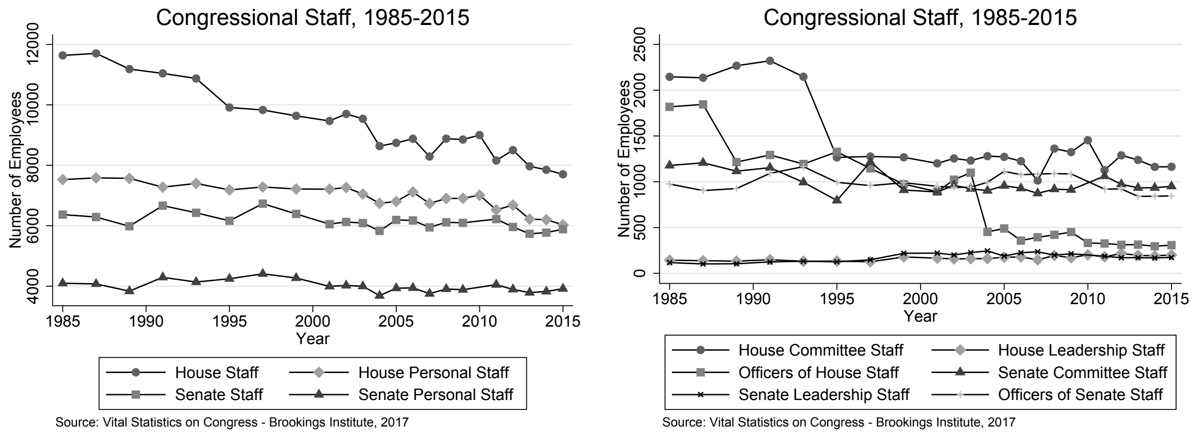
Supplemental Appendix – Online Only

"Strategic Legislative Subsidies: Informational Lobbying and the Cost of Policy" by Christopher J. Ellis and Thomas Groll published in *American Political Science Review.*

Empirical Trends



(a) Total and Personal Congressional Staff.

(b) Committee Staff, Leadership Staff, and Officers.

Figure 11: Congressional Staff.

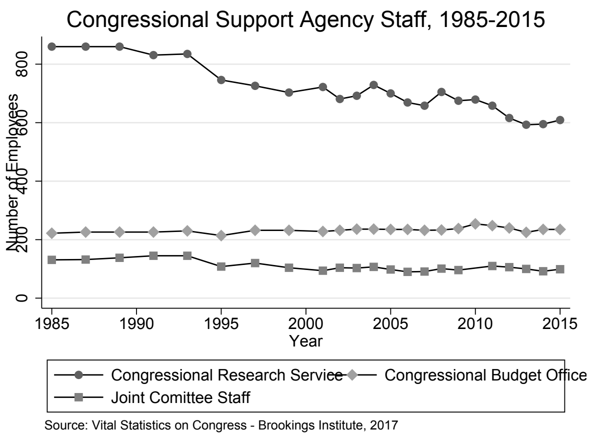
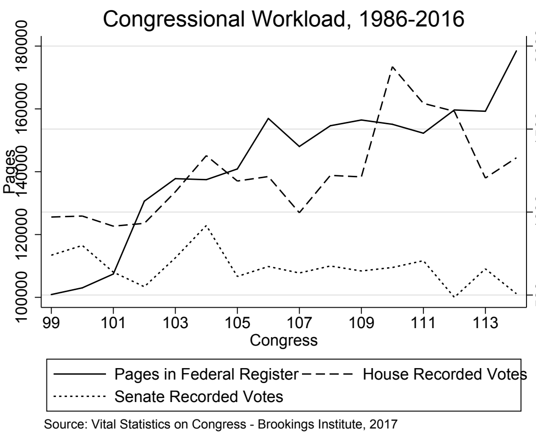
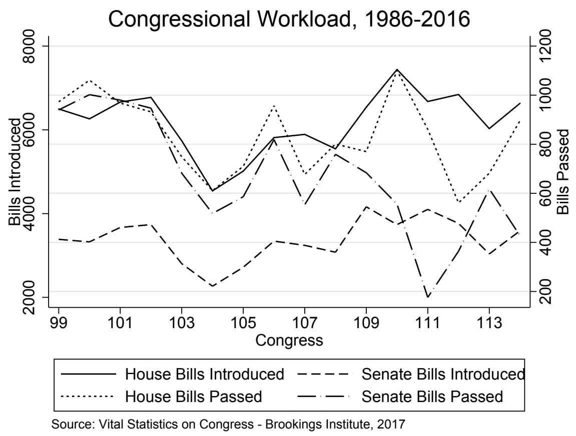


Figure 12: Congressional Support Agencies Staff.



(a) Federal Pages and Recorded Votes.



(b) Bills Introduced and Bills Passed.

Figure 13: Congressional Workload.

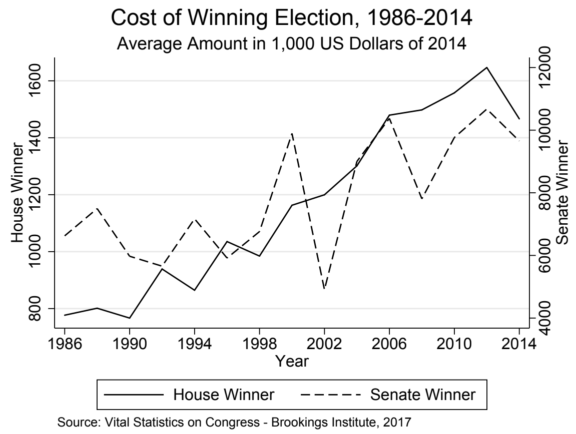


Figure 14: Electoral Costs for Congress.

Information Signals and Beliefs

Expected Information Signals

Given the sequence of play described in Figure 2 the players' information gathering choices depend on their expected signals conditional on their prior beliefs which may be written as

$$x^r(\lambda) \equiv Pr\left[x = x^r|\lambda\right] = \epsilon^L\lambda + (1-\eta^L)(1-\lambda) \quad \text{(B.1)}$$

$$\text{and } x^s(\lambda) \equiv Pr\left[x = x^s|\lambda\right] = \eta^L(1-\lambda) + (1-\epsilon^L)\lambda \quad \text{(B.2)}$$

and similarly for the policymaker as

$$z^r(\lambda^L) \equiv Pr[z = z^r|\lambda^L] = \epsilon^P\lambda^L + (1-\eta^P)(1-\lambda^L) \quad \text{(B.3)}$$

$$\text{and } z^s(\lambda^L) \equiv Pr[z = z^s|\lambda^L] = \eta^P(1-\lambda^L) + (1-\epsilon^P)\lambda^L, \quad \text{(B.4)}$$

where λ^L reflects the policymaker's and lobby's posterior belief and reflects the sequence of play in which policymakers gathers information after observing a lobby's signal. The corresponding policymaker's expected signals conditional on the lobby's observed information signal are

$$z^r(\lambda(x^j)) \equiv \epsilon^P\lambda^L(x^j) + (1-\eta^P)(1-\lambda^L(x^j)) \quad \text{(B.5)}$$

$$\text{and } z^s(\lambda(x^j)) \equiv \eta^P(1-\lambda^L(x^j)) + (1-\epsilon^P)\lambda^L(x^j) \text{ for } j = r, s. \quad \text{(B.6)}$$

