Online Appendix

For "A Logical Model for Predicting Minority Representation: Application to Redistricting and Voting Rights Cases"

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A Derivation of the Logical Model

This section presents the derivation of the logical model of minority candidate emergence:

$$P_{\text{run}} = \widehat{P}_{\text{win}} = \Phi\big(\tfrac{1}{2}(\widehat{V}_t^M - \widehat{V}_t^W)\big) = \Phi\big((MC)^{1/2} - 50\big).$$

In addition to the model derivation, this section proves the universality of the racial margin of victory in biracial elections.

A.1 Step I (Candidate Entry)

The first step of our model derivation is to show that the probability of minority candidate entry can be modeled as the probability of winning for minority candidates: $P_{\text{run}} = \widehat{P}_{\text{win}}$.

To do so, I fist consider a rational choice model of candidate emergence. Following the literature of political ambition (Black, 1972; Lazarus, 2008; Aldrich, 1995; Jacobson and Kernell, 1983; Stone, Maisel and Maestas, 2004), I apply the following model of strategic candidate entry to minority candidate emergence:

$$u_{it} = \widehat{P}_{it} B_{it} - C_{it} \tag{1}$$

Here, u_{it} is the utility that candidate i obtains from running for office, $\widehat{P}_{it}$ is the candidate's estimate of the probability of winning if she enters the race, B_{it} is the benefits of holding the office, and C_{it} is the cost of running, which is greater than zero, all at time t.[1] Black (1972) suggests that a rational office-seeking candidate enters the race if and only if $\widehat{P}_{it} B_{it} > C_{it} > 0$ and $u_{it} > u_{it}(A)$, where $u_{it}(A)$ expresses the utility that the candidate receives when staying out.[2]

While this model looks fairly simple at first glance, it is in fact a very complicated model where four variables and three inequalities collectively determine the outcome. As we often prefer parsimonious models to complex models in logical model building (Taagepera, 2007, 2008; Shugart and Taagepera, 2017), here, I only focus on the win probability term by taking other terms out of the equation.[3] Consequently, I directly model the probability of entry $P_{\text{run},it}$ as the perceived probability of winning:

$$P_{\text{run},it} = \widehat{P}_{it} \tag{2}$$

To simplify the notation and facilitate an intuitive interpretation, I suppress the indices and add a subscript in the right hand side:

$$P_{\text{run}} = \widehat{P}_{\text{win}} \tag{3}$$

Substantively, this implies that *individual-level* minority candidate emergence can be predicted by knowing about $\widehat{P}_{\text{win}}$, that is, how likely it is for minority candidates to win when they enter the race. By Assumption 4, I then model that *district-level* minority candidate emergence can be predicted by $\widehat{P}_{\text{win}}$ for the most

[1] In contrast to previous studies, I use the notation $\widehat{P}_{it}$ (as opposed to P_{it}) to make it clear that it is an estimate of the true probability of winning.

[2] Lazarus (2008) discusses that $u_{it}(A)$ is usually very different for experienced politicians and amateurs, which affect the required level of $\widehat{P}_{it}$ in the model; however, I do not distinguish the two types of politicians here.

[3] The resulting model is also obtained by assuming that $B_{it} = 1, C_{it} = 0.5$, and $u_{it}(A) \sim N(0, \sigma^2)$, where $\sigma^2 > 0$. Under these assumptions, the original model tells that minority candidates run if and only if $\widehat{P}_{it} > 0.5 + u_{it}(A)$. Since 0.5 and u_{it} are constant and random numbers, we can say that the probability of entry is a function of the probability of winning as $\Pr(E_{it}) = \widehat{P}_{it} + u'_{it}$, where u'_{it} combines the constant and the error term. If we ignore the random component, we obtain Equation (2). More specifically, Equation (2) is $P_{\text{run},it} = \mathbb{E}[\widehat{P}_{it}]$. While the model can be presented both in a deterministic form (without the error term) and stochastic form (with the error term), I choose to use the deterministic form due to its simplicity.

viable minority candidate in a given district.

A.2 Step II (Racial Margin of Victory)

The second step of our derivation is to logically model the probability of winning. To achieve this goal, I start by considering *what the logical consequence would be* if minority candidates know exactly how upcoming elections results will look like. Suppose that there will be a single viable minority and white candidates, respectively. Then, if the minority candidate knows that she can earn one more vote than her white counterpart, she knows that the probability of winning is 1 (i.e., she will win with certainty). Otherwise, she knows that the probability of winning is 0 (i.e., she will lose with certainty).

