Appendices

This supplementary Appendix is two parts. In part A, we present two extensions to the baseline model. The first considers a variant model in which bargaining failure results in a reversion to the legal *status quo*. The second consider a different variation, in which judges may change their dispositional votes after observing the proposed majority (and dissenting) opinions. As we make clear, the results from our baseline model continue to hold more-or-less under these variations. In part B, we present detailed proofs of the main results.

A. EXTENSIONS

A.1 Reversion to Status Quo

A legislature may propose changes to a given law repeatedly; however, unless one of those proposals is accepted, it is understood that the existing law continues to be in effect. The same cannot be said of courts. As we argued in Section 2, the mere fact that the court agrees to hear a case signals to the community that the legal landscape is apt to change, even if the court fails to implement that change in deciding the instant case. Thus, our preferred model specification does not include a *status quo* policy but instead requires that the court, through the bargaining process, eventually settle on a new policy.

Nevertheless, one might ask how our results would change if we instead assumed that failure to agree resulted in reversion to the *status quo ante*. The bargaining procedure would be amended as follows: in the event that a proposal is rejected, with probability δ a new proposer is selected and bargaining continues; however, with probability $1 - \delta$, the bargaining terminates (exogenously), and the policy reverts to the *status quo*.[21] This might represent the rare set of cases where no majority can be found to support any given opinion.

With this re-interpretation of the bargaining process, Proposition 1 (and all of the subsequent

[21]This is the bargaining protocol in Banks and Duggan (2006).

results) continue to hold true[22], replacing the disagreement utility with the utility of the *status quo* policy. Thus, our analysis is not only perfectly compatible with this alternative formulation, but demonstrates how to analyze it.

Of course, reversion to a *status quo* imposes different costs on different judges, depending on where the *status quo* stands in relation to their ideal policy. As such, the equilibrium policies will be different, even if the essential structure of the equilibrium is unchanged. One can show (see Banks and Duggan (2006)) that, if the *status quo* lies outside the core (i.e. $y_{sq} \notin [x^l, x^r]$), then with $\delta < 1$, there will be a range of equilibrium policies that are proposed in equilibrium, and that the social acceptance set becomes narrower as the likelihood that bargaining fails gets smaller (i.e. δ becomes larger). Moreover, as $\delta \to 1$, equilibrium proposals converge to a unique policy, characterized by the asymmetric Nash bargaining solution, as in Proposition 2. The analysis from section 3 carries through exactly as described.

However, if the *status quo* lies within the core (i.e. $y_{sq} \in [x^l, x^r]$), then for any δ, the only policy that is equilibrium consistent is the *status quo* itself. (It turns out that, in this case, the *status quo* policy exactly coincides with the asymmetric Nash bargaining solution, by construction, so Proposition 2 continues to hold, albeit trivially.)

Although we do not take up the issue of *certiorari* petitions in this paper, this last point may shed some light on the issue. Since whenever the *status quo* lies within the core, the court will fail to amend the existing rule, we should not expect the court to hear cases where such an outcome is likely to obtain. Moreover, since the core consists of the interval between the median judge's ideal, and the ideal policy of the other decisive judge (which, in the event of a unanimous dispositional

[22]A minor technical caveat: In the baseline framework, it was sufficient that the dispositional loss function l weakly satisfied the *IDID* property. Here, we strengthen that assumption, requiring the loss function to satisfy the *IDID* property strictly. Of particular interest, the loss function associated with the absolute value policy preferences that we highlighted in Example 1 only satisfies IDID weakly. However, the other cases presented all satisfy the strict condition.

vote, is also the median judge), it would be improvident for the court to grant *cert* on cases that where the *status quo ante* lies too close to the median judge's ideal policy. Furthermore, to the extent that the Court does agree to here a case, we should expect larger coalitions and more strategic voting, since the size of the core (which determines the likelihood of failing to amend the existing rule) is decreasing in the size of the majority coalition.

Even when the *status quo* policy lies outside of the core (so that policies are chosen through a genuine process of bargaining), its location affects the policies that will be chosen in equilibrium. Interestingly, as the status quo policy becomes more extreme, the policy that is implemented is likely to be more moderate (in the sense of being closer to the 'middle' of the core), *ceteris paribus* (see Parameswaran and Rendleman (2019)). Thus, policy-making by the court exhibits path dependence, with existing rules shaping the sorts of rules that courts can implement in the future.

A.2 Dissents and Competition for the Dispositional Majority

In the baseline model, we assumed that, once chosen, the composition of the dispositional majority remained fixed. Since there is little evidence that dispositional coalitions shift between the initial conference and the Court's rendering of it final decision, we hold this assumption to be reasonable, as an empirical matter. Nevertheless, it may be objected that this result ought to be a consequence of our model, rather than an assumption. In this sub-section, we consider a variant model in which stable dispositional coalitions arise in equilibrium.

Before outlining the variant model and results, let us briefly acknowledge the implications of our baseline approach. In the baseline model, since the dispositional majority was fixed in the first stage, the consequence of proposing a relatively 'extreme' policy in the second stage was simply that the policy would be rejected and a counter-proposal made. However, if dispositional coalitions were allowed to change, there may be an additional consequence; an extreme proposal might cause sufficiently many judges to switch their dispositional votes, such that the original majority is lost. The threat of such defection creates an additional incentive for judges to moderate their proposals. It is this additional incentive that we seek to explore.

We modify the model as follows: after the initial dispositional vote, the judges divide into majority and minority dispositional coalitions. As before, the most senior judge in the majority assigns to some judge in the majority, the task of writing a majority opinion, and this opinion may be refined through a sequence of counter-proposals. Similarly, the most senior judge in the minority assigns to some judge in the minority, the task of writing a dissent. Having observed the two opinions, the judges then take a second dispositional vote, with the understanding that whichever opinion receives a majority will automatically become the opinion of the court.[23] We retain the baseline assumption that policy-making is purely consequentialist —opinion location matters only insofar as it affects the judges' actual policy utility. Thus, the role of the dissent is not as an expression of the minority's ideal rule, but as a competing potential majority opinion. The location of the dissent affects utility only if it succeeds in causing the disposition of the court to switch.[24]

For concreteness, suppose $x_{med} < z$, so that the median judge's ideal disposition is $d = 1$. Consider the $d = 0$ and $d = 1$ dispositional coalitions. The former must agree on an opinion $y_0 \geq z$ and the latter must write an opinion $y_1 \leq z$.

We briefly note some features of incentives in this new setting. First, every judge who voted sincerely would rather moderate their side's opinion to guarantee that they were in the eventual majority, than write an opinion that results in the eventual majority going to the other side. This should be intuitive; the most moderate opinion consistent with one's ideal disposition is preferred to any opinion that rationalizes the opposite disposition. Thus, in the competition over opinions, there is a strong force that pushes each coalition to moderate its opinion in order to win (or retain) a majority.