Homogeneous Information Accuracies For the case of $\epsilon^L = \eta^L = \epsilon^P = \eta^P \equiv \mu$ we have

$$x^r(\lambda) \equiv Pr\left[x = x^r|\lambda\right] = z^r(\lambda^L) \equiv Pr\left[z = z^r|\lambda\right] = \mu\lambda + (1-\mu)(1-\lambda) \quad \text{(B.7)}$$

$$\text{and } x^s(\lambda) \equiv Pr\left[x = x^s|\lambda\right] = z^s(\lambda^L) \equiv Pr\left[z = z^s|\lambda\right] = \mu(1-\lambda) + (1-\mu)\lambda \quad \text{(B.8)}$$

as well as

$$z^r(\lambda(x^j)) \equiv \mu\lambda^L(x^j) + (1-\mu)(1-\lambda^L(x^j)) \quad \text{(B.9)}$$

$$\text{and } z^s(\lambda(x^j)) \equiv \mu(1-\lambda^L(x^j)) + (1-\mu)\lambda^L(x^j) \text{ for } j = r, s. \quad \text{(B.10)}$$

Posterior Beliefs

We denote the lobby's posterior belief and policymaker's belief given any information signals as $\lambda(x^r) \equiv Pr[\theta = \theta^r|\lambda, x^r]$ and $\lambda(x^s) \equiv Pr[\theta = \theta^r|\lambda, x^s]$ – i.e., also $\lambda^L \in \{\lambda, \lambda(x)\}$. Similarly, the policymaker's posterior is $\lambda^P \equiv Pr[\theta = \theta^r|\lambda^L, z]$ and depends on the history of information signals – i.e., also $\lambda^P \in \{\lambda, \lambda(x), \lambda(z), \lambda(x, z)\}$.

If the lobby gathers information, then the probabilities of signals x^r and x^s are as defined in (B.1) and (B.2) and the corresponding updated beliefs given the signals are

$$\lambda^L(x^r) \equiv Pr(\theta = \theta^r | \lambda, x^r) = \frac{\epsilon^L \lambda}{\epsilon^L \lambda + (1-\eta^L)(1-\lambda)} \tag{B.11}$$

$$\lambda^L(x^s) \equiv Pr(\theta = \theta^r | \lambda, x^s) = \frac{(1-\epsilon^L)\lambda}{\eta^L(1-\lambda) + (1-\epsilon^L)\lambda}. \tag{B.12}$$

If the policymaker also gathers information and receives either z^r and z^s, then using Bayes' rule again, these posteriors are either $\lambda^P(z^r)$ or $\lambda^P(z^s)$ depending on their received signal, and where

$$\lambda^P(z^r) \equiv Pr(\theta = \theta^r | \lambda^L, z^r) = \frac{\epsilon^P \lambda^L}{\epsilon^P \lambda^L + (1-\eta^P)(1-\lambda^L)} \tag{B.13}$$

$$\lambda^P(z^s) \equiv Pr(\theta = \theta^r | \lambda^L, z^s) = \frac{(1-\epsilon^P)\lambda^L}{\eta^P(1-\lambda^L) + (1-\epsilon^P)\lambda^L}. \tag{B.14}$$

Note that $\lambda^L = \lambda$ if the lobby did not gather information; otherwise we substitute (B.11) or (B.12).

Homogeneous Information Accuracies For the case of $\epsilon^L = \eta^L = \epsilon^P = \eta^P \equiv \mu$ we have

$$\lambda^L(x^r) \equiv Pr(\theta = \theta^r | \lambda, x^r) = \lambda^P(z^r) \equiv Pr(\theta = \theta^r | \lambda^L, z^r) = \frac{\mu\lambda}{\mu\lambda + (1-\mu)(1-\lambda)} \tag{B.15}$$

$$\lambda^L(x^s) \equiv Pr(\theta = \theta^r | \lambda, x^s) = \lambda^P(z^s) \equiv Pr(\theta = \theta^r | \lambda^L, z^s) = \frac{(1-\mu)\lambda}{\mu(1-\lambda) + (1-\mu)\lambda}. \tag{B.16}$$

Comparative Statics: Homogeneous and heterogeneous Signals and Costs

Comparative Statics: $\underline{\lambda}$, $\bar{\lambda}$, and Λ

Homogeneous Information Signals and Costs The quantitative comparative statics follow from the first-order derivatives of $\underline{\lambda}$ and $\overline{\lambda}$ with $\epsilon^P = \epsilon^L = \eta^P = \eta^L \equiv \mu$ and $e^P \equiv e$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \underline{\lambda}}{\partial \alpha} = \frac{2c(\mu-1)\mu - e}{(\alpha - 2c\mu + c)^2} < 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial \alpha} = \frac{2c(\mu-1)\mu + e}{(\alpha + c(2\mu-1))^2} \gtreqless 0; \tag{B.17}$$

$$\frac{\partial \underline{\lambda}}{\partial c} = \frac{e(2\mu-1) - 2\alpha(\mu-1)\mu}{(\alpha - 2c\mu + c)^2} > 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial c} = \frac{e(2\mu-1) - 2\alpha(\mu-1)\mu}{(\alpha + c(2\mu-1))^2} > 0; \tag{B.18}$$

$$\frac{\partial \underline{\lambda}}{\partial e} = \frac{1}{\alpha - 2c\mu + c} > 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial e} = \frac{1}{-\alpha - 2c\mu + c} < 0; \tag{B.19}$$

$$\frac{\partial \underline{\lambda}}{\partial \mu} = \frac{-\alpha^2 + c^2 + 2ce}{(\alpha - 2c\mu + c)^2} < 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial \mu} = \frac{\alpha^2 - c^2 + 2ce}{(\alpha + c(2\mu-1))^2} > 0. \tag{B.20}$$

The quantitative comparative statics follow from the first-order derivatives of the updating range Λ with $\epsilon^P = \epsilon^L = \eta^P = \eta^L \equiv \mu$ and $e^P \equiv e$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \Lambda}{\partial \alpha} = \frac{2\left(\alpha^2 e - c^2(2\mu-1)(4\alpha(\mu-1)\mu - 2e\mu + e)\right)}{(\alpha - 2c\mu + c)^2(\alpha + c(2\mu-1))^2} > 0; \tag{B.21}$$

$$\frac{\partial \Lambda}{\partial c} = \frac{4\alpha c(2\mu-1)(2\alpha(\mu-1)\mu - 2e\mu + e)}{(\alpha - 2c\mu + c)^2(\alpha + c(2\mu-1))^2} < 0; \tag{B.22}$$

$$\frac{\partial \Lambda}{\partial e} = -\frac{2\alpha}{(\alpha - 2c\mu + c)(\alpha + c(2\mu-1))} < 0; \tag{B.23}$$

$$\frac{\partial \Lambda}{\partial \mu} = -\frac{2\left(-\alpha^4 + c^4(1-2\mu)^2 - 4\alpha c^2(\alpha(\mu-1)\mu - 2e\mu + e)\right)}{(\alpha - 2c\mu + c)^2(\alpha + c(2\mu-1))^2} > 0. \tag{B.24}$$