To formally express this idea, I introduce a new concept of *racial margin of victory*. The racial margin of victory is based on a well-established notion of *margin of victory*. The general definition of the margin of victory is as follows:

Definition 1 (MARGIN OF VICTORY): *A margin of victory is the minimum number of vote shares that have to be modified in order to change the outcome (i.e., winner) of an election (Xia, 2012).*[4]

In first-past-the-post (FPTP) elections, the margin of victory is computed as half the difference in vote shares between the winner and the runner-up. That is,

$$\text{Margin of Victory}_{\text{FPTP}} \equiv \tfrac{1}{2}\left(\left|V^{\text{Winner}} - V^{\text{Runner-up}}\right|\right), \tag{4}$$

where V^{Winner} and $V^{\text{Runner-up}}$ represent the vote shares for the winner and runner-up (as in the most and second most vote-earning candidates), respectively, and $|\cdot|$ is an absolute value operator.

For example, when the winner received 60% and the runner-up earned 40% of the total ballots, the margin of victory is $\frac{1}{2}(|60 - 40|) = \frac{1}{2}(|20|) = 10$. This means that if I remove 10% of the total ballots from the winner and re-allocate them to the runner-up, the election will be a tie (50% v. 50%) and thus such re-allocation changes the outcome of the election.[5]

Building on this concept, I then define the *racial* margin of victory as follows:

Definition 2 (RACIAL MARGIN OF VICTORY): *A racial margin of victory is the minimum number of vote shares that have to be modified in order to change the* race *of the winner of an election.*

In FPTP elections, the racial margin of victory (assuming no ties and rounding the difference) is computed as the half the difference between the vote shares of the most vote-earning minority and white candidates. That is,

$$\text{Racial Margin of Victory}_{\text{FPTP}} \equiv \tfrac{1}{2}\left(\left|V^{M} - V^{W}\right|\right), \tag{5}$$

[4] More precisely, Xia (2012, 986) defines that "[g]iven a voting rule r and a profile P, the margin of victory (MoV) of P, denoted by Mov(P,r), is the small number k such that the set of winners can be changed by changing k votes in P, while keeping the other votes unchanged." In this article, I represent the margin of victory using the vote shares instead of raw vote counts for conceptual simplicity and generalizability.

[5] To compute the "tie-free" margin, one can simply add 1 to the above number (Xia, 2012; Magrino et al., 2011), while I assume that there is always a tie-breaker in my argument as discussed above.

where V^M represents the share of **v**otes received by the top **m**inort candidate (in percentage), V^W represents the share of **v**otes received by the top **w**hite candidate. To see the logic behind this, consider that the winner in our last example is a minority and the runner-up is white. Then, changing the winner necessarily changes the race of the winner. For proof, see Online Appendix A.4. Substantively, the racial margin of victory represents *how close an electoral contest was between the most viable minority and white candidates*.

An important property of the racial margin of victory is that it does not distinguish the direction of the change in the race of the winner. In other words, while the racial margin of victory tells how close the contest was it does not tell which group ultimately won. Since we want to quantify whether the most viable minority candidate earns one more vote than her white counterpart, we need to take one more step to modify the concept:

Definition 3 (SIGNED RACIAL MARGIN OF VICTORY): *A signed racial margin of victory is the minimum number of vote shares that are required to change the minority winner to the white winner*

In FPTP, the signed racial margin of victory becomes:

$$\text{Signed Racial Margin of Victory}_{\text{FPTP}} \equiv \frac{1}{2}\big(V^M - V^W\big) \tag{6}$$

Substantively, this quantity shows the *extent to which minority candidates safely secure their descriptive representation relative to their white counterparts*. While the technical distinction between the signed and unsigned racial margin of victories is important, I hereafter (and throughout the main text) use the term "margin of victory" to refer to the *signed* racial margin of victory for notational simplicity. Because $V^M \in (0, 100)$ and $V^W \in (0, 100)$, the logical range for the signed racial margin of victory is between -50 and 50: $\frac{1}{2}\big(V^M - V^W\big) \in (-50, 50)$. Technically, this can be in a closed set $[-50, 50]$; however, for conceptual simplicity, I consider that *all quantities derived in this article are elements of closed sets*.

