Supplemental Materials: Autocratic Stability in the Shadow of Foreign Threats*

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Contents

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A Aligned Leader

Since in the main model we only consider a Leader who is misaligned with Foreign, in this supplement we present results for the case of an aligned Leader. Notice that the benchmark does not change, regardless of Leader's alignment with Foreign, therefore we begin by considering F's intervention cutoff when Leader is aligned.

To make clear any differences between aligned and misaligned Leaders, we consider the domestic security threshold that makes Foreign indifferent between intervention and non-intervention that is a function of Leader's alignment, Opposition's alignment, and who is the Ruler of Home, i.e. $x(j_L, j_Z; r)$ for $j \in \{A, M\}$ and $r \in \{L, Z\}$.

We begin with the case where Leader is in power. Since Leader is aligned, Foreign has no incentive to intervene. Therefore, regardless of Opposition's alignment,

$$\overline{x}(A, j_Z; L) = \overline{x}(j_L, A; Z) = 0, \tag{A.1}$$

recognizing that this is the same threshold as in the main model for the case of a misaligned Leader when an aligned Opposition has successfully become ruler.

Considering Foreign's choice when Opposition has taken power, if Opposition is aligned, (A.1) shows that Foreign will not intervene. Lastly, if Opposition is misaligned and Leader is aligned, Foreign's tradeoff is identical to the case where misaligned Opposition is the ruler and Leader was also misaligned. Thus,

$$\overline{x}(A, M; Z) = \overline{x}(M, M; Z),$$

where $\overline{x}(M, M; Z)$ satisfies the condition for F's indifference in the proof of Lemma 2, part (2) in the main appendix.

Proceeding to Opposition's challenge decision, if both Leader and Opposition are aligned then Foreign has no effect and Z's challenge decision is identical to the benchmark. If Opposition is misaligned, Foreign has the exact same effect on opposition challenges when Leader is aligned as when Leader is misaligned. This is because Opposition's relevant consideration is whether Foreign would be willing to overthrow her should she take power, and Leader's alignment is no longer relevant.

Moving to the aligned Leader's choice, we want to assess the level of domestic security he chooses and his survival prospects relative to a misaligned Leader. If Leader is aligned there are two possibilities. First, when Opposition is also aligned, the presence of F has no effect. Second, when Opposition is misaligned, Leader chooses the minimal level of domestic security, $x^* = 0$, because this leads the threat of intervention by F to deter a challenge from Opposition, and hence, Leader's probability of retaining power is 1.

Therefore, an aligned Leader facing an aligned Opposition has the same survival prospects and chooses the same level of domestic security as in the benchmark case where Foreign is absent. Also, he chooses a weakly lower level of domestic security, and therefore has a weakly lower probability of survival than a misaligned Leader facing an aligned Opposition, while in terms of expected utility the aligned Leader is better off than his misaligned counterpart. Instead, an aligned Leader facing a misaligned Opposition has the same survival prospects as a misaligned Leader facing a misaligned Opposition, but chooses a strictly lower level of domestic security, thus implying he is better off than a misaligned Leader.

B Domestic Security and Political Instability

In this appendix, we consider the case where, by repressing Opposition, Leader affects the ability of Opposition to grab power following an intervention from Foreign. Assume that the parameter q is a function of x, so that the probability of taking power following intervention is $q(x)$, with q being strictly decreasing. That is, the higher is domestic security, the lower the probability that Opposition will take power after an intervention from Foreign. To ensure a meaningful comparison across models, we assume that $q(0) = q$.

Let us start from Foreign's decision. Consider first the case where Opposition is in power. If Opposition is aligned, once again Foreign has no incentive to intervene and we write $\overline{x}^{\dagger}(A;Z) = 0$.

If Opposition is misaligned, the expected utility of F from intervening is

$$-(1-p)d(x) - k$$

Since $q(x)$ does not enter the expression above, the cutoff is identical to the one in the text, that is $\overline{x}^{\dagger}(M;Z) = \overline{x}(M;Z)$

Consider next the case where Leader has retained control of Home. When Opposition is aligned, F's expected payoff from intervention is given by

$$-(1-q(x))(1-p)d(x) - k.$$

Rearranging the equation above we obtain

$$(1-p)q(x) + p = \frac{k}{d(x)}. \tag{B.1}$$

The left-hand side is decreasing in x and the right-hand side is increasing in x. Since we have that $\lim\limits_{x\to 0} \frac{k}{d(x)} = 0$, and $\lim\limits_{x\to X} \frac{k}{d(x)} = +\infty$, then there is a unique $\overline{x}^{\dagger}(A;L)$ that solves (B.1).