Heterogeneous Information Signals and Costs The quantitative comparative statics follow from the first-order derivatives of $\underline{\lambda}$ and $\overline{\lambda}$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \underline{\lambda}}{\partial \alpha} = \frac{2c\epsilon^P(\eta^P - 1) - e^P(1 + \epsilon^P - \eta^P)}{\left(c(\epsilon^P + \eta^P - 1) - \alpha(1 + \epsilon^P - \eta^P)\right)^2} < 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial \alpha} = \frac{2c(\epsilon^P - 1)\eta^P + e^P(1 - \epsilon^P + \eta^P)}{\left(\alpha(1 - \epsilon^P + \eta^P) + c(\epsilon^P + \eta^P - 1)\right)^2} \gtreqless 0; \tag{B.25}$$

$$\frac{\partial \underline{\lambda}}{\partial c} = \frac{2\alpha\epsilon^P(1 - \eta^P) + e^P(\epsilon^P + \eta^P - 1)}{\left(c(\epsilon^P + \eta^P - 1) - \alpha(1 + \epsilon^P - \eta^P)\right)^2} > 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial c} = \frac{2\alpha(1 - \epsilon^P) + e^P(\epsilon^P + \eta^P - 1)}{\left(\alpha(1 - \epsilon^P + \eta^P) + c(\epsilon^P + \eta^P - 1)\right)^2} > 0; \tag{B.26}$$

$$\frac{\partial \underline{\lambda}}{\partial e^P} = \frac{2\alpha\epsilon^P(1 - \eta^P) + e^P(\epsilon^P + \eta^P - 1)}{\alpha(1 + \epsilon^P - \eta^P) - c(\epsilon^P + \eta^P - 1)} > 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial e^P} = \frac{1}{\alpha(1 - \epsilon^P + \eta^P) + c(\epsilon^P + \eta^P - 1)} < 0; \tag{B.27}$$

$$\frac{\partial \underline{\lambda}}{\partial \epsilon^P} = \frac{(c-\alpha)(e^P + (\alpha + c)(1-\eta^P))}{(c(\epsilon^P + \eta^P - 1) - \alpha(1 + \epsilon^P - \eta^P))^2} < 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial \epsilon^P} = \frac{(\alpha - c)((\alpha + c)\eta^P - e^P)}{(\alpha(1 - \epsilon^P + \eta^P) + c(\epsilon^P + \eta^P - 1))^2} > 0; \tag{B.28}$$

$$\frac{\partial \underline{\lambda}}{\partial \eta^P} = \frac{(\alpha + c)(e^P + (c-\alpha)\epsilon^P)}{(c(\epsilon^P + \eta^P - 1) - \alpha(1 + \epsilon^P - \eta^P))^2} < 0 \text{ and } \frac{\partial \bar{\lambda}}{\partial \eta^P} = \frac{(\alpha + c)(e^P + (\alpha - c)(1 - \epsilon^P))}{(\alpha(1 - \epsilon^P + \eta^P) + c(\epsilon^P + \eta^P - 1))^2} > 0. \tag{B.29}$$

Comparative Statics: $\underline{\lambda}'$ and $\bar{\lambda}'$

Homogeneous Information Signals and Costs The quantitative comparative statics follow from the first-order derivatives of $\underline{\lambda}'$ with $\epsilon^P = \epsilon^L = \eta^P = \eta^L \equiv \mu$ and $e^P \equiv e$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \underline{\lambda}'}{\partial \alpha} = \frac{e\left(-2\mu^2 + 2\mu - 1\right) + 2(\mu - 1)\mu y^P}{(\alpha + (e + y^P)(2\mu - 1))^2} < 0; \tag{B.30}$$

$$\frac{\partial \underline{\lambda}'}{\partial c} = 0; \tag{B.31}$$

$$\frac{\partial \underline{\lambda}'}{\partial e} = \frac{\alpha\left(2\mu^2 - 2\mu + 1\right) - y^P(1 - 2\mu)}{\alpha + (e + y^P)(2\mu - 1))^2} \gtreqless 0; \tag{B.32}$$

$$\frac{\partial \underline{\lambda}'}{\partial \mu} = -\frac{\alpha^2 + e^2 - y^{P2}}{\alpha + (e + y^P)(2\mu - 1))^2} < 0; \tag{B.33}$$

$$\frac{\partial \underline{\lambda}'}{\partial y^P} = \frac{e(2\mu - 1) - 2\alpha(\mu - 1)\mu}{\alpha + (e + y^P)(2\mu - 1))^2} > 0. \tag{B.34}$$

The quantitative comparative statics follow from the first-order derivatives of $\overline{\lambda}'$ with $\epsilon^P = \epsilon^L = \eta^P = \eta^L \equiv \mu$ and $e^P \equiv e$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \bar{\lambda}'}{\partial \alpha} = \frac{-c - 2e(\mu - 1)\mu + \left(2\mu^2 - 2\mu + 1\right) y^P}{(\alpha + (2\mu - 1)(y^P - e))^2} \gtreqless 0; \tag{B.35}$$

$$\frac{\partial \bar{\lambda}'}{\partial c} = \frac{1}{\alpha + (2\mu - 1)(y^P - e^P)} > 0; \tag{B.36}$$

$$\frac{\partial \bar{\lambda}'}{\partial e} = \frac{2\alpha(\mu - 1)\mu + c(2\mu - 1) - 2\mu y^P + y^P}{(\alpha + (2\mu - 1)(y^P - e))^2} < 0; \tag{B.37}$$

$$\frac{\partial \bar{\lambda}'}{\partial \mu} = \frac{\alpha^2 + 2c(e - y^P) - e^2 + y^{P2}}{(\alpha + (2\mu - 1)(y^P - e))^2} > 0; \tag{B.38}$$

$$\frac{\partial \bar{\lambda}'}{\partial y^P} = \frac{\alpha\left(-2\mu^2 + 2\mu - 1\right) - 2c\mu + c + e(2\mu - 1)}{(\alpha + (2\mu - 1)(y^P - e))^2} < 0. \tag{B.39}$$

Heterogeneous Information Signals and Costs The quantitative comparative statics follow from the first-order derivatives of $\underline{\lambda}'$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \underline{\lambda}'}{\partial \alpha} = \frac{2y^P\epsilon^P(\eta^P - 1) + e^P(\epsilon^P + \eta^P - 1 - 2\epsilon^P\eta^P)}{(\alpha(1 + \epsilon^P - \eta^P) - (y^P - e^P)(\epsilon^P + \eta^P - 1))^2} < 0; \tag{B.40}$$

$$\frac{\partial \underline{\lambda}'}{\partial c} = 0; \tag{B.41}$$

$$\frac{\partial \underline{\lambda}'}{\partial e^P} = \frac{\alpha(1-\eta^P-\epsilon^P+2\eta^P\epsilon^P) - y^P(\epsilon^P+\eta^P-1)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} \gtreqless 0; \tag{B.42}$$

$$\frac{\partial \underline{\lambda}'}{\partial \epsilon^P} = \frac{(\alpha-y^P+e^P)((\eta^P-1)(y^P+\alpha)-\eta^P e^P)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0; \tag{B.43}$$

$$\frac{\partial \underline{\lambda}'}{\partial \eta^P} = \frac{(\alpha+y^P-e^P)(e^P(1-\epsilon^P)-(\alpha-y^P)\epsilon^P)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0; \tag{B.44}$$

$$\frac{\partial \underline{\lambda}'}{\partial y^P} = \frac{2\alpha\epsilon^P(\eta^P-1)+e^P(\epsilon^P+\eta^P-1)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} > 0. \tag{B.45}$$

The quantitative comparative statics follow from the first-order derivatives of $\overline{\lambda}'$ and the quotient rule with $0 \leq \lambda \leq 1$.

