1)\kappa_F(r - \mu^F) + \lambda$ (ignoring arguments whenever possible), with $\overline{e}_1 = \overline{v} + \kappa_F(\overline{V_2} + L)$ since the agents' problem is unchanged. Define

$$S_{PD}(\kappa_F, L) = -(1-\gamma)\big((1-\epsilon) + \epsilon(1-\overline{e}_1)\big)(r - \mu^F)\beta'\big(1-\mathcal{P}(L)\big) - C_0 - C_1\kappa_F(1-\overline{e}_1) \tag{E.4}$$

If the purge is partially discriminate, $\kappa_F^*(L)$ is defined as a solution to $S_{PD}(\kappa_F, L) = 0$ since the autocrat takes effort and violence as given at the time of her purging decision. Notice that for $\kappa_F = 1$, we obtain: $S_{PD}(1, L) = -(1-\gamma)\big((1-\epsilon) + \hat{\alpha}_F(L)\epsilon\big)(r - \widehat{\mu^F}(L))\beta'\big(1-\widehat{\mathcal{P}}(L)\big) - C_0 - C_1 \times \hat{\alpha}_F(L)$. From Equation D.6, if $\kappa_S \in (0,1)$, $\mathcal{P}(L) = r - \overline{e}_1(1-\kappa_S)(r - \mu^S)$, with $\overline{e}_1 = (1-\kappa_S)(\overline{v} + \overline{V_2} + L)$. Define

$$S_{SD}(\kappa_S, L) = -(1-\gamma)\big((1-\epsilon) + \epsilon(1-\overline{e}_1)\big)(r - \mu^S)\beta'\big(1-\mathcal{P}(L)\big) - C_0 - C_1(1-(1-\kappa_S)\overline{e}_1) \tag{E.5}$$

If the purge is partially discriminate, $\kappa_S^*(L)$ is defined as a solution to $S_{SD}(\kappa_S, L) = 0$ since the autocrat takes effort and violence as given at the time of her purging decision. Notice that for

$\kappa_S = 0$, we obtain: $S_{SD}(0, L) = -(1-\gamma)\big((1-\epsilon) + \hat{\alpha}_F(L)\epsilon\big)(r - \mu^S(L))\beta'\big(1 - \widehat{\mathcal{P}}(L)\big) - C_0 - C_1 \times \hat{\alpha}_F(L) < S_{PD}(1, L)$ since $\mu^S > \widehat{\mu^F}$. We now show that $\partial S_{SD}(\kappa_S, L)/\partial \kappa_S < 0$ for ϵ not too large. Using the definition of $\mathcal{P}(L)$ and since μ^S does not depend on κ_S, we obtain:

$$\begin{aligned}\frac{\partial S_{SD}(\kappa_S, L)}{\partial \kappa_S} =& -(1-\gamma)\epsilon(\overline{v} + \overline{V_2} + L)(r - \mu^S)\beta'\big(1 - \mathcal{P}(L)\big) \\ &+ (1-\gamma)\big((1-\epsilon) + \epsilon(1-\overline{e}_1)\big)(r - \mu^S)^2 2\overline{e}_1\beta''\big(1 - \mathcal{P}(L)\big) - 2C_1\overline{e}_1 \end{aligned} \tag{E.6}$$

Notice that under the assumption that $\beta''(\cdot) \leq 0$, the terms on the second line are both negative. Hence, when $\epsilon = 0$, $\frac{\partial S_{SD}(\kappa_S, L)}{\partial \kappa_S} < 0$. Since $\frac{\partial S_{SD}(\kappa_S, L)}{\partial \kappa_S}$ is continuous in ϵ, there exists $\overline{\epsilon}^\kappa(L)$ such that for all $\epsilon \leq \overline{\epsilon}^\kappa(L)$, $\frac{\partial S_{SD}(\kappa_S, L)}{\partial \kappa_S} \leq 0$.

Hence, if $\epsilon \leq \overline{\epsilon}^\kappa(L)$, the properties of $S_{SD}(\kappa_S, L)$ yield $S_{SD}(\kappa_S, L) < S_{PD}(1, L)$ for all $\kappa_S \geq 0$. We can then apply a similar reasoning as in the proof of Lemmas B.2 and D.2 to prove the claim. Note, however, that there may be multiple solutions to $S_{PD}(\kappa_F, L) = 0$ even if point (i) holds. In this case, recall that our equilibrium refinement criterion selects the highest solution. □

Our next two Lemmas establish that the purge incidence $\kappa^*_\omega(L)$, when interior, and purge breadth $\kappa^*(L)$ are strictly increasing with L in a discriminate purge ($\omega = F$) and semi-indiscriminate purge ($\omega = S$) when ϵ is not too large.

Lemma E.2. *There exists $\overline{\epsilon}^{PD}(L) > 0$ such that if $\epsilon < \overline{\epsilon}^{PD}(L)$, then in a discriminate purge with some failures surviving, the purge inference $\kappa^*_F(L)$ and breadth $\kappa^*(L)$ are strictly increasing with L.*

Proof. Since we select the highest purge inference, by a similar reasoning as in the proof of Proposition 1, $\partial S_{PD}(\kappa^*_F(L), L)/\partial \kappa_F < 0$ in a partially discriminate purge (i.e., condition (i) in Lemma E.1 holds). We thus just need to show that $\partial S_{PD}(\kappa^*_F(L), L)/\partial L > 0$ to prove that $\kappa^*_F(L) > 0$ (by the Implicit Function Theorem). Observe that (using subscript to denote partial derivative):

$$\begin{aligned}\frac{\partial S_{PD}(\kappa^*_F(L), L)}{\partial L} =& \epsilon\kappa^*_F(L)\beta'(1 - \mathcal{P}(L)) \\ &+ (1-\gamma)(1 - \epsilon + \epsilon(1 - \overline{e}_1))\mu^F_L\beta'(1 - \mathcal{P}(L)) \\ &- (1-\gamma)(1 - \epsilon + \epsilon(1 - \overline{e}_1))(r - \mu^F)\kappa^*_F(L)^2(r - \lambda)\beta''(1 - \mathcal{P}(L)) \\ &+ C_1\kappa^*_F(L)^2 \end{aligned} \tag{E.7}$$

Under the assumptions, given $\mu^F_L < 0$ (see Lemma B.3), the terms on the last three lines are strictly positive. Hence, when $\epsilon = 0$, $\frac{\partial S_{PD}(\kappa^*_F(L), L)}{\partial L} > 0$. Since $\frac{\partial S_{PD}(\kappa^*_F(L), L)}{\partial L}$ is continuous in ϵ, there exists $\overline{\epsilon}^{PD}_1(L) > 0$ (possibly equals 1) such that for all $\epsilon < \overline{\epsilon}^{PD}_1(L)$, $\frac{\partial S_{PD}(\kappa^*_F(L), L)}{\partial L} > 0$ and the purge incidence is strictly increasing with L.

Further, recall that the purge breadth is $(1-\overline{e}_1)\kappa_F$ so the purge breadth is strictly increasing with L whenever $\frac{\partial S_{PD}(\kappa_F^*(L),L)}{\partial L} - C_1\kappa_F^*(L)^2$ is strictly positive. This is guaranteed for $\epsilon = 0$ using Equation E.7 and the assumptions on $\beta(\cdot)$. Hence, there exists $\overline{\epsilon}_2^{PD}(L) > 0$ (possibly equals 1) such that for all $\epsilon < \overline{\epsilon}_2^{PD}(L)$, the purge breadth is strictly increasing with L. Since $C_1\kappa_F^*(L)^2 > 0$, $\overline{\epsilon}_2^{PD} \leq \overline{\epsilon}_1^{PD}$ (with strict inequality whenever $\overline{\epsilon}_2^{PD}(L) < 1$). Denote $\overline{\epsilon}^{PD}(L) := \overline{\epsilon}_2^{PD}(L)$ so that the claim holds. □

Lemma E.3. *There exists $\overline{\epsilon}^{SD}(L) > 0$ such that if $\epsilon < \overline{\epsilon}^{SD}$, then in a semi-indiscriminate purge, the purge incidence $\kappa_S^*(L)$ and breadth $\kappa^*(L)$ are strictly increasing with L.*

Proof. The proof proceeds along the same lines as the proof of Lemma E.2 noting that a necessary condition for the purge to be semi-indiscriminate is $r > \lambda$ and given Equation E.5

$$\begin{aligned}\frac{\partial S_{SD}(\kappa_S^*(L),L)}{\partial L} =&(1-\gamma)\epsilon(1-\kappa_S^*(L))\beta'(1-\mathcal{P}(L))\\ &+(1-\gamma)(1-\epsilon+\epsilon(1-\overline{e}_1))\mu_L^S\beta'(1-\mathcal{P}(L))\\ &-(1-\gamma)(1-\epsilon+\epsilon(1-\overline{e}_1))(r-\mu^S)(1-\kappa_S^*(L))^2(r-\lambda)\beta''(1-\mathcal{P}(L))\\ &+C_1(1-\kappa_S^*(L))^2,\end{aligned} \tag{E.8}$$

with $\mu_L^S < 0$. □

The following condition is the equivalent to the condition (2) in Assumption B.1 in this setting

$$C_0 + C_1\hat{\alpha}_F(\overline{L}) < -(1-\gamma)((1-\epsilon)+\epsilon\hat{\alpha}_F(\overline{L}))\beta'\big(1-\lambda-r\hat{\alpha}_F(\overline{L})\big) \tag{E.9}$$

We can now state a similar proposition as in the text linking violence and the nature of the purge (see Propositions 1 and 5) in this setting.

Proposition E.1. *There exists $\overline{\epsilon} > 0$ such that if $\epsilon < \overline{\epsilon}$, then*

1. If Equation E.9 does not hold, then for all intensity of violence, the purge is discriminate and some failures survive: $\kappa_F^*(L) \in [0,1)$.

2. If Equation E.9 holds, then there exist unique $L^{full} < \overline{L}$ and $L^{ind} \in (L^{full},\overline{L}]$ such that:

(i) For $L < L^{full}$, the purge is discriminate and some failures survive ($\kappa_F^*(L) \in [0,1)$);

(ii) For $L \in [L^{full}, L^{ind}]$, the purge is discriminate and all failures are purged ($\kappa_F^*(L) = 1$ and $\kappa_S^*(L) = 0$);

(iii) For $L > L^{ind}$, the purge is semi-indiscriminate ($\kappa_S^*(L) > 0$).

Proof. Denote $\overline{\epsilon} = \min_{L\in[0,\overline{L}]} \min\left\{\overline{\epsilon}^{\kappa}(L), \overline{\epsilon}^{PD}(L), \overline{\epsilon}^{SD}(L)\right\} > 0$. For all $\epsilon < \overline{\epsilon}$, we can then apply the same reasoning as in the proof of Proposition 1 and the relevant part of the proof of Proposition 5 using Lemmas E.1-E.3. □

Proposition E.2. *If $r \geq \lambda$ and $\epsilon < \overline{\epsilon}$ (with $\overline{\epsilon}$ defined in the text of Proposition E.1), then the relationship between the purge breadth $\kappa^*(L)$ and the intensity of violence L exhibits the following properties:*

(i) For $L < L^{full}$, $\kappa^*(L)$ is strictly increasing in L;

(ii) For $L \in [L^{full}, L^{ind}]$, $\kappa^*(L)$ is strictly decreasing in L;

(iii) For $L > L^{ind}$, $\kappa^*(L)$ is strictly increasing in L.

Proof. Follows directly from Lemmas E.2 and E.3 and Proposition E.1. □

The next proposition establishes that the positive effect of violence on effort holds in this setting as long as the survival probability is not too concave.