Moreover, since $q(x)$ is decreasing in x and $q(0) = q$, we have that $\overline{x}^{\dagger}(A;L) < \overline{x}(A;L)$, meaning that when domestic security has a negative effect on q, Foreign becomes less willing to intervene, since the probability that an aligned actor (Opposition) will take power following the intervention is lower.

Last, consider when Leader is still in power and Opposition is misaligned. In this case, F's expected payoff from intervention is given by

$$-q(x)d(x) - (1-q(x))(1-p)d(x) - k,$$

which after rearranging, we obtain

$$p(1-q(x)) = \frac{k}{d(x)}. \tag{B.2}$$

Both the left-hand side and the right-hand side are increasing in x. However notice that the left-hand side is bounded from above by p and from below by $p(1-q)$, while $\lim_{x\to 0} \frac{k}{d(x)} = 0$, and $\lim_{x\to X} \frac{k}{d(x)} = +\infty$. This implies that there is a unique $\overline{x}^\dagger(M;L)$ that solves (B.2). Foreign is *more* willing to intervene than in the model where q is constant in x, since the risk of regime change, namely, of ending up with a misaligned ruler, is now lower. It is also easy to see that we have $\overline{x}^\dagger(M;L) < \overline{x}^\dagger(M;Z) < \overline{x}^\dagger(A;L)$, just like in the main model, where $q(\cdot)$ was constant.

Moving on to the choice of Opposition to challenge Leader, notice that the only effect of introducing direct dependence of q on x is when $x < \overline{x}^\dagger(A;L)$ and $c < (1-q(x))B_Z(x)$. First, notice that this case is off the equilibrium path. Yet, we need to pin down Opposition's strategy. To this end, notice that if $\frac{B'_Z(x)}{B_Z(x)} < -\frac{(1-q'(x))}{(1-q(x))}$, then $\tau^*_A(x)$ is increasing in x, just like in the model analyzed in the main text. However, it is possible for $\tau^*_A(x)$ to be *decreasing* in x. This is the case if domestic security has such a negative impact on Opposition's ability to take power following Foreign's intervention that Opposition prefers to directly challenge Leader, rather than free ride on Foreign's possible intervention.[1]

Finally, the characterization of L's optimal choice of domestic security is similar to that in the main model, with the only difference being that some of the thresholds on x, from Foreign's intervention strategy, are different. Given that the ranking of the various thresholds on x is unchanged, the qualitative predictions of our model are unaffected by introducing a dependence of the probability Z gains power following intervention on the level of domestic security.

[1]However, recall that we are assuming that domestic security has no direct effect on τ, the strength of the opposition.

C Stochastic Intervention Success

In this supplement, we relax the assumption that intervention by F is able to overthrow whoever rules H with probability 1. Let $\gamma \in [0,1]$ be the probability that intervention by F is successful and $1-\gamma$ be the probability that intervention fails.[2] The expected payoffs for players, then, follow by taking expectations over the payoffs in the main model.

We now proceed through the main results accounting for F's inability to remove the ruler of H with certainty. Notice that the benchmark does not change, therefore we begin by considering Lemma **??** in light of stochastic intervention success. The cutoffs for F follow by an identical argument as in Lemma **??**, but where the left-hand side in each case is multiplied by γ. In particular, using the exact same argument and replacing η with $\frac{\eta_\gamma}{\gamma}$, Foreign's intervention choice rule follows. Consequently, we have that $\overline{x}^\gamma(A;Z) < \overline{x}^\gamma(M;L) < \overline{x}^\gamma(M;Z) < \overline{x}^\gamma(A;L)$ as in the main text. However, substitution of $\frac{\eta_\gamma}{\gamma}$ into Lemma **??** shows that when intervention fails with probability γ, the threshold levels of domestic security for F is lower than in the main model, reflecting that Foreign is less willing to intervene.

Proceeding to Opposition's challenge decision, when intervention can fail, fewer Opposition types challenge when Z is aligned, while more Opposition types challenge when Z is misaligned, relative to the main model.

