The Logic of Violence in Drug War

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Abstract

Drug traffickers sometimes share profits peacefully. Other times they fight. We propose a model to investigate this variation, focusing on the role of the state. Seizing illegal goods can paradoxically increase traffickers' profits, and higher profits fuel violence. Killing kingpins makes crime bosses short-sighted, also fueling conflict. Only by targeting the most violent traffickers can the state reduce violence without increasing supply. These results help explain empirical patterns of violence in drug war, which is less studied than interstate or civil war but often as deadly.

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Online Appendix

A Formal statements and proofs of propositions and lemmas

A.1 Properties of the production and survival functions

For clarity, we first state all assumptions we make on the production function:

Assumption 1. *The production function $q(x_i, R_i, e)$ satisfies the following properties:*

1. *It it is twice-differentiable, increasing in both factors of production, and decreasing in enforcement ($\frac{\partial q}{\partial x_i} > 0$, $\frac{\partial q}{\partial R_i} > 0$, and $\frac{\partial q}{\partial e} < 0$).*
2. *The marginal productivity of both factors of production is decreasing ($\frac{\partial^2 q}{\partial x_i^2} < 0$ and $\frac{\partial^2 q}{\partial R_i^2} < 0$).*
3. *Enforcement reduces the marginal productivity of both factors of production ($\frac{\partial^2 q}{\partial e \partial x_i} < 0$ and $\frac{\partial^2 q}{\partial e \partial R_i} < 0$)*
4. *It is homogeneous of degree one in (x_i, R_i) (i.e., it has constant returns to scale).*

The following lemma states several results that are useful throughout:

Lemma A. *The following properties of $q(x_i, R_i, e)$ are a consequence of assumption 1:*

1. *Routes and drugs are complementary production factors ($\frac{\partial^2 q}{\partial x_i \partial R_i} > 0$).*
2. *$q(\cdot)$ is concave in (x_i, R_i).*

The survival rate of drugs $w(x_i, R_i, e) = \frac{1}{x} q(x_i, R_i, e)$ can be written as $w(r_i, e)$, where $r_i = \frac{R_i}{x_i}$. It satisfies the following properties:

3. *It is increasing in the inverse route saturation rate r_i ($\frac{\partial w}{\partial r_i} > 0$).*

4. *The marginal productivity of r_i is decreasing ($\frac{\partial^2 w}{\partial r_i^2} < 0$).*

Proof. For 1, $\frac{\partial^2 q}{\partial x_i \partial R_i} = x \frac{\partial^2 w}{\partial x_i \partial R_i} + \frac{\partial w}{\partial R_i}$. By the chain rule, $\frac{\partial w}{\partial R_i} = \frac{\partial w}{\partial r_i}\frac{\partial r_i}{\partial R_i}$ and $\frac{\partial^2 w}{\partial x_i \partial R_i} = \frac{\partial^2 w}{\partial r_i^2}\frac{\partial r_i}{\partial R_i}\frac{\partial r_i}{\partial x_i} + \frac{\partial w}{\partial r_i}\frac{\partial^2 r_i}{\partial x_i \partial R_i}$. The derivatives of r_i can be readily calculated. Substituting everything in the initial expression for the cross derivative of q yields $\frac{\partial^2 q}{\partial x_i \partial R_i} = -\frac{R_i}{x_i^2}\frac{\partial^2 w}{\partial r_i^2}$, which is positive due to the decreasing marginal productivity of r_i. For 2, homogeneity and complementarity of routes and drugs imply that the function is quasiconcave. Quasiconcavity and homogeneity of degree one imply concavity. 3 and 4 can be easily checked by finding the derivatives of w with respect to R_i, holding x_i fixed. □

A.2 Functional form of the contest success function

We start by stating that the functional form for our contest success function can be motivated axiomatically as the only form that satisfies four characteristics:

Lemma B. *Consider a contest success function $R_i(g_i, G_{-i})$ that satisfies the following properties:*

1. *If cartel i spends zero and some other cartel spends a nonzero quantity, then $R_i = 0$.*

2. *The sum of routes held by all cartels is equal to one.*

3. *The functional form is symmetric across all cartels.*

4. *The functional form is homogeneous of degree zero.*

Then the contest success function takes the following form:

$$R_i(g_i, G_{-i}) = \frac{g_i}{g_i + G_{-i}}. \tag{11}$$

Proof. If m cartels spend g and every other cartel spends zero, $1 = \sum R_i(g_i, G_{-i}) = mR(g, (m-1)g) = mR(1/m, 1-1/m) \implies R(1/m, 1-1/m) = \frac{1}{m}$. If p cartels spend g and one cartel spends qg, the first cartels get $R(g, (p-1+q)g) = R(1/(p+q), 1-1/(p+q)) = 1/(p+q)$ routes, so the other cartel gets $R(q/(p+q), 1-q/(p+q)) = 1-p/(p+q) = q/(p+q)$. This pins down the function for every rational $s \in (0,1)$ as $R(s, 1-s) = s$. By continuity, this also has to be true for every real $s \in (0,1)$. And by homogeneity of degree zero, $R(g, G-g) = R(g/G, 1-g/G) = g/G$. □

A.3 Market shares don't affect supply

Lemma C. *Aggregate demand from the producer market and supply to the consumer market are determined by*

$$p_c \frac{\partial q(X,1,e)}{\partial x} = p_p \qquad Q = q(X,1,e), \tag{12}$$

which are independent of the distribution of routes.

The share of drugs bought and sold by each cartel is equal to the fraction of routes it controls:

$$x_i = R_i X \qquad q_i = R_i Q \tag{13}$$

Proof. Suppose that in some equilibrium cartel i controls a quantity of routes $\hat{R}_i$. Since the quantity of drugs that cartel i purchases from producers does not affect other cartels, cartel i chooses that quantity in order to maximize profit $x_i^* = \text{argmax}_{x_i} \left[p_c q(x_i, \hat{R}_i, e) - g_i - p_p x_i \right]$. The optimal quantity x_i^* can be found from the first-order condition $p_c \frac{\partial q}{\partial x_i} = p_p$, which equates marginal benefit and cost. We assume that the ratio of retail to wholesale prices, $\frac{p_c}{p_p}$, is large enough that there is an interior solution (otherwise, the illegal drug market would not exist). The solution is bounded since the marginal productivity drops to zero as x_i goes to infinity: using notation from lemma A, $\partial r / \partial x = -R/x^2$, and

$\partial q/\partial x = w + x \times (\partial w/\partial r)(\partial r/\partial x) = w - r \times (\partial w/\partial r)$. And w and r drop to zero as x_i goes to infinity, while $\partial w/\partial r$ converges to its value at $r = 0$, which is bounded.

For two different cartels i and j, $\frac{\partial q(x_i^*, R_i^*, e)}{\partial x_i} = \frac{\partial q(x_j^*, R_j^*, e)}{\partial x_j}$. Since q is homogeneous of degree one, its derivative is homogeneous of degree zero, so $\frac{\partial q(x_i^*/R_i^*, 1, e)}{\partial x_i} = \frac{\partial q(x_j^*/R_j^*, 1, e)}{\partial x_j}$, and since this derivative is strictly decreasing, $\frac{x_i^*}{R_i^*} = \frac{x_j^*}{R_j^*}$. Thus, $\frac{x_i^*}{x_j^*} = \frac{\hat{R}_i}{\hat{R}_j} = \frac{q_i^*}{q_j^*}$, where we used homogeneity of q again for the last step. Summing over i yields $\frac{X}{x_j^*} = \frac{1}{\hat{R}_j} = \frac{Q}{q_j^*}$, so $X = \frac{x_i^*}{\hat{R}_i}$. Substituting in the maximization problem, taking into account the homogeneity of q, yields the condition for X. For Q, $Q = \sum_i q(\hat{R}_i X, \hat{R}_i, e) = \sum_i \hat{R}_i q(X, 1, e) = q(X, 1, e)$. For the shares of production, $q_i = q(x_i, \hat{R}_i, e) = \hat{R}_i q(X, 1, e)$. $\square$

A.4 Lemma 1: Defining cartel profit

Proof. From lemma C, cartel i invests g_i in the conflict, buys an amount of drugs $x_i = R(g_i, G_{-i})X$ and sells an amount $q_i = R(g_i, G_{-i})Q$. Substituting in (2) yields $\pi_i = p_c q_i - p_p x_i - g_i = (p_c Q - p_p X)R(g_i, G_{-i}) - g_i$. $\square$

A.5 Proposition 1: Effect on supply and elasticity threshold

It might appear that seizing drugs in transit would reduce the total quantity reaching consumers. In fact, interdiction has two opposing effects. The effect of interdiction on the aggregate quantity of drugs purchased from producers can be found by applying the implicit function theorem to Equation (12):

$$\frac{\partial X}{\partial e} = \left(\frac{-\frac{\partial q}{\partial X}}{\frac{1}{Q\epsilon_c}\left(\frac{\partial q}{\partial X}\right)^2 + \frac{\partial^2 q}{\partial X^2}} \right) \left[\overbrace{\frac{1}{\epsilon_c} \frac{\partial \log q}{\partial e}}^{\text{(a)}} + \overbrace{\frac{\partial \log \frac{\partial q}{\partial X}}{\partial e}}^{\text{(b)}} \right] \tag{14}$$

The first term is positive, so the sign of the effect is determined by the term in square brackets.

Two mechanisms work in opposite directions. Term (a) captures the fact that interdiction decreases the supply of drugs, thereby increasing prices and encouraging cartels to buy more drugs to sell them to consumers. In term (b), interdiction reduces the marginal productivity of drugs purchased, with the opposite effect. Which effect dominates depends on whether demand is inelastic enough that effect (a) is larger.

While the effect of interdiction on drugs purchased (X) is ambiguous, the effect of interdiction on the supply of drugs to consumers (Q) is not. The intuition is that supply of drugs to consumers, Q, can only increase if cartels purchase more drugs from producers as interdiction intensifies. But if cartels purchase more drugs from producers as interdiction intensifies, it must be because prices are increasing—which implies that supply Q is falling. This can be seen by substituting in $\frac{\partial Q}{\partial e} = \frac{\partial q}{\partial e} + \frac{\partial q}{\partial x}\frac{\partial X}{\partial e}$, which yields

$$\frac{\partial Q}{\partial e} = \frac{\frac{\partial^2 q}{\partial X^2}\frac{\partial q}{\partial e} - \frac{\partial q}{\partial X}\frac{\partial^2 q}{\partial X \partial e}}{\frac{1}{Q\epsilon_c}\left(\frac{\partial q}{\partial X}\right)^2 + \frac{\partial^2 q}{\partial X^2}}. \tag{15}$$

Therefore it is always the case that:

Proposition 6.

- $\dfrac{\partial Q}{\partial e} < 0$*: Increasing interdiction reduces the supply of drugs.*
- $\dfrac{\partial Q}{\partial n} = 0$*: The number of cartels has no effect on the supply of drugs.*

Substituting both derivatives above in the expression for profits, and isolating ϵ_c after equating $\dfrac{\partial \pi^A}{\partial e}$ to one results in the threshold in Equation (4).

A.6 Proposition 7: Enforcement and violence in the SGNE

Cartels choose g_i such that:

$$\underbrace{\pi^A \frac{\partial R}{\partial g_i}}_{MgBg_i} = \underbrace{1}_{MgCg_i} \tag{16}$$

meaning that cartels equate the marginal benefit of conflict expenditure to its marginal cost (one).[41]

The Nash equilibrium of the stage game occurs when first-order conditions (16) are satisfied simultaneously for all cartels. The symmetry of the problem means that the unique one-period Nash solution has $g_i = g_j = g^N$ and $x_i = x_j = x^N \quad \forall i, j \in I$, implying that each cartel controls an equal share of routes $R_i = \frac{1}{n}$.[42]

Proposition 7. *Under a symmetric stage-game Nash equilibrium, the comparative statics on the level of violence are:*

a) If $\epsilon_c < \hat{\epsilon}_c$, then $\frac{\partial v^N}{\partial e} < 0$: If demand is sufficiently elastic, interdiction reduces the level of violence.

b) If $\epsilon_c > \hat{\epsilon}_c$, then $\frac{\partial v^N}{\partial e} > 0$: If demand is sufficiently inelastic, interdiction increases the level of violence.

c) $\frac{\partial v^N}{\partial n} > 0$: An increase in the number of cartels increases the level of violence.