$$\frac{\partial \bar{\lambda}'}{\partial \alpha} = \frac{y^P(1-\epsilon^P-\eta^P+2\epsilon^P\eta^P)-c(1-\epsilon^P+\eta^P)-2e^P(\epsilon^P-1)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} \gtreqless 0; \tag{B.46}$$

$$\frac{\partial \bar{\lambda}'}{\partial c} = \frac{1}{\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1)} > 0; \tag{B.47}$$

$$\frac{\partial \bar{\lambda}'}{\partial e^P} = \frac{2\alpha(\epsilon^P-1)\eta^P-(y^P-c)(\epsilon^P+\eta^P-1)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0; \tag{B.48}$$

$$\frac{\partial \bar{\lambda}'}{\partial \epsilon^P} = \frac{(y^P-e^P-\alpha)(y^P(1-\eta^P)-c+(e^P-\alpha)\eta^P)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0; \tag{B.49}$$

$$\frac{\partial \bar{\lambda}'}{\partial \eta^P} = \frac{(e^P-y^P-\alpha)(c-\alpha+\epsilon^P(\epsilon^P-1)+(\alpha-y^P)\epsilon^P}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0; \tag{B.50}$$

$$\frac{\partial \bar{\lambda}'}{\partial y^P} = \frac{(e^P-c)(\epsilon^P+\eta^P-1)+\alpha(\epsilon^P+\eta^P-1-2\epsilon^P\eta^P)}{(\alpha(1+\epsilon^P-\eta^P)-(y^P-e^P)(\epsilon^P+\eta^P-1))^2} < 0. \tag{B.51}$$

Comparative Statics – Summary for Heterogeneous Signals and Costs

Purely Informational Lobbying

Case		$d\epsilon^L$	$d\epsilon^P$	de^L	$d\eta^L$	$d\eta^P$	$d\lambda$
C1	$\lambda < \underline{\lambda} \leq \lambda(x^r) < \bar{\lambda}$	(+)	(+)	(−)	(−)	(−)	(+)
C2	$\lambda < \underline{\lambda} < \bar{\lambda} \leq \lambda(x^r)$	(+)	(0)	(−)	(−)	(0)	(+)
C3	$\underline{\lambda} \leq \lambda(x^s) \leq \lambda \leq \bar{\lambda} \leq \lambda(x^r)$	(+)	(−)	(−)	(−)	(+)	(+/−)
C4	$\lambda(x^s) \leq \underline{\lambda} \leq \lambda \leq \bar{\lambda} \leq \lambda(x^r)$	(+)	(−)	(−)	(−)	(+)	(+/−)

Table 3: Comparative Statics for the Lobby's Updating Net Payoff – Both Unconstrained.

Informational Lobbying and Policy Implementation Subsidies

Case		$d\epsilon^L$	$d\epsilon^P$	de^L	$d\eta^L$	$d\eta^P$	$d\lambda$	dy^P	de^P	dc
C1'	$\lambda < \underline{\lambda}' \leq \lambda(x^r) < \bar{\lambda}'$	(+)	(+)	(−)	(−)	(−)	(+)	(+)	(−) *Complements*	(−)
C2'	$\lambda < \underline{\lambda}' < \bar{\lambda}' \leq \lambda(x^r)$	(+)	(0)	(−)	(−)	(0)	(+)	(0)	(0) *Independent*	(0)
C3'	$\underline{\lambda}' \leq \lambda(x^s) \leq \lambda \leq \bar{\lambda}' \leq \lambda(x^r)$	(+)	(−)	(−)	(−)	(+)	(+/−)	(+/−)	(−/+) *Substitutes/Damage Ctrl.*	(−/+)
C4'	$\lambda(x^s) \leq \underline{\lambda}' \leq \lambda \leq \bar{\lambda}' \leq \lambda(x^r)$	(+)	(−)	(−)	(−)	(+)	(+/−)	(−)	(+) *Substitutes*	(+)

Table 4: Comparative Statics for Lobby's Updating Net Payoff – Constrained Policymaker.

Informational Lobbying or Policy Implementation Subsidies

Case		$d\epsilon^L$	$d\epsilon^P$	de^L	$d\eta^L$	$d\eta^P$	$d\lambda$	dy^P	de^P	dc
C2"	$\lambda < \underline{\lambda}' \leq \lambda^* < \lambda(x^r)$	(+)	(0)	(−)	(−)	(0)	(+)	(0)	(0) *Independent*	(0)
C4"	$\underline{\lambda}' \leq \{\lambda, \lambda^*\} < \{\lambda(x^r), \bar{\lambda}'\}$	(+)	(−)	(−)	(−)	(+)	(+/−)	(−)	(+) *Substitutes*	(+)
CS	$\underline{\lambda} < \lambda(z^s) < \lambda^* < \lambda(x^s) < \lambda < \{\bar{\lambda}, \lambda(x^r)\}$	(0)	(−)	(−)	(0)	(+)	(−)	(−)	(+) *Substitutes*	(+)

Table 5: Comparative Statics for Lobby's Updating Net Payoff – Both Constrained.

Extensions

Extension 1: No Informational Lobbying

Here we are solving the general lobbying game with $e^P \neq e^L$, $\epsilon^P \gtreqless \epsilon^L$, and $\eta^P \gtreqless \eta^L$ but $e^L > 1$. Varying the resource constraints for the policymaker and lobby, we are solving each game backward and derive only 1) the policymaker's policy choice, 2) the lobby's policy implementation subsidy, and 3) the policymaker's information choice as informational lobbying would not be beneficial for the lobby. In other words, we are considering the standard sequence of play presented in Figure 2 except for the first stage.

Policy Choice and Implementation Subsidy At the last stage of the lobbying game the policymaker has posterior belief λ^P and chooses whether to reform or to keep the status quo. Comparing the payoffs for each and deriving the policy threshold λ^*, we get as before

$$\begin{aligned} \lambda^P \alpha - c &= (1-\lambda^P)\alpha \\ \lambda^P &\geq \frac{1}{2} + \frac{c}{2\alpha} \equiv \lambda^*. \end{aligned} \tag{B.52}$$

The implementation subsidies for the various constraints follow from

i) $\tau^* = 0$ if the policymaker is unconstrained;

ii) $\tau^* = e^p + c - y^P$ if the policymaker is constrained, received signal z^r, and $e^P + c - y^P \leq 1$.

iii) $\tau^* = 0$ if the policymaker is constrained and did not receive signal z^r or $e^P + c - y^P > 1$.

This is similar to the characterization of τ^* in (4.1).

Unconstrained Policymaker's Information Choice The lower threshold $\underline{\lambda}$ follows from the policymaker's trade-off between the expected payoff from costly information with uncertain information signal and policy outcome and the expected payoff from the costless status quo without information choice. Note that the policymaker considers the posterior belief about the state of the world, $\lambda^P(z)$, for the expected payoff associated with the expected information signal, $z(\lambda)$. The policymaker gathers information if

$$z^r(\lambda)\left[y^P - c + \lambda^P(z^r)\alpha\right] + z^s(\lambda)\left[y^P + (1-\lambda^P(z^s))\alpha\right] - e^P \geq y^P + (1-\lambda)\alpha. \tag{B.53}$$

Applying each information signal's probability, the updating choice reduces to

$$\lambda \geq \frac{(1-\eta^P)(\alpha+c) + e^P}{\alpha(1+\epsilon^P - \eta^P) + c(1-\epsilon^P-\eta^P)} \equiv \underline{\lambda}, \tag{B.54}$$

which defines the lower updating threshold.