Proposition E.3. *There exists $\underline{\beta} < 0$ such that if $\epsilon < \overline{\epsilon}$ (with $\overline{\epsilon}$ defined in the text of Proposition E.1) and $\min_{z\in[1-r,1]} \beta''(z) > \underline{\beta}$, then:*

1. The total derivative of effort with respect to violence $\frac{d\overline{e}(L)}{dL}$ is always strictly positive.

2. Further, there exists $L^{eff} \leq L^{full}$ such that the derivative satisfies:

(i) $\frac{d\overline{e}(L)}{dL} > 1$ for all $L \in (L^{eff}, L^{full})$;

(ii) $\frac{d\overline{e}(L)}{dL} = 1$ for all $L \in [L^{full}, L^{ind})$;

(iii) $\frac{d\overline{e}(L)}{dL} < 1$ for all $L \geq L^{ind}$.

Proof. We consider the three types of purges in turn. First, in a discriminate purge with some failures surviving, equilibrium first-period performance is (ignoring superscript and subscript):

$$\overline{e}(L) = \overline{v} + \kappa_F^*(L)(\overline{V_2} + L)$$

As in the baseline model, we obtain: $\frac{d\overline{e}(L)}{dL} = \kappa_F^*(L) + \frac{\partial \kappa_F^*(L)}{\partial L}(\overline{V_2} + L) > 0$ since $\frac{\partial \kappa_F^*(L)}{\partial L} > 0$ under the assumption that $\epsilon < \overline{\epsilon}$ (Lemma E.2). Given that $\kappa_F^*(L)$ is not necessarily continuous, we need to consider two cases. Case 1: there is an intensity of violence L such that $\kappa_F^*(L) + \frac{\partial \kappa_F^*(L)}{\partial L}(\overline{V_2} + L) > 1$ (this is the case if $\kappa_F^*(L)$ is continuous in L). In this case, denote $L' = \max\left\{L : \kappa_F^*(L) + \frac{\partial \kappa_F^*(L)}{\partial L}(\overline{V_2} + L) = 1\right\}$. If for all $L \in (L', L^{full}]$, $\kappa_F^*(L) + \frac{\partial \kappa_F^*(L)}{\partial L}(\overline{V_2} + L) > 1$ then denote $L' = L^{eff}$.[7] Otherwise,

[7]Observe that in this environment, we do not know whether $\kappa_F^*(L)$ is convex in L. The statement of the proposition and the proof do not exclude intervals $[L^1, L^2]$, $L^1 < L^2 < L^{full}$ such that $d\overline{e}_1/dL > 1$ for all $L \in [L^1, L^2]$.

$L^{eff} = L^{full}$. Case 2: there is no L such that $\kappa_F^*(L) + \frac{\partial \kappa_F^*(L)}{\partial L}(\overline{V_2} + L) > 1$. In this case denote $L^{eff} := L^{full}$.

Let us now turn to a discriminate purge with all failures purged. In this case, the equilibrium first-performance is:

$$\overline{e}(L) = \overline{v} + \overline{V_2} + L$$

and $\frac{d\overline{e}(L)}{dL} = 1$ as claimed.

Finally, in a semi-indiscriminate purge ($\kappa_S^*(L) > 0$), the equilibrium first-period performance is:

$$\overline{e}(L) = (1 - \kappa_S^*(L))(\overline{v} + \overline{V_2} + L)$$

Using $S_{SD}(\kappa_S^*(L), L) = 0$, we obtain (ignoring all superscripts and arguments whenever possible)

$$\frac{d\overline{e}(L)}{dL} = (1 - \kappa_S) + \frac{\frac{\partial S_{SD}(\kappa_S(L),L)}{\partial L}}{\frac{\partial S_{SD}(\kappa_S(L),L)}{\partial \kappa_S}}(\overline{v} + \overline{V_2} + L)$$

Given $\frac{\partial S_{SD}(\kappa_S(L),L)}{\partial \kappa_S} < 0$ (since we select the highest purge inference) and $\frac{\partial S_{SD}(\kappa_S(L),L)}{\partial L} > 0$ (Lemma E.3), clearly $\frac{d\overline{e}(L)}{dL} < 1$. Further, $\frac{d\overline{e}(L)}{dL}$ has opposite sign than (using Equation E.6 and Equation E.8)

$$\begin{aligned}
SD =& -(1-\gamma)\epsilon\overline{e}(r-\mu^S)\beta'(1-\mathcal{P}(L)) - 2C_1\overline{e}_1(1-\kappa_S) \\
&+ (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))(r-\mu^S)^2 2\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L)) \\
&+ \Big((1-\gamma)\epsilon\overline{e}(r-\mu^S)\beta'(1-\mathcal{P}(L)) + (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\mu_L^S(\overline{v}+\overline{V_2}+L)\beta'(1-\mathcal{P}(L)) \\
&- (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))(r-\mu^S)\overline{e}(1-\kappa_S)(r-\lambda)\beta''(1-\mathcal{P}(L)) + C_1\overline{e}(1-\kappa_S)\Big) \\
=& -C_1\overline{e}(1-\kappa_S) + (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\mu_L^S(\overline{v}+\overline{V_2}+L)\beta'(1-\mathcal{P}(L)) \\
&+ (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))(r-\mu^S)\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L))\big(2(r-\mu^S)-(r-\lambda)\big)
\end{aligned}$$

Using $S_{SD}(\kappa_S^*(L), L) = 0 \Leftrightarrow C_0 + C_1 - C_1\overline{e}(1-\kappa_S^*(L)) = -(1-\gamma)\big((1-\epsilon)+\epsilon(1-\overline{e}_1)\big)(r-\mu^S)\beta'\big(1-\mathcal{P}(L)\big)$, we obtain:

$$\begin{aligned}
SD =& (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\beta'(1-\mathcal{P}(L))\big(\mu_L^S(\overline{v}+\overline{V_2}+L) - (r-\mu^S)\big) - C_0 - C_1 \\
&+ (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))(r-\mu^S)\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L))\big(r+\lambda-2\mu^S\big)
\end{aligned}$$

Given $\mu^S = \lambda\frac{v(S,c)+V_2(c)+L}{\overline{v}+\overline{V_2}+L}$, we obtain $(\overline{v}+\overline{V_2}+L)\mu_L^S = -\lambda(1-\lambda)\frac{v(S,c)-v(S,nc)+V_2(c)-V_2(nc)}{\overline{v}+\overline{V_2}+L} = \lambda - \mu^S$. Further, using the assumption $C_0 + C_1 > -(1-\gamma)r\beta'(1-r) > -(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))r\beta'(1-r)$, we have:

$$\begin{aligned}
SD < (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\Big(&(\lambda-r)\beta'(1-\mathcal{P}(L)) + r\beta'(1-r) \\
&+ (r-\mu^S)\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L))\big(r+\lambda-2\mu^S\big)\Big)
\end{aligned}$$

Observe that if $\beta''(z) = 0$ for all $z \in [1-r, 1]$, then $(\lambda - r)\beta'(1-\mathcal{P}(L)) + r\beta'(1-r) + (r-\mu^S)\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L))\big(r+\lambda-2\mu^S\big) = \lambda\beta'(1-r) < 0$ so $SD < 0$ and $\frac{d\overline{e}(L)}{dL} > 0$. We now show that there exists a strictly positive lower bound on the second derivative such first-period performance is strictly increasing with violence.

Suppose $\widehat{\beta} = \min_{z\in[1-r,1]} \beta''(z) < 0$ and note that $\beta'(1-\mathcal{P}(L)) - \beta'(1-r) = \int_{1-r}^{1-\mathcal{P}(L)} \beta''(z)dz \geq \widehat{\beta} \times (1-\mathcal{P}(L)-(1-r)) = \widehat{\beta} \times (r-\mu^S)(1-\kappa_S)\overline{e}_1$. Assume $r+\lambda-2\mu^S < 0$ (a similar reasoning holds if the inequality is reversed), we hence have:

$$\begin{aligned}
SD &< (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\Big((\lambda-r)\beta'(1-\mathcal{P}(L)) + r\beta'(1-r) \\
&\qquad + (r-\mu^S)\overline{e}(1-\kappa_S)\beta''(1-\mathcal{P}(L))\big(r+\lambda-2\mu^S\big)\Big) \\
&< (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\Big(\lambda\beta'(1-r) - 2(r-\mu^S)(\mu^S-\lambda)\overline{e}_1(1-\kappa_S)\widehat{\beta}\Big) \\
&< (1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))\Big(\lambda\beta'(0) - 2(r-\mu^S)(\mu^S-\lambda)\widehat{\beta}\Big)
\end{aligned}$$

So for $\widehat{\beta} \geq \max_{L\in[L^{ind},\overline{L}]} \left\{\frac{\lambda\beta'(0)}{2(r-\mu^S)(\mu^S-\lambda)}\right\}$, $SD < 0$ and $d\overline{e}_1/dL > 0$. Hence, there exists $\underline{\beta} < 0$ such that if $\min_{z\in[1-r,1]} \beta''(z) > \underline{\beta}$, the fear effect holds for all $L \geq L^{ind}$. □

Our last proposition establishes that an increase in violence can also worsen selection in this setting.

Proposition E.4. *There exists $\underline{\underline{\beta}} < 0$ such that if $\epsilon < \overline{\epsilon}$ (with $\overline{\epsilon}$ defined in the text of Proposition E.1) and $\min_{z\in[1-r,1]} \beta''(z) > \underline{\underline{\beta}}$, then:*
(i) The proportion of congruent types among surviving subordinates of the purge strictly increases with L if and only if $L < L^{full}$, and decreases otherwise (strictly if $r > \lambda$);
(ii) If $r \in (\lambda, 2\lambda]$, the proportion of congruent types among subordinates in the second period strictly increases with L for $L < L^{full}$ and strictly decreases otherwise.

Proof. Point (i) for $r = \lambda$ follows directly from a similar reasoning as the proof of Proposition 4 since the purge inference is strictly increasing with L under the assumption $\epsilon < \overline{\epsilon}$.