In other words, in the stage game, the effect of interdiction on violence mirrors the effect of interdiction on aggregate productive profit π^A: if in-

[41] Note that there is no corner solution with $g_i = 0$ and $x_i > 0$, since the marginal productivity of conflict expenditure tends to infinity if all cartels set $g_i = 0$. Likewise, there cannot be a solution with unbounded g_i because the marginal productivity of conflict expenditure goes to zero as g_i goes to infinity. Thus, every cartel arrives at an interior solution. We check the second-order conditions in Appendix B.

[42] The solution is unique because the marginal productivity of g_i is strictly decreasing.

terdiction increases the prize π^A, cartels invest more in fighting over it; if interdiction shrinks the prize, violence declines. As in other one-period models, the conflict intensifies as the stakes increase (Garfinkel and Skaperdas, 2007, p. 661) and as the number of cartels grows.

The conflict also intensifies as the number of cartels increases. Splitting a given conflict expenditure G among more cartels means that each cartel has lower conflict expenditure g_i—which implies higher marginal returns, leading each cartel to invest more in the conflict. However, we need an additional assumption about the violence function $v(g_1 \ldots g_n)$ in order to analyze how violence changes with the number of cartels. Informally, we also need to assume that violence is nondecreasing in total conflict expenditure $G = \sum_i g_i$. More specifically, we need only a somewhat weaker assumption, which is that for all g and n, violence is at least as high if $n+1$ cartels each spend $g\frac{n}{n+1}$ on the conflict as if n cartels each spends g. This intuitive property would hold if, for instance, violence were equal to $\sum_i g_i$, total conflict expenditure.

Proof. Individual expenditure is determined by (16). Enforcement has no direct effect on $\frac{\partial R_i}{\partial g_i}$, whereas g^N has no direct effect on π^A. This, and the implicit function theorem, lead to $\frac{\partial g^N}{\partial e} = -\frac{\frac{\partial \pi^A}{\partial e}}{\frac{\partial}{\partial g^N}\left(\frac{\partial R}{\partial g_i}\right)}$. The denominator is negative: $\frac{\partial^2 R}{\partial g^N \partial g_i} = \frac{\partial^2 R}{\partial g_i^2} + (n-1)\frac{\partial^2 R}{\partial G_{-i}\partial g_i} < 0$. Thus, the sign of the effect of enforcement is the same as the sign of the effect on π^A, and its sign is determined by the same threshold as in the SGNE.

For the effect of the number of cartels, also take (16). Since π^A does not depend on the number of cartels, $\frac{\partial R_i}{\partial g_i}$ cannot depend on it either, so its derivative with respect to n must be zero: $\frac{\partial^2 R}{\partial n \partial g_i} = \left[\frac{\partial^2 R}{\partial g_i^2} + (n-1)\frac{\partial^2 R}{\partial g_i \partial G_{-i}}\right]\frac{\partial g^N}{\partial n} + g^N \frac{\partial^2 R}{\partial g_i \partial G_{-i}} = 0$. Isolating $\frac{\partial g^N}{\partial n}$ yields $\frac{\partial g^N}{\partial n} = -g^N \frac{\partial^2 R}{\partial g_i \partial G_{-i}} \left[\frac{\partial^2 R}{\partial g_i^2} + (n-1)\frac{\partial^2 R}{\partial g_i \partial G_{-i}}\right]^{-1}$.

In order to find the comparative statics on aggregate violence G, we use the fact that $\frac{\partial G^N}{\partial n} = g^N + n\frac{\partial g^N}{\partial n}$ to obtain

$$\frac{\partial G^N}{\partial n} = g^N \left[\frac{\partial^2 R}{\partial g_i^2} - \frac{\partial^2 R}{\partial g_i \partial G_{-i}}\right] \left[\frac{\partial^2 R}{\partial g_i^2} + (n-1)\frac{\partial^2 R}{\partial g_i \partial G_{-i}}\right]^{-1}$$

whose sign is undetermined, since it depends on whether $\frac{\partial^2 R}{\partial g_i^2}$ or $\frac{\partial^2 R}{\partial g_i \partial G_{-i}}$ is greater. However, it is negative in a symmetric equilibrium. $R(g,(n-1)g) = \frac{1}{n}$, so

$$\begin{aligned}\frac{d^2 R(g,(n-1)g)}{\partial dg^2} &= 0 \\ &= \frac{\partial^2 R(g,(n-1)g)}{\partial g_i^2} + 2(n-1)\frac{\partial^2 R(g,(n-1)g)}{\partial g_i \partial G_{-i}} \\ &\quad + (n-1)^2 \frac{\partial^2 R(g,(n-1)g)}{\partial G_{-i}^2}\end{aligned}$$

Let $S(g_i, G_{-i}) = 1 - R(g_i, G_{-i})$, the fraction of routes held by all cartels other than i. In a symmetric equilibrium $S = (n-1)R$, and $G_{-i} = (n-1)g_i$, so $\frac{\partial^2 R}{\partial G_{-i}^2} = \frac{\partial^2(1-S)}{\partial G_{-i}^2} = -\frac{\partial^2((n-1)R)}{\partial((n-1)g_i)^2} = -\frac{1}{n-1}\frac{\partial^2 R}{\partial g_i^2}$. Substituting above yields $(n-2)\frac{\partial R(g,(n-1)g)}{\partial g_i^2} = 2(n-1)\frac{\partial R(g,(n-1)g)}{\partial G_{-i}^2}$, so $\frac{\partial^2 R}{\partial g_i^2} < \frac{\partial^2 R}{\partial g_i \partial G_{-i}}$ and $\frac{\partial G^N}{\partial n} > 0$. $\square$

A.7 New notation

We use some new notation in the remaining proofs. $\pi^a(\bar{g}) = \pi_i(\bar{g})$ are profits when all cartels comply with a treaty with violence $\bar{g}$. $\pi^d(\bar{g}) = \max_{g_i} \pi_i(g_i, \bar{g}_{-i})$ are the profits of a cartel that deviates from $\bar{g}$ by maximizing its one-period profits.

$\pi^p(\tilde{g}) = \max_{g_i} \pi_i(g_i, \tilde{g}^i_{-i})$ are the profits of a cartel that is punished by all other cartels—all of whom invest a punishment level $\tilde{g}$—and who responds by maximizing its one-period profits. When that is the case, $\pi^e(\tilde{g}) = \pi_j(\tilde{g}^i)$ are the profits obtained by all remaining cartels, who enforce the punishment.

Finally, $\pi^c(\tilde{g}) = \max_{g_j} \pi_j(g_j, \tilde{g}^i_{-j})$ is the maximum profit a punishing cartel can obtain when reneging on the punishment.

A.8 Proposition 2: Conditions for peace

Proposition 2 in the main text stated only that peace can be sustained if $\beta \geq \bar{\beta}^p(n)$, where $\bar{\beta}^N(n) > \bar{\beta}^m(n)$. Here we prove this result, as well as the result that the threshold $\bar{\beta}^p(n)$ increases in the number of cartels n and does not depend on enforcement. We omit the result on the number of cartels from the main text because it requires additional assumptions.

Proposition. *For punishment strategy $p \in \{N, m\}$, a peaceful equilibrium can be sustained if $\beta \geq \bar{\beta}^p(n)$, where $\bar{\beta}^N(n) > \bar{\beta}^m(n)$, and where $\bar{\beta}^p(n)$ increases in n and does not depend on enforcement. For all $n \in \{2, 3, \dots\}$, the thresholds can be ordered $\bar{\beta}^N(n) > \bar{\beta}^m(n)$.*

Proof. For Nash reversion, $\frac{\pi^N}{\pi^A}$ is independent of β. Thus, the threshold is simply $\bar{\beta}^p(n) = \frac{n-1}{n\left(1-\frac{\pi^p}{\pi^A}\right)}$. For maximal punishment, we will show that π^p is decreasing in β, in which case the left hand side of the inequality $\beta \geq \frac{n-1}{n\left(1-\frac{\pi^p}{\pi^A}\right)}$ is decreasing, which means that it is satisfied if and only if β is greater or equal than some threshold.

The key idea is that an increase in β gives some slack to the IC2. A harsher punishment can thus be sustained, decreasing punishment profits. Formally, the implicit function theorem on the IC2 yields $\frac{\partial \tilde{g}}{\partial \beta} = \frac{\pi^p - \pi^c}{\frac{\partial \pi^e}{\partial \tilde{g}} - (1-\beta)\frac{\partial \pi^c}{\partial \tilde{g}} - \beta \frac{\partial \pi^p}{\partial \tilde{g}}}$. The numerator is clearly negative. We will now show that the denominator is also negative, which means that $\frac{\partial \tilde{g}}{\partial \beta} > 0$.

Explicit expressions for all three individual profits are $\pi^e = \sqrt{\frac{\tilde{g}}{n-1}} - \tilde{g}$, $\pi^c = 1 - \tilde{g} - 2\sqrt{-\tilde{g} + \sqrt{\tilde{g}(n-1)}} + \sqrt{\tilde{g}(n-1)}$ and $\pi^p = 1 + \tilde{g}(n-1) - 2\sqrt{\tilde{g}(n-1)}$. This means that the lhs of the IC2 is concave whereas the rhs is convex. Note that $\left.\frac{\partial \pi^p}{\partial \tilde{g}}\right|_{\tilde{g}=g^N} = -1$: as all cartels increase their expenditure equally, profits decrease as much as expenditure increased. The cartel will reoptimize, but by the envelope theorem that will not have an effect on earnings. After some algebra, $\left.\frac{\partial \pi^e}{\partial \tilde{g}}\right|_{\tilde{g}=g^N} = \left.\frac{\partial \pi^c}{\partial \tilde{g}}\right|_{\tilde{g}=g^N} = -\frac{n-2}{n-1} > -1$. The intuition for the derivative of π^e being less negative is that if all cartels increase their expenditure earnings would decrease as much as expenditure increased. But the punished cartel will reoptimize with lower expenditure, and punishers will get some extra routes. The same intuition holds for π^e, except that the deviator can reoptimize, but that will not have any effect on profits by the envelope theorem.

Thus, the LHS crosses the RHS from below at the Nash equilibrium, and the concavity and convexity imply that the LHS crosses the RHS again from above. This is guaranteed to happen with positive profit since at $\tilde{g} = \frac{1}{n-1}$ both π^e and π^p are equal to zero but $\pi^c > 0$. This is the point at which maximal punishment takes place. And the fact that the LHS crosses from below implies that the denominator $\frac{\partial \pi^e}{\partial \tilde{g}} - (1-\beta)\frac{\partial \pi^c}{\partial \tilde{g}} - \beta \frac{\partial \pi^p}{\partial \tilde{g}}$ is negative.

Higher $\tilde{g}$ reduces the punished cartel's profits directly. The cartel can reoptimize, but by the envelope theorem that has no effect on profits, so earnings decrease. This, in turn, implies that $\frac{\partial \pi^p}{\partial \beta} < 0$, which is what we wanted to show.

We now prove that the thresholds are ordered as in the proposition. For Nash reversion, $\frac{\pi^p}{\pi^A} = \frac{1}{n^2}$. Substituting in the inequality shows that the threshold is $\frac{n}{n+1}$, which is also increasing in n. For maximal punishment, note that, regardless of β, $0 < \pi^m < \pi^N$. This implies that $\frac{n}{n+1} > \frac{n-1}{n\left(1-\frac{\pi^p}{\pi^A}\right)} > \frac{n-1}{n}$. Since, as n increases by one, the upper bound in the previous inequality becomes the lower bound, $\frac{n-1}{n\left(1-\frac{\pi^p}{\pi^A}\right)}$ is increasing in n. The threshold is determined implicitly by $\beta \geq \frac{n-1}{n\left(1-\frac{\pi^p}{\pi^A}\right)}$, so any increase in n must be offset in by an increase in β to restore equality. $\square$

A.9 Proposition 3: Set of sustainable equilibria

Proof. If each cartel spends $\bar{g} < g^N$ on conflict expenditure, each cartel's profit is $\pi^a(\bar{g}) = \pi^A R^a - \bar{g} = \frac{1}{n}\pi^A - \bar{g}$. Comparing the profit from colluding to the profit in the SGNE yields $\pi^a(\bar{g}) = \pi^N + g^N - \bar{g}$, which means that $\frac{\partial \pi^a}{\partial \bar{g}} = -1$.