If Opposition is aligned, her expected payoff for challenging is

$$\rho(\tau,x)B_Z(x) + \mathbb{1}_{\{x<\overline{x}^\gamma(A;L)\}}\Big(\big(1-\rho(\tau,x)\big)q\gamma B_Z(x)\Big) - c,$$

while if she supports she receives

$$\mathbb{1}_{\{x<\overline{x}^\gamma(A;L)\}}q\gamma B_Z(x).$$

Combining these, aligned Opposition is indifferent between challenging and supporting if

[2] Note that the main model is achieved in this extension by setting $\gamma = 1$.

and only if her type τ satisfies,

$$\rho(\tau, x)\Big[1 - \mathbb{1}_{\{x<\overline{x}^{\gamma}(A;L)\}} q\gamma\Big] B_Z(x) = c. \tag{C.1}$$

Observe that if $x \geq \overline{x}^{\gamma}(A; L)$ the fact that intervention may fail has no effect on Opposition's strategy. For the case where $x < \overline{x}^{\gamma}(A; L)$, (C.1) reduces to

$$\rho(\tau, x)(1 - q\gamma)B_Z(x) = c.$$

Since $1 - q\gamma > 1 - q$, fewer Opposition types will challenge when compared to the main model, i.e. $\tau_A^{\gamma}(x) > \tau_A^{*}(x)$.

For misaligned Opposition, the expected payoff of challenging is

$$\mathbb{1}_{\{x\geq\overline{x}^{\gamma}(M;Z)\}}\Big(\rho(\tau, x)B_Z(x)\Big) + \mathbb{1}_{\{x<\overline{x}^{\gamma}(M;Z)\}}\bigg(\rho(\tau, x)(1-\gamma)B_Z(x) + \Big(1-\rho(\tau, x)\Big)q\gamma B_Z(x)\bigg) - c,$$

while her payoff for supporting Leader is

$$\mathbb{1}_{\{x<\overline{x}^{\gamma}(M;Z)\}} q\gamma B_Z(x).$$

These imply that misaligned Opposition is indifferent between challenging and supporting if and only if her type τ satisfies,

$$\mathbb{1}_{\{x\geq\overline{x}^{\gamma}(M;Z)\}}\Big(\rho(\tau, x)B_Z(x)\Big) + \mathbb{1}_{\{x<\overline{x}^{\gamma}(M;Z)\}}\bigg(\rho(\tau, x)\Big(1 - (1-q)\gamma\Big)B_Z(x)\bigg) = c. \tag{C.2}$$

If $x \geq \overline{x}^{\gamma}(M; Z)$ then stochastic intervention failure has no effect on Opposition challenges. If $x < \overline{x}^{\gamma}(M; Z)$, (C.2) reduces to

$$\rho(\tau, x)\Big[1 - (1-q)\gamma\Big]B_Z(x) = c.$$

The left-hand side may be positive, therefore, some types will challenge. Since no types of misaligned Opposition challenge in the main model, stochastic intervention failure increases the risk to Leader from domestic challenges.

Finally, moving on to Leader, L chooses the level of domestic security to solve:

$$\max_{x \in [0,X]} \lambda_j^{\gamma}(x) B_L(x),$$

for $j \in \{A, M\}$. Equilibrium existence follows by an identical argument as Proposition **??** from the main model. Of interest is whether the level of domestic security, and the outcomes for Leader, change when intervention by Foreign may fail. Even when Foreign's intervention may fail, it is never a best response for Leader to choose a level of domestic security below F's sequentially rational intervention threshold unless the probability of intervention success is 0.

Since $\overline{x}^{\gamma}(j; R) < x^{*}(j; R)$ for $R \in \{L, Z\}$ and $B_L(x)$ is strictly decreasing, the level of domestic security selected by Leader may be less than the level of domestic security in the main model. That is, $x^{\gamma}(j; R) \leq x^{*}(j; R)$, depending on whether the level of domestic security necessary to deter intervention (when intervention success is stochastic) also deters domestic challenges sufficiently to maximize Leader's probability of retaining office.

Lastly, relative to the main model, Leader's hazard of holding power increases and his benefit of retaining office increases. This implies $\lambda_j^{\gamma}(x_j^{\gamma}) \leq \lambda_j^{*}(x_j^{*})$. Leader will choose $x_j^{\gamma} \geq \overline{x}^{\gamma}(j; R)$, but this generates two cases to consider. If $x_j^{\gamma} \geq \overline{x}(j; R)$ from the main model, then F's stochastic intervention success has no effect on Leader's hazard of holding power, or payoff from retaining office. If instead, $x_j^{\gamma} \in \left[\overline{x}^{\gamma}(j; R), \overline{x}(j; R)\right)$ then Leader's hazard may increase. In this case, Foreign does not intervene, eliminating that threat to Leader's power. However, with a lower level of domestic security, Leader may be more vulnerable to Opposition challenges compared to the main model. If Leader does survive in office, he does so having invested less in domestic security, increasing his payoff.

D Multiple Optima

The probability of Leader's survival in office, $\lambda_0(x)$, is quasiconcave but not necessarily strictly concave in x. Therefore, there may be multiple solutions to Leader's choice problem. In this supplement we address this possibility but note initially that the possibility of multiple optimal x^* does not qualitatively change the outcomes—either for foreign intervention or challenges against Leader—from the main model.