[23] In principle, if the dispositional coalitions change, we could allow for new majority and dissenting opinions to be drafted, and for this process to continue *ad infinitum*, until a pair of opinions arise for which the dispositional coalitions are stable. It suffices, however, in equilibrium, that there be a single additional round of dispositional voting.

[24] As we noted in footnote 16, in a dynamic model, there might be a role for a dissent that has no immediate policy consequence, but which sets the basis for a different policy to be adopted if the court revisits the issue in the future.

Second, since the support of the median judge is sufficient to win a majority, both sides will 'moderate' their opinions with a view to earning the support of the median judge. Notice that the $d = 1$ coalition has a distinct advantage in this regard. They can always offer the median judge her ideal policy $y_1 = x_{med}$, whereas dispositional consistency restricts the $d = 0$ coalition to at best offer $y_0 = z > x_{med}$. Thus, in equilibrium, the $d = 1$ coalition will always prevail —the disposition of the court will coincide with the ideal disposition of the median judge.

Third, although the majority opinion must be close to the median judge's ideal, it need not coincide with the median's ideal policy. A majority opinion is *incentive compatible* if it is weakly preferred by the median judge to the dissenting opinion. In equilibrium, the median judge must do at least as well by joining the $d = 1$ coalition, as if she joined a $d = 0$ coalition offering the most moderate policy satisfying dispositional consistency (i.e. when the dispositional consistency requirement is binding on the dissent). Let $\zeta(z)$ be the policy (with $\zeta(z) < x_{med} < z$) having the property that $u_P(\zeta(z), x_{med}) = u_P(z, x_{med}) + \alpha u_D(z, x_{med})$. The median judge would be indifferent between voting sincerely and endorsing opinion $\zeta(z)$, and voting strategically and endorsing opinion z (the most moderate policy that the rationalizes the opposite disposition). Any policy in the interval $[\zeta(z), z]$ is thus equilibrium incentive compatible for the $d = 1$ coalition.

Recall, $d(z)$ denotes the disposition of the court if all judges voted sincerely, and $M(z)$ denotes the majority coalition when there is sincere voting. By construction, the sincere disposition must coincide with the ideal disposition of the median judge. The above points, taken together, imply the following:

Proposition 4. *The game with competing opinions admits a unique CCPAE (d^*, M^*) satisfying:*

1. *The equilibrium disposition coincides with the sincere disposition, i.e. $d^* = d(z)$.*
2. *All judges who sincerely agree with the median will vote sincerely, while some judges who sincerely disagree may vote strategically, i.e. $M(z) \subseteq M^*$.*
3. *The policies proposed in the policy-making stage are given by a modified version of Proposition 1, in which proposals must additionally satisfy the incentive compatibility condition. (Formally, an*

equilibrium proposal y must satisfy: $u_P(y, x_{med}) \geq u_P(z, x_{med}) + \alpha u_D(z, x_{med})$.)

4. *The equilibrium is sustained by a (threatened) dissent,* $y_{diss} = z$.

A few comments are worth noting. First, we stress that most of the results from the baseline model continue to hold, even after adding competing dissents and allowing the composition of dispositional coalitions to change. The policy-making results (section 3) are qualitative unchanged, and require only a minor modification in the addition of the incentive compatibility constraint. Proposition 1, appropriately modified, will continue to imply that whenever $\delta < 1$, there will be range of potential majority opinions, reflecting a degree of agenda control by the opinion authors. Furthermore, per Proposition 2, as $\delta \to 1$, these opinions all converge to a unique policy that generically *does not* coincide with the median judge's ideal, and which is characterized by the (incentive compatibility constrained) Nash bargaining solution. Most of the results from section 4 (dispositional voting) also carry over, including Proposition 3 and Lemmas 2 and 3. The median judge remains dispositionally pivotal (although with competing dissents, she is guaranteed to vote sincerely), and judges whose ideal disposition coincides with the median judge's will always vote sincerely. Moreover, judges who sincerely disagree may vote strategically to participate in policy-making, and the likelihood of strategic voting decreases as expressive utility becomes more salient.

Equilibrium in the model with competing opinions differs from the baseline in two ways. First, at the adjudication stage, there is now a unique CCPAE, which renders the results in Corollary 1 moot. Moreover, the disposition in this unique CCPAE coincides with median judge's ideal. Second, at the policy-making stage, there is an additional constraint (incentive compatibility) that affects the set of profile of policies that may be offered in equilibrium. Indeed, adding this constraint is sufficient to cause the results of the baseline and variant models to coincide.

Second, we briefly note that the equilibrium does not require that a dissent actually be constructed as described —simply that the minority can credibly threaten to write such a policy (which they can).

Finally, the equilibrium with competing dissents may be thought of as a 'median voter theorem with frictions'. There is clearly a (Bertrand-competition-like) force that pushes the equilibrium policy closer to the median judge's ideal. However, the requirement that the dissent be dispositionally consistent, along with the expressive cost of voting insincerely, make the dissenting opinion an imperfect substitute to the majority opinion, from the perspective of the median judge. This allows other judges in the dispositional majority to pull the majority opinion slightly away from the median's ideal, subject to incentive compatibility. Hence, a range of majority opinions are equilibrium consistent and policy needn't converge all the way to the median's ideal.

B. PROOFS

Proof of Proposition 1. The proof is similar to that in Parameswaran and Murray (2019). Since u_P is non-concave, we must first establish that equilibria must be in no-delay pure strategies. Let

$$\begin{aligned} v_P(F(y); F(x)) &= u_P\left(F^{-1}(F(y)); F^{-1}(F(x))\right) = u_P(y, x) \\ &= -\left|\int_{F^{-1}(F(x))}^{F^{-1}(F(y))} l(z-x)dF(z)\right| \end{aligned}$$

be the policy utility after re-scaling the policy space. Notice that v_P is concave in $F(y)$:

$$\frac{\partial^2 v_P}{\partial F(y)^2} = -\left|l'(y-x)\cdot\frac{1}{f(F(y))}\right| < 0$$

Now, take any (possibly mixed) profile of strategies in the continuation game. Let $\sigma(y,t)$ be the implied distribution over outcomes, where $\sigma(y,t)$ is the probability that policy y is agreed to at time t. Let $\Delta u_P(y,x) = u_P(y,x) - u_P(D,x)$ be the utility gain over disagreement of policy y for a judge with ideal policy x. Similarly, define $\Delta v_P(F(y), F(x))$. Let $\hat{y}$ be the policy defined by: $F(\hat{y}) = \sum_{t=0}^{\infty}\int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t)\cdot\delta^t F(y)dy$.