The profit obtained by the deviator i is $\pi^d(\bar{g}) = \max_{g_i}\left[\pi^A R(g_i,(n-1)\bar{g}) - g_i\right]$. The first order condition is the same as for the SGNE, Equation (16), but with expenditure by other cartels evaluated at $g_{-1} = n\bar{g}$. From the envelope theorem, $\frac{d\pi^d}{d\bar{g}} = \pi^A(n-1)\frac{\partial R}{\partial G_{-i}}$. From the first order condition, $\pi^A\frac{\partial R}{\partial g_i} = 1$, so $\frac{d\pi^d}{d\bar{g}} = \pi^A\left(\frac{\partial R}{\partial g_i} + (n-1)\frac{\partial R}{\partial G_{-i}}\right) - 1$. For $\bar{g} = g^N$ the term in parentheses is $\frac{\partial R(\bar{g},(n-1)\bar{g})}{\partial \bar{g}}$, which is zero because all cartels increase their expenditure by the same amount. Thus, $\left.\frac{\partial \pi^d}{\partial \bar{g}}\right|_{\bar{g}=g^N} = -1$.

Setting $\bar{g} = g^N$, $\pi^a(g^N) = \pi^N$, since all cartels spend the amount corresponding to the SGNE. A deviator's one-period optimal response is g^N, so $\pi^d(g_N) = \pi^N$. Thus, with Nash reversion the IC is satisfied with equality at $\bar{g} = g^N$. This means that at $\bar{g} = g^N$ the derivative on the left hand side of the IC is -1, which is lower than the derivative on the right hand side, $-(1-\beta)$. So the IC holds strictly for some $\bar{g} < g^N$ with Nash reversion.[43]

The deviator's profits can be written as $(n-1)\bar{g} + \pi^A - 2\sqrt{\pi^A(n-1)\bar{g}}$, which is convex. This means that, other than at π^N, profits from deviating and complying can be the same at most once more for some $0 \leq \hat{g}^{a,N} < g^N$. If there exists such $\hat{g}^{a,N}$, all other levels of expenditure in $[\hat{g}^{a,N}, g^N]$ are sustainable. If not, all levels of expenditure in $[0, g^N]$ are sustainable. Then define $g^{a,N} = \hat{g}^{a,N}$ if $\hat{g}^{a,N}$ nonnegative exists, and $g^{a,N} = 0$ otherwise.

With maximal punishment, the same result holds as with targeted

[43] This is a particular case of a general theorem in Mas-Colell et al. (1995), chapter 12 appendix A, that states that any SGNE can be improved by using Nash reversion strategies.

punishment as long as $\pi^p < \pi^N$, which is the case as long as there exists a punishment level of conflict $\tilde{g}$ that is higher than g^N and that can be sustained in a subgame-perfect equilibrium (e.g., as long as IC2 $\pi^e(\tilde{g}) \geq (1-\beta)\pi^c(\tilde{g}) + \beta\pi^p(\tilde{g})$ holds). Note, in particular, that $\pi^e(g^N) = \pi^c(g^N) = \pi^p(g^N)$, which means that IC2 holds with equality at the Nash-equilibrium level of conflict expenditure.

Now note that $\frac{\partial \pi^p}{\partial \tilde{g}}\Big|_{\tilde{g}=g^N} = -1$: as all cartels increase their expenditure equally, profits decrease as much as expenditure increased. The cartel will reoptimize, but by the envelope theorem that will not have an effect on earnings. After some algebra, $\frac{\partial \pi^e}{\partial \tilde{g}}\Big|_{\tilde{g}=g^N} = \frac{\partial \pi^c}{\partial \tilde{g}}\Big|_{\tilde{g}=g^N} = -\frac{n-2}{n-1} > -1$. The intuition for the derivative of π^e being less negative is that if all cartels increase their expenditure earnings would decrease as much as expenditure increased. But the punished cartel will reoptimize with lower expenditure, and punishers will get some extra routes. The same intuition holds for π^e, except that the deviator can reoptimize, but that will not have any effect on profits by the envelope theorem.

Based on those derivatives, there exists $\tilde{g} > g^N$ that leads to IC2 being satisfied with strict inequality. $\pi^p(\tilde{g})$ is then lower than π^N. This also means that some equilibria with $g < g^{a,N}$ can be sustained unless $g^{a,N} = 0$. □

A.10 Propositions 4 and 5: Comparative statics on violence in a collusive equilibrium

Intuition for Proposition 5. In the main text, 5 states that violence increases with interdiction if demand is sufficiently inelastic. Before proving this result, we provide some additional intuition.

In the repeated game, when demand is sufficiently inelastic, interdiction raises the total profits under deviating *and* the total profits under complying with the agreement. The question, then, is whether profits under deviating increase *more* than profits under complying. How, then, does interdiction affect cartels' profits under compliance with the low-violence agreement, relative to profits under deviation? This is the key to understanding how conflict expenditure must change in order for the incentive constraint in Equation 7 to hold with equality.

To answer this question, recall the expressions for each cartel's profit under complying with a low-violence agreement (π^a) and for the profit under deviating (π^d): $\pi^a = R^a \pi^A - \bar{g}$ and $\pi^d = R^d \pi^A - g^d$. Because the contest success function has diminishing returns, when a deviating cartel increases conflict expenditure from $\bar{g}$ to g^d, its share of routes increases by a smaller proportion: $\frac{R^d}{R^a} < \frac{g^d}{\bar{g}}$. This implies that the profit margin is lower when deviating than when complying. So while any increase in aggregate productive profit π^A causes the same percent increase in *revenues* for deviators and for compliers, that same increase in π^A causes a larger percent increase in *profits* for deviators than for compliers. *This* is the mechanism that links interdiction to higher violence (results a) and b) in Proposition 5), *not* the simple fact of interdiction increasing the stakes of the conflict.

To illustrate, imagine that there are ten cartels, and that initially total productive profit π^A is \$100. Imagine further that the deterrent expenditure required to sustain a low-violence equilibrium is \$1; since the equilibrium is symmetric, each cartel controls 1/10 of routes, earning \$10 in productive profit ($R^a \pi^A$) and \$9 in overall profits ($\pi^a = R^a \pi^A - \bar{g}$).

Now consider a cartel weighing whether to deviate, increasing conflict expenditure in order to double its share of routes from 1/10 to 1/5. Because of the declining returns to conflict expenditure, the would-be deviator must more than double expenditure in order to double its route share—for the sake of example, say that it must spend \$5 to control 1/5 of routes. For one glorious period, then, the deviator would earn $\pi^d = \$20 - \$5 = \$15$, a 67% increase over its complying profits of $\pi^a = \$9$;

it would then earn zero profits thereafter. Since we defined \$1 as sufficient deterrent expenditure to sustain a low-violence equilibrium, we already know that the cartel values the future sufficiently not to deviate. The one-period 67% increase in profits just isn't worth it.

Now imagine that total productive profit increases to $\pi^A = \$200$. In this new world, our cartel earns $\pi^a = \$20 - \$1 = \$19$ by complying. But by deviating, the cartel would earn $\pi^d = \$40 - \$5 = \$35$. Just as before, revenues double—but *profits* from deviating now increase by 84%, significantly more than 66%. Even with the same discount factor β, then, this might be enough to tempt deviators—in which case deterrent expenditure, and thus violence, would have to increase in order to sustain a new agreement.

Proofs. Proposition 4 states that violence increases as the shadow of the future shortens (as β declines) when peace cannot be sustained, and Proposition 5 states how violence changes with interdiction in a collusive equilibrium when peace cannot be sustained. Here, we prove these results along with an additional comparative static: that violence increases with the number of cartels n. This last result is omitted from the main text because it requires additional functional form assumptions.

Proposition. *Under punishment strategy $p \in \{N, m, t\}$, if the discount factor is such that peace cannot be sustained (i.e., $\beta < \bar{\beta}^p(n)$), the comparative statics on the level of violence under maximal punishment are:*

a) If $\epsilon_c < \hat{\epsilon}_c$, then $\dfrac{\partial v^{a,p}}{\partial e} < 0$: If demand is sufficiently elastic, interdiction reduces violence.

b) If $\epsilon_c > \hat{\epsilon}_c$, then $\dfrac{\partial v^{a,p}}{\partial e} > 0$: If demand is sufficiently inelastic, interdiction increases violence.

c) $\dfrac{\partial v^{a,p}}{\partial n} > 0$: *An increase in the number of cartels increases the level of violence.*

d) $\dfrac{\partial v^{a,p}}{\partial \beta} < 0$: *More forward-looking cartels decreases the level of violence.*

We prove this proposition based on three lemmas that give stronger results.

Lemma D. *Consider some punishment strategy p such that $\pi^p = P\pi^A$, where P is some constant. If $\epsilon_c < \hat{\epsilon}_c$, then $\dfrac{\partial G^{a,p}}{\partial e} < 0$. If $\epsilon_c > \hat{\epsilon}_t$, then $\dfrac{\partial G^{a,p}}{\partial e} > 0$.*

Proof. Consider $g^{a,p}$ under some level of enforcement e which leads to profits π^A. The IC is then $\pi^a(g^{a,p}) \geq (1-\beta)\pi^d(g^{a,p}) + \beta P\pi^A$, and is satisfied with equality. Consider a new level of enforcement e', which leads to profits $b\pi^A$, where b is some constant.

Note that $\pi^a(bg^{a,p}) = b\pi^A R(bg^{a,p}, (n-1)bg^{a,p}) - bg^{a,p} = b(\pi^a(g^{a,p}))$ by homogeneity of R. If the deviator's expenditure is g^d, the new expenditure is bg^d: the old FOC was $\pi^A R_g(g^d, (n-1)g^{a,p}) = 1$, and since R_g is homogeneous of degree -1, $b\pi^A R_g(bg^d, b(n-1)g^{a,p}) = \pi^A R_g(g^d, (n-1)g^{a,p}) = 1$. And this also means that $\pi^d(bg^{a,p}) = b\pi^A R(bg^d, b(n-1)g^{a,p}) - bg^{a,p} = b(\pi^d(g^{a,p}))$ by homogeneity of R.

Putting it all together, expenditure $bg^{a,p}$ satisfies the new IC with equality, since $\pi^a(bg^{a,p}) - (1-\beta)\pi^d(bg^{a,p}) - \beta P b\pi^A = b(\pi^a(g^{a,p}) - (1-\beta)\pi^d(g^{a,p}) - \beta P\pi^A) = 0$. Thus, any increase in π^A leads to an increase in $g^{a,p}$ and subsequently in $G^{a,p}$. The main result follows from the elasticity threshold (4). □

Nash reversion and maximal punishment,[44] both satisfy the condition $\pi^p = P\pi^A$. But this result is much more general than these two punish-

[44]The exact same idea from the above proof can be used to show that if $\tilde{g}$ satisfies IC2 with aggregate productive profit π^A, then $b\tilde{g}$ satisfies IC2 when aggregate productive profit is $b\pi^A$. Homogeneity of R then means that π^o is proportional to π^A.

ment strategies. And the condition $\pi^p = P\pi^A$ has an intuitive interpretation: as the size of the conflict increases, punishment increases proportionally.

Lemma E. *Consider some punishment strategy p such that $\pi^p \leq \pi^N$ and $\frac{\partial \pi^p}{\partial \beta} \leq 0$. Then $\dfrac{\partial G^{a,p}}{\partial \beta} \leq 0$, with equality only when violence is zero.*

Proof. The implicit function theorem on the IC yields $\frac{\partial g^{a,p}}{\partial \beta} = \frac{\pi^p - \pi^d + \beta \frac{\partial \pi^p}{\partial \beta}}{\frac{\partial \pi^a}{\partial \bar{g}} - (1-\beta)\frac{\partial \pi^d}{\partial \bar{g}}}$ for $g^{a,p} > 0$. Under the assumptions of the lemma, the numerator is negative. And the denominator is positive, since the deviators' profit crosses the complier's profit from above, as shown in figure 2. For zero violence, greater β gives more slack to the IC, which means that zero is still sustainable after increases in β. $\square$

Nash reversion has $\pi^p \leq \pi^N$ (trivially) and $\frac{\partial \pi^p}{\partial \beta} \leq 0$. For maximal punishment, it is evident that $\pi^p < \pi^N$, and we already showed in the proof of proposition 2 that $\frac{\partial \pi^p}{\partial \beta} \leq 0$.