The upper threshold $\overline{\lambda}$ follows from the policymaker's trade-off between the expected payoff from costly information with uncertain information signal and policy outcome and the expected payoff from a costly reform without gathering information. The policymaker gathers information if

$$z^r(\lambda)\left[y^P - e^P - c + \lambda^P(z^r)\alpha\right) + z^s(\lambda)\left[y^P - e^P + (1-\lambda^P(z^s))\alpha\right) \geq y^P - c + \lambda\alpha. \tag{B.55}$$

Solving for λ, we can write

$$\lambda \leq \frac{\eta^P(\alpha + c) - e^P}{\alpha(1-\epsilon^P+\eta^P) + c(\epsilon^P + \eta^P - 1)} \equiv \overline{\lambda}, \tag{B.56}$$

which defines the upper updating threshold.

Note that these thresholds are similar to the homogenous case's thresholds presented in (3.2) and similar to the ones of the general model presented in (A.25) and (A.27), with the only difference being $\lambda^L = \lambda$.

Constrained Policymaker's Information Choice The lower threshold $\underline{\lambda}'$ follows again from the policymaker's trade-off between the expected payoff from costly information with uncertain information signal and policy outcome and the expected payoff from the costless status quo without gathering information. Note that the policymaker anticipates a policy implementation subsidy of $\tau = e^P + c - y^P$ in the case of a reform signal and zero otherwise. The policymaker gathers information if

$$\begin{aligned} &z^r(\lambda)\left[y^P - e^P - c + \tau(z^r) + \lambda^P(z^r)\alpha\right] + z^s(\lambda)\left[y^P - e^P + \tau(z^s) + (1-\lambda^P(z^s))\alpha\right] \geq y^P + (1-\lambda)\alpha \\ &\Rightarrow z^r(\lambda)\lambda^P(z^r)\alpha + z^s(\lambda)\left[y^P - e^P + (1-\lambda^P(z^s))\alpha\right] \geq y^P + (1-\lambda^L)\alpha. \end{aligned} \tag{B.57}$$

Applying each information signal's probability, the updating choice reduces to

$$\lambda \geq \frac{(1-\eta^P)(\alpha + y^P) + \eta^P e^P}{(1-\eta^P)(\alpha + y^P - e^P) + \epsilon^P(\alpha + e^P - y^P)} \equiv \underline{\lambda}', \tag{B.58}$$

which defines the new lower updating threshold.

The upper threshold $\overline{\lambda}'$ follows again from the policymaker's trade-off between costly information and costly reform without gathering information. The policymaker gathers information if

$$z^r(\lambda)\left[y^P - e^P - c + \tau(z^r) + \lambda^P(z^r)\alpha\right] + z^s(\lambda)\left[y^P - e^P + \tau(z^s) + (1-\lambda^P(z^s))\alpha\right] \geq y^P - c + \lambda\alpha$$

$$\Rightarrow z^r(\lambda)\lambda^P(z^r)\alpha + z^s(\lambda)\left[y^P - e^P + (1-\lambda^P(z^s))\alpha\right] \geq y^P - c + \lambda\alpha. \tag{B.59}$$

Solving for λ, we can write

$$\lambda \leq \frac{(\alpha + y^P - e^P)\eta^P + c - y^P}{(\alpha + y^P - e^P)\eta^P + (\alpha - y^P + e^P)(1-\epsilon^P)} \equiv \overline{\lambda}', \tag{B.60}$$

which defines the new upper updating threshold.

Note that these thresholds are similar to the homogenous case's thresholds underlying Proposition 2 and similar to the ones of the general model presented in (A.34) and (A.36), with the only difference being $\lambda^L = \lambda$.

Extension 2: Contributions in Stages

Suppose now that the policymaker's resource constraint binds even more tightly such that they can afford neither information nor to implement a reform. There are two possible cases here. One case involves policy subsidies as before; the other case independent subsidies for information gathering and implementing reforms.[50] We focus our analysis here on the general case with $e^P \gtreqless e^L$, $\epsilon^P \gtreqless \epsilon^L$, and $\eta^P \gtreqless \eta^L$.

In the first case, which is illustrated in Figure 15, the lobby must first decide whether to gather information, then whether to offer a policy implementation subsidy. Hence, if a reform signal is generated, they then offer a policy implementation subsidy and the lobbying instruments are complements, if a status quo signal is generated they are independent.

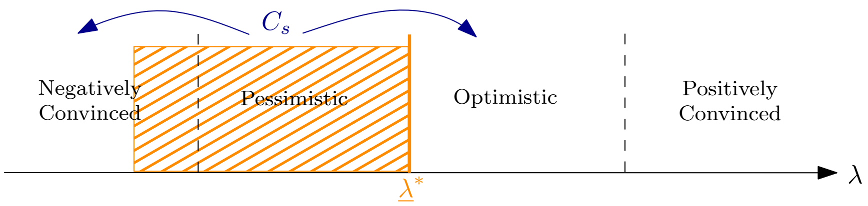


Figure 15: Cases for Lobby's Information Choices – Very Constrained Policymaker.

In the second case, a lobby may benefit from the additional flexibility of providing subsidies in stages. Suppose that $y^P < \min\{c, e^P\}$ and $y^L \geq e^L + e^P + c - y^P$ and that the lobby can contribute before and after a policymaker's information stage. The policymaker's lack of resources implies that a lobby has to provide a policy implementation subsidy whenever the policymaker wants to implement a reform. Whether to lobby engages in information gathering, or subsidizes information

[50]Mathematically, the policymaker's resource constraint is $y^P < \min\{c, e^P\}$. Information subsidies from the lobby to the policymaker may arise when the policymaker cannot afford neither to investigate or implement a policy (presented as an illustration), or when the policymaker has sufficient resources for a policy change but not for gathering information, $e^P > y^P \geq c$. The implications of the latter are very similar and we discuss here the more extensive case.

gathering by the policymaker, or both, determines the various possible cases.

Proposition 5. *If the policymaker's constraint is very binding and the lobby contributes in stages, then the lobby's choices of gathering and subsidizing information follow the same patterns of complements, substitutes, and independence as when the policymaker can afford information.*

The lobby's trade-offs between informational lobbying, subsidizing information, and policy implementation subsidies follow from the same strategic considerations as before. The differences are that; i) a lobby's policy subsidy is always complementary to its other two instruments, and, ii) a lobby can induce a reform the lower standard, $\lambda^L \geq \lambda^*$ instead of $\lambda^L \geq \overline{\lambda}'$. The lobby can achieve the latter by strategically withholding an information subsidy from the policymaker, leaving them with the decision to implement reform or not given their belief.

The lobbying instruments of information subsidies and informational lobbying follow the same pattern and illustrate when the lobby chooses to use its own information technology, prefers to subsidize the use of the policymaker's, or does both.[51] For example, when the policymaker is negatively convinced, reform may require two reform signals, and the lobby may engage in informational lobbying and subsidizing the policymaker's information gathering as complimentary instruments. Similarly, when a policymaker would like to gather information, then the lobby may either gather information on its own and de facto prevent a policymaker gathering information by denying an information subsidy or subsidize the policymaker to gather information and implement a reform – which would imply that both lobbying instruments are substitutes.