For point (ii), the claim holds directly for a discriminate purge with all failures purged (see Equation B.11). We thus focus on a semi-discriminate purge for which $\mathcal{P}(L) = r - (1-\kappa_S)\overline{e}(r-\mu^S)$ (ignoring subscripts, superscripts, and arguments). So as before $\frac{d\mathcal{P}(L)}{dL} = -\frac{d(1-\kappa_S)\overline{e}}{dL}(r-\mu^S) + \mu_L^S(1-\kappa_S)\overline{e}$ (with μ_L^S the partial derivative of μ^S with respect to L). From Equation E.5 and $S_{SD}(\kappa_S^*(L), L) = 0$, treating $(1-\kappa_S)\overline{e}$ as our variable of interest (and again ignoring subscripts, superscripts, and arguments whenever possible), we obtain:

$$\begin{aligned}&(1-\gamma)\epsilon(1-\kappa_S)(r-\mu^S)\beta'(1-\mathcal{P}(L))\\&+(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))\mu_L^S\big(\beta'(1-\mathcal{P}(L))+(1-\kappa_S)\overline{e}(r-\mu^S)\beta''(1-\mathcal{P}(L))\big)\\&+\frac{d(1-\kappa_S)\overline{e}}{dL}\big(C_1-(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))(r-\mu^S)^2\beta''(1-\mathcal{P}(L))\big)=0\end{aligned}$$

Hence, we obtain that $d\mathcal{P}(L)/dL$ has the same sign as:

$$\begin{aligned}\Upsilon :=&(1-\gamma)\epsilon(1-\kappa_S)\beta'(1-\mathcal{P}(L))(r-\mu^S)^2\\&+(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))\mu_L^S\big(\beta'(1-\mathcal{P}(L))+(1-\kappa_S)\overline{e}(r-\mu^S)\beta''(1-\mathcal{P}(L))\big)(r-\mu^S)\\&+C_1\mu_L^S\overline{e}(1-\kappa_S)-(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))(r-\mu^S)^2\beta''(1-\mathcal{P}(L))\mu_L^S\overline{e}(1-\kappa_S)\\=&(1-\gamma)\epsilon(1-\kappa_S)\beta'(1-\mathcal{P}(L))(r-\mu^S)^2\\&+\mu_L^S\big((1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))\beta'(1-\mathcal{P}(L))(r-\mu^S)+C_1\overline{e}(1-\kappa_S)\big)\end{aligned}$$

Given Equation E.5, we can rewrite the equality as:

$$\begin{aligned}\Upsilon =&(1-\gamma)\epsilon(1-\kappa_S)\beta'(1-\mathcal{P}(L))(r-\mu^S)^2\\&+\mu_L^S\big(2(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))\beta'(1-\mathcal{P}(L))(r-\mu^S)+C_0+C_1\big)\end{aligned}$$

Using $\mu_L^S<0$ and $C_0+C_1>-(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}))r\beta'(1-r)$, we obtain:

$$\begin{aligned}\Upsilon <&(1-\gamma)\epsilon(1-\kappa_S)\beta'(1-\mathcal{P}(L))(r-\mu^S)^2\\&+\mu_L^S(1-\gamma)((1-\epsilon)+\epsilon(1-\overline{e}_1))\big(2\beta'(1-\mathcal{P}(L))(r-\mu^S)-r\beta'(1-r)\big)\end{aligned}$$

Observe that if $\beta''(z)=0$ for all $z\in[1-r,1]$, $2\beta'(1-\mathcal{P}(L))(r-\mu^S)-r\beta'(1-r)=\beta'(1-r)(r-2\mu^S)>0$ under the assumption that $r\leq 2\lambda$. This implies $\Upsilon<0$. By a similar reasoning as in the proof of Proposition E.3, there exists $\underline{\underline{\beta}}<0$ such that if $\min_{z\in[1-r,1]}\beta''(z)\geq\underline{\underline{\beta}}$, then $\Upsilon<0$ and $d\mathcal{P}(L)/dL<0$ as claimed. □

Since the consequences of increased intensity of violence are still present in this setting, a similar reasoning as in Appendix C and D yields conditions such that the purge is discriminate with or without all failures purged or semi-indiscriminate. There are, however, two important differences. First, we do not have a condition such that $\kappa_F^*(L)$ is continuous so Remark C.1 applies. Second, without imposing additional conditions on $\beta(\cdot)$, we cannot determine whether the benefit of investing in the infrastructure of violence is convex for $L\leq L^{full}$ so Remark 1 does not necessarily apply in this setting.

This last section concludes the proofs of the results in the text. In the next appendix we prove some additional results. Specifically, we consider in turn the following extensions or modifications to our set-up:

1. Relaxing the assumption that the autocrat observes all project outcomes (Appendix F.1);
2. Introducing the possibility for the autocrat to commit to the purge inferences as well as the intensity of violence at the beginning of the game (Appendix F.2);
3. Applying a version of the model to repression (Appendix F.3);
4. Looking at a one-agent set-up (Appendix F.4).

F Additional results

F.1 Partial observation of project outcomes

In this subsection, we suppose that the autocrat observes an agent i's project outcome with exogenous probability $\gamma \in (0,1)$ (the baseline model corresponds to the case when $\gamma = 1$). More specifically, before making her purging decision, the autocrat receives a signal $s^i \in \{\emptyset, \omega^i\}$ for each subordinate i, with $Pr(s^i = \omega^i) = \gamma$. The autocrat then chooses to purge a proportion κ_F of agents who failed, $\kappa_\emptyset$ of agents for which she does not know ω, and κ_S of agents who succeeded. The rest of the model is as described in the main text (in particular, the autocrat's payoffs are given by Equation 3 and Equation 4). We throughout suppose that $r \geq \lambda$.

A natural extension of Lemma 1 holds in this setting. The autocrat first purges from the pool of failure, then from the pool with no information and from the success pool. Hence, we have $\kappa_\emptyset > 0$ only if $\kappa_F = 1$ and $\kappa_S = 1$ only if $\kappa_\emptyset = 1$. Indeed, as before, success is a positive signal of congruence so her posterior, μ^S, satisfies $\mu^S > \lambda$, failure a negative signal so the autocrat's posterior satisfies $\mu^F < \lambda$. In turn, after learning nothing, the autocrat's posterior, denoted $\mu^\emptyset$ equals the prior: $\mu^\emptyset = \lambda$.

A type-$\tau \in \{c, nc\}$ agent then exerts effort:

$$e_1^i(\tau) = \begin{cases} v(S,\tau) + \gamma\kappa_F(V_2(\tau) + L) & \text{if } \kappa_\emptyset = 0 \\ (\gamma + (1-\gamma)(1-\kappa_\emptyset))v(S,\tau) + \gamma(V_2(\tau) + L) & \text{if } \kappa_\emptyset > 0 \text{ and } \kappa_S = 0 \\ \gamma(1-\kappa_S)(v(S,\tau) + V_2(\tau) + L) & \text{if } \kappa_S > 0 \end{cases} \tag{F.1}$$

Comparing Equation F.1 with Equation 5, it is clear that all the (partial and full equilibrium) comparative statics characterized in Appendix B still hold in this context. Hence, there exists $L^{full}(\gamma) \geq 0$ such that if $L < L^{full}$, some failures survive and the purge breadth is increasing with the intensity of violence. In what follows, we assume that $L^{full} < \overline{L}$

Suppose now the autocrat considers purging from the 'no-information pool.' In this case, denoting $\alpha_F(\kappa_\emptyset, L, \gamma) = 1 - \overline{e} = 1 - \big((\gamma + (1-\gamma)(1-\kappa_\emptyset))\overline{v} + \gamma(\overline{V_2} + L)\big)$ the proportion of failure, her purging decision is driven by comparing the marginal cost of purging an additional member of this pool $c'(\gamma\alpha_F(\kappa_\emptyset, L, \gamma) + (1-\gamma)\kappa_\emptyset)$ and the marginal benefit $(r-\lambda)(W_2(c) - W_2(nc))$. Note that the marginal benefit is fixed since the autocrat has no information about the 'no-information pool.' In turn, the marginal cost, fixing $\kappa_\emptyset$ is strictly decreasing with L. This has two consequences:
(i) there exists $L^\emptyset(\gamma) > L^{full}(\gamma)$, with $L^\emptyset < \overline{L}$ only if $r > \lambda$, such that $\kappa_\emptyset > 0$ if and only if

$L > L^{\emptyset}(\gamma)$;

(ii) for all $L > L^{\emptyset}(\gamma)$, the equilibrium purge inference $\kappa^*_{\emptyset}(L)$ is strictly increasing with L.

As a result, there exists $L^{tot}(\gamma) \in (L^{\emptyset}(\gamma), \overline{L}]$, with $L^{tot}(\gamma) < \overline{L}$ only if r and γ are both sufficiently large, such that for all $L \geq L^{tot}(\gamma)$, $\kappa^*_{\emptyset}(L) = 1$. We can then apply the same reasoning as in Appendix D to show that there exists $L^{ind}(\gamma) > L^{tot}(\gamma)$, with $L^{ind}(\gamma) < \overline{L}$ only if r and γ are both sufficiently large, such that for all $L > L^{ind}(\gamma)$, $\kappa^*_S(L) > 0$.

From the reasoning above, as in the baseline model, we thus obtain that the nature of the purge fully depends on the intensity of violence

Proposition F.1. *There exist $L^{full}(\gamma), L^{\emptyset}(\gamma), L^{tot}(\gamma), L^{ind}(\gamma)$, unique if the threshold is strictly less than $\overline{L}$, such that:*

(i) For $L < L^{full}(\gamma)$, some failures survive ($\kappa^*_F(L) \in (0,1)$);

(ii) For $L \in [L^{full}(\gamma), L^{\emptyset}(\gamma))$, all failures are purged ($\kappa^*_F(L) = 1$ and $\kappa^*_{\emptyset}(L) = 0$);

(iii) For $L \in [L^{\emptyset}(\gamma), L^{tot}(\gamma))$, the autocrat targets the no-information pool ($\kappa^*_F(L) = 1$ and $\kappa^*_{\emptyset}(L) \in (0,1)$);

(iv) For $L \in [L^{tot}(\gamma), L^{ind}(\gamma))$, only agents who succeed survive the purge ($\kappa^*_F(L) = 1$ and $\kappa^*_{\emptyset}(L) = 1$);

(v) For $L \in [L^{ind}(\gamma), \overline{L}]$ (possibly an empty interval), the purge is semi-indiscriminate ($\kappa^*_F(L) = 1$, $\kappa^*_{\emptyset}(L) = 1$, and $\kappa^*_S(L) = 0$).

In what follows, we suppose throughout that $L^{ind}(\gamma) < \overline{L}$ to facilitate the exposition. We note, however, that the conditions to satisfy this inequality are more stringent than in the main text (especially, it is necessary that γ is sufficiently large as highlighted above).

How does the purge breadth vary with the intensity of violence? When the purge is discriminate but some failures survive (case (i) of Proposition F.1) and in a semi-indiscriminate purge (case (v)), a similar reasoning as in the main text yields that the purge breadth increases with L. In turn, by simple observation of Equation F.1, effort is increasing with L and the purge breadth thus decreasing in L in a discriminate purge with all failures purged ($\kappa^*_F(L) = 1$, $\kappa^*_{\emptyset}(L) = 0$) or when only successful agents survive the purge ($\kappa^*_{\emptyset}(L) = 1$, $\kappa^*_S(L) = 0$).

It remains to characterize the effect of the intensity of violence on the purge breadth when part of the no-information pool is purged ($\kappa^*_{\emptyset}(L) \in (0,1)$). As noted above, the marginal benefit of purging is constant and equals $\beta(r-\lambda)(W_2(c) - W_2(nc))$ then. Since the equilibrium purge breadth is the solution to $c'(\kappa) = \beta(r-\lambda)(W_2(c) - W_2(nc))$, we obtain that $\kappa^*(L)$ is constant in L then. As such, including the possibility that the autocrat does not observe all project outcomes, if anything, reinforces the non-monotonic relationship between purge breadth and the intensity of violence.

Proposition F.2. *The relationship between the equilibrium purge breadth $\kappa^*(L)$ and the intensity of violence L satisfies:*
(i) For $L < L^{full}(\gamma)$, the purge breadth is strictly increasing in L;
(ii) For $L \in [L^{full}(\gamma), L^{\emptyset}(\gamma))$, the purge breadth is strictly decreasing in L;
(iii) For $L \in [L^{\emptyset}(\gamma), L^{tot}(\gamma))$, the purge breadth is constant in L;
(iv) For $L \in [L^{tot}(\gamma), L^{ind}(\gamma))$, the purge breadth is strictly decreasing in L;
(v) For $L \in [L^{ind}(\gamma), \overline{L}]$, the purge breadth is strictly increasing in L.

We now look at the consequences of violence on effort. As in the main text, greater intensity of violence always raises effort. The direct payoff effect still dominates any negative indirect effect through the purge inference when the no information pool is targeted or in a semi-indiscriminate purge. One noticeable difference is that the increase in the work rate is now relative to the autocrat's probability of learning project outcomes γ.