Then, the judge i's continuation payoff (over disagreement) if the current proposal is rejected is:

$$
\begin{aligned}
\delta \Delta U(x^i) &= \delta \sum_{t=0}^{\infty} \int_{\underline{x}}^{\overline{x}} \sigma(y,t) \cdot \delta^t \Delta u_P(y, x^i) dy \\
&= \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right) \cdot \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \frac{\sigma(F(y),t) \cdot \delta^t}{\left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right)} \Delta v_P(F(y), F(x^i)) dy \\
&\leq \delta \left(\sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \right) \cdot \Delta v_P(F(\hat{y}), F(x^i)) \\
&< \Delta u_P(\hat{y}, x^i)
\end{aligned}
$$

where we use the facts that v_P is concave, and that $\delta \sum_{t=0}^{\infty} \int_{F(\underline{x})}^{F(\overline{x})} \sigma(F(y),t) \cdot \delta^t dy \leq \delta < 1$. Hence, there is a policy $\hat{y}$ that is strictly preferred by every judge to the continuation game. It is immediate, then, that there is a proposal for every judge that is socially acceptable and preferable to the continuation game. Moreover, since u_P is strictly quasi-concave, this policy is unique. Hence, every equilibrium must be in pure strategies and no-delay.

The acceptance set for any judge i is $A_i = \{y \in [\underline{x}, \overline{x}] \,|\, \Delta u_P(y, x^i) \geq \delta \Delta U(x^i)\}$. Since $u_P(y; x^i)$ is strictly quasi-concave in y, each individual acceptance set is an interval $A_i = [\underline{y}_i, \overline{y}_i]$. Let $C \subset \{1, ..., m\}$ be any coalition containing at least k members. Then, the coalitional acceptance set $A_C = \cap_{i \in C} A_i$ is also an interval. Moreover, since each A_i (and thus each A_C) contains $\hat{y}$, the social acceptance set $A = \cup_C A_C$ must be an interval as well. Denote $A = \left[\underline{y}, \overline{y}\right]$.

Given this social acceptance set, the optimal offers for each agent are:

$$
y_i = \begin{cases} \underline{y} & x^i \leq \underline{y} \\ x^i & x^i \in \left(\underline{y}, \overline{y}\right) \\ \overline{y} & x^i \geq \overline{y} \end{cases}
$$

For notational convenience, we often denote $u_P(y, x^i)$ by $u_i(y)$. For any $x \in X$, let $P(x) = \sum_{x_i \leq x} p_i$.

(The proof allows for p_i's to be different, although we typically focus on the case of $p_i = \frac{1}{m}$.) Then, given social acceptance set $\left[\underline{y}, \overline{y}\right]$, the expected utility of each judge i is:

$$U_i\left(\underline{y}, \overline{y}\right) = P\left(\underline{y}\right) u_i\left(\underline{y}\right) + \sum_{j:\, x^j \in \left(\underline{y}, \overline{y}\right)} p_j u_i\left(x^j\right) + \left(1 - P\left(\overline{y}\right)\right) u_i\left(\overline{y}\right)$$

The remainder of the proof proceeds in two steps. First, we show that in any equilibrium, $\underline{y} = \underline{y}_r$ and $\overline{y} = \overline{y}_l$. Next, using this fact, we show that the equilibrium is a fixed point of a mapping, and that the mapping admits a unique fixed point. This suffices to prove uniqueness of the equilibrium.

Step 1. For any player i, suppose $u_i\left(\underline{y}\right) \leq (1-\delta)\, u_i(D) + \delta U_i\left(\underline{y}, \overline{y}\right)$ — i.e. that $\Delta u_i(\underline{y}) < \delta \Delta U_i(\underline{y}, \overline{y})$. Since policy preferences satisfy the single crossing property, it must be that: $\Delta u_j\left(\underline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ for any j with $x^j > x^i$. To see this, suppose not; i.e. suppose $\Delta u_j\left(\underline{y}\right) \geq \delta U_j\left(\underline{y}, \overline{y}\right)$. Then:

$$\Delta u_i(\underline{y}) - \Delta u_j(\underline{y}) < \delta[\Delta U_i\left(\underline{y}, \overline{y}\right) - \Delta U_j\left(\underline{y}, \overline{y}\right)]$$

Recall, by the single crossing condition, that $x^i < x^j$ implies $\frac{\partial}{\partial y}(\Delta u_i - \Delta u_j) \leq 0$ (see footnote 8). Then:

$$\begin{aligned}
\Delta U_i\left(\underline{y}, \overline{y}\right) - \Delta U_j\left(\underline{y}, \overline{y}\right) &= P\left(\underline{y}\right)\left[\Delta u_i\left(\underline{y}\right) - \Delta u_j\left(\underline{y}\right)\right] + \sum_{j:\, x^j \in \left(\underline{y}, \overline{y}\right)} p_j \left[\Delta u_i\left(x^j\right) - \Delta u_j\left(x^j\right)\right] \\
&\quad + \left(1 - P\left(\overline{y}\right)\right)\left[\Delta u_i\left(\overline{y}\right) - \Delta u_j\left(\overline{y}\right)\right] \\
&\leq \Delta u_i\left(\underline{y}\right) - \Delta u_j\left(\underline{y}\right)
\end{aligned}$$

But by assumption, $\Delta u_i\left(\underline{y}\right) - \Delta u_j\left(\underline{y}\right) < \delta\left[\Delta U_i - {}_j\right] \leq \delta\left(\Delta u_i(\underline{y}) - \Delta u_j(\underline{y})\right)$, which is a contradiction. Hence, $\Delta u_i\left(\underline{y}\right) \leq \delta \Delta U_i\left(\underline{y}, \overline{y}\right)$ implies that $\Delta u_j\left(\underline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ whenever $x^j > x^i$. We can similarly show that $\Delta u_i\left(\overline{y}\right) \leq \delta \Delta U_i\left(\underline{y}, \overline{y}\right)$ implies $\Delta u_j\left(\overline{y}\right) < \delta \Delta U_j\left(\underline{y}, \overline{y}\right)$ whenever $x^j < x^i$.

Suppose $\underline{y} < \underline{y}_r$, then any proposal $y \in \left[\underline{y}, \underline{y}_r\right)$ will be rejected by agent r and all agents $j > r$. But since $r = k$, this implies that fewer than k agents will accept the proposal, which means it cannot be in the acceptance set. Hence $\underline{y} \geq \underline{y}_r$. Suppose $\underline{y} > \underline{y}_r$. Take any proposal $y \in \left(\underline{y}_r, \underline{y}\right)$. By construction $\Delta u_r(y) > \delta \Delta U\left[\underline{y}, \overline{y}\right]$, and so $u_j(y) > \delta \Delta U\left[\underline{y}, \overline{y}\right]$ for all agents $j < r$. But since $r = k$, this implies that at least k agents will accept proposal y. But this contradicts the assumption that y is outside the acceptance set. Hence $\underline{y} = \underline{y}_r$. We can similarly show that $\overline{y} = \overline{y}_l$.