This expression again holds for more general equilibria than for the three types of punishment strategies we analyze. The key conditions, $\pi^p \leq \pi^N$ and $\frac{\partial \pi^p}{\partial \beta} \leq 0$, are very reasonable. The first one holds unless punishment is more lenient than Nash reversion, in which case cooperation could just break down and the market would go to Nash reversion. The second condition reflects the intuition that stronger punishment strategies should be sustainable in a subgame-perfect equilibrium as agents become more forward-looking.

Lemma F. *Consider some punishment strategy s such that $g^{a,p}(n) \leq g^{a,N}(n)$ for all n and, as the number of cartels increases, $\pi^p(n+1) - \pi^p(n) \geq \pi^N(n+1) - \pi^N(n)$. Then $G^{a,p}(n+1) \geq G(n)$, for all n, where equality only holds when both quantities are zero.*

Proof. The lemma holds trivially when $G^{a,p}(n) = 0$. Now consider the case $G^{a,p}(n) > 0$. Suppose that as the number of cartels moves from n to $n+1$, the level of violence moves to $\hat{g} = g^{a,p}\frac{n-1}{n}$. The profits of the deviator can be written as $\pi^d = (n-1)g^{a,p} + \pi^A - 2\sqrt{\pi^A(n-1)}g^{a,p}$, which means that π^d stays constant. The IC constraint with n cartels was $\frac{\pi^A}{n} - g^{a,p} - (1-\beta)\pi^d - \pi^p(n) = 0$. The new IC is $\frac{\pi^A}{n+1} - g^{a,p}\frac{n-1}{n} - (1-\beta)\pi^d \geq 0$. Substituting the original IC and some algebra leads to the conclusion that as long as $\pi^p(n+1) - \pi^p(n) > \frac{1}{n\beta}\left(\bar{g} - \frac{\pi^A}{n+1}\right)$ the new IC will not be satisfied, meaning that $g^{a,p}(n+1) > \frac{n-1}{n}g^{a,p}(n)$ will be necessary to deter deviation, which implies that $G^{a,p}(n+1) > G^{a,p}(n)$.

Note that $\frac{1}{n\beta}\left(g^{a,p} - \frac{\pi^A}{n+1}\right) < \frac{1}{n\beta}\left(\frac{n-1}{n^2}\frac{(\beta-n+n\beta)^2}{(\beta+n-n\beta)^2} - \frac{1}{n+1}\right)\pi^A$. It is straightforward to show that $\frac{1}{n\beta}\left(\frac{n-1}{n^2}\frac{(\beta-n+n\beta)^2}{(\beta+n-n\beta)^2} - \frac{1}{n+1}\right)\pi^A < \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)\pi^A = \pi^N(n+1) - \pi^N(n)$ as long as $\beta < \frac{n}{n+1}$, which is a condition for $g^{a,p}$ to be nonzero (otherwise even Nash reversion can sustain a peaceful equilibrium). So the condition above holds, meaning that $G^{a,p}(n+1) > G^{a,p}(n)$. $\square$

Under any circumstances, one would expect $\pi^p(n) \leq \pi^N(n)$, since in the worst case cartels can punish with Nash reversion and no cooperation. Also note that $\pi^N = \frac{\pi^A}{n^2}$, which means that π^N decays very fast with n (the intuition being that with no cooperation the entry of new cartels is very harmful to new cartels). These two facts suggest that one should expect any other punishment strategy to satisfy $\pi^p(n+1) - \pi^p(n) \geq \pi^N(n+1) - \pi^N(n)$; otherwise it would have to decay even faster than $\frac{1}{n^2}$.

This key condition $\pi^p(n+1) - \pi^p(n) \geq \pi^N(n+1) - \pi^N(n)$ holds trivially for Nash reversion. It is hard to check analytically for maximal punishment (since solving for $\tilde{g}$ requires solving a quartic equation), but it is easy to check numerically. The intuition above holds, especially since π^p is much lower with maximal punishment than with Nash reversion.

A.11 Punishment for a limited number of periods

If we consider punishments that last for a limited number of periods T and then return to the original agreement, IC1 becomes

$$\frac{1-\beta^{T+1}}{1-\beta}\pi^a(\bar{g}) \geq \pi^d(\bar{g}) + \beta\frac{1-\beta^T}{1-\beta^{T+1}}\pi^p$$

which can be simplified as $\pi^a(\bar{g}) \geq (1-w)\pi^d(\bar{g}) + w\pi^p$, where $w = \beta\left(1-\beta^T/1-\beta^{T+1}\right)$ is the relative weight of punishment periods. The same substitution can be done with IC2. The weight w increases in β, which means that all our results hold. Also note that w increases in T, which means that lengthening the punishment phase has the same effect as increasing the discount factor.

A.12 Conditional repression

This section states formally and proves propositions behind the informal discussion in Section 6 of the main text.

A.12.1 Indiscriminate conditional interdiction.

The state sets a baseline level of interdiction e as long as violence is less or equal to $\bar{v}$; if violence ever goes above $\bar{v}$, the state sets interdiction at a different level $\tilde{e}$. This policy sustains a level of expenditure g with $\bar{v} = v(g, g, \ldots)$ if the following IC is satisfied:

$$\pi^a(e,g) \geq (1-\beta)\pi^d(e,g) + \beta\pi^p(\tilde{e}). \tag{17}$$

This equation makes explicit the dependence of cartels' profits on interdiction. For Nash reversion, $\pi^p(\tilde{e})$ simply arises from the SGNE with interdiction $\tilde{e}$. For maximal punishment, it arises from an IC2 that ensures that enforcing the punishment is subgame perfect.

Let $g^{I,p}(\tilde{e})$ be the lowest level of violence that is sustainable under punishment strategy p when the state sets a certain level of indiscriminate conditional interdiction $\tilde{e}$. Similarly, let $\beta^{I,p}(\tilde{e}, n)$ be the maximum discount factor that sustains a peaceful equilibrium with n cartels. If $\tilde{e} = e$, we are back to our main results, so $g^{I,p}(e) = g^{a,p}$ and $\beta^{I,p}(\tilde{e}, n) = \beta^{p}(n)$. The principal question, then, is how $g^{I,p}(\tilde{e})$ and $\beta^{I,p}(\tilde{e}, n)$ vary with $\tilde{e}$, the level of conditionality. The key issue is how $\tilde{e}$ affects punishment profits $\pi^{p}(\tilde{e})$, which are equal to $\pi^{A}(\tilde{e})$ times a constant of proportionality that depends on the punishment strategy p. Taking into account Proposition 1, this immediately leads to the following result:

Proposition 8. *a) If $\epsilon_c < \hat{\epsilon}_c$, then $\dfrac{\partial g^{I,p}}{\partial \tilde{e}} < 0$ and $\dfrac{\beta^{I,p}(\tilde{e}, n)}{\partial \tilde{e}} < 0$: If demand is sufficiently elastic, conditional interdiction reduces violence and makes a peaceful equilibrium more likely.*

b) If $\epsilon_c > \hat{\epsilon}_c$, then $\dfrac{\partial g^{I,p}}{\partial \tilde{e}} > 0$ and $\dfrac{\beta^{I,p}(\tilde{e}, n)}{\partial \tilde{e}} > 0$: If demand is sufficiently inelastic, conditional interdiction increases violence and makes a peaceful equilibrium less likely.

Proof. Proposition 1 determines how aggregate profits change with interdiction. For Nash reversion, punishment profit is $\frac{\pi^{A}(\tilde{e})}{n^2}$. For maximal punishment, punishment profits are determined by IC2, which now takes the form $\pi^{e}(\tilde{e}, \tilde{g}) \geq (1-\beta)\pi^{c}(\tilde{e}, \tilde{g}) + \beta\pi^{p}(\tilde{e}, \tilde{g})$. All profits in the inequality are homogeneous of degree 1 in (π^{A}, g), which means that punishment profits are also proportional to $\pi^{A}(\tilde{e})$. In either case, the level of violence is determined by IC1, which takes the form $\pi^{a}(\bar{g}) \geq (1-\beta)\pi^{d}(\bar{g}) + \beta P \pi^{A}(\tilde{e})$, where P is some constant of proportionality.

The implicit function theorem on the IC with equality yield $\frac{\partial g^{I,p}}{\partial \tilde{e}} = \frac{\beta P}{\frac{\partial \pi^{a}}{\partial \bar{g}} - (1-\beta)\frac{\partial \pi^{d}}{\partial \bar{g}}} \frac{\partial \pi^{A}(\tilde{e})}{\partial \tilde{e}}$, which has the same sign as $\frac{\partial \pi^{A}(\tilde{e})}{\partial \tilde{e}}$ (the denominator is positive since the profits when deviating cross the profits when complying from above). And setting $\bar{g} = 0$ on the IC and applying the implicit function theorem results in $\frac{\partial \beta^{I,p}}{\partial \tilde{e}} = \frac{\beta P}{\pi^d - P\pi^A}\frac{\partial \pi^A(\tilde{e})}{\partial \tilde{e}}$, which again has the same sign as $\frac{\partial \pi^A(\tilde{e})}{\partial \tilde{e}}$.

□

A.12.2 Indiscriminate conditional beheading

The discount factor β can be written as the product of the monetary discount factor δ times the probability p that the cartel is still in charge next period. Suppose that the state conditions efforts at capturing or killing cartel leaders, so that the probability of being in charge goes down to $\tilde{p} < p$ as soon as an equilibrium breaks down. This change affects all cartels, both the deviator and every other cartel. The IC can then be written as

$$\pi^a(\bar{g}) \geq (1-\delta p)\pi^d(\bar{g}) + (1-\delta p)\frac{\delta \tilde{p}}{1-\delta \tilde{p}}\pi^p \tag{18}$$

The key term is the factor $\Phi = (1-\delta p)\frac{\delta \tilde{p}}{1-\delta \tilde{p}} < \delta p$ that now multiplies π^p.[45] The profits from deviating are now lower, since continuation payoffs decrease. This leads to a reduction in violence.

Let $g^{B,p}(\tilde{p})$ be the lowest level of violence that is sustainable under punishment strategy p when the probability of the deviator being in charge is $\tilde{p}$. Similarly, let $\delta^{B,p}(\tilde{p}, n)$ be the maximum monetary discount factor that sustains a peaceful equilibrium with n cartels. The following proposition states the previous result formally:

Proposition 9. *For Nash reversion,* $\dfrac{\partial g^{B,p}(\tilde{p})}{\partial \tilde{p}} > 0$ *and* $\dfrac{\partial \delta^{B,p}(\tilde{p}, n)}{\partial \tilde{p}} > 0$*: Indiscriminate conditional beheading reduces violence and makes a peaceful equilibrium more likely. For maximal punishment, on the other hand, the sign of both derivatives is ambiguous.*

[45] With an alternate strategy in which the probability goes down to $\tilde{p}$ only during the *next* period, the factor is simply $\delta\tilde{p}$, which has the same properties.

Proof. We do comparative statics on equation (18) using the implicit function theorem, which yields $\frac{\partial g^{B,p}}{\partial \tilde{p}} = \frac{\frac{\partial \Phi}{\partial \tilde{p}}\pi^p + \Phi \frac{\partial \pi^p}{\partial \tilde{p}}}{\frac{\partial \pi^a}{\partial \tilde{g}} - (1-\delta\tilde{p})\frac{\partial \pi^d}{\partial \tilde{g}} - \Phi \frac{\partial \pi^p}{\partial \tilde{g}}}$. By the same arguments as in the proof of Proposition 3, the deviator's profit crosses the complier's profit from above. The denominator is thus negative, and the sign of this derivative is determined by the sign of the numerator.

We now show that the numerator is positive for Nash reversion. First of all, it is clear that Φ increases with $\bar{p}$. Also note that $\frac{\partial \pi^p}{\partial \tilde{p}} = 0$, which means the numerator is positive, and so is the derivative. For maximal punishment, note that the sign of the numerator is the same as the sign of $\frac{\partial \log \Phi}{\partial \log p} - \frac{\partial \log \pi^p}{\partial \log p}$. The right hand side is determined by the IC2 for the continuation game after punishment takes the form $\pi^e(\tilde{g}) = (1-\delta\tilde{p})\pi^c(\tilde{g}) + \delta\tilde{p}\pi^p(\tilde{g})$. It is not hard to find examples in which $\frac{\partial \log \Phi}{\partial \log p} - \frac{\partial \log \pi^p}{\partial \log p}$ is positive and negative, which means the sign of the derivative is ambiguous.