In the benchmark, as mentioned in the main text, there may be multiple, payoff-equivalent x_0^* that characterize multiple equilibria to the game without a foreign threat. In each of these possible equilibria, Opposition challenges if her type is sufficiently high, thus the possibility for multiple solutions to Leader's problem does not generate qualitatively different results.

Now consider Leader's problem in the presence of a foreign threat. For any alignment of Z, if Leader's problem, given in (**??**), has one or more solutions below the threshold $\overline{x}(j;R)$, for $j \in \{A,M\}$ and $R \in \{L,Z\}$, Leader's probability of retaining power should she choose any $x_j^* < \overline{x}(j;R)$ is zero. Thus any such solution is not a best response, given Foreign's choice rule. Leader will instead choose $x_j^* \geq \overline{x}(j;R)$ to avoid foreign intervention.

Opposition's alignment is consequential if there are multiple optima above the threshold $\overline{x}(j;R)$. If Z is misaligned, Leader will choose $x_M^* = \overline{x}(M;L)$, even if there is another (or multiple) solution $x_{M'}^* > x_M^*$, because Leader can guarantee a probability of survival $\lambda_M(x) = 1$ for either solution and $B_L(x)$ is decreasing in x. If Opposition is aligned and there are multiple possible solutions above the threshold $\overline{x}(A;L)$, there may be multiple payoff-equivalent equilibria that are observationally equivalent to the main model.

E Cooperation

In this section, we consider an extension of our main model where Foreign can provide logistical assistance or other support to Opposition ex ante, i.e. prior to the start of the game comprising the main analysis. To capture this possibility, suppose that at the beginning of the game, Foreign has a support decision, $\theta \in [0, \bar{\theta}]$, where $\theta > 0$ is the decision to grant support to Z, and $\theta = 0$ signifies the decision to not offer support. Support influences the the cost for Z of challenging Leader for control of Home. In particular, challenging L costs $y = c_0 - \theta$, where c_0 is the cost of challenging without support from F. Support is costly, captured by the cost function $\kappa(\theta)$, which is strictly increasing and strictly convex.

We begin with the following, which connects the extension of this section with the main model.

Lemma E.1 *Given any fixed level of support, θ, there is an equilibrium to the subsequent subgame beginning with Leader's domestic security choice.*

Proof: For any fixed θ, this result follows by setting $c = c_0 - \theta$ in the main model and applying Proposition **??**. ■

Lemma E.1 exploits that every aspect of the game, with the exception of the initial support decision, corresponds to the main model. To detail how support connects to our main model, we present the following intermediate result.

Lemma E.2 *The probability L retains power, $\lambda_0(x)$, is strictly increasing in the cost of challenges c.*

Proof: From Lemma **??**, τ_0^* is increasing in c. By Liebniz's rule,

$$\frac{d\lambda_0(x)}{d\tau_0^*} = \psi\Big(\tau_0^*(x)\Big) - \Big(1 - \rho(x, \tau_0^*)\Big)\psi\Big(\tau_0^*(x)\Big) = \rho(x, \tau_0^*)\psi(\tau_0^*(x)) > 0,$$

implying that the probability Leader survives is increasing in Opposition's cutoff τ_0^*. Then, by combining these two facts and using the chain rule, we have that

$$\frac{d\lambda_0(x)}{dc} < 0.$$

■

Holding fixed the level of domestic security, Lemma **??** establishes that the kind of logistical support represented by θ indeed corresponds to an increase in the willingness of Opposition to challenge Leader. Moreover, Lemma E.2 shows that increasing support corresponds to a reduction in the equilibrium probability that Leader retains power.

We now move on when Leader chooses the level of domestic security. When the alignment of Opposition is $j \in \{A, M\}$, denote the level of support chosen by F as θ_j.

Lemma E.3 *If Z is misaligned, then in equilibrium, F will not give support, i.e. $\theta^*_M = 0$.*

Proof: Proceeding by contradiction, suppose F offers support $\theta \in (0, \bar{\theta}]$ and pays cost $\kappa(\theta)$ for this cooperation. When Z is misaligned, then Proposition **??** establishes that the probability L retains power is 1, and the level of domestic security chosen by L is $\overline{x}(M, L)$, which is independent of the cost of challenging c. Consequently, because $\kappa(\cdot)$ is increasing in θ, F is strictly better off choosing $\theta = 0$. ■