Now, how is the (signed) racial margin of victory tied back to the argument of minority candidate entry? The idea is here that, under Assumptions 1-4, the probability of winning can be expressed as a deterministic function of the racial margin of victory. Thus, if minority candidates are fully short-term instrumentally rational with complete information, the probability of winning becomes a function of the vote shares of the top minority and white candidates in *upcoming* elections (at time t):

$$\widehat{P}_{t\text{win}} = \mathbb{1}\Big(\frac{1}{2}\big(V_t^M - V_t^W\big) > 0\Big), \tag{7}$$

where $\mathbb{1}$ is an indicator function that takes 1 if the inside condition is satisfied and 0 otherwise. To emphasize the idea that both quantities are unknown future vote shares, I put back the index t.

Equation 7 means that the probability of winning for the most viable black candidate is 1 (i.e., winning with certainty) if the top minority candidate (herself) obtains higher vote shares (or one more vote) than the top white candidate and 0 (i.e., losing with certainty) otherwise.[6] Hence, if such future vote shares are *ex-ante* known, the probability of winning is a deterministic function of the (signed) racial margin of victory, and the rational minority candidate decides to run (or not to run) for office with certainty.[7] This is

[6] Here, I assume that there is always a *tie-breaker* and do not consider the case where the two quantities are the same.

[7] Note that the argument presented so far is not surprising at all; rather, it simply clarifies a well known rational choice-type explanation that candidates decide to run for office when they see higher chances of winning. And if they can foresee their electoral fortune, they will either enter the race or abstain. However, a logical model building requires a clear definition and measurement of each concept and this section serves this purpose by connecting minority candidate emergence with a well-discussed notion of the margin of victory in political science.

the argument that I laid out at the beginning of this subsection.

Although the above argument makes *logical* sense, in reality, it is impossible for any candidate to perfectly calculate future vote shares due to many reasons, including the cognitive burden, lack of enough information, time restriction, and fundamental uncertainty around voters' behaviors. To get around this theoretical problem, I then theorize that the most viable minority candidate can calculate the probability of winning is based on some educated guess about future vote shares. To formalize this idea, I consider the following *estimated* probability of winning:

$$\widehat{P}_{t\text{win}} = \Phi\Big(\tfrac{1}{2}\big(\widehat{V}_t^M - \widehat{V}_t^W\big)\Big), \tag{8}$$

where the additional "hat symbols" ($\widehat{\cdot}$) imply that the vote shares inside the (signed) racial margin of victory are now *estimated* quantities as opposed to known values. Here, Φ is a Cumulative Distribution Function (CDF) of the standard normal distribution, which returns 0.5 or increasingly higher value when the (signed) racial margin of victory is positive and 0.5 or increasingly lower value when it is negative.

The key idea here is that since these vote shares are estimated quantities they can be far from the true (unknown future) vote shares. To account for this uncertainty, the standard normal CDF replaced the indicator function in the last equation. While a cumulative distribution function (CDF) of any *elliptical* distribution (e.g., symmetrically spiked distributions such as normal and t-distributions) which maps the input values onto the probabilistic space (values between 0 and 1) can be selected, I choose the standard normal CDF for several reasons. First, I choose the normal standard CDF because it is well known (also known as the probit function) in the discipline, and choosing a well-known function allows readers to understand the model easily. Second, I choose the standard normal CDF because it is centered around 0 and is consistent with the theoretical argument made in Equation 7. Third, I choose the standard normal CDF because its variance is relatively small ($\sigma = 1$) and it properly models the uncertainty around some educated guess about future vote shares. I assume that even if there would be some miscalculation the degree of miscalculation is not as big as 10 or larger. The standard normal CDF returns a value that is very close to 0 when the input value is less than about -4 and returns a value that is very close to 1 when the input is greater than about 4. This property fits my assumption about the degree of miscalculation and thus the CDF is an appropriate choice for my theoretical argument.