The lack of resources implies that a lobby can achieve a reform with greater probability than when the policymaker's constraint is less binding. The possibility of contributions in stages allows the lobby more flexibility such that it has the option of subsidizing the policymaker to gather information and then choosing whether or not to subsidize a reform. However, this comes at two costs: i) the lobby has to provide greater resources to achieve its policy goals, and, ii) the lobby's choices may not involve the socially optimal level of information gathering.[52]

Proof of Proposition 5 Solving the game backward, we consider 1) the policymaker's policy choice, 2) the lobby's policy implementation subsidy, 3) the policymaker's information choice, 4) the lobby's information subsidy, and 5) the lobby's information choice.

1. The policymaker's policy choice follows from the expected payoffs from keeping the status

[51] Table 6 in the Supplemental Appendix online illustrates details on the lobbying patterns.

[52] The welfare implications follow from our analysis that can be found in the Supplemental Appendix online.

quo or implementing a reform. We have

$$\begin{aligned} \lambda^P \alpha - c &\gtreqless (1-\lambda^P)\alpha \\ \lambda^P &\gtreqless \frac{1}{2} + \frac{c}{2\alpha}, \end{aligned} \tag{B.61}$$

which is identical to the other cases.

2. The lobby's policy implementation subsidy can be described by

$$\tau(\lambda^P) = \begin{cases} c - \bar{y}^p & \text{if } \lambda^P \geq \lambda^* \text{ and } 1 > c - \bar{y}^p \\ 0 & \text{otherwise,} \end{cases} \tag{B.62}$$

where $\bar{y}^p = 0$ if the policymaker gathered information and $\bar{y}^p = y^P$ if she did not.

3. The policymaker cannot gather information if the lobby did not provide an information subsidy – i.e., when $\tau(\lambda^L) = 0$. However, if the lobby provided an information subsidy, $\tau(\lambda^L) = e^P - y^P$, then the policymaker can either choose no information gathering and status quo, no information gathering and reform, or information gathering and π based on z^j with $j = r, s$. The information choice thresholds, for which a policymaker would anticipate a policy implementation subsidy later, follow the same logic as before: the policymaker chooses between information and status quo or information and reform. The lower threshold with applied subsidies follows from

$$z^r(\lambda^L)\alpha\lambda^L(z^r) + z^s(\lambda^L)\alpha(1-\lambda^L(z^s)) \geq \alpha(1-\lambda^L) + y^P \tag{B.63}$$

$$\Rightarrow \lambda^L \geq \frac{y^P + \alpha(1-\eta^P)}{\alpha(1+\epsilon^P-\eta^P)} \equiv \underline{\lambda}''. \tag{B.64}$$

The upper threshold with applied subsidies follows from

$$z^r(\lambda^L)\alpha\lambda^L(z^r) + z^s(\lambda^L)\alpha(1-\lambda^L(z^s)) \geq \alpha\lambda^L + \overbrace{y^P - c + \tau(\lambda^P)}^{=0}.$$

$$\Rightarrow \lambda^L \leq \frac{\eta^P}{1-\epsilon^P+\eta^P} \equiv \overline{\lambda}''. \tag{B.65}$$

4. The lobby's information subsidy follows the rationale that a policymaker's information gathering would benefit the lobby in expected terms. If $\lambda^* < \lambda^L$, then a policymaker, who could not gather information, would implement a reform with probability one and the lobby would not gain from a policymaker updating. If $\lambda^L < \underline{\lambda}'$, then the policymaker would

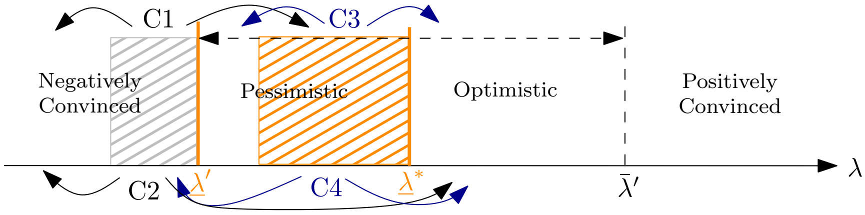


Figure 16: Lobby's Updating Cases – Entirely Constrained Policymaker and Contributions in Stages.

not use an information subsidy for gathering information and the lobby has no rationale to make a contribution. Hence, the lobby would only consider an information subsidy if $\underline{\lambda}' < \lambda^L < \lambda^* \leq \lambda^L(z^r)$. The lobby's information subsidy can be described by

$$\tau(\lambda^L) = \begin{cases} e^P - y^P & \text{if } \underline{\lambda}' < \lambda^L < \lambda^* \leq \lambda^L(z^r) \text{ and } z^r(\lambda^L)(1-c) \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{B.66}$$

5. The lobby's updating choice follows from the rationale that an information signal could induce the policymaker to gather subsidized information or could induce the policymaker to reform. Because of the lobby's ability to constrain the policymaker's information gathering, the lobby will not gather information if $\lambda^* < \lambda$. The four cases of interest are then i) $\lambda < \underline{\lambda}' \leq \lambda(x^r) < \lambda^*$, ii) $\lambda < \underline{\lambda}' < \lambda^* \leq \lambda(x^r)$, iii) $\underline{\lambda}' \leq \lambda(x^s) \leq \lambda \leq \lambda^* \leq \lambda(x^r)$, and iv) $\lambda(x^s) \leq \underline{\lambda}' \leq \lambda \leq \lambda^* \leq \lambda(x^r)$, all of which are illustrated in Figure 16.

For i) $\lambda < \underline{\lambda}' \leq \lambda(x^r) < \lambda^*$ the lobby compares the expected payoff from gathering information and a potential, subsidized policymaker information gathering with the certain payoff from the status quo. The lobby gathers information if

$$\begin{aligned} x^r(\lambda)\left(z^r(\lambda(x^r))(1-c) - e^P + y^P\right) &\geq e^L \\ \left(\epsilon^P\epsilon^L\lambda + (1-\eta^P)(1-\eta^L)(1-\lambda)\right)(1-c) - \left(1-\eta^L-\lambda(1-\epsilon^L-\eta^L)\right)(e^P - y^P) &\geq e^L. \end{aligned} \tag{B.67}$$

For ii) $\lambda < \underline{\lambda}' < \lambda^* \leq \lambda(x^r)$ the lobby compares the expected payoff from gathering information and no policymaker information gathering with the certain payoff from the status quo. The lobby gathers information if

$$\begin{aligned} x^r(\lambda)(1-c+y^P) &\geq e^L \\ \left(1-\eta^L-\lambda(1-\epsilon^L-\eta^L)\right)(1-c+y^P) &\geq e^L. \end{aligned} \tag{B.68}$$

iii) For $\underline{\lambda}' \leq \lambda(x^s) \leq \lambda \leq \lambda^* \leq \lambda(x^r)$ the lobby has to choose whether it would subsidize a policymaker's information gathering and whether it would subsidize a second information signal. Suppose $z^r(\lambda(x^s))(1-c) \geq e^P - y^P$, which implies $z^r(\lambda)(1-c) \geq e^P - y^P$, then the lobby would subsidize both information signals. The lobby compares the expected payoff from gathering information and either a reform or a subsidized policymaker information gathering with the expected payoff from a subsidized policymaker information gathering. The lobby gathers information if

$$x^r(\lambda)(1-c+y^P) - x^s(\lambda)(e^P - y^P) + x^s(\lambda)z^r(\lambda)(1-c) - e^L \geq z^r(\lambda)(1-c) - e^P + y^P$$
$$(1-c)((\eta^L-1)\eta^P(\lambda-1) - \lambda\epsilon^L(\epsilon^P-1)) - e^L + e^P(\eta^L(\lambda-1) + \lambda(\epsilon^l-1)+1) \geq 0 \quad \text{(B.69)}$$

Now suppose $z^r(\lambda(x^s))(1-c) < e^P - y^P$ but $z^r(\lambda)(1-c) \geq e^P - y^P$, then the lobby would subsidize a policymaker information gathering but not a second information signal. The lobby compares the expected payoff from information gathering and either a reform or status quo with the expected payoff from a subsidized policymaker information gathering. The lobby gathers information if

$$\begin{aligned} x^r(\lambda)(1-c+y^P) - e^L &\geq z^r(\lambda)(1-c) - e^P + y^P \\ \left(\epsilon^L\lambda + (1-\eta^L)(1-\lambda)\right)(1-c+y^P) - e^L &\geq \left(\epsilon^L\lambda + (1-\eta^L)(1-\lambda)\right)(1-c) - e^P + y^P \quad \text{(B.70)} \end{aligned}$$

Finally, suppose $z^r(\lambda)(1-c) < e^P - y^P$ and the lobby would not subsidize any policymaker information gathering, then the comparison would be the one of (B.68).