Given this result, we can focus the remainder of our analysis on the case where Opposition is aligned. To detail the incentives of F when considering whether to cooperate, we need to consider F's payoff for different levels of support. Once the level of support θ is chosen, Lemma E.1 establishes the existence of an equilibrium for the corresponding challenge cost y. Thus, when Foreign makes its choice θ, it accounts for the equilibrium values $w^* = 0$, x^*_A, and $\tau^*_A(x)$.[3] F's payoff is given by,

$$\mathbb{E}[U_F(\theta)]\Big|_{\{x^*_A, \tau^*_A, w^*\}} = -d(x^*_A)\left[\Psi\Big(\tau^*_A(x^*_A)\Big) + \int_{\tau^*_A(x^*_A)}^{\infty} \left[\Big(1 - \rho\Big(x^*_A, \tau\Big)\Big)\right]\psi(\tau)d\tau\right] - \kappa(\theta),$$

Differentiating with respect to θ,

$$-d'(x^*_A)\lambda_0(x^*_A)\left(\frac{dx^*_A}{dy}\right)\left(\frac{dy}{d\theta}\right) - d(x^*_A)\lambda'_0(x^*_A)\left(\frac{dx^*_A}{dy}\right)\left(\frac{dy}{d\theta}\right) - \kappa'(\theta) = 0,$$

whose first term is negative and second term is positive if $\frac{dx^*_A}{dy}$ is positive.[4] Rearranging,

[3] Recall that in the main model intervention is never sequentially rational for F given L's equilibrium level of domestic security.

[4] Notice that x^*_A is differentiable almost everywhere.

and given that $\frac{dy}{d\theta} = -1$,

$$\frac{dx_A^*}{dy}\Big(d'(x_A^*)\lambda_0(x_A^*) + d(x_A^*)\lambda_0'(x_A^*)\Big) = \kappa'(\theta).$$

This expression makes clear that F makes its support decision to balance the marginal cost of cooperation with the effect cooperation has on the level of domestic security. Thus, to understand when F will choose to cooperate with an aligned Opposition, it is key to establish how F's support choice affects the level of domestic security chosen by L.

Proposition E.1 *For each $j \in \{A, M\}$, there is a unique Perfect Bayesian equilibrium, $(\theta_j^*, x_j^*, \tau_j^*(x), \overline{x}(j; R))$, where Opposition challenges Leader if and only if $\tau \geq \tau_j^*(x)$; and Foreign intervenes if and only if R is misaligned and $x < \overline{x}(j; R)$. Moreover,*

(i) when Opposition is misaligned ($j = M$), Leader chooses $x_M^* = \overline{x}(M; L)$, and $\theta_M^* = 0$;

(ii) when Opposition is aligned ($j = A$), Leader chooses $x_A^* = \max\{x_0^*, \overline{x}(A; L)\}$, and θ_A^*, if interior, solves

$$\frac{dx_0^*}{dy}\Big(d'(x_0^*)\lambda_0(x_0^*) + d(x_0^*)\lambda_0'(x_0^*)\Big) = \kappa'(\theta).$$

if and only if

$$\mathbb{E}[U_F(\theta)]\Big|_{\theta=0} < \mathbb{E}[U_F(\theta)]\Big|_{\theta\in(0,\bar{\theta}]},$$

and $\theta_A^* = 0$ otherwise.

Proof: Lemma E.1 ensures existence of the equilibrium for a given θ. By Lemma E.3, x_M^* follows immediately from Proposition **??** and $\theta_M^* = 0$. If Z is aligned, Foreign's support decision depends on x_0^*. There are two cases.

First, consider $x_0^* < \overline{x}(A; L)$. Then $x_A^* = \overline{x}(A; L)$ and we can reassess Foreign's optimal choice of θ, which solves

$$\frac{d\overline{x}(A;L)}{dy}\Big(d'(\overline{x}(A;L))\lambda_0(\overline{x}(A;L)) + d(\overline{x}(A;L))\lambda_0'(\overline{x}(A;L))\Big) = \kappa'(\theta).$$

At $\overline{x}(A;L)$, $\frac{d\overline{x}(A;L)}{dy} = 0$. Leader has no incentive to increase the level of domestic security from $x_A^* = \overline{x}(A;L)$, because doing so reduces his payoff. Therefore, any support from F to Z has no effect on the optimal level domestic security. Since support is costly, F will choose $\theta_A^* = 0$.