By the implicit function theorem on the IC with zero conflict expenditure, $\frac{\partial \delta^{B,p}(\tilde{p},n)}{\partial \tilde{p}} = \frac{\frac{\partial \Phi}{\partial \tilde{p}}\pi^p + \Phi \frac{\partial \pi^p}{\partial \tilde{p}}}{p\pi^d(0) - \frac{\partial \Phi}{\partial \delta}\pi^p}$. It is not hard to check that $\frac{\partial \Phi}{\partial \tilde{p}} \leq p$, and $\pi^d(0) > \pi^p$, so the denominator is positive and the sign is determined by the numerator. This is the same numerator as in the comparative statics for g, which proves the desired result. □

A.12.3 Targeted conditional interdiction

The state first sets interdiction level e for all cartels. Whenever an agreement breaks down, the state punishes the deviator i by setting $e_i = \bar{e}$, and by setting the level of interdiction for other cartels to $e_{-i} = \underline{e}$.

The IC that must be satisfied for a level of conflict expenditure g to be sustained under punishment policy p is

$$\pi^a(e,g) \geq (1-\beta)\pi^d(e,g) + \beta\pi^p(\bar{e},\underline{e}). \tag{19}$$

With Nash reversion, $\pi^p(\bar{e},\underline{e})$ is simply the punished cartel's profits in the

SGNE that occurs when it gets a level of interdiction $\bar{e}$ and every other cartel gets a level of interdiction $\underline{e}$. With maximal punishment, $\pi^p(\bar{e}, \underline{e})$ arises from an IC2. To have a well-defined IC2, we assume that whenever a cartel deviates from a punishment strategy, the state targets its interdiction efforts with level $\bar{e}$ to the new deviator and sets interdiction at $\underline{e}$ for every other cartel.

As mentioned in the main text, this kind of policy is significantly more difficult to analyze than a symmetric change in interdiction. The complication is that lemma 1 no longer holds: different levels of interdiction for different cartels lead to different efficiencies in route usage, so cartels no longer split the pie of aggregate profits into shares proportional to the number of routes they control. Cartels' productive behavior can thus no longer be decoupled from the conflict, as explained in Section 6.

To simplify our analysis, we assume that production q takes the functional form $q(x, R, e) = \tilde{q}(x, \theta(e)R)$, where $\theta(e)$ is a decreasing function and $\tilde{q}$ satisfies the properties of q on its derivatives and homogeneity. This functional form has an intuitive interpretation: a cartel with R routes and interdiction e controls an *effective* number of routes $\theta(e)R$, which are then used to smuggle drugs i, resulting in $\tilde{q}(x, \theta(e)R)$ drugs arriving at the destination.[46]

This assumption leads to the following lemma, which is the basic building block for our next results:

Lemma G. *Cartel i's profit can be restated as*

$$\pi_i = \hat{\pi}(p_c)\theta(e_i)R(g_i, G_{-i}) - g_i, \tag{20}$$

[46]Our original specification with a completely free dependence of $q(x, R, e)$ on e was not problematic in our main results where interdiction is homogeneous across cartels. But now that enforcement is different across cartels, a free dependence could mean that changes in prices affect cartels with different levels of enforcement in radically different ways. Expressions that are similar, but much more complicated and far less intuitive than equations (22) and (25) still hold. Additionally, if we allow free dependence of $q(x, R, e)$, both equations still hold locally, for infinitesimal changes from a policy with $\bar{e} = \underline{e} = e$.

where $\hat{\pi}(p_c) = \max_y p_c\tilde{q}(y,1) - p_p y$ is increasing in p_c and decreasing in p_p. The value of drug purchases y that achieves $\hat{\pi}(p_c)$, which we denote by $y(p_c)$, is unique. Let $\tilde{q}(p_c) = \tilde{q}(y(p_c),1)$. Then $\frac{\partial \hat{\pi}}{\partial p_c} = \tilde{q}(p_c)$.[47]

This result is a generalization of Lemma 1. Cartels still fight over routes, but these routes are now more efficient for some cartels than for others. $\hat{\pi}(p_c)$ measures how much productive profit a cartel can get per unit of effective routes.

We need to define some additional notation for our next result. Let $\bar{\theta}$ and $\underline{\theta}$ be the levels of $\theta(e_i)$ corresponding to levels of interdiction $\bar{e}$ and $\underline{e}$, which are the relevant quantities when a cartel deviates and the state changes its interdiction levels. Also let $\bar{R}^p$ and $\underline{R}^p$ be the amount of routes controlled by the deviator and by every other cartel during punishment with strategy p, and let $\Theta = \sum_i R_i\theta_i$ be the average of all productivities, weighted by how many routes are controlled by each cartel. After deviation, $\Theta = \bar{\theta}\bar{R}^p + (n-1)\underline{\theta}\underline{R}^p$. The total drug supply is then $Q = \tilde{q}(p_c)\Theta$. Also let $\bar{E}^p$ and $\underline{E}^p$ be the values of $\frac{\pi_i}{\hat{\pi}(p_c)}$ that are achieved under punishment strategy p for deviators and for the rest of the cartels. Finally, let $\epsilon_{\tilde{q}} = \frac{\partial \log \tilde{q}}{\partial \log p_c}$ be the elasticity of supply per unit of effective route.

Our main results on targeted conditional interdiction, Propositions 10 and 11 below, depend on the following result:

Property 1. *For $p \in \{N, m\}$, the following four properties are true if $\bar{e} > e > \underline{e}$:*

a) $\frac{\partial \log \bar{E}^p}{\partial \log \bar{\theta}} > 1$

b) $\frac{\partial \log \bar{E}^p}{\partial \log \underline{\theta}} < 0$

c) $\frac{\partial \Theta^p}{\partial \bar{\theta}} < 1$

d) $\frac{\partial \Theta^p}{\partial \underline{\theta}} > 0$

[47]Two notes on notation: We redefine $q(p_c)$ in a slight abuse of notation. We also drop the dependence on p_p, since we assume cartels are price takers in the upstream market.

Property 1 holds for both Nash reversion and maximal punishment. The key to show it is to note that both E^p and Θ^p are independent of $\hat{\pi}(p_c)$ (which we state as Lemma H below). The main idea is that although the conflict is asymmetric, it still scales up homogeneously as productive profits increase. We can then analyze a conflict with $\hat{\pi}(p_c) = 1$ to check that these properties hold. We compute the punishment levels of expenditure from incentive constraints, and then find E^p and Θ by substitution. For Nash reversion, this results in long expressions from which the properties above are clear. We omit the algebra, but it is available upon request. It is not possible to find closed expressions for maximal punishment—since it involves solving cubic equations–but we check numerically that all four results above are true.

The four expressions in Property 1 are intuitive. Property a) states that when the punished cartel becomes more productive, its profits increase more than its productivity. The additional increase in profits arises from two channels. First, productivity directly affects productive profits but not conflict expenditure. Second, the cartel reoptimizes by increasing expenditure, which leads to a reduction in other cartels' expenditure. Property b) simply states that punished cartels are hurt if other cartels become more productive. Those other cartels increase expenditure, thus hurting the original cartel.

Property c) means that decreasing the punished cartel's productivity decreases overall productivity less than proportionally. This arises from the fact that route control shifts towards cartels with greater productivity. Finally, d) states that increasing other cartels' productivity increases overall productivity, a fairly intuitive result.

Lemma H. *For $p \in \{N, m\}$, E^p and Θ^p are independent of $\hat{\pi}(p_c)$.*

Proof. We start by showing this for Nash reversion. The FOCs are $\hat{\pi}(p_c)\bar{\theta}R_{g_i}(g_i, G_{-i}) = 1$. Since $R_{g_i}(g_i, G_{-i})$ is homogeneous of degree -1, it becomes clear that changes in $\hat{\pi}(p_c)$ lead to proportional changes in g, which in turn means that $\pi_i = \hat{\pi}(p_c)\left[\text{argmax}_{g_i}\,\theta(e_i)R(g_i, G_{-i}) - g_i\right]$. This makes it clear that the vector $(E_i)_{i=1}^{n}$ arises from simultaneous maximizations of the form $E_i = \text{argmax}_{g_i}\,\theta(e_i)R(g_i, G_{-i}) - g_i$. This also means that the fraction of routes R_i controlled by each cartel are independent of $\hat{\pi}(p_c)$ and so is Θ.

For maximal punishment, the conflict is determined by equality in the IC2 $\pi^e(\tilde{e}, \tilde{g}) \geq (1-\beta)\pi^c(\tilde{e}, \tilde{g}) + \beta\pi^p(\tilde{e}, \tilde{g})$. Every profit function in this expression is homogeneous of degree 1 in $\hat{\pi}(p_c)$ and the vector of all cartels' expenditures, which that changes in $\hat{\pi}(p_c)$ lead to proportional changes in g. Note that we are assuming that cartels are price takers, and thus do not take into account the effect on prices that takes place if they change the number of routes they control. The IC2 can then be written as

$$\underline{\theta}\frac{\underline{g}}{\bar{g}+(n-1)\underline{g}} - \underline{g} \geq (1-\beta)\max_{g}\left(\underline{\theta}\frac{g}{g+g^{*}+(n-2)\tilde{g}} - g\right) + \beta\max_{g}\left(\bar{\theta}\frac{g}{g+(n-1)\tilde{g}} - g\right) \tag{21}$$

where $g^{*} = \max_g\left(\bar{\theta}\frac{g}{g+(n-1)\tilde{g}} - g\right)$. It then becomes clear that E_i, R_i, and Θ are independent of $\hat{\pi}(p_c)$. □

Our main result about the effect of changes in $\bar{e}$ is the following:

Proposition 10. *For $p \in \{N, m\}$, the change in a punished cartel's profit caused by a change in $\bar{e}$ is given by*

$$\frac{\partial \pi^p}{\partial \bar{e}} = \theta\hat{\pi}\frac{\partial \bar{E}^p}{\partial \bar{e}} + \frac{1}{\epsilon_{\tilde{q}} - \epsilon_c}\frac{p\tilde{q}\bar{\theta}\bar{R}}{\Theta}\frac{\partial \Theta}{\partial \bar{e}}. \tag{22}$$

An increase in $\bar{e}$ reduces violence if and only if

$$\frac{1}{\epsilon_{\tilde{q}} - \epsilon_c} > \frac{S^R}{\bar{s}} \frac{\frac{\partial \log \bar{E}^p}{\partial \log \bar{\theta}}}{\frac{\partial \Theta^p}{\partial \bar{\theta}}}, \tag{23}$$

where $S^R = \frac{\hat{\pi}}{p_c \tilde{q}}$ is the share of production going to routes[48] *and $\bar{s} = \frac{\bar{\theta}}{\Theta}$ is the share of supply provided by the punished cartel.*

Proof. Equation (22) follows from lemma G and comparative statics on equation (20) . Equation (22) leads to equation (23) after some straightforward algebra. □

Equation (22) decomposes the effect on the targeted cartel into two channels. The first term is the effect on productive profit through the conflict outcome. This is all captured by $\frac{\partial \bar{E}^p}{\partial \bar{e}}$. This effect is negative, as we can see from the properties of $\bar{E}^p$. The second term captures an indirect effect through prices. As the punished cartel becomes less productive, it takes less drugs to downstream markets. This is offset to some extent by routes being redistributed from the punished, low productivity cartel to other cartels, but the net effect is still a decrease in supply. This increases prices, increasing profit.

The properties above state that $\frac{\partial \log \bar{E}^p}{\partial \log \bar{\theta}} \geq 1$ and $\frac{\partial \Theta^p}{\partial \bar{\theta}} \leq 1$. This means that, for targeted interdiction to increase violence, $\frac{1}{\epsilon_{\tilde{q}} - \epsilon_c}$ cannot be much lower than n. Equivalently, the sum of the elasticities of supply and demand cannot be much higher than $\frac{1}{n}$. The elasticity of supply $\epsilon_{\tilde{q}}$ is sometimes pretty low for certain functional forms for $\tilde{q}$, but demand still has to be very inelastic for the price effect of interdiction to overcome the effect through the conflict.