A.3 Step III (Two Logical Bounds)

Finally, how does the most viable minority candidate make educated guesses about the unknown future vote shares? I theorize that she can estimate the future racial margin of victory by considering two logical bounds or limit cases for that quantity. One limit scenario is that the future (unknown) racial margin of victory will converge to the (known) margin of victory in the most recent election:

$$M_0 = \tfrac{1}{2}\big(V_{t-1}^M - V_{t-1}^W\big), \tag{9}$$

where $t-1$ signifies that M_0 is the (signed) racial margin of victory in the most recent election. Importantly, it does not have any hat symbol because these vote shares are observed quantities instead of estimated vote shares. Naturally, the logical range of this quantity is between -50 and 50: $M_0 \in (-50, 50)$.

The other extreme case is that it converges to the *expected* margin of victory based on the district's racial composition, which I denote by C_0. Suppose that there are a single minority and white candidates, respectively. Suppose also that minority and white voters turnout at the same rate and they fully concentrate their votes on their co-ethnic candidates. C_0 is then an expected margin of victory under the assumptions that there are the strongest minority bloc voting and no white crossover. To formalize this idea, let ρ be the percentage of minority voters in the electorate. Let β_M and β_W be the proportions at which minority and white voters support the minority candidate, respectively. The above assumptions state that all minority voters support the minority candidate and so $\beta_M = 1$ (the strongest minority bloc voting), and no white voter votes for that candidate and thus $\beta_W = 0$ (no white crossover). Under the above assumptions, the expected vote share for the top minority candidate will be:

$$V_t^{M*} = \rho\beta_M + (100 - \rho)\beta_W \tag{10a}$$
$$= \rho. \tag{10b}$$

Similarly, the expected vote share for the top white candidate will be:

$$V_t^{W*} = \rho(1 - \beta_M) - (100 - \rho)(1 - \beta_W) \tag{11a}$$
$$= 100 - \rho, \tag{11b}$$

where $*$ indicate that they are logically expected quantities instead of observed vote shares.

By substituting these into the formula of the (signed) racial margin of victory, we obtain:

$$C_0 = \frac{1}{2}\left(V_t^{M*} - V_t^{W*}\right) \tag{12a}$$
$$= \frac{1}{2}(\rho - (100 - \rho)) \tag{12b}$$
$$= \frac{1}{2}(\rho - 100 + \rho)) \tag{12c}$$
$$= \frac{1}{2}(2\rho - 100) \tag{12d}$$
$$= \rho - 50 \tag{12e}$$

Consistent with the definition of the (signed) racial margin of victory, C_0 takes values between -50 and 50: $C_0 \in (-50, 50)$. It is also evident as ρ (percentage of minority voters in the electorate) ranges from 0 to 100.

Finally, I argue that the (unknown) future racial margin of victory will be located *somewhere* between the two logical bounds. Formally, I model that:

$$C_0 \leq \frac{1}{2}(\widehat{V}_t^M - \widehat{V}_t^W) \leq M_0 \quad \text{or} \quad M_0 \leq \frac{1}{2}(\widehat{V}_t^M - \widehat{V}_t^W) \leq C_0. \tag{13}$$

I model that the most likely results (or "best guess") for *average* elections can be represented by the *geometric mean* of these two extreme cases. Importantly, it is our best guess "*in the absence of* other information" (Shugart and Taagepera, 2017, 105) and it could be off for actual (future) results due to election or candidate-specific factors (e.g., racial incidents, scandals, celebrity candidates). I use the geometric mean over other metrics such as the arithmetic mean because while M_0 may take extreme values in some elections (i.e., minority candidates may do extremely well or poorly), the geometric mean is robust to such extreme values (Taagepera, 2008, 120-127). Since M_0 and C_0 may contain negative values and the geometric mean cannot be computed for negative values, however, I introduce slightly adjusted quantities without changing any substantive meaning:

$$M = M_0 + 50 \tag{14a}$$
$$C = C_0 + 50 \tag{14b}$$

I then take the geometric mean of these adjusted quantities while subtracting 50 to account for the above transformation:

$$\tfrac{1}{2}\big(\widehat{V}_t^M - \widehat{V}_t^W\big) = (MC)^{1/2} - 50 \tag{15}$$

The reason for subtracting 50 is to make sure that the values from Equation 15 are centered around 0 as the (signed) racial margin of victory should be.

Another advantage of the above transformation is that C is algebraically equivalent to the percentage of minority voters in the electorate. It is easy to confirm this because:

$$C = C_0 + 50 \tag{16a}$$
$$= (\rho - 50) + 50 \tag{16b}$$
$$= \rho \tag{16c}$$

This algebraic equivalence is rather powerful because ρ has been the most important determinant of minority descriptive representation in the literature *and* is what redistricting manipulates as an "institutional variable."