Proposition F.3. *The total derivative of average effort with respect to violence $\frac{d\overline{e}(L)}{dL}$ is always strictly positive. Further, there exists a unique $L^{eff}(\gamma) \leq L^{full}(\gamma)$ such that the derivative satisfies:*
(i) $\frac{d\overline{e}(L)}{dL} > \gamma$ for all $L \in (L^{eff}(\gamma), L^{full}(\gamma))$;
(ii) $\frac{d\overline{e}(L)}{dL} \leq \gamma$ for all $L \in [L^{full}(\gamma), \overline{L}]$.

Proof. For all $L \notin [L^{\emptyset}(\gamma), L^{tot}(\gamma))$, the result follows from a similar reasoning as in the proof of Proposition 3 or 6 or from direct observation of Equation F.1 (for corner purge inferences). It only remains to show the result for $L \in [L^{\emptyset}(\gamma), L^{tot}(\gamma))$.
To see that greater intensity of violence strictly increases effort, we proceed by contradiction. Suppose average effort weakly decreases. Given that $c'(\gamma(1-\overline{e}(L))+(1-\gamma)\kappa^*_{\emptyset}(L)) = (r-\lambda)(W_2(c)-W_2(nc))$, we then have $\kappa^*_{\emptyset}(L)$ weakly decreases with L. Using Equation F.1, it must then be that $\overline{e}(L)$ is strictly increasing with L. A contradiction. Hence, $\frac{d\overline{e}(L)}{dL} > 0$.
To show that $\frac{d\overline{e}(L)}{dL} < \gamma$, note that given that average effort strictly increases, it must be that $\frac{d\kappa^*_{\emptyset}(L)}{dL} > 0$. Average effort being $\overline{e}(L) = (\gamma + (1-\gamma)(1-\kappa^*_{\emptyset}(L))\overline{v} + \gamma(\overline{V_2} + L)$, it must be that $\frac{d\overline{e}(L)}{dL} < \gamma$. □

Finally, looking at selection, an increase in the intensity of violence strictly raises the proportion of congruent types among second-period subordinates if and only if $L < L^{full}(\gamma)$ so that some observed failures survive. The reasoning is the same as in the main text. When all observed failures are purged, under the same conditions as in the original model, selection deteriorates with violence because less agents are purged and replaced by new subordinates of better quality (for $L \in [L^{full}(\gamma), L^{ind}(\gamma)))$ or/and the pool of successful first-period agents becomes more tainted (for $L \geq L^{ind}(\gamma)$). Hence, our results on selection survive in this extension.

Proposition F.4. *For $r \in [\lambda, 2\lambda]$, the proportion of congruent types among second-period agents:*
(i) strictly increases with L if and only if $L < L^{full}(\gamma)$;
(ii) decreases with L if and only if $L \geq L^{full}(\gamma)$, strictly if $r > \lambda$.

Proof. For $L < L^{full}(\gamma)$, the proof proceeds exactly as in the proof of Proposition 4. This proves point (i).

For point (ii), for $L > L^{ind}(\gamma)$ (assuming $L^{ind}(\gamma)$), a similar reasoning as in the proof of Proposition 6 proves that the proportion of second-period congruent subordinates decreases with L. Hence, it only remains to prove the claim for $L \in [L^{full}(\gamma), L^{ind}(\gamma))$. To do so, denote again $\mathcal{P}(L)$ the proportion of congruent types among second-period agents. We obtain (ignoring arguments):

$$\mathcal{P}(L) = \gamma(1-\overline{e})r + \gamma\overline{e}\mu^S + (1-\gamma)(\kappa_\emptyset r + (1-\kappa_\emptyset)\lambda). \tag{F.2}$$

Using the value for effort (Equation F.1) and $\mu^S = \lambda\frac{e(c)}{\overline{e}}$, we obtain:

$$\mathcal{P}'(L) = \left(\gamma^2 - (1-\gamma)\frac{\partial\kappa_\emptyset(L)}{\partial L}\right)(\lambda - r). \tag{F.3}$$

Whenever $\kappa^*_\emptyset(L)$ is a corner, $\mathcal{P}'(L) \leq 0$, strictly if $r > \lambda$. Consider then the case when $\kappa^*_\emptyset(L) \in (0,1)$ ($\Leftrightarrow L \in (L^\emptyset(\gamma), L^{tot}(\gamma))$). We know from Proposition F.2 that the purge breadth is constant in L in this interval. Using Equation F.1 and $\kappa = \gamma(1-\overline{e}) + (1-\gamma)\kappa_\emptyset$, we have:

$$-\gamma^2 + (1-\gamma)(1+\gamma\overline{v})\frac{\partial\kappa^*_\emptyset(L)}{\partial L} = 0$$

Hence, $\gamma^2 - (1-\gamma)\frac{\partial\kappa^*_\emptyset(L)}{\partial L} = \frac{\gamma^2\overline{v}}{1+\overline{v}} > 0$ and $\mathcal{P}'(L) < 0$ (since $L^\emptyset(\gamma) < \overline{L}$ only if $r > \lambda$). ☐

F.2 Full commitment

In this appendix, we establish that most of our results remain unchanged when the autocrat can commit at the beginning of the game to the intensity of violence L and the purge incidences κ_F and κ_S (which determine the purge breadth κ). We refer to this case as 'full commitment' (in contrast to the original model where we study a partial commitment problem, the autocrat can only choose the intensity of violence at stage 1). Throughout, as above, we suppose that the replacement pool satisfies $r \geq \lambda$. Further, we assume that the payoff from a successful project for a type $\tau \in \{c, nc\}$—$v(\tau)$—is low relative to the insider benefits R: $\frac{v(c)v(nc)}{2} \leq R$. This assumption is only sufficient for all our results to go through, and we conjecture that our findings hold if it is fully relaxed (though the proof of this point proves difficult). It also should be noted that we do not look for conditions such that the purge is of one nature or the other (as in the main text, this would depend on the costs of purging C_0 and C_1 and the costs of investing in violence ζ_0 and ζ_1).[8] Instead, we look at the properties of the different type of purges. Finally, we use superscript fc to denote equilibrium values and use appropriate subscript to denote partial derivative whenever possible.

As in the main text, a type-τ agent's effort is defined by Equation A.2

$$e(\tau) = (1 - \kappa^S)v(S, \tau) + (\kappa^F - \kappa^S)(V_2(\tau) + L) \tag{F.4}$$

with the difference being that purge inferences are announced rather than (correctly) anticipated. As before, it is increasing in the risk of being purged condition on failure (κ_F) and decreasing in the probability of being purged conditional on success (κ_S). With this, we can now turn to the autocrat's problem who is different than in the main text. Nonetheless, we find several important similarities.

In a first preliminary result (Lemma F.1), we recover a generalized version of Lemma 1. With full commitment, intuitively, the autocrat first purges agents from the failure pool.[9] As the cost of violence diminishes, the autocrat purges a greater proportion of failures. The reason is the same as in the main text. With greater intensity of violence, congruent and non-congruent agents exit the failure pool at higher rate. Since there are less congruent agents in the failure pool to begin with, the quality of this pool deteriorates leading the autocrat to increase the purge inference κ_F. When parameters are such that violence is sufficiently high, all failures are purged. Greater violence then diminishes the quality of the success pool, which raises the autocrat's incentive to purge successful agents. Thus, much like the baseline model, the nature of the purge is fully linked to the level of violence. Some failures survive in mild purges, all failures are purged for intermediate violence, and some successful subordinates are removed when violence is very high.

[8]Existence is easily shown given that the choice set of the autocrat is compact.

[9]While the result is intuitive, its proof turns out to be more difficult than in the baseline model. The reason is that the autocrat could simultaneously balance κ_F and κ_S to achieve a certain level of selection. We show that the selection gain from such balancing act is always dominated by the loss from diminished effort due to $\kappa_S > 0$.

We further recover the non-monotonic relationship between the intensity of violence and purge breadth. In a discriminate purge with some failures surviving, the purge breadth and violence are positively correlated (since both quantities are chosen in the first stage, we state all our results in term of correlation). Greater violence is associated with more effort and better selection, absent higher purge breadth the first order condition for the purge incidences cannot be satisfied. In turn, in a discriminate purge with all failures purged, breadth and violence are negatively correlated as the size of the failure pool decreases with violence. Importantly, the relationship between violence and breadth is ambiguous in a semi-indiscriminate purge. While the purge inference κ_S^{fc} is positively linked with the intensity of violence L^{fc}, the autocrat may commit not to increase the purge incidence too much so as to limit the negative consequences of κ_S on effort. We thus obtain:

Proposition F.5. *There exist $L^{full\text{-}fc}, L^{ind\text{-}fc} \in \mathbb{R}_+^2$ such that if the equilibrium violence satisfies*
(a) $L^{fc} < L^{full\text{-}fc}$, then some failures survive and there exists a positive correlation between the equilibrium intensity of violence and purge breadth;
(a) $L^{fc} \in (L^{full\text{-}fc}, L^{ind\text{-}fc})$, then all failures are purged and there exists a negative correlation between the equilibrium intensity of violence and purge breadth;
(a) $L^{fc} > L^{ind\text{-}fc}$, then some successful subordinates are purged and the correlation between the equilibrium intensity of violence and purge breadth is ambiguous.

Turning to effort and selection, we find that the results of our baseline model holds in this setting, at least for discriminate purges. Effort and violence are positively correlated when only failures are targeted (but not necessarily in a semi-indiscriminate purge). Selection improves with violence if some failures survive, but decrease with L if all failures are purged, depending on the quality of the replacement pool. The results are again ambiguous in a semi-indiscriminate purge. Indeed, since we cannot determine the correlation between purge breadth and violence, we cannot extend the analysis to effort and selection which relies on the interaction effect between the purge inference κ_S and the intensity of violence.

To state our next result, it is useful to denote $\mathcal{A} = (L, \kappa_S, \kappa_F)$ the vector of autocrat's choice. We obtain:

Proposition F.6. *Consider two equilibrium strategies $\mathcal{A}^l$ and $\mathcal{A}^h$ such that $L^l < L^h \leq L^{disc\text{-}fc}$, then*
(i) The associated average efforts satisfy $\overline{e}^l < \overline{e}^h$;
(ii) The proportion of congruent types among second-period subordinates is strictly higher in equi-

librium $\mathcal{A}^h$ *than* $\mathcal{A}^l$ *whenever* $L^h \leq L^{full\_fc}$*;*

(iii) The proportion of congruent types among second-period subordinates is strictly lower in equilibrium $\mathcal{A}^h$ *than* $\mathcal{A}^l$ *whenever* $L^l > L^{full\_fc}$ *and* $r > \lambda$ *(constant if* $r = \lambda$*).*

Overall, our results hold in full when it comes to discriminate purge (as such, we can recover all our results from the baseline model). The main and only difference between the original model and the case of full commitment regards semi-indiscriminate purge, for which we cannot prove a positive correlation between violence and breadth or effort or a negative correlation with selection. Further, it should be noted that a semi-indiscriminate purge only occurs under specific stringent conditions.[10] Indeed, with full commitment, the autocrat fully takes into account the negative effect of κ_S on effort. Since she can commit, she may choose a high level of violence and not to purge the failure pool to avoid depressing effort. In the main text, in contrast, a semi-indiscriminate purge is an inescapable spillover of incentivizing effort through violence.

Proofs

Before proceeding we (re)introduce some notation. Recall that $\overline{e}$ denotes the total effort. We further denote $\mu^F(\kappa_F, \kappa_S)$, $\mu^S(\kappa_F, \kappa_S)$ the posteriors an agent is congruent conditional on failure and success, respectively as a function of the purge incidences, with the autocrat's anticipating agents' effort (when possible we omit arguments). The purge breadth is still $\kappa = \kappa^F(1-\overline{e}) + \kappa^S\overline{e}$.

Lemma F.1. *In equilibrium,* $\kappa_S^{fc} > 0 \Rightarrow \kappa_F^{fc} = 1$.