This can be stated formally as:

[48] Revenue per unit of effective routes is $p_c \tilde{q}$. The share of this revenue that goes to downstream drugs is $p_p y(p_c)$. Since $\tilde{q}$ is homogenous of degree one, the remaining profit is equal to the share to routes.

Corollary 1. *If property 1 holds, a necessary condition for violence to increase with $\bar{e}$ is:*

$$\epsilon_c > -\frac{\bar{s}}{S^R}. \tag{24}$$

Result 1 is a restatement of this corollary, taking into account that property 1 holds.

An additional tool that is available to the state is changing interdiction on cartels that did not deviate. The next result analyzes this mechanism:

Proposition 11. *For $p \in \{N, m\}$, the change in a punished cartel's profit caused by a change in $\underline{e}$ is given by*

$$\frac{\partial \pi^p}{\partial \underline{e}} = \theta\pi\frac{\partial \bar{E}^p}{\partial \underline{e}} + \frac{1}{\epsilon_{\tilde{q}} - \epsilon_c}\frac{p\tilde{q}\bar{\theta}\bar{R}}{\Theta}\frac{\partial\Theta}{\partial \underline{e}}. \tag{25}$$

Proof. The result follows from Lemma G and comparative statics on Equation (20). □

This lemma also breaks down the effect in two. The first term, a conflict channel, captures the fact that increasing interdiction on cartels that did not deviate makes them less productive. They end up grabbing fewer routes, benefiting the punished cartel. The second term measures a price effect. Lower productivity leads to less supply and higher prices, also benefitting the punished cartel.

Property 1 makes it clear that this effect is negative. The state can thus combine being harsher on the deviating cartel and being more lenient on every other cartel. The following proposition states that there always exists one such combination that decreases violence, while having no effect on supply or prices:

Proposition 12. *For every $\bar{e} > e$, there exists some $\underline{e} < e$ such that supply is the same as with homogeneous interdiction at the original level e. Setting these two levels of interdiction leads to a decrease in violence.*

Just increasing interdiction on the deviating cartel might be difficult if the state has constraints on the resources it uses for interdiction. Similarly, it might be difficult to be lenient on non-deviating cartels if the public dislikes being soft on cartels. An appealing feature of the combination from this proposition is that the state can simply shift its interdiction resources towards the deviating cartel. That way it does not have to use any additional resources, and it can tell the public that it is not being soft, but rather focusing its efforts strategically.

A.12.4 Targeted conditional beheading

The government only sets $\tilde{p}$ for the cartel that deviates, while the probability remains at p for every other cartel. Let $g^{Bt,p}(\tilde{p})$ be the lowest level of violence that is sustainable under punishment strategy p when the probability of the deviator being in charge is $\tilde{p}$. Similarly, let $\beta^{Bt,p}(\tilde{p}, n)$ be the maximum discount factor that sustains a peaceful equilibrium with n cartels.

Proposition 13. $\dfrac{\partial g^{Bt,p}(\tilde{p})}{\partial \tilde{p}} > 0$ *and* $\dfrac{\beta^{Bt,p}(\tilde{p}, n)}{\partial \tilde{p}} > 0$*: Targeted conditional beheading reduces violence and makes a peaceful equilibrium more likely.*

Proof. The IC is identical to the IC under indiscriminate conditional beheading, and so is the SGNE, which proves the result for Nash reversion. For maximal punishment, the derivative is $\frac{\partial g^{Bt,p}}{\partial \tilde{p}} = \frac{\frac{\partial \Phi}{\partial \tilde{p}}\pi^p + \Phi\frac{\partial \pi^p}{\partial \tilde{p}}}{\frac{\partial \pi^a}{\partial \tilde{g}} - (1-\delta\tilde{p})\frac{\partial \pi^d}{\partial \tilde{g}} - \Phi\frac{\partial \pi^p}{\partial \tilde{g}}}$ by the implicit function theorem. We know that Φ increases in $\tilde{p}$, and $\frac{\partial \pi^p}{\partial \tilde{p}}$ is determined by the IC2, which is now $\pi^a(\tilde{g}) \geq (1 - \delta p)\pi^d(\tilde{g}) + \Phi\pi^p(\tilde{g})$. The implicit function theorem gives $\frac{\partial \tilde{g}}{\partial \tilde{p}} = \frac{\pi^p \frac{\partial \Phi}{\partial \tilde{p}}}{\frac{\partial \pi^e}{\partial \tilde{g}} - (1-\delta\tilde{p})\frac{\partial \pi^c}{\partial \tilde{g}} - \delta\tilde{p}\frac{\partial \pi^p}{\partial \tilde{g}}}$. The numerator is positive, and the reasoning in the proof of Proposition 2 shows that the denominator is negative, which means a stronger punishment can be sustained in equilibrium. π^p is thus decreasing in $\tilde{p}$, so $\frac{\partial g^{B,m}(\tilde{p})}{\partial \tilde{p}} > 0$.

By the implicit function theorem on the IC with zero conflict expenditure, $\frac{\partial \delta^{Bt,p}(\tilde{p},n)}{\partial \tilde{p}} = \frac{\pi^p \frac{\partial \Phi}{\partial \tilde{p}}}{p\pi^d(0) - \frac{\partial \Phi}{\partial \delta}\pi^p}$. The denominator is positive so the sign is determined by the numerator. This is the same numerator as in the comparative statics for g, which proves the desired result.

□

B Second order conditions for the cartel

The cartel's problem is $\max_{(g_i, x_i)} \pi_i = p_c q(x_i, R(g_i, G_{-i}), e) - g_i - p_p x_i$, with first order conditions

$$p_c \frac{\partial q}{\partial x_i} = p_p \qquad p_c \frac{\partial q}{\partial R_i} \frac{\partial R_i}{\partial g_i} = 1 \tag{26}$$

The second-order conditions are $\frac{\partial^2 \pi_i}{\partial x_i^2} = p_c \frac{\partial^2 q_i}{\partial x_i^2} < 0$, $\frac{\partial^2 \pi_i}{\partial g_i^2} = p_c \left[\frac{\partial q_i}{\partial R_i} \frac{\partial^2 R_i}{\partial g_i^2} + \frac{\partial^2 q_i}{\partial R_i^2} \left(\frac{\partial R_i}{\partial g_i} \right)^2 \right] < 0$, and $\frac{\partial^2 \pi_i}{\partial x_i^2} \frac{\partial^2 \pi_i}{\partial g_i^2} - \left(\frac{\partial^2 \pi_i}{\partial x_i \partial g_i} \right)^2 = p_c^2 \left[\frac{\partial^2 q_i}{\partial x_i^2} \frac{\partial q_i}{\partial R_i} \frac{\partial^2 R_i}{\partial g_i^2} + \left(\frac{\partial R_i}{\partial g_i} \right)^2 \left(\frac{\partial^2 q_i}{\partial x_i^2} \frac{\partial^2 q_i}{\partial R_i^2} - \left(\frac{\partial^2 q_i}{\partial x_i \partial R_i} \right)^2 \right) \right] > 0$. Strict concavity of R and concavity of q_i ensure that all three conditions are satisfied.

It still remains to show that (16) and (26) are equivalent. Homogeneity of degree one means that derivatives are homogeneous of degree zero, so $\frac{\partial q}{\partial R_i}$ is the same if it is evaluated at (x_i, R_i, e) or $(X, 1, e)$. Euler's theorem means that $Q = X \frac{\partial q(X,1,e)}{\partial X} + \frac{\partial q(X,1,e)}{\partial R}$, and from (12), $p_c \frac{\partial q}{\partial R_i} = p_c Q - p_p X = \pi^A$, so both first order conditions are indeed equivalent.

C Elasticity threshold for more general production functions q

C.1 Production functions with constant returns to scale in x_i and R_i

As an example, define the *survival rate* $w_i = w(x_i, R_i, e)$ as the fraction of cartel i's drugs that reach consumer markets (so that $q(x_i, R_i, e) = w(x_i, R_i, e)x_i$), and consider a production function in which the survival rate depends on (a) inverse route saturation $r_i = \frac{R_i}{X_i}$ and (b) interdiction e:

$$w(r, e) = \frac{r}{r + \varphi e} \tag{27}$$

In this case, the threshold is $\hat{\epsilon}_c = -\left(1 + \frac{\sqrt{\gamma}}{2(1-\sqrt{\gamma})}\right)$, where $\gamma = \frac{p_p}{p_c}$. (Appendix C graphs $\hat{\epsilon}_c$ as a function of γ). This expression makes clear that $\hat{\epsilon}_c < -1$, meaning that profit is *more* sensitive to interdiction than revenue is to interdiction. Figure C.3 plots this threshold as a function of γ. As the gap between consumer and producer prices widens, and γ goes down to zero, costs become a smaller share of revenues and the threshold gets closer to -1. For high γ, the threshold goes well below -1, but empirical evidence shows that each step in the production chain of drugs, from producers to consumers, implies a large increase in prices (Mejía and Rico, 2010), and the results for high values of γ are therefore not very relevant. Plugging in the numbers provided in Reuter (2004) (p. 130) yields a corrected threshold of -1.1.

We now look at more general functional forms. Suppose that w depends on the ratio of effective routes r to enforcement e. In order to allow for different efficiencies and increasing or decreasing returns to scale, we assume that w is a function of $\rho = \frac{r}{\varphi e^{\eta}}$: φ is a parameter that captures the relative efficiency of enforcement, and η is a parameter that captures whether the returns to scale of enforcement decrease faster than the returns to scale of

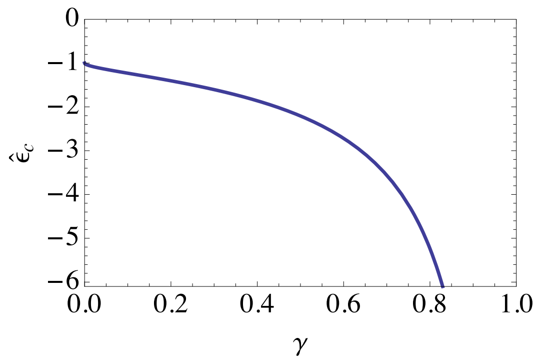


Figure C.3: Variation of the threshold as a function of γ

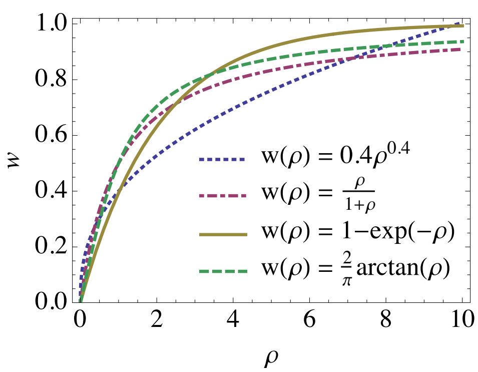
effective routes. Thus, $w(r,e) = w(\rho)$. The conditions set on the derivatives of q in section 2 mean that $w' > 0$ and $w'' < 0$. This kind of function includes a variety of production technologies. For instance, if $w(\rho) = \rho^{1-\alpha}$ the production function is $q = e^{\eta(1-\alpha)}x^{\alpha}R^{1-\alpha}$, a Cobb-Douglas function, and the same CSF used previously results if $w(\rho) = \frac{\rho}{1+\rho}$ with $\eta = 1$.