Step 2. We now show that the equilibrium exists and is unique. For each i, define $\underline{\zeta}_i(z) = \min_{y \in X}\{y \leq x_i | \, \Delta u_i(y) \geq \delta \Delta U_i(y, z)\}$ and $\overline{\zeta}_i(z) = \max_{y \in X}\{y \geq x_i | \, \Delta u_i(y) \geq \delta \Delta U_i(z, y)\}$. Since u_i is continuous and X compact, then $\underline{\zeta}_i$ and $\overline{\zeta}_i$ are both continuous. Note also that:

$$\underline{\zeta}'_j(y) = \begin{cases} \frac{\delta(1-P(y))}{1-\delta P\left(\underline{\zeta}_j(y)\right)} \cdot \frac{u'_j(y)}{u'_j\left(\underline{\zeta}_j(y)\right)} & \underline{\zeta}_j(y) > \underline{x} \\ 0 & \underline{\zeta}_j(y) = \underline{x} \end{cases}$$

and:

$$\overline{\zeta}'_i(y) = \begin{cases} \frac{\delta P(y)}{1-\delta+\delta P\left(\overline{\zeta}_i(y)\right)} \cdot \frac{u'_i(y)}{u'_i\left(\overline{\zeta}_i(y)\right)} & \overline{\zeta}_i(y) < \overline{x} \\ 0 & \overline{\zeta}_i(y) = \overline{x} \end{cases}$$

By the previous step, we know that $\overline{y} = \overline{y}_l$ and $\underline{y} = \underline{y}_r$. Hence, $\overline{y} = \overline{\zeta}_l\left(\underline{y}\right)$ and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Let $H(y) = \overline{\zeta}_l\left(\underline{\zeta}_r(y)\right)$. H is continuous since $\underline{\zeta}_r$ and $\overline{\zeta}_l$ are both continuous. It follows that if $\left[\underline{y}, \overline{y}\right]$ is an equilibrium acceptance set, then $\overline{y}$ is a fixed point of H, and $\underline{y} = \underline{\zeta}_r(\overline{y})$. Since X is compact and H is continuous and onto X, it follows by Brouwer's fixed point theorem that H admits a fixed point $\overline{y}$. Hence, an equilibrium of the bargaining exists.

To establish that H has a unique fixed point, it suffices to show that $H'(\overline{y}) < 1$ for any $\overline{y}$ that is a fixed point. (If there exist multiple fixed points, then $H' \geq 1$ for at least one fixed point.) By construction:

$$H'(\overline{y}) = \begin{cases} A(\overline{y}) \cdot \frac{u'_l\left(\underline{y}\right)}{u'_l(\overline{y})} \cdot \frac{u'_r(\overline{y})}{u'_r\left(\underline{y}\right)} & \underline{x} < \underline{y} \leq \overline{y} < \overline{x} \\ 0 & \underline{y} = \underline{x} \text{ or } \overline{y} = \overline{x} \end{cases}$$

where $\underline{y} = \underline{\zeta}_r(\overline{y}) < \min\{x_r, \overline{y}\}$, and $A(y) = \frac{\delta P\left(\underline{\zeta}_r(\overline{y})\right)}{1-\delta+\delta P(\overline{y})} \frac{\delta(1-P(\overline{y}))}{1-\delta P\left(\underline{\zeta}_r(\overline{y})\right)} \in (0,1)$.

Suppose $H(\overline{y}) \geq 1$. Then at least one of $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})}\right| > 1$ or $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. There are several cases to consider. First, suppose $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. Since $\underline{y} < \min\{x_r, \overline{y}\}$ then $u_r'\left(\underline{y}\right) > 0$. If $\underline{y} \leq \overline{y} \leq x_r$, then $0 \leq u_r'(\overline{y}) \leq u_r'\left(\underline{y}\right)$, which contradicts $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| > 1$. Hence $\underline{y} < x_r < \overline{y}$, and so $u_r'(\overline{y}) < 0$. Suppose additionally $x_l \leq \underline{y} < \overline{y}$. Then $u_l'\left(\underline{y}\right) < 0$ and $u_l'(\overline{y}) < 0$. Hence $\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} < -1$, and $\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} > 0$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$, and so:

$$\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} \cdot \frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} = \frac{-l\left(\underline{y} - x_l\right)}{l(\overline{y} - x_l)} \cdot \frac{l(\overline{y} - x_r)}{-l\left(\underline{y} - x_r\right)} \leq 1$$

since $l(z)$ is weakly increasing for $z < 0$ and weakly decreasing for $z > 0$. Hence $H < 1$, which cannot be, and so $\left|\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)}\right| \leq 1$.

Next, suppose that $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})}\right| > 1$.Since $\overline{y} > \max\left\{x_l, \underline{y}\right\}$, then $u_l'(\overline{y}) < 0$. If $x_l \leq \underline{y} \leq \overline{y}$, then $u_l'(\overline{y}) \leq u_l\left(\underline{y}\right) \leq 0$, which contradicts that $\left|\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})}\right| > 1$. Hence $\underline{y} < x_l < \overline{y}$, and so $u_l'\left(\underline{y}\right) > 0$. Suppose additionally that $\underline{y} < \overline{y} \leq x_r$. Then $u_r'\left(\underline{y}\right) > 0$ and $u_r'(\overline{y}) > 0$. Hence $\frac{u_r'(\overline{y})}{u_r'\left(\underline{y}\right)} > 0$, and $\frac{u_l'\left(\underline{y}\right)}{u_l'(\overline{y})} < -1$, and so $H < 0$, which cannot be. Hence $\underline{y} < x_l \leq x_r < \overline{y}$. But we know that this implies $H < 1$, which also cannot be. Hence our initial supposition was wrong; $H'(\overline{y}) \ngeq 1$. Hence, $H' < 1$ and so H admits a unique fixed point. □

Proof of Lemma 1 . Recall, the acceptance set is $A = [\underline{y_r}, \overline{y_l}]$, where $\underline{y_r} = \min\{y \geq \underline{x} \,|\, \Delta u_r(y) \geq \delta \Delta U_r(y, \overline{y_l})\}$, and $\overline{y_l} = \max\{y \leq \overline{x} \,|\, \Delta u_l(y) \geq \delta \Delta U_l(\underline{y_r}, y)\}$. Now, by construction $\Delta u_l(\underline{y_r}) \geq \Delta u_l(\overline{y_l})$, since l will accept $\underline{y_r}$. Then, since u is strictly quasi-concave, $\Delta u_l(y) > \Delta u_l(\overline{y_l})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Similarly, $\Delta u_r(y) > \Delta u_r(\underline{y_r})$ for all $y \in (\underline{y_r}, \overline{y_l})$. Hence $\Delta U_l(\underline{y_r}, \overline{y_l}) > \Delta u_l(\overline{y_l})$ and

$\Delta U_r(\underline{y_r}, \overline{y_l}) > \Delta u_r(\underline{y_r})$ whenever $\underline{y_r} < \overline{y_l}$.