For iv) $\lambda(x^s) \leq \underline{\lambda}' \leq \lambda \leq \lambda^* \leq \lambda(x^r)$ the lobby has to choose whether it would subsidize a policymaker's information signal. Suppose $z^r(\lambda)(1-c) \geq e^P - y^P$. The lobby then compares the expected payoff from information and either reform or status quo with the expected payoff from a policymaker information gathering. Then the comparison would be the one of (B.70). Finally, suppose the lobby would not subsidize a policymaker's information gathering, $z^r(\lambda)(1-c) < e^P - y^P$, then the comparison would be the one of (B.68).

Table 6 illustrates the qualitative comparative statics and relationships between costly informational lobbying and information subsidy as well as informational lobbying and policy implementation subsidy.

We summarize the results in Table 6.

Lobby gathers information for...		$d\epsilon^L$	$d\epsilon^P$	de^L	$d\eta^L$	$d\eta^P$	$d\lambda$	dy^P	de^P	dc
Case S1	$\lambda < \underline{\lambda}' \leq \lambda(x^r) < \lambda^*$	$(+/-)$	$(+)$	$(-)$	$(+/-)$	$(-)$	$(+/-)$	$(+)$	$(-)$ *Compl./Compl.*	$(-)$
Case S2	$\lambda < \underline{\lambda}' < \lambda^* \leq \lambda(x^r)$	$(+)$	(0)	$(-)$	$(-)$	(0)	$(+)$	$(+)$	(0) *Ind./Compl.*	$(-)$
Case S3	$\underline{\lambda}' \leq \lambda(x^s) \leq \lambda \leq \lambda^* \leq \lambda(x^r)$									
	$\tau(\lambda(x^s)) > 0,\ \tau(\lambda) > 0$	$(+)$	$(-)$	$(-)$	$(-)$	$(+)$	$(+/-)$	(0)	$(+)$ *Subs./Dam.Ctrl.*	$(+/-)$
	$\tau(\lambda(x^s)) = 0,\ \tau(\lambda) > 0$	$(+)$	$(-)$	$(-)$	$(-)$	$(+)$	$(+/-)$	$(-)$	$(+)$ *Subs./Dam.Ctrl.*	$(+/-)$
	$\tau(\lambda(x^s)) = 0,\ \tau(\lambda) = 0$	$(+)$	(0)	$(-)$	$(-)$	(0)	$(+)$	$(+)$	(0) *Ind./Compl.*	$(-)$
Case S4	$\lambda(x^s) \leq \underline{\lambda}' \leq \lambda \leq \lambda^* \leq \lambda(x^r)$									
	$\tau(\lambda) > 0$	$(+)$	$(-)$	$(-)$	$(-)$	$(+)$	(i)	$(-)$	$(+)$ *Substitutes*	$(+/-)$
	$\tau(\lambda) = 0$	$(+)$	(0)	$(-)$	$(-)$	(0)	$(+)$	$(+)$	(0) *Ind./Compl.*	$(-)$

Table 6: Comparative Statics for Lobby's Updating Net Payoff – Contributions in Stages.

Extension 3: Detailed Social Welfare Implications

To consider the welfare implications of our analysis, we restrict the social planner's choices to those made by the lobbyist and the policymaker. Then a deviation of the described market solution from the welfare optimum can occur for only three reasons; firstly, they arise because the lobbyist's and planner's objectives differ; secondly, the constraints faced by the planner are less binding than either those faced by the policymaker or lobbyist; or finally, a cost may not be internalized by the lobbyist or policymaker but will be by the planner. Employing (2.1) and (2.1), we write the social welfare function as

$$E[U^P(\pi,\theta)] + \sum_{k=L,P} y^k - fe^P - he^L - gc, \tag{B.71}$$

where we have assumed that the lobbyist's benefit from reform is a pure transfer. Hence, $E[U^L(\pi,\theta)]$ does not appear in the social welfare function. Notice immediately that the incentives of the policymaker and social planner are perfectly aligned and therefore only the three distortions mentioned above may occur. We begin by assuming that neither budget constraint would independently bind on either a lobby or policymaker; then we will consider the scenario in which the policymaker's budget constraint is binding.

Neither Budget Constraint is Binding In the case where neither of the budget constraints binds, and given that the incentives of the policymaker and planner are aligned, it follows that the deviation of the market outcome from the welfare optimum arises from differences in the lobbyist's and planner's objectives, and the lobbyist's failure to internalize the policymaker's information and policy implementation cost. Given that for any common prior the policymaker and planner would both make the same information choices and subsequent choice between reform and status quo, it follows that any deviation of the market outcome from the welfare optimum follows from differences between the lobbyist's and planner's choices over purchasing the signal x.

We explore this question by asking what initial priors are required for the lobbyist and planner to gather x. We employ the case where two reform signals x^r and z^r are required for the policymaker or planner to choose reform. That is

$$\lambda(x^r, z^r) > \frac{1}{2} + \frac{c}{2\alpha} > \text{Max}\{\lambda(x^r), \lambda(z^r)\}. \tag{B.72}$$

In this case (A.29) tells us that a lobbyist will choose to gather the signal x iff

$$\epsilon^P \epsilon^L \lambda + (1-\eta^P)(1-\eta^L)(1-\lambda) \geq e^L. \tag{B.73}$$

Which is satisfied if

$$\lambda \geq \frac{e^L - (1-\eta^P)(1-\eta^L)}{\epsilon^P\epsilon^L - (1-\eta^P)(1-\eta^L)} \equiv \lambda_x^L. \tag{B.74}$$

Whereas employing (B.1)-(B.4) and (B.13)-(B.14) as well as (B.11)-(B.12), we may show that a planner will choose to gather information if

$$\begin{aligned} & \alpha\left[\epsilon^P\epsilon^L\lambda - (1-\eta^P)(1-\eta^L)(1-\lambda)\right] \\ \geq \quad & e^L + e^P\left[\epsilon^L\lambda + (1-\eta^L)(1-\lambda)\right] + c\left[\epsilon^P\epsilon^L\lambda + (1-\eta^P)(1-\eta^L)(1-\lambda)\right], \end{aligned} \tag{B.75}$$

which in turn is satisfied if

$$\lambda \geq \frac{e^L + e^P(1-\eta^L) + c(1-\eta^P)(1-\eta^L)}{\alpha\left[\epsilon^P\epsilon^L + (1-\eta^P)(1-\eta^L)\right] + e^P\left[(1-\eta^L) - \epsilon^L\right] + c\left[(1-\eta^P)(1-\eta^L) - \epsilon^P\epsilon^L\right]} \equiv \lambda_x^s. \tag{B.76}$$

Notice that λ_x^L is invariant with respect to α, e^P, and c whereas λ_x^s is increasing in each of these variables (since signals are informative). It then follows that for α, e^P, or c sufficiently large the social planner will gather less information than the lobbyist. This is just the lobby failing to internalize all costs and benefits. We may isolate the different incentives of the lobbyist and planner to gather x arising purely from their valuation of information by letting $e^P = c = 0$ and $\alpha = 1$, that is effectively "turning-off" the other sources of distortions. Notice that we can interpret $\alpha = 1$ as the lobbyist capturing all the rents from a reform when it is socially desirable.