We will now show that such functions result in a correction that lowers the elasticity threshold $\hat{\epsilon}_c$. In terms of w, the threshold is $\hat{\epsilon}_c = -1 - \frac{\left(w - r\frac{\partial w}{\partial r}\right)^2}{r^2 \frac{\partial w}{\partial e}\frac{\partial^2 w}{\partial r^2}}\left(\frac{\frac{\partial w}{\partial e}}{w} - \frac{\frac{\partial w}{\partial e} - r\frac{\partial^2 w}{\partial r \partial e}}{w - r\frac{\partial w}{\partial r}}\right)$ The term in parentheses, which determines its sign, is now $\frac{rww''\rho_r\rho_e + rww'\rho_{re} - r(w')^2\rho_r\rho_e}{w(w - rw'\rho_r)}$. The denominator is positive, and by substituting the derivatives of ρ, its numerator is $-\rho w w'' - ww' + \rho(w')^2$, which is positive if

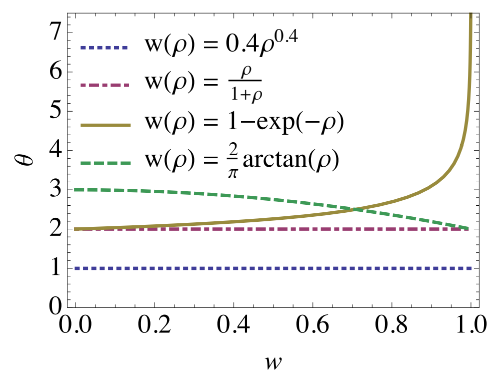
$$\theta = \frac{w''}{\frac{(w')^2}{w} - \frac{w'}{\rho}} > 1 \tag{28}$$

The numerator is clearly negative, and the numerator is also negative since the conditions on w imply that $w > \rho w'$. If (28) is satisfied, the effect of enforcement on marginal productivity is greater than the effect on productivity, so the threshold is lower than -1. Condition (28) has the advantage that it is scale free: θ does not change by substituting $w(\rho)$ with $\hat{w}(\rho) = w(a\rho)$, where a is an arbitrary constant. It is also independent of η.

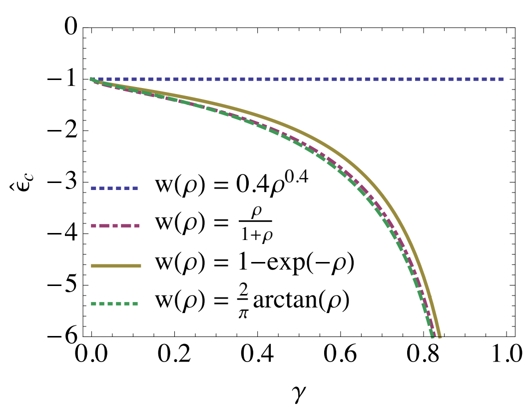
Setting $w_{CD} = \rho^{1-\alpha}$, a Cobb-Douglas technology, yields $\theta_{CD} = 1$. But as we argued in the main text, this is not a very reasonable form for w since it increases without bound. For it to be bounded above, given some value $w = w_{CD}$ and some value of $w' = w'_{CD}$, w'' should be less than for a Cobb-Douglas function ($w'' < w''_{CD}$) so that the function curves downward fast enough that it does not go past $w = 1$. This implies that $\theta > 1$. The relevance of θ being scale-free now becomes clear: the scale parameter a can be chosen so that $w = w_{CD}$ and $w' = w'_{CD}$, allowing comparison of θ and θ_{CD} only in terms of w'' and w''_{CD}.



(a) Four functional forms for $w(\rho)$.



(b) Relation between derivatives (θ).



(c) Elasticity threshold.

Figure C.4: Comparison of different functional forms.

Figure C.4 illustrates our argument graphically with three functions that fulfill the conditions for $w(\rho)$:[49] $w = \frac{\rho}{1+\rho}$, $w = 1 - \exp(-\frac{1}{2}\rho)$, and $w = \frac{2}{\pi}\arctan\rho$. We also show $w = 0.4\rho^{0.4}$ for comparison. The particular values of the parameters were chosen so that the functions are relatively similar, although this does not change our conclusions. Figure C.4a shows the general form of the functions. Figure C.4b shows how θ behaves as a function of the value of w, and, in particular, that for all three functional forms $\theta > \theta_{CD}$. Finally, figure C.4c shows the threshold that results for each functional form in terms of $\gamma = \frac{p_p}{p_c}$. Comparison with figure C.3 shows that the conclusions from section 3 are not a peculiarity of the functional form that we chose for w.

C.2 Production functions with increasing or decreasing returns to scale

The baseline analysis assumes that the production function $q(x_i, R_i, e)$ has constant returns to scale in drug purchases x_i and route ownership R_i. In the text, we briefly note the consequences of relaxing this assumption; this section elaborates the discussion.

Stage-game Nash equilibrium. In the stage game, if we relax constant returns by assuming that production is homogenous of degree $\alpha \neq 1$, the elasticity threshold above which interdiction increases profits remains unchanged—but the threshold above which interdiction increases *violence* changes. However, the *sign* of the correction (relative to -1) remains the same. For instance, if we follow Appendix C.1 and assume that $\hat{\epsilon}_c < -1$, and if we let $\hat{\epsilon}'$ denote the new threshold above which interdiction increases violence, then $\hat{\epsilon}' < -1$. With decreasing returns to scale ($\alpha < 1$), the *magnitude* of the correction is larger (if $\hat{\epsilon}_c < -1$, then $\hat{\epsilon}' < \hat{\epsilon}_c < -1$). With increasing returns to scale ($\alpha > 1$), the magnitude of the correction is smaller (if $\hat{\epsilon}_c < -1$, then $\hat{\epsilon}_c < \hat{\epsilon}' < -1$).

[49] $w(0) = 0$, $w > 0$, $\lim_{\rho\to\infty} w(\rho) = 1$, $w' > 0$, and $w'' < 0$

To understand this, note from Equation (16) that the level of violence in the SGNE depends on the marginal productivity of routes R_i. Under the assumption of constant returns to scale, the marginal productivity of routes is proportional to productive profits: by Euler's homogeneous function theorem, $p_c q_R R = p_c q - p_p x = \pi$. Under the more general assumption that production is homogenous of degree α, this equation becomes:

$$p_c q_R R = \pi + (\alpha - 1) p_c q \tag{29}$$

meaning that the marginal product of routes could exceed or fall short of productive profits. This is why the threshold above which interdiction increases productive profits is no longer the threshold above which interdiction increases violence.

Equation 29 illustrates why, with increasing returns to scale, the marginal product of route ownership exceeds productive profit: specifically, the marginal productivity of route ownership is proportional to profits *plus* a fraction of revenue. The elasticity threshold above which interdiction increases this quantity ($p_c q_R R$) thus falls in between the threshold for productive profits ($\hat{\epsilon}_c$) and the threshold for revenues (-1). Specifically, if we assume that $\hat{\epsilon}_c < -1$—as Appendix C.1 suggests—then $\hat{\epsilon}_c < \hat{\epsilon}' < -1$. With decreasing returns to scale, on the other hand the new threshold moves in the other direction, *away* from the -1 threshold above which interdiction increases revenue: $\hat{\epsilon}' < \hat{\epsilon}_c < -1$.

Repeated game. In the stage game, assuming that production was homogeneous of degree α was sufficient to pin down the revised elasticity threshold above which interdiction increases violence. In the repeated game, we need a second assumption, about the relative efficiency of enforcement against cartels who deviate and cartels who comply.

In particular, if $\alpha > 1$, cartels that control more routes will have a higher ratio of drug purchases to routes $\frac{x_i}{R_i}$ (i.e., a higher route saturation). This means that route saturation $\frac{x_i}{R_i}$ is higher for cartels that deviate from a low-violence agreement than for cartels that comply. To reach conclusions about the elasticity threshold above which interdiction increases violence, we require an assumption about how the effectiveness of interdiction changes with route saturation $\frac{x_i}{R_i}$. We can assume either that (a) interdiction is more effective against cartels with high route saturation, or (b) interdiction is less effective against cartels with high route saturation.

In the former case—interdiction especially hurts cartels with high route saturation $\frac{x_i}{R_i}$—then, if $\alpha > 1$, interdiction hurts deviators more than compliers, weakening the temptation to deviate. This shrinks the range of elasticities for which interdiction increases violence, i.e., $\hat{\epsilon}' > \hat{\epsilon}_c$. In the latter case—interdiction especially hurts cartels with low route saturation—then, if $\alpha > 1$, interdiction hurts compliers more than deviators, strengthening the temptation to deviate. This widens the range of elasticities for which interdiction increases violence, i.e., $\hat{\epsilon}' < \hat{\epsilon}_c$.

The logic is reversed under decreasing returns to scale $\alpha < 1$, because in that case, cartels that control more routes will have *lower* route saturation $\frac{x_i}{R_i}$. In that case, if interdiction is more effective against cartels with high route saturation, interdiction will hurt compliers more than deviators, strengthening the temptation to deviate, and shifting the elasticity threshold downward, i.e., $\hat{\epsilon}' < \hat{\epsilon}_c$. Similarly, if $\alpha < 1$ and interdiction is more effective against cartels with low route saturation, interdiction will hurt deviators more than compliers, weakening the temptation to deviate and shifting the elasticity threshold upward, i.e., $\hat{\epsilon}' > \hat{\epsilon}_c$.

D Variable prices in the producer market

In this section we relax the assumption that prices in the producer market are fixed. Since cartels are price takers, their individual behavior does not change in any way, and their maximization problem is the same, both in the SGNE and with repeated games. The comparative statics, however, must now take into account that changes in policy will have an effect in the producer market, thus changing p_p. This effect is described by the elasticity of supply ϵ_p.

D.1 Aggregate productive behavior

From proposition C, the number of cartels has no effect on productive behavior, which means that it does not affect the amount of drugs bought from the producer region, and p_p. Thus, $\frac{\partial Q}{\partial n}$ stays the same. On the other hand, $\frac{\partial X}{\partial e}$ and $\frac{\partial X}{\partial e}$ do change. The analysis must now take into account that prices in producer markets are increasing in X, so marginal cost is increasing. The implicit function theorem yields the following expression, which replaces (14):

$$\frac{\partial X}{\partial e} = -\frac{\frac{\partial^2 q}{\partial X \partial e} + \frac{1}{Q\epsilon_c}\frac{\partial q}{\partial X}\frac{\partial q}{\partial e}}{\frac{1}{Q\epsilon_c}\left(\frac{\partial q}{\partial X}\right)^2 + \frac{\partial^2 q}{\partial X^2} - \frac{1}{X\epsilon_p}\frac{p_p}{p_c}} \tag{30}$$

The only change is a new term in the denominator, which does not change the sign, although the magnitude of the effect is less. From the chain rule, the new expression that replaces (15) is

$$\frac{\partial Q^e}{\partial e} = \frac{\frac{\partial^2 q}{\partial X^2}\frac{\partial q}{\partial e} - \frac{\partial q}{\partial X}\frac{\partial^2 q}{\partial X \partial e} - \frac{1}{X\epsilon_p}\frac{p_p}{p_c}\frac{\partial q}{\partial e}}{\frac{1}{Q\epsilon_c}\left(\frac{\partial q}{\partial X}\right)^2 + \frac{\partial^2 q}{\partial X^2} - \frac{1}{X\epsilon_p}\frac{p_p}{p_c}} \tag{31}$$

The sign of this expression does not change either. The comparative statics thus remains the same.

D.2 Threshold for the elasticity of demand

The effect of enforcement on violence depends on the effect it has on the aggregate productive profit. The new dependence of producer prices on quantities means that $\frac{\partial \pi^A}{\partial e} = \frac{\partial p_c}{\partial Q}\frac{\partial Q}{\partial e} Q + p_c \frac{\partial Q}{\partial e} - \frac{\partial p_p}{\partial X}\frac{\partial X}{\partial e} X - p_p \frac{\partial X}{\partial e}$. Rewriting $\frac{\partial p_c}{\partial Q}$ and $\frac{\partial p_p}{\partial X}$ in terms of elasticities leads to

$$\frac{\partial \pi^A}{\partial e} = p_c \left(1 + \frac{1}{\epsilon_c}\right) \frac{\partial Q}{\partial e} - p_p \left(1 + \frac{1}{\epsilon_p}\right) \frac{\partial X}{\partial e} \tag{32}$$

instead of (**??**). Substituting $\frac{\partial Q}{\partial e}$ and $\frac{\partial X}{\partial e}$ from (30) and (31) and isolating ϵ_c yields the following threshold for the elasticity of demand:

$$\hat{\epsilon}_c = -1 - \frac{\left(1 + \frac{1}{\epsilon_p}\right)\left(\frac{\partial q}{\partial X}\right)^2}{\frac{\partial^2 q}{\partial X^2}\frac{\partial q}{\partial e} + \frac{1}{\epsilon_p}\frac{\partial q}{\partial X}\left(\frac{\partial^2 q}{\partial X \partial e} - \frac{1}{X}\frac{\partial q}{\partial e}\right)} \left(\frac{\frac{\partial q}{\partial e}}{Q} - \frac{\frac{\partial^2 q}{\partial X \partial e}}{\frac{\partial q}{\partial X}}\right) \tag{33}$$

Two new terms arise. First, the correction is smaller, since increasing marginal cost means that changes in X are smaller (the new term in the denominator)[50]. On the other hand, any change in X induces a larger change in costs, since p_p changes with X (see $\left(1 + \frac{1}{\epsilon_p}\right)$ in the numerator). The sign of the correction is still determined by the sign of $\frac{\partial \log q}{\partial e} - \frac{\partial \log \frac{\partial q}{\partial X}}{\partial e}$.