Proof. The proof proceeds by contradiction. The autocrat maximizes the following objective function:

$$\begin{aligned}
\mathcal{F}(L,\kappa^F,\kappa^S) &= \overline{e} + \beta(1-\overline{e})\Big(\kappa^F\big(rW_2(c)+(1-r)W_2(nc)\big) + (1-\kappa^F)\big(\mu^F W_2(c) + (1-\mu^F)W_2(nc)\big)\Big)\\
&\quad + \beta\overline{e}\Big(\kappa^S\big(rW_2(c)+(1-r)W_2(nc)\big) + (1-\kappa^S)\big(\mu^S W_2(c) + (1-\mu^S)W_2(nc)\big)\Big) - C(\kappa) - \zeta(L)\\
\mathcal{F}(L,\kappa^F,\kappa^S) &= \overline{e} + \beta\big(\lambda W_2(c) + (1-\lambda)W_2(nc)\big)\\
&\quad + \beta(1-\overline{e})\kappa^F(r-\mu^F)(W_2(c)-W_2(nc)) + \beta\overline{e}\kappa^S(r-\mu^S)(W_2(c)-W_2(nc)) - C(\kappa) - \zeta(L)
\end{aligned} \tag{F.5}$$

Suppose there exists an equilibrium in which the autocrat's strategy denoted $\mathcal{A}^{fc} = (L^{fc}, \kappa_F^{fc}, \kappa_S^{fc})$ satisfies $\kappa_S^{fc} > 0$ and $\kappa_F^{fc} < 1$. We show that the autocrat's expected utility is higher when she

[10] For example, a necessary condition is that $(r-\lambda)(v(c)-v(nc)) > \frac{1}{2}$.

instead chooses $\widehat{A} = (L^{fc}, \kappa_F^{fc}, 0)$. Throughout, we suppose that $\overline{v}+\kappa_F^{fc}(\overline{V_2}+L^{fc})-\kappa_S^{fc}(\overline{v}+\overline{V_2}+L) > 0$, but the proof can be extended to the case when effort is 0 in equilibrium.

Note that the purge breadths in the supposed equilibrium—denoted κ^{fc}—and in the proposed deviation—denoted $\widehat{\kappa}$—satisfy $\kappa^{fc} > \widehat{\kappa}$. Further, the posteriors satisfy $\mu^F(\kappa_F^{fc}, 0) < \mu^F(\kappa_F^{fc}, \kappa_S^{fc})$ since $\mu^F_{\kappa_S}(\kappa_F, \kappa_S) > 0$ (using a similar reasoning as in Lemma B.1). Therefore, we obtain

$$\begin{aligned}
&\mathcal{F}(L^{fc}, \kappa_F^{fc}, 0) - \mathcal{F}(L^{fc}, \kappa_F^{fc}, \kappa_S^{fc}) \\
&\qquad = \kappa_S^{fc}(\overline{v} + \overline{V_2} + L^{fc}) - \kappa_S^{fc}(\overline{v} + \overline{V_2} + L)\kappa_F^{fc}\beta(r - \mu^F(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) \\
&\qquad - (\overline{v} - \kappa_S^{fc}(\overline{v} + \overline{V_2} + L) + \kappa_F^{fc}(\overline{v} + L^{fc}))\kappa_S^{fc}\beta(r - \mu^S(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) \\
&\qquad + (1 - (\overline{v} + \kappa_F^{fc}(\overline{V_2} + L))\kappa_F^{fc}\beta(\mu_F(\kappa_F^{fc}, \kappa_S^{fc}) - \mu^F(\kappa_F^{fc}, 0))(W_2(c) - W_2(nc)) \\
&\qquad + C(\kappa^{fc}) - C(\widehat{\kappa}) \\
&\qquad > \kappa_S^{fc}(\overline{v} + \overline{V_2} + L^{fc}) - \kappa_S^{fc}(\overline{v} + \overline{V_2} + L)\kappa_F^{fc}\beta(r - \mu^F(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) \\
&\qquad - (1 - \kappa_S^{fc})(\overline{v} + \overline{V_2} + L)\kappa_S^{fc}\beta(r - \mu^S(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc))
\end{aligned}$$

Given $\beta(r - \mu^F(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) < 1$, $\mathcal{F}(L^{fc}, \kappa_F^{fc}, 0) - \mathcal{F}(L^{fc}, \kappa_F^{fc}, \kappa_S^{fc}) > 0$ if $(r - \mu^S(\kappa_F^{fc}, \kappa_S^{fc})) \leq 0$. We thus focus in what follows on the case when $(r - \mu^S(\kappa_F^{fc}, \kappa_S^{fc})) > 0$. Further, we can rewrite the lower bound on $\mathcal{F}(L^{fc}, \kappa_F^{fc}, 0) - \mathcal{F}(L^{fc}, \kappa_F^{fc}, \kappa_S^{fc})$ as

$$\begin{aligned}
&\kappa_S^{fc}(\overline{v} + \overline{V_2} + L^{fc}) \begin{pmatrix} 1 - \kappa_F^{fc}\beta(r - \mu^F(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) \\ -(1 - \kappa_S^{fc})\beta(r - \mu^S(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc)) \end{pmatrix} \\
&> \kappa_S^{fc}(\overline{v} + \overline{V_2} + L^{fc})\Big(1 - (\kappa_F^{fc} + (1 - \kappa_S^{fc}))\beta(r - \mu^F(\kappa_F^{fc}, \kappa_S^{fc}))(W_2(c) - W_2(nc))\Big)
\end{aligned}$$

So if $\kappa_F^{fc} < \kappa_S^{fc}$, the lower bound is strictly positive, implying $\mathcal{F}(L^{fc}, \kappa_F^{fc}, 0) - \mathcal{F}(L^{fc}, \kappa_F^{fc}, \kappa_S^{fc}) > 0$, contradicting the claim that $\mathcal{A}^{fc}$ is an equilibrium strategy.

Suppose therefore that $\kappa_F^{fc} \geq \kappa_S^{fc}$. Taking the derivative of $\mathcal{F}(\cdot)$ with respect to L, κ_F, and κ_S, we obtain:

$$\begin{aligned}\mathcal{F}_L(L,\kappa_F,\kappa_S) =&\overline{e}_L - \beta\overline{e}_L\kappa_F(r-\mu^F)(W_2(c)-W_2(nc)) + \beta\overline{e}_L\kappa_S(r-\mu^S)(W_2(c)-W_2(nc))\\ &-\beta(1-\overline{e})\kappa_F\mu^F_L(W_2(c)-W_2(nc)) - \beta\overline{e}\kappa_S\mu^S_L(W_2(c)-W_2(nc)) - \zeta'(L) \qquad \text{(F.6)}\\ \mathcal{F}_{\kappa_F}(L,\kappa_F,\kappa_S) =&\overline{e}_{\kappa_F} + \beta((1-\overline{e})-\overline{e}_{\kappa_F}\kappa_F)(r-\mu^F)(W_2(c)-W_2(nc)) + \beta\overline{e}_{\kappa_F}\kappa_S(r-\mu^S)(W_2(c)-W_2(nc))\\ &-\beta(1-\overline{e})\kappa_F\mu^F_{\kappa_F}(W_2(c)-W_2(nc)) - \beta\overline{e}\kappa_S\mu^S_{\kappa_F}(W_2(c)-W_2(nc))\\ &-((1-\overline{e})+(\kappa_S-\kappa_F)\overline{e}_{\kappa_F})C'(\kappa) \qquad \text{(F.7)}\\ \mathcal{F}_{\kappa_S}(L,\kappa_F,\kappa_S) =&\overline{e}_{\kappa_S} - \beta\overline{e}_{\kappa_S}\kappa_F(r-\mu^F)(W_2(c)-W_2(nc)) + \beta(\overline{e}+\overline{e}_{\kappa_S}\kappa_S)(r-\mu^S)(W_2(c)-W_2(nc))\\ &-\beta(1-\overline{e})\kappa_F\mu^F_{\kappa_S}(W_2(c)-W_2(nc)) - \beta\overline{e}\kappa_S\mu^S_{\kappa_S}(W_2(c)-W_2(nc))\\ &-(\overline{e}+(\kappa_S-\kappa_F)\overline{e}_{\kappa_S})C'(\kappa) \qquad \text{(F.8)}\end{aligned}$$

Note that the derivatives with respect to the purge incidences take into account the effect on the autocrat's posteriors since the autocrat fully anticipates (i) agents' effort and (ii) the effect of κ_F and κ_S on efforts. In particular, we have: $\mu^F_{\kappa_F} < 0 < \mu^F_{\kappa_S}$. Further, given the effort described in Equation A.2, $\mu^S_{\kappa_S} \propto \lambda(1-\lambda)(v(c)-v(nc))\left(R+L-\frac{v(c)v(nc)}{2}\right)(1-\kappa_F)$, which is positive under the assumption. And $\mu^S_{\kappa_F} \propto -\lambda(1-\lambda)(v(c)-v(nc))\left(R+L-\frac{v(c)v(nc)}{2}\right)(1-\kappa_S)$, which is negative under the assumption.

Rearranging a bit Equation F.7 and Equation F.8, we obtain:

$$\begin{aligned}\mathcal{F}_{\kappa_F}(L,\kappa_F,\kappa_S) =&\overline{e}_{\kappa_F}\Big(1-\kappa_F\beta(r-\mu^F)(W_2(c)-W_2(nc)) + \kappa_S\beta(r-\mu^S)(W_2(c)-W_2(nc)) + (\kappa_F-\kappa_S)C'(\kappa)\Big)\\ &+(1-\overline{e})\big(\beta(r-\mu^F)(W_2(c)-W_2(nc)) - C'(\kappa)\big)\\ &-\beta(1-\overline{e})\kappa_F\mu^F_{\kappa_F}(W_2(c)-W_2(nc)) - \beta\overline{e}\kappa_S\mu^S_{\kappa_F}(W_2(c)-W_2(nc))\\ \mathcal{F}_{\kappa_S}(L,\kappa_F,\kappa_S) =&\overline{e}_{\kappa_S}\Big(1-\kappa_F\beta(r-\mu^F)(W_2(c)-W_2(nc)) + \kappa_S\beta(r-\mu^S)(W_2(c)-W_2(nc)) + (\kappa_F-\kappa_S)C'(\kappa)\Big)\\ &+\overline{e}\big(\beta(r-\mu^S)(W_2(c)-W_2(nc)) - C'(\kappa)\big)\\ &-\beta(1-\overline{e})\kappa_F\mu^F_{\kappa_S}(W_2(c)-W_2(nc)) - \beta\overline{e}\kappa_S\mu^S_{\kappa_S}(W_2(c)-W_2(nc))\end{aligned}$$

Now, we have established above that if both purges inferences are interior, they must satisfy $\kappa_F^{fc} \geq \kappa_S^{fc}$. Further, $\mu^F > 0$ and $\mu^S < 1$ imply that $1-\kappa_F\beta(r-\mu^F)(W_2(c)-W_2(nc))+\kappa^S\beta(r-\mu^S)(W_2(c)-W_2(nc)) > 1-\kappa^F\beta r(W_2(c)-W_2(nc)) - \kappa^S\beta(1-r)(W_2(c)-W_2(nc)) > 1-\beta(W_2(c)-W_2(nc)) > 0$. Both combined imply that at equilibrium values, $1-\kappa_F\beta(r-\mu^F)(W_2(c)-W_2(nc)) + \kappa^S\beta(r-\mu^S)(W_2(c)-W_2(nc)) + (\kappa_F-\kappa_S)C'(\kappa) > 0$. But then (a) if $\beta(r-\mu^F)(W_2(c)-W_2(nc)) - C'(\kappa) < 0$ (which implies $\beta(r-\mu^S)(W_2(c)-W_2(nc)) - C'(\kappa) < 0$), then $\mathcal{F}_{\kappa^S}(L,\kappa^F,\kappa^S) < 0$ or (b) if $\beta(r-\mu^F)(W_2(c)-W_2(nc)) - C'(\kappa) \geq 0$, then $\mathcal{F}_{\kappa^F}(L,\kappa^F,\kappa^S) > 0$. In both cases, one of the

FOC is not satisfied, a contradiction.