Now, for every $\delta < 1$, $\frac{\Delta u_l(\overline{y_l})}{\Delta U_l(\underline{y_r}, \overline{y_l})} = \delta = \frac{\Delta u_r(\underline{y_l})}{\Delta U_r(\underline{y_r}, \overline{y_l})}$, and so as $\delta \to 1$, we need $\Delta U_l(\underline{y_r}, \overline{y_l}) - \Delta u_l(\overline{y_l}) \to 0$ and $\Delta U_r(\underline{y_r}, \overline{y_l}) - \Delta u_r(\underline{y_r}) \to 0$. But this requires $\overline{y_l} - \underline{y_r} \to 0$. Hence $A = [\underline{y_r}, \overline{y_l}] \to [\mu, \mu]$ as $\delta \to 1$. □

Proof of Proposition 2 . Take any $i \in \{1, \dots, m\}$, and suppose $\mu \in \left(x^{i-1}, x^i\right)$. Then, by Lemma 1, there exists $\bar{\delta} < 1$ s.t. for $\delta > \bar{\delta}$, $x^{i-1} < \underline{y_r}(\delta) < \overline{y_l}(\delta) < x^i$. (For clarity, we make explicit the dependence of $\underline{y_r}$ and $\overline{y_l}$ on δ.) Then, by Proposition 1, all judges $j \in \{1, .., i-1\}$ will propose $\underline{y_r}$ and all judges $j \in \{i, .., n\}$ will propose $\overline{y_l}$. Again by Proposition 1, this implies that:

$$\Delta u_r\left(\underline{y_r}\right) = \delta\left[(1-P_i)\,\Delta u_r\left(\underline{y_r}\right) + P_i \Delta u_r\left(\overline{y_l}\right)\right] \tag{1}$$

$$\Delta u_l\left(\overline{y_l}\right) = \delta\left[(1-P_i)\,\Delta u_l\left(\underline{y_r}\right) + P_i \Delta u_l\left(\overline{y_l}\right)\right] \tag{2}$$

where $P_i = \sum_{j \geq i} p_j$. By the implicit function theorem, this system of equations pins down $\underline{y_r}$ and $\overline{y_l}$ in terms of the model parameters.

Now, let $E[y] = (1-P_i)\,\underline{y_r} + P_i \overline{y_l}$. Note, by construction, that $\underline{y_r} < E[y] < \overline{y_l}$. Then $\overline{y_l} - E[y] = \frac{1-P_i}{P_i}\left(E[y] - \underline{y_r}\right)$. We affect the following change of variables: Let $\varepsilon = E[y] - \underline{y_r}$. Then, we have: $\underline{y_r} = E[y] - \varepsilon$ and $\overline{y_l} = E[y] + \frac{1-P_i}{P_i}\varepsilon$. Equations (1) and (2) become:

$$(1 - \delta(1-P_i))\,\Delta u_r\left(E[y] - \varepsilon\right) = \delta P_i \Delta u_r\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) \tag{3}$$

$$(1 - \delta P_i)\,\Delta u_l\left(E[y] + \frac{1-P_i}{P_i}\varepsilon\right) = \delta(1-P_i)\,\Delta u_l\left(E[y] - \varepsilon\right) \tag{4}$$

By the implicit function theorem, and since u is continuously differentiable, we have:

$$\begin{bmatrix} (1-\delta(1-P_i))\,u_r'\left(\underline{y_r}\right) - \delta P_i u_r'\left(\overline{y_l}\right) & -(1-\delta(1-P_i))\,u_r'\left(\underline{y_r}\right) - \delta(1-P_i)\,u_r'\left(\overline{y_l}\right) \\ (1-\delta P_i)\,u_l'\left(\overline{y_l}\right) - \delta(1-P_i)\,u_l'\left(\underline{y_r}\right) & \left(\frac{1-P_i}{P_i} - \delta(1-P_i)\right) u_l'\left(\overline{y_l}\right) + \delta(1-P_i)\,u_l'\left(\underline{y_r}\right) \end{bmatrix} \begin{pmatrix} \frac{\partial E[y]}{\partial \delta} \\ \frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} =$$

$$\begin{pmatrix} (1-P_i)\,\Delta u_r\left(\underline{y_r}\right) + P_i \Delta u_r\left(\overline{y_l}\right) \\ P_i \Delta u_l\left(\overline{y_l}\right) + (1-P_i)\,\Delta u_l\left(\underline{y_r}\right) \end{pmatrix}$$

Taking limits as $\delta \longrightarrow 1$, we have:

$$\begin{bmatrix} 0 & -u_r'(\mu) \\ 0 & \frac{1-P_i}{P_i} u_l'(\mu) \end{bmatrix} \begin{pmatrix} \lim_{\delta\to 1} \frac{\partial E[y]}{\partial \delta} \\ \lim_{\delta \to 1} \frac{\partial \varepsilon}{\partial \delta} \end{pmatrix} = \begin{pmatrix} u_r(\mu) \\ u_l(\mu) \end{pmatrix}$$

These imply that:

$$\lim_{\delta\to 1} \frac{\partial \varepsilon}{\partial \delta} = -\frac{u_r(\mu)}{u_r'(\mu)} = \frac{P_i}{1-P_i}\frac{u_l(\mu)}{u_l'(\mu)}$$

The second equality provides an equation that uniquely defines the limit equilibrium.

Next, we note that equation defining μ_i coincides with the first order condition of the i^{th} Nash Bargaining problem. Recall, that problem was: $\max_{y\in X} (\Delta u_l(y))^{1-P_i} (\Delta u_r(y))^{P_i}$. Since utilities are concave (after rescaling the space), the maximizer must be the solution to the first order condition: $(1-P_i)\frac{u^l_{P,y}(b_{i-1,i})}{u^l_P(b_{i-1,i})} + P_i \frac{u^r_{P,y}(b_{i-1,i})}{u^r_P(b_{i-1,i})} = 0$. Re-arranging gives the desired result.

Notice that $b_{i-1,i}$ is increasing in P_i. (To see this, re-arrange the first order condition to give: $\frac{u_l'(b_{i-1,i})}{u_r'(b_{i-1,i})} \cdot \frac{u_r(b_{i-1,i})}{u_l(b_{i-1,i})} = -\frac{P_i}{1-P_i}$. We know that $b \in \left[x^l, x^r\right]$. By single-peakedness, over this region we know that $u_l(b)$ is strictly decreasing in b and $u_r(b)$ is strictly increasing in b, and so $\frac{u_r(b)}{u_l(b)}$ is strictly decreasing in b. Similarly, by concavity (after transformation) of u, $u_l'(b)$ is decreasing in b and $u_r'(b)$ is increasing in b, and so $\frac{u_r'(b)}{u_l'(b)}$ is weakly decreasing in b. Hence, the left hand term is strictly decreasing in b. The right hand term is also strictly decreasing in P. Hence, as P increases, so must b.) Then, since P_i is decreasing in i, it follows that $b_{i-1,i}$ is decreasing is as well.