Second, consider $x_0^* > \overline{x}(A;L)$, such that $x_A^* = x_0^*$, and Foreign's choice of θ, solves

$$\max_{\theta\in[0,\bar{\theta}]} -d(x_0^*)\left[\Psi\Big(\tau_A^*(x_0^*)\Big) + \int_{\tau_A^*(x_0^*)}^{\infty}\left[\Big(1-\rho\Big(x_0^*,\tau\Big)\Big)\right]\psi(\tau)d\tau\right] - \kappa(\theta).$$

The first-order condition associated with this problem is

$$\frac{dx_0^*}{dy}\Big(d'(x_0^*)\lambda_0(x_0^*) + d(x_0^*)\lambda_0'(x_0^*)\Big) = \kappa'(\theta).$$

Foreign's problem has a solution $\hat{\theta}$ since $[0,\bar{\theta}]$ is compact and $\mathbb{E}[U_F(\theta)]$ is continuous in θ.[5] F will provide support of $\theta_A^* = \hat{\theta}$ to Z if $\mathbb{E}[U_F(\hat{\theta})] > \mathbb{E}[U_F(0)]$, and otherwise, $\theta_A^* = 0$. ■

Foreign will only provide support ex ante to Opposition under very limited circumstances. It is not generally the case that it is easier or more cost effective for Foreign to cooperate with Opposition and thus avoid a costly intervention in Home. This is because the threat of intervention is powerful—the possibility of intervention deters challenges by Opposition and pushes Leader to choose a high level of domestic security. Further, in equilibrium, F does not actually intervene, it is merely the threat of intervention that changes L and Z's decisions, and the threat itself is not costly for F. Therefore, only when Opposition is aligned, and offering support could increase the level of domestic security, is Foreign willing to pay the cost of supporting Z, in exchange for lower levels

[5]Foreign's problem may have multiple payoff-equivalent solutions. This is not problematic for our analysis by the same argument in Appendix D.

of damage imposed by Home.

F Ego Rents

In the main text, we note that if Leader or Opposition are driven by "ego rents," then more Opposition types will challenge, and Leader's choice of domestic security will depend on the value of these rents. In this appendix, we more explicitly represent the value of ego rents and describe their effect on domestic security.

Let e represent the strength of ego rents, which is taken from a complete lattice where higher e corresponds to higher levels of ego rents. Let the benefit of office, $B_i(x, e)$ for $i \in \{L, Z\}$, be a function of e, and assume B_i is strictly increasing in e, and that B_i has strict increasing differences in (x, e). This latter feature implies that the marginal return from x increases in e. Notice also that because the benefit of office is different for Leader and Opposition, both L and Z value ego rents, but may not value such rents equally.

First consider the effect of an increase in ego rents on Opposition's willingness to challenge.

Remark F.1 *The type cutoff $\tau_0^*(x)$ is strictly decreasing in ego rents, e.*

Proof: The characterization follows by Lemma **??** and the result follows by noticing that increases in ego rents correspond to pointwise increases in B_Z. ■

As ego rents increase, more Opposition types are willing to challenge because the benefit of holding office is greater for a given level of domestic security chosen by Leader. Because B_Z has strict increasing differences, the effect of an increase in ego rents on Z's willingness to challenge is stronger when the level of domestic security is high.

Proceeding to the effect of ego rents on Leader. Recall that the elasticity of Leader's benefit of holding power following from a unit change in the level of domestic security is defined in Equation (4) in the main text. Given that the elasticity is negative, increases in e make B_i more inelastic.

Remark F.2 *The elasticity of the function $B_i(x, e)$ is strictly decreasing in ego rents.*

Proof: We can write the elasticity of $B_i(x, e)$ with respect to the level of domestic security as

$$\mathcal{E}_{B_i}(x) = \frac{d\ln(B_i(x,e))}{d\ln(x)} = \frac{d\ln(B_i(x,e))}{dx}x = \frac{x}{B_i(x,e)}\frac{\partial B_i(x,e)}{\partial x} < 0,$$

because B_i is strictly decreasing in x. Differentiating this expression in e gives

$$\frac{x}{(B_i(x,e))^2}\bigg(\frac{\partial^2 B_i(x,e)}{\partial x \partial e}B_i(x,e) - \frac{\partial B_i(x,e)}{\partial x}\frac{\partial B_i(x,e)}{\partial e}\bigg).$$

Since $\frac{\partial^2 B_i(x,e)}{\partial x \partial e} > 0$, because of increasing differences, and $\frac{\partial B_i(x,e)}{\partial x} < 0$ we see that $\mathcal{E}_{B_i}(x)$ is strictly increasing in e. ■

Remark F.2 implies that as ego rents increase, Leader's benefit function is less responsive to changes in the level of domestic security. This does not, however, give the change in the equilibrium level of domestic security. We now consider how a change in ego rents affects Leader's chosen level of domestic security.