Hence, we cannot have $\kappa_F^{fc} \geq \kappa_S^{fc}$ if both purge inferences are interior and so we necessarily have $\mathcal{F}(L^{fc}, \kappa_F^{fc}, 0) - \mathcal{F}(L^{fc}, \kappa_F^{fc}, \kappa_S^{fc}) > 0$ which contradicts $\mathcal{A}^{fc}$ is an equilibrium strategy. By contrapositive, it must be that $\kappa_S^{fc} > 0$ implies $\kappa_F^{fc} = 1$ as claimed. □

Given Lemma F.1, we can focus on three cases:

1) some failures survive: $\kappa_F^{fc} \in (0,1)$ and $\kappa_S^{fc} = 0$;

2) all failures are purged: $\kappa_F^{fc} = 1$ and $\kappa_S^{fc} = 0$;

3) some successful subordinates are purged: $\kappa_F^{fc} = 1$ and $\kappa_S^{fc} \in (0,1)$.

We now establish properties of purge inferences and breadth in these three cases. First, we show the correlation between the intensity of violence and the purge inference and breadth in case 1). Given that both violence and inference are chosen simultaneously, we can only express the relationship as correlation rather than comparative statics like in the main text.

Lemma F.2. *Consider two equilibrium strategies $\mathcal{A}^l = (L^l, \kappa_F^l, 0)$ and $\mathcal{A}^h = (L^h, \kappa_F^h, 0)$ such that $\kappa_F^l, \kappa_F^h \in (0,1)^2$. If $L^l < L^h$, then $\kappa_F^l < \kappa_F^h$ and the associated purge breadth also satisfies $\kappa^l < \kappa^h$.*

Proof. Assume some failures survive in both equilibria. Using Equation F.6 and Equation F.7 after imposing $\kappa_S = 0$ and rearranging, we obtain:

$$\begin{aligned}\mathcal{F}_L(L, \kappa_F, \kappa_S) =& \bar{e}_L\big(1 - \beta\kappa_F(r - \mu^F)(W_2(c) - W_2(nc)) + \kappa_F C'(\kappa)\big) \\ & - \beta(1-\bar{e})\kappa_F \mu_L^F (W_2(c) - W_2(nc)) - \zeta'(L)\end{aligned} \tag{F.9}$$

$$\begin{aligned}\mathcal{F}_{\kappa_F}(L, \kappa_F, \kappa_S) =& \bar{e}_{\kappa_F}(1 - \kappa_F(r - \mu^F)(W_2(c) - W_2(nc)) + \kappa_F C'(\kappa)\big) \\ & + (1-\bar{e})\big(\beta(r-\mu^F)(W_2(c) - W_2(nc)) - C'(\kappa)\big) - \beta(1-\bar{e})\kappa_F \mu_{\kappa_F}^F (W_2(c) - W_2(nc))\end{aligned} \tag{F.10}$$

We first prove that purge inference and purge breadth move in similar directions. Since $\beta\kappa_F(r - \mu^F)(W_2(c) - W_2(nc)) < 1$, $1 - \beta\kappa_F(r-\mu^F)(W_2(c) - W_2(nc)) + \kappa_F C'(\kappa) > 0$. At equilibrium values, it must then be that $(r - \mu^F)(W_2(c) - W_2(nc)) - C'(\kappa) < 0$ (all other quantities are positive). Rearranging $\mathcal{F}_{\kappa_F}(\cdot)$ as

$$\begin{aligned}\mathcal{F}_{\kappa_F}(L, \kappa_F, \kappa_S) =& \bar{e}_{\kappa_F} + \big((1-\bar{e}) - \bar{e}_{\kappa_F}\kappa_F\big)\big(\beta(r-\mu^F)(W_2(c) - W_2(nc)) - C'(\kappa)\big) \\ & - \beta(1-\bar{e})\kappa_F \mu_{\kappa_F}^F (W_2(c) - W_2(nc))\end{aligned}$$

it further implies that $(1-\bar{e}) - \bar{e}_{\kappa^F}\kappa^F = \frac{d(1-\bar{e})\kappa_F}{d\kappa_F} = \frac{d\kappa}{d\kappa_F} > 0$ at equilibrium values (otherwise the FOC can never be satisfied contradicting the assumption that some failures survive).

We now show that the intensity of violence and the purge inference are strategic complement. For this, it is sufficient to show that $\mathcal{F}_{\kappa_F L}(\cdot) > 0$.[11] Using Equation F.4 and Equation F.10, we obtain

$$\begin{aligned}\mathcal{F}_{\kappa_F L}(L, \kappa_F, 0) =& 1 - 2\kappa_F\big(\beta(r-\mu^F)(W_2(c) - W_2(nc)) - C'(\kappa)\big) \\ &+ \big((1-\overline{e}) - \overline{e}_{\kappa_F}\kappa_F\big)\big(-\mu^F_L\beta(W_2(c) - W_2(nc)) + \overline{e}_L\kappa_F C''(\kappa)\big) \\ &+ \beta\overline{e}_L\kappa_F\mu^F_{\kappa_F}(W_2(c) - W_2(nc)) - \beta(1-\overline{e})\kappa_F\mu^F_{\kappa_F L}(W_2(c) - W_2(nc)) \end{aligned} \tag{F.11}$$

Using Equation F.4 and Bayes' rule,

$$\begin{aligned}\mu^F_{\kappa_F} =& -\lambda(1-\lambda)\frac{(v(c) - v(nc))\left(R + L + v(c) + v(nc) - \frac{v(c)v(nc)}{2}\right)}{(1-\overline{e})^2} \\ \mu^F_{\kappa_F L} =& -\lambda(1-\lambda)\frac{(v(c) - v(nc))(1-\overline{e}) + 2\overline{e}_L(v(c) - v(nc))\left(R + L + v(c) + v(nc) - \frac{v(c)v(nc)}{2}\right)}{(1-\overline{e})^3} \\ =& -\lambda(1-\lambda)\frac{(v(c) - v(nc))}{(1-\overline{e})^2} + 2\frac{\overline{e}_L}{1-\overline{e}}\mu^F_{\kappa_F}\end{aligned}$$

This implies that the last line of Equation F.11 is strictly positive. Further, $\mu^F_L < 0$ (Lemma B.3) and $C''(\kappa) > 0$ so the second line of Equation F.11 is strictly positive since we have established above that $(1-\overline{e}) - \overline{e}_{\kappa_F}\kappa_F > 0$. Finally, at equilibrium values, $\beta(r-\mu^F)(W_2(c) - W_2(nc)) - C'(\kappa) < 0$ so the first line is also strictly positive. Hence, at equilibrium values, $\mathcal{F}_{\kappa_F L} > 0$, which proves that $L^l < L^h$ implies $\kappa^l_F < \kappa^h_F$.

We now show that $L^l < L^h$ also implies $\kappa^l < \kappa^h$. Suppose not. Slightly abusing notation denote $\mu^l = \mu^F\big|_{L=L^l, \kappa_F=\kappa^l}$ and similarly $\mu^h = \mu^F\big|_{L=L^h, \kappa_F=\kappa^h}$. Notice that since $\mu^F_L < 0$, $\mu^F_{\kappa_F} < 0$, and $\mu^F_{\kappa_F L} < 0$, then $\mu^h < \mu^l$. Further using the formula for $\mu^F_{\kappa_F}$ above, note that $-\beta(1-\overline{e})\mu^F_{\kappa_F}$ is strictly increasing with κ_F and L. Hence, denoting $K^j := -\beta(1-\overline{e})\kappa_F\mu^F_{\kappa_F}(W_2(c) - W_2(nc))\big|_{L=L^j, \kappa_F=\kappa^j_F}$, $j \in \{1, 2\}$, we obtain $K^l < K^h$.

Now,

$$\begin{aligned}\mathcal{F}_{\kappa_F}(L^j, \kappa^j_F, 0) =& \overline{v} + \kappa^j_F(\overline{V_2} + L^j) + K^j \\ &+ \big(1 - \overline{v} - \kappa^j_F(\overline{V_2} + L^j)\big)(\beta(r-\mu^j)(W_2(c) - W_2(nc)) - C'(\kappa^j))\end{aligned}$$

Now if $\kappa^h < \kappa^l$, then $C'(\kappa^h) < C'(\kappa^l)$, which implies $(\beta(r-\mu^l)(W_2(c) - W_2(nc)) - C'(\kappa^l)) < (\beta(r-\mu^h)(W_2(c) - W_2(nc)) - C'(\kappa^h)) < 0$ given $\mu^h < \mu^l$. Given $1 - \overline{v} - \kappa^l_F(\overline{V_2} + L^l) > 1 - \overline{v} - \kappa^h_F(\overline{V_2} + L^h)$

[11] Take a parameter value which only affects the intensity of violence directly, say ζ_0. Then by the Implicit Function Theorem, $\mathcal{F}_{\kappa_F\kappa_F}\frac{d\kappa_F}{d\zeta_0} + \mathcal{F}_{\kappa_F L}\frac{dL}{d\zeta_0} = 0$ (ignoring superscript and arguments). Since we assume κ_F interior, it must be that $\mathcal{F}_{\kappa_F\kappa_F} < 0$ evaluated at equilibrium values. Hence, $\frac{d\kappa_F}{d\zeta_0}$ has the same sign as $\mathcal{F}_{\kappa_F L}\frac{dL}{d\zeta_0}$.

(given $\kappa_F^l < \kappa_F^h$ and $L^l < L^h$) and $\beta(r-\mu^j)(W_2(c)-W_2(nc))-C'(\kappa^j) < 0$ (a necessary condition for the FOC to hold), then $\big(1-\overline{v}-\kappa_F^l(\overline{V_2}+L^l)\big)(\beta(r-\mu^l)(W_2(c)-W_2(nc))-C'(\kappa^l)) < \big(1-\overline{v}-\kappa_F^h(\overline{V_2}+L^h)\big)(\beta(r-\mu^h)(W_2(c)-W_2(nc))-C'(\kappa^h))$. Since $\overline{v}+\kappa_F^l(\overline{V_2}+L^l)+K^l < \overline{v}+\kappa_F^h(\overline{V_2}+L^h)+K^h$, this implies $\mathcal{F}_{\kappa_F}(L^l,\kappa_F^l,0) < \mathcal{F}_{\kappa_F}(L^h,\kappa_F^h,0)$. But this contradicts that both κ_F^l and κ_F^h are interior. Hence, it must be that $\kappa^l < \kappa^h$. □

Using Lemma F.2, we can establish that the purge nature changes with the intensity of violence under a simple condition.

Remark F.1. *If $\mathcal{F}_{\kappa_F}(\overline{L},1,0) > 0$, then there exists $L^{full\_fc} \in [0,\overline{L})$ such that $\kappa_F^{fc} < 1$ if and only if $L^{fc} < L^{full\_fc}$.*

Proof. Follows from the proof of Lemma F.2. □

We now turn to the case when $\mathcal{F}_{\kappa_F}(\overline{L},1,0) > 0$ and parameter values (i.e., ζ_0 and ζ_1) are such that $L^{fc} > L^{full\_fc}$.