Since we conjectured $\mu \in \left(x^{i-1}, x^i\right)$, then the limit equilibrium policy coincides with i^{th} Nash Bargaining solution provided that $x^{i-1} < b_{i-1,i} < x^i$. Now, since x^i is increasing and $b_{i-1,i}$ is decreasing in i, then by the definition of i^*, $x^i < b_{i,i+1}$ for all $i < i^*$ and $x^i \geq b_{i,i+1}$ for all $i \geq i^*$. Moreover, for $i < i^*$, $x^{i-1} \leq x^i < b_{i,i+1} \leq b_{i-1,i}$, which is inconsistent. Similarly, for $i > i^*$, $b_{i-1,i} \leq x^{i-1} \leq x^i$, which is inconsistent. Hence, if $b_{i-1,i} \in \left(x^{i-1}, x^i\right)$, then $i = i^*$. Note however, that the converse need not be true. Setting $i = i^*$ gives two possibilities: (i) $x^{i^*-1} < b_{i^*-1,i^*} < x^{i^*}$, or (ii) $x^{i^*-1} \leq x^{i^*} \leq b_{i^*-1,i^*}$ (with at least one inequality strict). The former case is equilibrium consistent, and since the equilibrium is unique, we have $\mu = b_{i^*-1,i^*}$.

Suppose the latter case prevails. It follows that the limit equilibrium is not contained in any of the open intervals $\left\{(x^{i-1}, x^i)\right\}_{i=1}^{m-1}$, and so $\mu \in \left\{x^1, ..., x^m\right\}$. (In fact, since $\underline{y_r} < x^r$ and $\overline{y_l} > x^l$ for all δ, and since $\lim_{\delta\to 1} \underline{y_r} = \mu = \lim_{\delta\to 1} \overline{y_l}$, then $x^l \leq \mu \leq x^r$, and so $\mu \in \left\{x^l, ..., x^r\right\}$.) Suppose $\mu = x^i$ for some $i \in \{l, ..., r\}$. Let $I = \{j | x^j = x^i\}$ and denote $I = \{i^-, ..., i^+\}$, where $i^- \leq j \leq i^+$ for all $j \in I$. (Obviously, I may be a singleton, in which case $i^- = i = i^+$.) Let $\Pi_i^- = \sum_{j<i^-} p_j$ and $\Pi_i^+ = \sum_{j>i^+} p_j$ and $\Pi_i = \sum_{j\in I} p_j$. Then, for δ sufficiently large, (1) becomes:

$$u_r(\underline{y}_r) = \delta\left[\Pi_i^- u_r(\underline{y}_r) + \Pi_i u_r(x^i) + \Pi_i^+ u_r(\overline{y}^l)\right]$$

Since $\underline{y}_r < x^i < \overline{y}^l$, there exists $\tau \in (0, 1)$ s.t. $x^i = \tau\underline{y}^r + (1 - \tau)\overline{y}^l$. We can write (1) as:

$$\begin{aligned} u_r(\underline{y}^r) = \delta &\left[(\Pi_i^- + \Pi_i\tau)u_r(\underline{y}^r) + (\Pi_i^+ + \Pi_i(1-\tau))u_r(\overline{y}_l)\right] \\ &+ \delta\left[\Pi_i\tau(u_r(\underline{y}_r) - u_r(x^i)) + \Pi_i(1-\tau)(u_r(\overline{y}^l) - u_r(x^i))\right] \end{aligned} \tag{5}$$

Notice (5) is the sum of two terms, with the first term being analogous to the expression in (1), and the second term being a 'correction' term.

We repeat the procedure for equation (2), and then apply the change of basis method above, and take limits as $\delta \to 1$. Since $\underline{y}^r \to x^i$ and $\overline{y}^l \to x^i$, the 'correction' term in (5) goes to zero. It follows that $\mu = b(\rho^*)$, where $\rho^* = \Pi_i^+ + \Pi_i(1 - \lim_{\delta\to 1} \tau(\delta))$. Now, there must be some k s.t. $b_{k,k+1} < b(\rho^*) = x^i < b_{k-1,k}$. Moreover, it must be that $k \in I$, since $b_{i^+,i^++1} < b(\rho^*) < b_{i^--2,i^--1}$, by construction. But then, we can choose i appropriately s.t. $b_{i,i+1} < x^i < b_{i-1,i}$. But this requires $i = i^*$. $\square$

Proof of Lemma 2. Let z be an arbitrary case. Suppose $d^* = 0$. (The other scenario is analogous.)

Recall $M^0 = \{j | x^j > z\}$. Moreover, all feasible second stage policies must satisfy $y \geq z$. Suppose there is a j, such that $j \in M^0$ and $j \notin M^*$. Then the payoff to j of choosing $d = 1$ must exceed that of choosing $d = 1$, which implies:

$$\left[u_P(\gamma(M), x^j) - u_P\left(\gamma(M \cup \{j\}), x^j\right)\right] + \alpha l(z - x^j) > 0$$

By assumption 1, the term in square brackets is non-positive, since joining the coalition cannot make the policy worse from j's perspective. Moreover, the second term is negative by construction. Hence the LHS is negative, which is a contradiction. Hence $j \in M^*$. □

Lemma 4. *Let (d, M) and (d, M') both be adjudication (Nash) equilibria, and suppose $M \subset M'$. Then (d, M) is not coalition-proof.*

Proof of Lemma 4. Suppose (d, M) and (d, M') are both adjudication (Nash) equilibria, with $M \subset M'$. Since M and M' are both equilibrium coalitions, it (generically) must be that $|M'| \geq |M+2|$, where $|X|$ denotes the cardinality of set X. (To see this, note that if $M' = M \cup \{i\}$ where $i \notin M$, then it must be that judge i is exactly indifferent between joining the majority coalition or not; otherwise, i would have a strictly improving unilateral deviation. This indifference is non-generic and requires an exact alignment of the case, the equilibrium policies chosen by the respective coalitions, and the salience parameter α.)

Note by Lemma 2 that $M^d(z) \subseteq M \subset M'$. WLOG, suppose $d = 1$. Then, by part 1 of Assumption 1, $\gamma(M) \leq \gamma(M' \setminus \{j\}) \leq \gamma(M')$ for every $j \in M' \setminus M$, since $M \subset M' \setminus \{j\}$. Moreover, for all $j \in M' \setminus M$, $\gamma(M) \leq \gamma(M') \leq z < x^j$. Now, since M' is a Nash equilibrium coalition, then $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M' \setminus \{j\}), x^j)$ for each $j \in M' \setminus M$, and given the above ordering, we know that $u_P(\gamma(M' \setminus \{j\}), x^j) \geq u_P(\gamma(M), x^j)$. Hence $u_P(\gamma(M'), x^j) + \alpha l(z, x^j) \geq u_P(\gamma(M), x^j)$ for all $j \in M' \setminus M$, and this inequality will generically be strict for some j. Hence, the joint deviation from M to M' is Pareto improvement within the deviating coalition.