The condition $\lambda_x^L \gtreqless \lambda_x^s$ may then be written

$$2e^L - \epsilon^L\epsilon^P - (1-\eta^L)(1-\eta^P) \gtreqless 0. \tag{B.77}$$

From which we observe that if there are almost equal chances of reform signals and status quo when the state is reform $\epsilon^L\epsilon^P \to 1/4$ and/or there are almost always status quo signals when the state is status quo $(1-\eta^L)(1-\eta^P) \to 0$, then there is a tendency for the lobbyist to gather less information than the planner. Conversely, if there are almost always reform signals when the state is reform $\epsilon^L\epsilon^P \to 1$ and/or there are almost equal status quo signals and reform when the state is status quo $(1-\eta^L)(1-\eta^P) \to 1/2$, then there is a tendency for the lobbyist to gather more information than the planner.

The Policymaker's Budget Constraint is Binding The immediate implication of this configuration of constraints is that the lobbyist can choose to gather information by purchasing the signal x, in which case there does not exist a transfer that allows the policymaker to both gather information in the form of the signal z and finance the cost of reform c. This provides an example of how the lobbyist may strategically gather information so as to limit the transfer they can afford, and hence manipulate the policymaker's choices via their budget constraint. A planner who makes all of the choices will in certain circumstances prefer to purchase the signal z rather than the signal x.

Employing (2.1) and (2.2), we write the social welfare function as

$$E[U^P(\pi,\theta)] + y^P - e^P - c + y^L - e^L, \tag{B.78}$$

where we have assumed that the lobby's benefit from reform is a pure transfer. Hence, $E[U^L(\pi,\theta)]$ does not appear in the social welfare function. We begin by assuming that neither budget constraint would independently bind on either a lobby or policymaker.

Here we consider the case

$$\text{Max}\{c+e^L, c+e^P\} < y^L + y^P < e^L + e^P + c \text{ and } y^P < e^P + c. \tag{B.79}$$

The immediate implication of this configuration of constraints is that the lobby can choose to strategically gather the signal x, in which case there does not exist a transfer that allows the policymaker to both gather the signal z and finance the cost of reform c. This provides an example of how the lobbyist may gather a signal so as to limit the transfer they can afford and hence manipulate the policymaker's choices via their budget constraint. A planner who makes all of the choices will in certain circumstances prefer to gather the signal z rather than the signal x. In this case the lobbyist will choose to gather x if (A.43) is satisfied, which may be rewritten as

$$\epsilon^L\lambda + (1-\eta^L)(1-\lambda) - e^L \quad \geq \quad \left(\epsilon^P\lambda + (1-\eta^P)(1-\lambda)\right)\left(1-c-e^P+y^P\right), \tag{B.80}$$

which again may be rewritten as a condition on the initial common prior

$$\lambda \geq \frac{e^L + (1-c-e^P+y^P)(1-\eta^P) - (1-\eta^L)}{\epsilon^L - (1-\eta^L) - (1-c-e^P+y^P)(\epsilon^P - (1-\eta^P))} \equiv \lambda_x^L. \tag{B.81}$$

In a similar fashion the planner will choose to gather z rather than x if

$$\begin{aligned} & z^r(\lambda)\left[\alpha\lambda(z^r) - e^P - c\right] + z^s(\lambda)\left[\alpha(1-\lambda(z^s)) - e^P\right] \\ & \geq x^r(\lambda)\left[\alpha\lambda(x^r) - e^L - c\right] + x^s(\lambda)\left[\alpha(1-\lambda(x^s)) - e^L\right] \\ & \Leftrightarrow \alpha\epsilon^P\lambda - \left[\epsilon^P + (1-\eta^P)(1-\lambda)\right]c + \alpha(1-\epsilon^P)\lambda - e^P \\ & \geq \alpha\epsilon^L\lambda - \left[\epsilon^L + (1-\eta^L)(1-\lambda)\right]c + \alpha(1-\epsilon^L)\lambda - e^L, \end{aligned} \tag{B.82}$$

which reduces to the condition on the initial common belief

$$\lambda \geq \frac{e^P - e^L + c(\eta^L - \eta^P)}{\epsilon^L - \epsilon^P + c(\eta^L - \eta^P)} \equiv \lambda_z^s. \tag{B.83}$$

We can gain some insights into this distortion by choosing parameter values that "turn-off" the lobby's informational incentives to gather a signal by assuming $\eta^L = \epsilon^L \to \frac{1}{2}$ hence any signal the lobbyist receives is uninformative. Further, we assume $\epsilon^P \to \frac{1}{2}$ and $\eta^P \to 1$ which provides some incentives for a planner to gather a signal, but is a "worst-case-information-scenario" for the lobby as this maximizes the likelihood of a status quo signal if z is gathered. With these assumptions (B.81) and (B.83) reduce to

$$\lambda \geq \frac{2e^L - 1}{c + e^P - y^P - 1} \equiv \lambda_x^L \quad \text{and} \quad \lambda \geq 1 + \frac{2(e^L - e^P)}{c} \equiv \lambda_z^s. \tag{B.84}$$

It can be shown that a simple sufficient condition for there to exist a range of values of the initial prior, λ, that satisfy both conditions in (B.84) is $e^P \leq \frac{1}{2}$.

We summarize these findings in Table 7.

	Unconstrained Policymaker		**Constrained Policymaker**		
Case	Beliefs	Informational Lobbying	Beliefs	Informational Lobbying	Implementation Subsidy
C1	$\lambda \leq \underline{\lambda} \leq \lambda(x^r) \leq \bar{\lambda}$	yes	$\lambda \leq \underline{\lambda}' \leq \lambda(x^r) \leq \bar{\lambda}'$	both	
C2	$\lambda \leq \underline{\lambda} \leq \bar{\lambda} \leq \lambda(x^r)$	yes	$\lambda \leq \underline{\lambda}' \leq \bar{\lambda}' \leq \lambda(x^r)$	yes	no
C3(a)	$\underline{\lambda} < \lambda < \lambda^*$	no	$\underline{\lambda}' < \lambda < \lambda^*$	no	yes
C3(b)	$\underline{\lambda} < \lambda^* < \lambda \leq \overline{\lambda}$	yes	$\underline{\lambda}' < \lambda^* < \lambda \leq \overline{\lambda}'$	either or both	
C4	$\lambda(x^s) \leq \underline{\lambda} \leq \lambda \leq \bar{\lambda} \leq \lambda(x^r)$	no	$\lambda(x^s) \leq \underline{\lambda}' \leq \lambda \leq \bar{\lambda}' \leq \lambda(x^r)$	no	yes

Table 7: Comparison of Possibilities for Informational Lobbying and Implementation Subsidies.