First, we show that if the equilibrium is such that $\kappa_S^{fc} > 0$ and the purge is semi-indiscriminate, the intensity of violence and the purge inference are positively correlated. We also highlight that a similar relationship between violence and breadth does not necessarily hold.

Lemma F.3. *Consider two equilibrium strategies $\mathcal{A}^l = (L^l,1,\kappa_S^l)$ and $\mathcal{A}^h = (L^h,1,\kappa_S^h)$ such that $\kappa_S^l,\kappa_S^h \in (0,1)^2$. If $L^l < L^h$, then $\kappa_S^l < \kappa_S^h$, the ranking of the associated purge breadth κ^l and κ^h is ambiguous.*

Proof. Like in the baseline model (see Equation D.7), it is useful to rewrite the objective function as

$$\mathcal{F}(L,1,\kappa_S) = \overline{e}+W_2(nc)+\beta\mu^S(W_2(c)-W_2(nc))+\beta((1-\overline{e})+\kappa_S\overline{e}))(r-\mu^S)(W_2(c)-W_2(nc))-C(\kappa)-\zeta(L),$$

with $\overline{e} = (1-\kappa_S)(\overline{v}+\overline{V_2}+L)$ and $\kappa = (1-\overline{e})+\overline{e}\kappa_S$.

Taking the derivative with respect to κ_S and L, we obtain (since $\mu^S_{\kappa_S} = 0$):

$$\begin{aligned}\mathcal{F}_L(L,1,\kappa_S) =& \overline{e}_L + \frac{\partial((1-\overline{e})+\kappa_S\overline{e})}{\partial L}\big(\beta(r-\mu^S)(W_2(c)-W_2(nc))-C'(\kappa)\big)\\ &+\beta\mu^S_L\big(1-((1-\overline{e})+\overline{e}\kappa_S)\big)(W_2(c)-W_2(nc))-\zeta'(L)\\ \mathcal{F}_{\kappa_S}(L,1,\kappa_S) =& \overline{e}_{\kappa_S} + \frac{\partial((1-\overline{e})+\kappa_S\overline{e})}{\partial \kappa_S}\big(\beta(r-\mu^S)(W_2(c)-W_2(nc))-C'(\kappa)\big)\end{aligned}$$

Now taking the cross derivative, we have:

$$\begin{aligned}\mathcal{F}_{\kappa_S L}(L,1,\kappa_S) =& -1 + 2(1-\kappa_S)\big(\beta(r-\mu^S)(W_2(c)-W_2(nc)) - C'(\kappa)\big)\\ &+ 2(1-\kappa_S)(\overline{v}+\overline{V_2}+L)\big(-\beta\mu^S_L(W_2(c)-W_2(nc))\big) + (1-\kappa_S)^2 C''(\kappa)\big)\\ \mathcal{F}_{\kappa_S L}(L,1,\kappa_S) =& \frac{\mathcal{F}_{\kappa_S}(L,1,\kappa_S)}{\overline{v}+\overline{V_2}+L}\\ &+ 2(1-\kappa_S)(\overline{v}+\overline{V_2}+L)\big(-\beta\mu^S_L(W_2(c)-W_2(nc))\big) + (1-\kappa_S)^2 C''(\kappa)\big)\end{aligned}$$

At equilibrium values, it must be that $\mathcal{F}_{\kappa_S}(L,1,\kappa_S) = 0$ and so $\mathcal{F}_{\kappa_S L}(L,1,\kappa_S) > 0$ since $\mu^S_L < 0$. By a similar reasoning as in the proof of Lemma F.2, purge inference and violence are positively correlated then.

We now highlight that the ranking of purge breadth is ambiguous. To see that, using $\overline{e} = (1-\kappa_S)(\overline{v}+\overline{V_2}+L)$, the FOC with respect to κ_S and L are, respectively (after slight rearranging):

$$\begin{aligned}(1-\kappa_S)\big(\beta(r-\mu^S)(W_2(c)-W_2(nc)) - C'(\kappa)\big) =& \frac{1}{2}\\ \lambda(1-\lambda)\beta\frac{(v(c)-v(nc))^2(1+\frac{v(c)+v(nc)}{2})}{\overline{v}+\overline{V_2}+L}(1-\kappa_S)^2 + \zeta'(L) =& \frac{1-\kappa_s}{2}\end{aligned}$$

Absent additional assumptions, the following an increase in both κ_S and L, the first FOC can be satisfied either with an increase in κ or with a decrease in κ. Hence, the overall effect is ambiguous. □

We then obtain the following result:

Remark F.2. *If $\mathcal{F}_{\kappa_S}(\overline{L},1,0) > 0$, then there exists $L^{disc\_fc} \in [0,\overline{L})$ such that $\kappa_S^{fc} > 0$ if and only if $L^{fc} < L^{disc\_fc}$.*

Proof. Follows from the proof of Lemma F.3. □

For ease of exposition, let $L^{full\_fc} > \overline{L}$ if $\mathcal{F}_{\kappa_F}(L,1,0) \leq 0$ and $L^{disc\_fc} > \overline{L}$ if $\mathcal{F}_{\kappa_S}(L,1,0) \leq 0$. Before stating our main result regarding the link between violence and purge incidence/breadth, we establish that $L^{disc\_fc} > L^{full\_fc}$ whenever $L^{full\_fc} < \overline{L}$.

Remark F.3. *If $L^{full\_fc} < \overline{L}$, then $L^{disc\_fc} > L^{full\_fc}$.*

Proof. To show the result, it is sufficient to establish that $\mathcal{F}_{\kappa_F}(L,1,0) = 0 \Rightarrow \mathcal{F}_{\kappa_S}(L,1,0) < 0$ and $\mathcal{F}_{\kappa_S}(L,1,0) = 0 \Rightarrow \mathcal{F}_{\kappa_F}(L,1,0) > 0$. Using Equation F.7 and Equation F.8 evaluated at $\kappa_F = 1$

and $\kappa_S = 0$, we obtain:

$$\begin{aligned}
\mathcal{F}_{\kappa_F}(L,1,0) =& \overline{e}_{\kappa_F}\Big(1-\beta(r-\mu^F)(W_2(c)-W_2(nc))+C'(\kappa)\Big)\\
&+(1-\overline{e})\big(\beta(r-\mu^F)(W_2(c)-W_2(nc))-C'(\kappa)\big)\\
&-\beta(1-\overline{e})\mu^F_{\kappa_F}(W_2(c)-W_2(nc))\\
\mathcal{F}_{\kappa_S}(L,1,0) =& \overline{e}_{\kappa_S}\Big(1-\beta(r-\mu^F)(W_2(c)-W_2(nc))+C'(\kappa)\Big)\\
&+\overline{e}\big(\beta(r-\mu^S)(W_2(c)-W_2(nc))-C'(\kappa)\big)\\
&-\beta(1-\overline{e})\mu^F_{\kappa_S}(W_2(c)-W_2(nc))
\end{aligned}$$

Now, given $\overline{e}_{\kappa_S} < 0 < \overline{e}\kappa_F$ and $\mu^F_{\kappa_F} < 0 < \mu^F_{\kappa_S}$, we obtain:

(a) $\mathcal{F}_{\kappa_F}(L,1,0)=0 \Rightarrow \beta(r-\mu^F)(W_2(c)-W_2(nc))-C'(\kappa)<0 \Rightarrow \beta(r-\mu^S)(W_2(c)-W_2(nc))-C'(\kappa) < 0 \Rightarrow \mathcal{F}_{\kappa_S}(L,1,0)<0$ and

(b) $\mathcal{F}_{\kappa_S}(L,1,0)=0 \Rightarrow \beta(r-\mu^S)(W_2(c)-W_2(nc))-C'(\kappa)>0 \Rightarrow \beta(r-\mu^F)(W_2(c)-W_2(nc))-C'(\kappa) > 0 \Rightarrow \mathcal{F}_{\kappa_F}(L,1,0)>0$. □

If parameter values are such that $L^{fc} \in (L^{full\_fc}, L^{disc\_fc})$, then all failures are purged ($\kappa_F^{fc}=1$ and $\kappa_S^{fc}=0$). L^{fc} must be the solution to:

$$\max_L \overline{e}+\beta(1-\overline{e})(rW_2(c)+(1-r)W_2(nc))+\beta\overline{e}(\mu^S W_2(c)+(1-\mu^S)W_2(nc))-C(1-\overline{e})-\zeta(L),$$

with $\overline{e}=\overline{v}+\overline{V_2}+L$ like in the baseline model (see Lemma C.2). Hence, there is a negative correlation between violence and breadth follows in this case.

Proof of Proposition F.5

Follows directly from all the results above. □

Proof of Proposition F.6

For $L < L^{disc\_fc}$, the result follow from a similar reasoning as in the baseline model since $L^l < L^h$ implies $\kappa_F^l \leq \kappa_F^h$ (with strict inequality if $\kappa_F^l < 1$) . So we recover the same direct and indirect effects as in the baseline model. In particular, the case of discriminate purge with all failures purged is absolutely identical. □

F.3 Repression

In this subsection, we return to the case of partial commitment. We also suppose that the autocrat seeks to survive (and gets a payoff of 1 if she does so). Unlike Appendix E.3, we no longer assume that purged agents are replaced. Instead, we simply suppose that the autocrat's survival probability depends negatively on the mass of non-congruent subordinates $\mathcal{N}(L)$. That is, the survival probability is:

$$P(survives) = \gamma\bar{e}_1 + (1-\gamma)\big((1-\epsilon) + \epsilon(1-\bar{e}_1)\big) \times \beta\big(\mathcal{N}(L)\big), \tag{F.12}$$

with $\beta(\cdot)$ decreasing and weakly concave.

The mass of non-congruent subordinates after the repression is:

(i) $\mathcal{N}(L) = (1-\bar{e}_1)(1-\kappa_F)(1-\mu^F) + \bar{e}_1(1-\mu^S)$ when some failures survive;

(ii) $\mathcal{N}(L) = \bar{e}_1(1-\mu^S)$ when all failures are purged;

(iii) $\mathcal{N}(L) = \bar{e}_1(1-\kappa_S)(1-\mu^S)$ when some successful citizens are purge (i.e., repression is semi-indiscriminate).

Define

$$R_{PD}(\kappa_F, L) = -(1-\gamma)\big((1-\epsilon) + \epsilon(1-\bar{e}_1)\big)(1-\mu^F)\beta'\big(\mathcal{N}(L)\big) - C_0 - C_1\kappa_F(1-\bar{e}_1) \tag{F.13}$$

When some failures survive, $\kappa_F^*(L)$ is defined as a solution to $R_{PD}(\kappa_F, L) = 0$ since the autocrat takes effort and violence as given at the time of her purging decision.

Define

$$R_{SD}(\kappa_S, L) = -(1-\gamma)\big((1-\epsilon) + \epsilon(1-\bar{e}_1)\big)(1-\mu^S)\beta'\big(\mathcal{N}(L)\big) - C_0 - C_1(1-(1-\kappa_S)\bar{e}_1) \tag{F.14}$$

When repression is semi-indiscriminate, $\kappa_S^*(L)$ is defined as a solution to $R_{SD}(\kappa_S, L) = 0$ since the autocrat takes effort and violence as given at the time of her purging decision.

Comparing Equation F.13-F.14 and Equation E.4-E.5, it can be checked that we can apply a similar reasoning as Section E.3 to show that as long as ϵ is sufficiently small:

(i) discriminate repression tends to be mild and semi-indiscriminate repression violent (Proposition E.1);

(ii) the size of repression is non-monotonic in violence (Proposition E.2);

(iii) as long as $\beta(\cdot)$ is not "too concave," effort always increases with violence, but selection strictly improves with the intensity of violence only if L is low to begin with (Propositions E.3 and E.4).