We must also show that this deviation is stable. Suppose not. Then there exists a (strict) sub-coalition

$C \subset M' \setminus M$ that would deviate back to voting sincerely. It must be that C contains at least two judges, since otherwise it is a unilateral deviation, which cannot be, since M' is a Nash equilibrium coalition. (This implies that $M' \setminus M$ contains at least 3 judges.) Take some $k \in C$. By construction, $M' \setminus C \subset M' \setminus \{k\} \subset M'$, and so $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) \leq \gamma(M')$. Since the deviation from the deviation is profitable, we have: $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M', x^k)) + \alpha l(z, x^k) \geq u_P(\gamma(M' \setminus \{k\}), x^k)$, where the second inequality follows from the fact that M' is an equilibrium coalition. Hence $u_P(\gamma(M' \setminus C), x^k) > u_P(\gamma(M' \setminus \{k\}), x^k)$, which cannot be since $\gamma(M' \setminus C) \leq \gamma(M' \setminus \{k\}) < x^k$. Hence, the deviation is stable. □

Lemma 5. *Let (d, M) be an adjudication (Nash) equilibrium. There exists a connected coalition M' with $|M'| = |M|$ and such that (d, M') is also an adjudication (Nash) equilibrium coalition. Moreover, (d, M') can be sustained as an adjudication equilibrium over a (weakly) larger range of values of α than (d, M).*

Proof of Lemma 5. Let (d, M) be an adjudication (Nash) equilibrium, and suppose M is not connected. WLOG, suppose $d = 1$, so that, by Lemma 2, $M^1(z) \subset M$. Since M^1 is a connected coalition and M is disconnected, M must contain members of $M^0(z)$. Then there exists $i < j$ with $i, j \in M^0(z)$, $i \notin M$ and $j \in M$. Then $z < x^i \leq x^j$. Let M' be identical to M except that judge j is replaced by judge i. By part 2 of Assumption 1, it must be that $\gamma(M) = \gamma(M')$. (To see this, note that replacing judge i with j causes the social acceptance set to be unchanged, since both judges will make the same proposal $\overline{y}$.) Since M is an equilibrium, it must be that:

$$u_P(\gamma(M), x^j) + \alpha l(z - x^j) \geq u_P(\gamma(M - \{j\}), x^j) \tag{6}$$

and:

$$u_P(\gamma(M \cup \{i\}), x^i) + \alpha l(z - x^i) < u_P(\gamma(M), x^i) \tag{7}$$

We seek to show that M' is also an equilibrium coalition. It suffices to show that:

$$u_P(\gamma(M'), x^i) + \alpha l(z - x^i) \geq u_P(\gamma(M' - \{i\}), x^i) \tag{8}$$

and:

$$u_P(\gamma(M' \cup \{j\}), x^j) + \alpha l(z - x^j) < u_P(\gamma(M'), x^j) \tag{9}$$

If $x^i = x^j$, it is trivial to do so, since i and j have identical preferences. Suppose $x^i < x^j$. Note that:

$$\begin{aligned}
&\left\{\left[u_P(\gamma(M), x^j) - u_P(\gamma(M - \{j\}), x^j)\right] + \alpha l(z - x^j)\right\} - \left\{\left[u_P(\gamma(M'), x^i) - u_P(\gamma(M' - \{i\}), x^i)\right] + \alpha l(z - x^i)\right\} \\
=&\left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^j)dy + \alpha l(z - x^j)\right) - \left(\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x^i)dy + \alpha l(z - x^i)\right) \\
=&\int_{x^i}^{x^j} \frac{\partial}{\partial x}\left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l(y - x)dy + \alpha l(z - x)\right] dx \\
=&-\int_{x^i}^{x^j}\left[\int_{\gamma(M-\{j\})}^{\gamma(M)} l'(y - x)dy + \alpha l'(z - x)\right] dx \\
\leq& 0
\end{aligned}$$

where the final line follows from the fact that $\gamma(M - \{j\}) < \gamma(M) \leq z < x^i < x^j$ and that, by the IDID property, $l'(y - x) > 0$ for all $y < x$. It follows that (6) implies (8). By a similar argument, we can show that (7) implies (9). Hence, M' is an equilibrium coalition as well.

Moreover, if $x^i < x^j$, then the inequality above is strict, and continues to be so for some $\alpha' > \alpha$ and even for some $\gamma(M') < \gamma(M)$. □

Proof of Proposition 3. The existence of an adjudication (Nash) equilibrium follows by standard game theoretic results. We now establish the existence of a CCPAE. Let (d_0, M_0) be an adjudication (Nash) equilibrium, and suppose it is a candidate to be a CCPAE. By Lemma 4, we know that there is no larger adjudication equilibrium with the same case disposition (i.e. there is no M' with $M_0 \subset M'$ s.t. (d_0, M') is an adjudication equilibrium). If (d_0, M_0) is not a CCPAE, then there must exist some other coalition C_0 and induced disposition d'_0 s.t. all the members of $M_0 \cap C_0$ prefer to deviate from (d_0, M_0) to (d'_0, C_0). Moreover, no subset of the deviators $M_0 \cap C_0$ should have a strict incentive to deviate from C_0. Immediately, this implies that (d'_0, C_0) is an adjudication (Nash) equilibrium.

By construction, it cannot be that $d'_0 = d_0$, since any smaller coalition inducing the same case disposition must be inferior for the deviating judges (by Lemma 4). Hence $d'_0 = 1 - d_0$. Using the same logic as in Lemma 5, if C_0 is disconnected, we can always find some other coalition C'_0 that is connected and which implies a strictly favorable deviation for the judges in $M_0 \cap C'_0$. Hence, it is WLOG to focus on deviations by connected coalitions. Hence (d_1, C_0) is a connected adjudication (Nash) equilibrium, where $d_1 = 1 - d_0$. Let (d_1, M_1) be the largest connected coalition that implements case disposition $d_1 = 1 - d_0$. Clearly $C_0 \subseteq M_1$. (d_1, M_1) is the only other candidate for a CCPAE. Suppose it is not. Then, by the same argument, there must be some connected $C_1 \subseteq M_0$, s.t. (d_0, C_1) is preferred by all judges in the deviating coalition $M_1 \cap C_1$, and this deviating coalition is stable.