Remark F.3 *The level of domestic security is strictly increasing in ego rents.*

Proof: Equilibrium existence follows by Proposition **??**. The optimal level of domestic security in the first stage is

$$x_0^*(e) = \underset{x\in[0,X]}{\operatorname{argmax}}\ \lambda_0(x)B_L(x,e).$$

Let $\tilde{e} > e$ and $\tilde{x} > x$, we have that

$$\lambda_0(\tilde{x})B_L(\tilde{x},\tilde{e}) - \lambda_0(\tilde{x})B_L(\tilde{x},e) = \lambda_0(\tilde{x})\Big(B_L(\tilde{x},\tilde{e}) - B_L(\tilde{x},e)\Big)$$

Differentiating this difference with respect to x, and taking $\tilde{e} \to e$, this difference goes to

$$\lambda_0'(\tilde{x})\frac{\partial B_L(\tilde{x},e)}{\partial e} + \lambda_0(\tilde{x})\frac{\partial^2 B_L(\tilde{x},e)}{\partial x \partial e}.$$

Notice that by Remark **??**, $\lambda_0(x)$ is strictly increasing in x, and because B_L is strictly increasing in e the first term is positive. The second term is positive because $\lambda_0(x)$ is always positive and B_L has strict increasing differences. Hence $\lambda_0(x)B_L(x,e)$ has strict increasing differences in (x,e), so by Edlin and Shannon (1998), $x_0^*(e)$ is strictly increasing in e. This implies that increases in ego rents lead to an increase in the level of domestic security. ■

Ego rents essentially act as a substitute for the spoils of office. By Remark F.2, as ego rents increase, Leader's benefit of holding power increases, making remaining in power more valuable. Remark F.3 establishes that increasing ego rents prompts an increase in the level of domestic security, making it more likely Leader will remain in power. This investment reduces the benefits from spoils but ego rents compensate for this decrease in spoils benefits. Because the level of ego rents enjoyed by Leader do not affect F, and as established in the main text, F's incentives determine the level of domestic security in many circumstances, our main results are unaffected by changes in Leader's ego rents.

G Propaganda

In this supplement, to see how propaganda could be incorporated into our framework, we consider an extension to our model in which we introduce asymmetric information regarding the alignment of Opposition and Foreign. The setup in this section allows us to answer two related questions. First, does propaganda change the conditions under which Z is willing to challenge L? Second, in what way does Leader benefit from propaganda in our model? We make the following modifications of our framework:

- Opposition does not know whether she is aligned or misaligned with Foreign;
- Z's prior belief that she is aligned is μ;
- Both Leader and Foreign know Opposition's alignment;
- For tractability, L is restricted to two possible levels of domestic security: $\overline{x}(M;L)$

or $\overline{x}(A; L)$.

Note that all other aspects are the same as in the main model.

Because Opposition does not know if she is aligned or misaligned with F, Leader's choice of domestic security potentially serves as a signal of Z's alignment. To focus the analysis of this section on informational issues, we restrict Leader's choice of domestic security to two levels, $\overline{x}(M; L)$ or $\overline{x}(A; L)$, since they are the equilibrium levels for misaligned and aligned Z in our main model. We make this simplification to avoid common technical challenges which typically arise in signaling games when the space of signals is continuous (see, e.g., Manelli (1996) or Mas-Colell, Whinston and Green (1995, Section 13.C)).

Upon observing the level of domestic security, and updating her beliefs in concordance with Bayes' rule, Opposition decides whether to challenge Leader for control of Home. Finally, upon observing the level of domestic security and Z's challenge decision, Foreign, who knows Z's alignment, chooses whether to intervene. The solution concept we employ in this extension is Perfect Bayesian Nash equilibrium.

A separating equilibrium in the context of this extension is the same as in the main model (where Z knows her alignment). Because of this, we focus our analysis in this section on pooling equilibria. By Proposition **??**, Leader benefits from Opposition being misaligned, hence, Leader prefers Z to believe that she is likely misaligned, regardless of whether this is actually the case. So if L can keep Z sufficiently confident that she is misaligned, she can choose a lower level of domestic security while simultaneously deterring challenges (due to Proposition **??**). In our main result of this section, we explore under what conditions a pooling equilibrium exists where Leader chooses $\overline{x}(M; L)$ for both aligned and misaligned Opposition.[6]

Remark G.1 *There is a nonempty set of prior beliefs and off the path beliefs such that*

[6]There may also be a pooling equilibrium in which L chooses $\overline{x}(A; L)$. However, we focus on the case where L chooses $\overline{x}(M; L)$ since this is the case where L would benefit from manipulating the informational environment.

there exists a pooling Perfect Bayesian Nash equilibrium where Leader chooses domestic security level $\overline{x}(M;L)$, regardless of the alignment of Z.