F.4 Single agent set-up

In this last subsection, we study a model with a single agent rather than a mass of agents. As our goal is to illustrate the differences with our baseline model, we only perform a comparative statics on violence and do not consider the autocrat's problem of choosing the optimal intensity of violence. Our single-agent model imposes a binary level of effort 0 or 1. The reason is that with continuous effort, the purge is semi-indiscriminate only for a set of parameter values of measure 0 (details available upon request).

Consider a variant of our model with with three players: an autocrat (A), a single incumbent subordinate (I), and a potential new subordinate (N). At the end of period 1, the autocrat decides whether to purge the current subordinate I ($k \in \{0, 1\}$, with $k = 1$ denoting I being purged). If agent I is purged, then N becomes the autocrat's subordinate. Further, if purged, agent I suffers a loss $L \geq 0$. Each period, whomever is the autocrat's agent works on a project. A project can be a success $\omega = S$ or a failure $\omega = F$. The probability a project is successful depends on the agent's costly effort, which takes value $e \in \{0, 1\}$. The cost of effort is $c(e) = \rho \times e$, with $c > 0$ and the probability that the project is successful is $Pr(\omega = S) = q \times e$, with $q \in (0, 1)$ (and q common knowledge).

The incumbent agent I is either congruent ($\tau_I = c$) or non-congruent ($\tau_I = nc$). I's type is his private information. However, it is common knowledge that there is a probability $\lambda \in (0, 1)$ that I is congruent: $Pr(\tau_I = c) = \lambda$. Similarly, N is either congruent or non-congruent. His type is his private information and the probability that N is congruent is r: $Pr(\tau_N = c) = r \in [\lambda, 1)$ (as in our original model). All types enjoy a payoff $R > 0$ from being a regime insider. In addition, a type $\tau \in \{c, nc\}$ gets a payoff $v(F, \tau) = 0$ from a non-successful project and $v(S, \tau)$, with $v(S, c) > 0$ and $v(S, nc) \in [0, v(S, c))$ from a successful project. I's payoff in period 1 is

$$u_1^I(e; \tau) = R + (1 - k) \times v(\omega, \tau) + k(-L) - \rho \times e \tag{F.15}$$

In the second period the payoff of subordinate $J \in \{I, N\}$ is:

$$u_2^J(e; \tau) = R + v(\omega, \tau) - \rho \times e \tag{F.16}$$

The autocrat cares about the success of the agent's project. She gets a payoff of 1 when the project is successful and 0 otherwise. In addition, the autocrat pays a cost $C_1 > 0$ when she purges the agent S at the end of period 1. Her utility function can thus be represented as:

$$U_A(\kappa) = \mathbb{I}_{\{\omega_1 = S\}} + \mathbb{I}_{\{\omega_2 = S\}} - C_1 \times k, \tag{F.17}$$

To summarize, the timing of the game is:

Period 1:

1. I and N privately observe their type $\tau \in \{c, nc\}$;
2. I decides whether to exert effort on his project: $e \in \{0, 1\}$;
3. The autocrat A observes $\omega_1 \in \{S, F\}$. She decides whether to purge I;
4. First-period payoffs are realised;

Period 2:

1. The subordinate (I if not purged, N if purged) chooses effort level;
2. ω_2 and second-period payoffs are realized, the game ends.

The equilibrium concept is Perfect Bayesian Equilibrium. Notice that the autocrat observes only the outcome of the project in period 1 (not I's effort) before deciding whether to purge I. We impose D1 equilibrium refinement to facilitate comparison with the baseline model.

Throughout, we use the same notation as in the baseline model. $V_2(\tau)$ denotes an agent's expected payoff in period 2 as a function of his type. The (ex-ante) average payoffs are denoted by $\overline{v} = \lambda v(S, c) + (1 - \lambda)v(S, nc)$ and $\overline{V_2} = \lambda V_2(c) + (1 - \lambda)V_2(nc)$.

The agent's strategy is a mapping from his type τ to an effort level $e \in \{0, 1\}$ denoted with slight abuse of notation $e(\tau) \in \{0, 1\}$. A mixed strategy is denoted $\alpha : \{c, nc\} \to \Delta(\{0, 1\})$. For the autocrat, her purging strategy is a mapping from outcome ω to a purge decision $k \in \{0, 1\}$. In particular, we denote the probability I is purged after outcome $\omega \in \{F, S\}$ κ_ω (the equivalent of the purge incidence in the baseline model). Finally, denote $\mu^\omega(\alpha(c), \alpha(nc))$ the autocrat's posterior that I is congruent after observing $\omega \in \{F, S\}$) when she anticipates (correctly in equilibrium) that I plays the tuple of strategies $(\alpha(c), \alpha(nc))$. Denote $\alpha_F = \lambda(1 - \alpha(c)q) + (1 - \lambda)(1 - \alpha(nc)q)$ the probability I fails and $\alpha_S = 1 - \alpha_F$, the probability it succeeds. We further denote $\mu^F(\alpha(c), \alpha(nc))$ and $\mu^S(\alpha(c), \alpha(nc))$ the posteriors as a function of the (anticipated) subordinate's strategy.

To make the problem interesting, we impose two assumptions on parameter values. First, we suppose that only congruent agents exert effort in period 2 ($qv(S, nc) - \rho < 0 < qv(S, c) - \rho$). This implies that $V_2(c) = R + qv(S, c) - \rho$ and $V_2(nc) = R$. Further, the autocrat's gain from replacing a non-congruent type with a congruent type is $\mathcal{D}^{c,nc} := q$. Using this result, we assume that the autocrat has some incentive to purge when her agent plays a separating strategy $C_1 < (r - \mu^F(1, 0))\mathcal{D}^{c,nc}$. Observe that absent the first condition, a purge does not occur in this set-up.

First, observe that general version of Lemma 1 (i.e., $\kappa^*_S(L) > 0 \Rightarrow k^*_F(L) = 1$) holds in this setting due to the D1 equilibrium refinement. Second, a no effort equilibrium does not exist because of the D1 refinement. Third, there is no equilibrium in which a congruent type randomizes between effort and no effort. If so, $\kappa_S = 0$ (since success perfectly reveals congruence) and a congruent type's expected payoff from effort is $qv(S,c) - \rho + q\kappa_F(V_2(c) + L) + (1-\kappa_F)V_2(c) + \kappa_F(-L)$. If he does not exert effort, his expected payoff is $(1-\kappa_F)V_2(c) + \kappa_F(-L)$. Under our assumption that $qv(S,c) - \rho > 0$, a congruent type is never indifferent.

We thus look for three types of equilibria:

(i) Discriminate purge with some failures surviving ($\kappa^*_F(L) \in (0,1)$) in which a non-congruent type randomizes between effort and no effort;

(ii) Discriminate purge with all failures purged ($\kappa^*_F(L) = 1$ and $\kappa^*_S(L) = 0$) in which a non-congruent type plays a possibly degenerate mixed strategy;

(iii) Semi-indiscriminate purge ($\kappa^*_S(L) > 0$) in which a non-congruent S randomizes between effort and no effort.

Type (i) equilibrium.

The equilibrium features:

(a) $\kappa_F = \frac{\rho - qv(S,nc)}{q(V_2(nc)+L)}$ and $\kappa_S = 0$;

(b) $\alpha(c) = 1$ and $\alpha(nc)$ is the solution to $(r - \mu^F(1,\alpha(nc)))\mathcal{D}^{c,nc} = C_1$, with $\mu^F(1,\alpha(nc)) = \frac{\lambda(1-q)}{\lambda(1-q)+(1-\lambda)(1-q\alpha(nc))}$.

This equilibrium exists if and only if $(r-\lambda)\mathcal{D}^{c,nc} < C_1$ and $qL > \rho - q(v(S,nc) + V_2(nc))$.

In this equilibrium, the ex-ante probability a subordinate is congruent in the second period is $\mathcal{P}(L) = \alpha_F\big(\kappa_F \times r + (1-\kappa_F) \times \mu^F(1,\alpha(nc))\big) + \alpha_S\mu^S(1,\alpha(nc))$. Since κ_F strictly decrease with L and other quantities do not depend on L, $\mathcal{P}'(L) < 0$.

Type (ii) equilibrium.

The equilibrium a.e features:

(a) $\kappa_F = 1$ and $\kappa_S = 0$;

(b) $\alpha(c) = 1$ and $\alpha(nc) = 0$;

This equilibrium exists if and only if $qL < \rho - q(v(S,nc) + V_2(nc))$.

To see this, suppose that $\alpha(nc) = 1$, then $\mu^F(1,1) = \mu^S(1,1) = \lambda$ and the autocrat either always purges or never purges except if $(r-\lambda)\mathcal{D}^{c,nc} = C_1$ (a knife-edge condition). A contradiction with the assumed equilibrium type. Suppose $\alpha(nc) \in (0,1)$, then given $\kappa_F = 1$, it must be that $q(V_2(nc) + L) + qv(S, nc) - \rho = 0$ again a knife-edge condition. Hence, almost always, the equilibrium is as described above.

In this equilibrium, the ex-ante probability a subordinate is congruent in the second period is $\mathcal{P}(L) = \alpha_F r + \alpha_S$, with $\mathcal{P}'(L) = 0$.

Type (iii) equilibrium.

The equilibrium features:

(a) $\kappa_F = 1$ and $\kappa_S = 1 - \frac{\rho}{q(v(S,c)+V_2(c)+L)}$;

(b) $\alpha(c) = 1$ and $\alpha(nc)$ is the solution to $(r - \mu^S(1, \alpha(nc))\mathcal{D}^{c,nc} = C_1$ with $\mu^S(1, \alpha(nc)) = \frac{\lambda q}{\lambda q+(1-\lambda)q\alpha(nc)}$.

The equilibrium exists if and only if: $(r - \lambda)\mathcal{D}^{c,nc} > C_1$ and $qL > \rho - q(v(S, nc) + V_2(nc))$.

In this equilibrium, the ex-ante probability a subordinate is congruent in the second period is $\mathcal{P}(L) = \alpha_F r + \alpha_S(\kappa_S r + (1 - \kappa_S)\mu^S(1, \alpha(nc))$. Since κ_S strictly increase with L and other quantities do not depend on L, $\mathcal{P}'(L) > 0$.

Using the results above, we can observe major differences with our baseline model.

1. A purge is discriminate with all failures purged (type (ii) equilibrium) only if violence is low rather than intermediary like in the baseline model.

2. The nature of the purge does not depend on the intensity of violence unlike in the baseline model since for $qL > \rho - q(v(S, c) + V_2(c))$, it is fully determined by the quality of the replacement pool.

3. Fixing the nature of the purge, effort does not depend on violence.

4. The effect of increased violence on selection is monotonic (weakly) even if $r > \lambda$. Indeed, consider the two possible cases.

Case (i) $(r - \lambda)\mathcal{D}^{c,nc} < C_1$: For low intensity ($qL < \rho - q(v(S, nc) + V_2(nc))$), all failures are purged and selection does not depend on L; for high intensity ($qL > \rho - q(v(S, nc) + V_2(nc))$), some failures survive and selection worsens with the intensity of violence (the exact opposite of our results).

Case (ii) $(r - \lambda)\mathcal{D}^{c,nc} > C_1$: For low intensity ($qL < \rho - q(v(S, nc) + V_2(nc))$), all failures are purged and selection does not depend on L; for high intensity ($qL > \rho - q(v(S, nc) + V_2(nc))$), some successful subordinates are purged and selection improves with the intensity of violence (again the exact opposite of our model).

These four major differences imply that the many-to-one accountability problem we study in the main text is fundamentally different than a one-to-one accountability problem. The latter cannot be used to approximate the former.