[50]The sign of the correction could actually change if supply is very inelastic and $\frac{\partial^2 q}{\partial X \partial e} > \frac{1}{X}\frac{\partial q}{\partial e}$, but expanding this in terms of the derivatives of w shows that this would imply $\frac{\partial^2 w}{\partial e \partial r} > 0$.

E Empirical analysis of Mexico before and after 2006

In this section we extend the analysis in Castillo et al. (2018) to show that, although violence in Mexico responded to cocaine supply changes from seizures in Colombia after Calderón's crackdown, the data shows no evidence of such response before December 2006. The results are consistent with the idea that, in the absence of aggressive enforcement policies, repeated interaction creates incentives for traffickers to divide the business peacefully.

Using month-level time-series data on the Mexican homicide rate and cocaine seizures in Colombia, we estimate:

$$\begin{aligned} \ln h_t =& \alpha \ln S_t \times D_t + \beta \ln S_t \times (1 - D_t) + F_t(X_t; \gamma) \times D_t \\ & + F_t(X_t; \delta) \times (1 - D_t) + \epsilon_t \end{aligned} \tag{34}$$

where h_t is the homicide rate in Mexico in month t (as reported by the Mexican statistics agency, INEGI), S_t are cocaine seizures in Colombia (as reported by the Colombian Ministry of Defense), D_t is an indicator variable that takes a value of one from December 2006 onwards, and F_t includes (a) a cubic polynomial in t, (b) year dummies, and (c) time-varying controls X_t. If we were to assume that $E(\epsilon_t | S_t, D_t, F_t) = 0$, we could interpret α as the effect of cocaine seizures in Colombia on violence in Mexico prior to December 2006, and β as the effect of seizures in Colombia on violence in Mexico after December 2006. Castillo et al. (2018) explains in detail why this assumption might be justified; in brief, they contend that short-term (monthly) fluctuations in seizures are largely determined by chance. If this assumption were not justified, however, such that $E(\epsilon_t | S_t, D_t, F_t) \neq 0$, we would interpret α and β as the partial correlation between seizures in Colombia and violence in Mexico, conditional on F_t, before and after December 2006.

Table E.4 shows our estimates of α and β. The first three columns show regressions for all of Mexico. The rest of the columns focus on municipalities in the first two quintiles of distance to the US, where the effect measured in Castillo et al. (2018) is strongest. In some specifications we control for the unemployment rate, by the Índice Global de la Actividad Económica, an early indicator of GDP that is computed every month, and for weather using dummies for the rainy and hurricane season in Mexico.

Table E.4: Relationship between cocaine seizures in Colombia and violence in Mexico (Estimates of Equation 34)

	All of Mexico			Quintiles 1 and 2			First quintile		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Dependent variable: Log homicide rate.								
Before Dec. 2006 (α)	-0.015	-0.017*	-0.019**	-0.009	-0.010	-0.013	-0.000	-0.003	-0.019
	(0.012)	(0.010)	(0.010)	(0.015)	(0.013)	(0.012)	(0.019)	(0.019)	(0.015)
Dec. 2006 onward (β)	0.059**	0.056**	0.050*	0.082**	0.079**	0.073**	0.119***	0.120***	0.109***
	(0.027)	(0.025)	(0.026)	(0.033)	(0.032)	(0.034)	(0.033)	(0.034)	(0.037)
Dif ($\beta - \alpha$)	0.073**	0.074***	0.069**	0.091**	0.089***	0.086**	0.119***	0.122***	0.128***
	(0.030)	(0.027)	(0.028)	(0.036)	(0.034)	(0.036)	(0.038)	(0.039)	(0.040)
Observations	96	96	96	96	96	96	96	96	96
R-squared	0.957	0.964	0.967	0.957	0.963	0.965	0.964	0.965	0.968
Controls:									
Unemployment rate		✓	✓		✓	✓		✓	✓
Economic activity		✓	✓		✓	✓		✓	✓
Weather			✓			✓			✓

Note: Errors are robust to heteroskedasticity. Coefficients with *** are significant at the 1% level, with ** at the 5% level and with * at the 10% level.

The estimates for β simply replicate the findings in Castillo et al. (2018): After December 2006, cocaine seizures in Colombia caused an increase in homicide rates, especially in municipalities close to the U.S. border. The estimates for α reveal a different pattern before Calderón's term: no strong relationship between cocaine seizures in Colombia and violence in Mexico.

These results are consistent with the predictions of our theoretical model. Before Calderón began aggressively targeting kingpins, cartels operated in a peaceful equilibrium—and, therefore, violence did not respond to supply shocks. Calderón's crackdown broke the peace treaties, pushing Mexico into a new equilibrium in which violence responded to supply shocks. While the data themselves do not identify the timing of the switch from one equilibrium to the other (instead, we select December 2006 as a cutoff, since that is when Calderón took office), the contrast between the pre- and post- periods suggests that our model helps explain violence in Mexico during this period.

F Microfoundation of constant returns to scale in smuggling

Consider a continuum of routes $y \in [0,1]$. In each route, the government spends an effort $\lambda(y)$ in enforcement and the cartel that controls it tries to smuggle an amount of drugs $x(y)$. This results in an amount of drugs $\tilde{q}(y, \lambda(y), x(y))$ arriving the final destination, which is decreasing in $\lambda(y)$, and increasing and concave in $x(y)$. Note that this function is allowed to vary by y, which means that some routes can be more productive for cartels or easier to monitor by the government.

Let $e = \int_0^1 \lambda(y)dy$ be total enforcement, $X = \int_0^1 x(y)dy$ be the total number of drugs bought in upstream markets, and $Q = \int_0^1 \tilde{q}(y, \lambda(y), x(y))dy$ be the total amount of drugs sold in downstream markets. Note here the key assumption in this specification: total production is additive across routes, which means no complementarities or substitutabilities.

Suppose cartel i controls $Y_i \subseteq [0,1]$ and has an amount x_i of drugs to sell in downstream markets, and has to decide how many drugs to ship in each route. Then it solves the following problem:

$$\max_{x(\cdot)} p_c \int_{Y_i} \tilde{q}(y, \lambda(y), x(y))dy - p_p \int_{Y_i} x(y)dy \quad \text{s.t.} \quad \int_{Y_i} x(y)dy = x_i. \quad (35)$$

From the calculus of variations, the first order conditions are

$$p_c \tilde{q}_x(y, \lambda(y), x(y)) = p_p + \eta \quad \forall y \in Y_i, \tag{36}$$

where η is a Lagrange multiplier for the constraint. These first order conditions implicitly define optimal quantities $x^*(y)$ and $q^*(y)$. Let optimal drug demand and supply for cartel i be $x_i^* = \int_{Y_i} x^*(y)dy$ and $q_i^* = \int_{Y_i} q^*(y)dy$.

Our first result arises from a simple functional form for q that illustrates the connection between route independence and constant returns to scale:

Proposition 14. *Suppose $\tilde{q}(y, \lambda(y), x(y)) = \frac{\mu(y)}{\lambda(y)} f\left(\frac{\lambda(y)}{\mu(y)} x(y)\right)$, where $f(\cdot)$ is increasing and concave. Given the sets of routes controlled by each cartel Y_i and the level of enforcement in each route $\lambda(y)$, the optimal amount of drugs sold by cartel i is given by some CRS function $q(x_i, R_i)$, where $R_i = \frac{\int_{Y_i} \frac{\mu(y)}{\lambda(y)} dy}{\int_{Y} \frac{\mu(y)}{\lambda(y)} dy}$. If $\lambda(y)$ increases weakly for all $y \in Y$, then $q(x_i, R_i)$ decreases weakly for all (x_i, R_i).*

Proof. First, let $I = \int_Y \frac{\mu(y)}{\lambda(y)} dy$. The FOC can be written as $f'\left(\frac{\lambda(y)}{\mu(y)} x^*(y)\right) = p_p + \eta$, which means that $x^*(y) = \bar{x}\frac{\mu(y)}{\lambda(y)}$ and $q^*(y) = \frac{\mu(y)}{\lambda(y)} f(\bar{x})$, where $\bar{x}$ is some constant defined by $\bar{x} = \frac{x_i}{R_i I}$ (since $\int_{Y_i} \bar{x}\frac{\mu(y)}{\lambda(y)} dy = \bar{x} R_i I = x_i$). This, in turn, implies that $q^* = R_i f(\bar{x}) = R_i I f\left(\frac{x_i}{R_i I}\right)$, which is homogeneous of degree 1.

As $\lambda(y)$ increases weakly for all Y, I decreases weakly. Concavity of f then means that $q^* = R_i I f\left(\frac{x_i}{R_i I}\right)$ decreases. $\square$

The idea behind the functional form in this proposition is that $\mu(y)$ measures how productive route y is. This proposition clarifies the two assumptions needed for the production function in our main model to satisfy CRS: the production technology must have no complementarities or substitutabilities across routes, and some additional functional form assumption is necessary.

We now show that, although the functional form assumption simplifies the results throughout this paper, it is in no way necessary for our main results. Let local profits be $\pi(y) = p_c q^*(y) - x^*(y)$, and total profits be $\pi^A = \int_0^1 \pi(y)dy$. Also define optimal drug demand and supply for cartel i $x_i^* = \int_{Y_i} x^*(y)dy$ and $q_i^* = \int_{Y_i} q^*(y)dy$, and total optima $X^* = \int_0^1 x^*(y)dy$ and $Q^* = \int_0^1 q^*(y)dy$.

Suppose cartel i controls $Y_i \subseteq [0,1]$, and must decide how much drugs to carry through each route. Unlike the previous problem in equation (35), it is not constrained to some value of x_i. Then it solves the following problem:

$$\max_{x(\cdot)} p_c \int_{Y_i} q(y, \lambda(y), x(y))dy - p_p \int_{Y_i} x(y)dy, \tag{37}$$

with first order conditions

$$p_c q_x(y, \lambda(y), x(y)) = p_p \quad \forall y \in Y_i, \tag{38}$$

which implicitly define $x^*(y)$ and $q^*(y)$. Let local profits be $\pi(y) = p_c q^*(y) - x^*(y)$, and total profits be $\pi^A = \int_0^1 \pi(y)dy$, and a cartel's profits be $\pi_i = \int_{Y_i} \pi(y)dy$. Also define weights $\nu(y) = \frac{\pi(y)}{\pi^A}$, which are proportional to $\pi(y)$ and add up to one. We can then define $R_i = \int_{Y_i} \nu(y)dy$. The following proposition is the link that allows us to show that the main results in our paper hold with this general specification:

Proposition 15. *Suppose $\tilde{q}(y, \lambda(y), x(y))$ is decreasing in $\lambda(y)$, increasing and concave in $x(y)$, and more enforcement increases the marginal productivity of x ($\frac{\partial^2 \tilde{q}}{\partial \lambda \partial x} < 0$). Given the sets of routes controlled by each cartel Y_i and the level of enforcement in each route $\lambda(y)$, cartel i's profits are given by $\pi_i = R_i \pi^A$. Drug demand X and supply Q do not depend on the distribution of routes. If $\lambda(y)$ increases weakly for all $y \in Y$, then Q decreases.*

Proof. The definition of R_i and $\nu(y)$ directly yield $\pi_i = R_i \pi^A$. Supply and demand are given by $X = \int_0^1 x^*(y)dy$ and $Q = \int_0^1 \tilde{q}(y, \lambda(y), x(y))dy$, which do not depend on how routes are distributed. By the same argument as in section A.5, supply from every route decreases weakly, meaning that total supply decreases. □

The only result in section 3 that does not have a direct analogy in this proposition is equation 13 in proposition C. Note, however, that the main results in our paper in sections about conflict equilibrium do not depend on equation 13.