Proof: Opposition's expected utility of challenging, given prior μ is

$$\mathbb{E}[U_Z(s=1)|\overline{x}(M;L)] = \mu\Big(\rho\Big(\tau,\overline{x}(M;L)\Big)B_Z\Big(\overline{x}(M;L)\Big) - c\Big) + (1-\mu)\Big(0-c\Big),$$

while her expected utility of supporting is $\mathbb{E}[U_Z(s=0)|\overline{x}(M;L)] = 0$.

For an arbitrary level of domestic security x, the unique threshold level of strength at which Opposition is indifferent between supporting and challenging, $\tau^\dagger_\mu(x)$, solves

$$\mu\rho(\tau^\dagger_\mu(x),x)B_Z(x) = c.$$

Notice that $\tau^\dagger_\mu(x)$ is decreasing in μ, meaning that the higher the probability Opposition is aligned, the more types of Opposition challenge Leader.

What remains is to check that Leader finds choosing $x = \overline{x}(M;L)$ optimal, regardless of the true alignment of Opposition. L's payoff from choosing $x = \overline{x}(M;L)$ is

$$\mathbb{E}[U_L(\overline{x}(M;L)) \mid \mu] = B_L(\overline{x}(M;L))\left[\Psi(\tau^\dagger_\mu(\overline{x}(M;L))) + \int_{\tau^\dagger_\mu(\overline{x}(M;L))}^{\infty}(1-\rho(\overline{x}(M;L),\tau))\psi(\tau)d\tau\right].$$

Suppose that, off the equilibrium path, if Opposition observes $x = \overline{x}(A;L)$, then her belief that she is aligned is $\tilde{\mu}$. Then, L's payoff to choosing $\overline{x}(A;L)$ is

$$\mathbb{E}[U_L(\overline{x}(A;L)) \mid \tilde{\mu}] = B_L(\overline{x}(A;L))\left[\Psi(\tau^\dagger_{\tilde{\mu}}(\overline{x}(A;L))) + \int_{\tau^\dagger_{\tilde{\mu}}(\overline{x}(A;L))}^{\infty}(1-\rho(\overline{x}(A;L),\tau))\psi(\tau)d\tau\right].$$

Note that since $\tau^\dagger_\mu$ is decreasing in μ, and because L's survival probability is increasing in $\tau^\dagger_\mu$ the probability of survival is increasing in $\tau^\dagger_\mu$, Leader's survival probability is decreasing in μ. Consequently, $\mathbb{E}[U_L(\overline{x}(A;L)) \mid \tilde{\mu}^*]$ is monotone in $\tilde{\mu}$, and that $\mathbb{E}[U_L(\overline{x}(M;L)) \mid \mu]$

is constant in $\tilde{\mu}$. There are two cases. First, if μ is such that

$$\mathbb{E}[U_L(\overline{x}(M; L)) \mid \mu] > \mathbb{E}[U_L(\overline{x}(A; L)) \mid 0],$$

then there does not exist a pooling equilibrium. Second, if instead,

$$\mathbb{E}[U_L(\overline{x}(A; L)) \mid 1] \leq \mathbb{E}[U_L(\overline{x}(M; L)) \mid \mu] \leq \mathbb{E}[U_L(\overline{x}(A; L)) \mid 0]$$

then, by the Intermediate Value Theorem, for any prior belief, μ, there exists an off the path belief $\tilde{\mu}^*$ such that

$$\mathbb{E}[U_L(\overline{x}(M; L)) \mid \mu] = \mathbb{E}[U_L(\overline{x}(A; L)) \mid \tilde{\mu}^*],$$

where the proscribed pooling equilibrium exists for any off the path belief $\tilde{\mu} \geq \tilde{\mu}^*$. Combining these two cases establishes the result. ■

Remark G.1 shows that Leader is only able to sustain lower levels of domestic security, exploiting the likely misalignment of Z, when the prior probability that Z is misaligned is sufficiently high. Whether the pooling equilibrium outlined in Remark G.1 exists depends on the prior belief, μ, and the off the path belief of Opposition, $\tilde{\mu}$. Consequently, Leader, if he could affect these beliefs, has an incentive to manipulate information in a manner that keeps Z sufficiently confident that she is misaligned with F. To the extent that propaganda is a tool that manipulates these aspects of the informational environment, μ and $\tilde{\mu}$, Remark G.1 illustrates a way Leader can remain in power with certainty, exploiting an uncertain informational environment. Propaganda can thus enable Leader to deter all challenges and remain in power at a lower level of domestic security than is possible without such tools.

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