Since each deviation flips the case disposition, and coalitions are connected, then the median judge must be a member of the deviating coalition in each case. WLOG, suppose $d_0 = 0$ and $d_1 = 1$. We have:

$$u_P(\gamma(C_0), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med}) > u_P(\gamma(M_0), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med}) \quad (10)$$

and

$$u_P(\gamma(C_1), x^{med}) + \mathbf{1}[z > x^{med}]l(z - x^{med}) > u_P(\gamma(M_1), x^{med}) + \mathbf{1}[z < x^{med}]l(z - x^{med}) \quad (11)$$

Suppose $x^{med} < z$. By assumption 1, $\gamma(C_0) \leq \gamma(M_1) \leq z \leq \gamma(M_0) \leq \gamma(C_1)$. It cannot be that $x^{med} \leq \gamma(M_1)$, otherwise $u_P(\gamma(M_1), x^{med}) > u_P(\gamma(C_1), x^{med})$, which contradicts (10). Hence: $\gamma(C_0) \leq \gamma(M_1) < x^{med} < z \leq \gamma(M_0) \leq \gamma(C_1)$. But then, by the strict quasi-concavity of u_P, $u_P(\gamma(M_0), x^{med}) \geq u_P(\gamma(C_1), x^{med}) > u_P(\gamma(M_1), x^{med}) \geq u_P(\gamma(C_0), x^{med})$. But (10) implies that

$u_P(\gamma(C_0), x^{med}) > u_P(\gamma(M_0), x^{med})$. We have a contradiction. By a symmetric argument, we can show that a contradiction arises in the scenario that $x^{med} > z$. Hence, it cannot be that both (d_0, M_0) and (d_1, M_1) are both *not* CCPAE. Existence is established.

Establishing the equilibrium properties is straight-forward. Fix a case z. Suppose (d, M) is a CCPAE. By Lemma 2, $M^d \in M$. Suppose $M^d \neq \emptyset$. Then, by the ordering over judges, $1 \in M^d$ if $d = 1$ and $n \in M^d$ if $d = 0$. Since M is connected and contains at least $k = \frac{n+1}{2}$ agents, then $\frac{n+1}{2} \in M$. Hence either $\{1, ..., \frac{n+1}{2}\} \subset M$ or $\{\frac{n+1}{2}, ..., n\} \subset M$. (If $M^d = \emptyset$, then the result follows provided that we rule out equilibria that relies upon a majority of judges voting strategically, but not those judges with the lowest cost of doing so.) $\square$

Proof of Corollary 1. To show part (1), let (d, M) and (d', M') be distinct CCPAE, and suppose that $d = d'$. Then, by Lemma 2, $M^d(z) \subset M$ and $M^{d'}(z) \subset M'$. Since M and M' are connected, this implies (WLOG) that $M \subset M'$. But then, by Lemma 4, M cannot be coalition-proof, which is a contradiction. Hence, $d \neq d'$. Since distinct CCPAE must have distinct dispositions, and there are only two possible dispositional values, then there can be at most two CCPAE.

Next, we establish part (2). Fix some case z. For $j = \{1, .., \frac{n-1}{2}\}$, define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{j+1, ..., n\}), x^j) - u_P(\gamma(\{j, ..., n\}), x^j)}{l(z - x^j)}$$

If $x^j < z$, so that j's ideal disposition is $d = 1$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the left-most judge who votes strategically. Similarly, for $j = \{\frac{n+3}{2}, ..., n\}$ define:

$$\alpha_j(z) = \frac{u_P(\gamma(\{1, , ..., j-1\}), x^j) - u_P(\gamma(\{1, ..., j\}), x^j)}{l(z - x^j)}$$

If $x^j > z$, so that j's ideal disposition $d = 0$, then whenever $\alpha > \alpha_j(z)$, there cannot be an adjudication equilibrium in which j is the right-most judge who votes strategically. Finally, define:

$$\alpha_{\frac{n+1}{2}}(z) = \frac{u_P(\gamma(\{1, ..., \frac{n+1}{2}\}), x^{med}) - u_P(\gamma(\{\frac{n+1}{2}, ..., n\}), x^{med})}{l(z - x^{med}})$$

Recall $M^1(z)$ and $M^0(z)$ are the coalitions that arise if judges vote sincerely. Since n is odd, one of these will be larger than the other. We refer to the larger coalition as the 'sincere majority coalition' and the smaller coalition as the 'sincere minority coalition'.

We consider two scenarios. First, suppose $|M^1(z)| - |M^0(z)| \geq 2$. This implies that if judges vote sincerely, the size of the majority and minority coalitions will differ by at least two. Then, for all $\alpha \geq 0$, there exists an adjudication (Nash) equilibrium in which all members of the sincere majority coalition vote sincerely. (To see why, note that if all judges in the sincere majority coalition vote sincerely, then no judge is pivotal over the case disposition. The result is then an immediate consequence of Lemma 2. Note, of course, that judges in the sincere minority might nevertheless have an incentive to vote strategically.)

We show that, for α sufficiently large, there cannot be an adjudication (Nash) equilibrium which implements the opposite disposition. Suppose there is. By Lemma 5, we know that it suffices to focus on connected equilibria. Suppose $M^1(z) > M^0(z) + 1$, so that the sincere disposition is $d = 1$. The connected majority coalitions that implement the opposite disposition ($d = 0$) and satisfy Lemma 2 are of the form: $\{j, , , n\}$, where $j \in \{1, .., \frac{n+1}{2}\} \subseteq M^1(z)$. Define $\alpha(z) = \max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}$. By construction, if $\alpha > \alpha(z)$, then none of these coalitions is consistent with an adjudication equilibrium. Hence, if $\alpha > \alpha(z)$, there cannot be any adjudication equilibria that implement the sincere minority's preferred disposition. Hence, any adjudication equilibrium must implement the sincere majority's preferred disposition. By previous arguments, there is a unique CCPAE that achieves this.

Suppose instead that $M^0(z) > M^1(z) + 1$, so that the sincere disposition is $d = 0$. Then the result obtains by defining $\alpha(z) = \max\{\alpha_{\frac{n+1}{2}}, ..., n\}$.

Next, consider the scenario where $|M^1(z) - M^0(z)| = 1$, so that, if all judges vote sincerely, the median is pivotal. This scenario differs from the previous one only insofar as the median judge may have an incentive to vote strategically for α low enough, even if all other judges in the sincere majority vote sincerely. Again, first suppose that $x^{med} < z$, so that the sincere disposition is $d = 1$. Define:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n+1}{2}}\}, \max\{\alpha_{\frac{n+3}{2}}, ..., \alpha_n\}\right\}$$

Following the same logic, there is a unique equilibrium provided that $\alpha > \alpha(z)$. Supposing instead that $x^{med} > z$, then the result obtains by defining:

$$\alpha(z) = \min\left\{\max\{\alpha_1, ..., \alpha_{\frac{n-1}{2}}\}, \max\{\alpha_{\frac{n+1}{2}}, ..., \alpha_n\}\right\}$$

□

Proof of Lemma 3. Follows immediately from the proofs of Proposition 3 and Corollary 1. □