By connecting Equations 3, 8, and 15, I obtain the logical model of minority candidate emergence presented in the main text:

$$P_{\text{run}} = \widehat{P}_{\text{win}} = \Phi\big(\tfrac{1}{2}(\widehat{V}_t^M - \widehat{V}_t^W)\big) = \Phi\big((MC)^{1/2} - 50\big). \tag{17}$$

A.4 Proof of the Racial Margin of Victory

In Online Appendix A.2, I introduced the new concept of the racial margin of victory and stated that, under FPTP, it is *computed* as half the difference between the vote shares of the most vote-earning minority and white candidates ($\frac{1}{2}|V^M - V^W|$) (Equation 5). This subsection provides proof for this quantity, regardless of the number of minority and white candidates.

Recall that the racial margin of victory is *defined* as the minimum amount of vote shares one needs to modify in order to change the race of the winner in an election. Let M denote the top minority candidate and W denote the top white candidate on a ballot. Similarly, Let m denote any other (non-top) minority candidate and w denote any other (non-top) white candidate on the ballot. Finally, let V^M and V^W be the vote shares obtained by M and W, respectively. Below, I consider eight possible patterns of biracial elections and show that Equation 5 fits the definition of the racial margin of victory in all configuration of biracial elections under FPTP.

A.4.1 One minority candidate & one white candidate

When there are only one minority candidate and one white candidate, all possible patterns of electoral results are as follows:

1st	W	M
2nd	M	W

Here, *1st* means that the listed candidate in the 1st position (i.e., winner) and *2nd* represents that the candidate in the 2nd position (i.e., runner-up). In other words, there are only two patterns: (1) the minority candidate wins and the white candidate takes second place, or (2) the white candidate wins and the minority candidate ends in second place. In order to change the race of the winner in these elections, one needs to change the order of the winner and the runner-up. Hence, the racial margin of victory is $\frac{1}{2}|(V^M - V^W)|$.

A.4.2 One minority candidate & multiple whites candidates

When there are one minority candidate and *two* white candidates, all possible patterns of electoral results are as follows:

1st	W	W	M
2nd	w	M	W
3rd	M	w	w

Following the above notation, *3rd* means that the listed candidate finishes in the 3rd place (A similar notation will be used with 4th and lower places below).

To change the race of the winner in these elections, one only needs to consider the vote shares obtained by the single minority and the top white candidates. Thus, the racial margin of victory is $\frac{1}{2}|(V^M - V^W)|$. Since this fact does not change even when there are more than two white candidates, this result applies to a more general case of elections with one minority and *multiple* white candidates.

A.4.3 Multiple minoritys candidates & one white candidate

Similarly, in elections with *two* minority candidates and only one white candidate, all possible orderings of candidates are as follows:

1st	W	M	M
2nd	M	W	m
3rd	m	m	W

Following the same logic as A.4.2, the racial margin of victory is computed as $\frac{1}{2}|(V^M - V^W)|$. Since this fact does not change even when there are more than two minority candidates, this applies to a more general case of elections with *multiple* minority candidates and one white candidate.

A.4.4 Multiple minority candidates & multiple white candidates

Now consider elections with more than one minority and white candidates, respectively. When there are *two* minority and *two* white candidates on a ballot, the following covers all the possible electoral outcomes.

1st	W	W	W	M	M	M
2nd	w	M	M	W	W	m
3rd	M	w	m	m	w	W
4th	m	m	w	w	m	w

In these elections, it again requires only $\frac{1}{2}|(V^M - V^W)|$ to change the race of the winner, regardless of how votes are distributed among minority candidates and white candidates. This result can be extended to elections with more than two minority or white candidates and thus applies to a more general case of *multiple* minority candidates and *multiple* white candidates.

A.4.5 No minority candidate & one white candidate

Now consider elections in which no minority candidate emerged and one white candidate wins unopposed. When elections are unopposed with a white winner, the only possible electoral result is as follows:

1st	W

A remarkable result is that this can be considered as a *special case* of A.4.1. (One minority candidate & one white candidate), where the minority candidate received zero votes. Thus,

1st	W	$(V^W = 100)$
2nd	M	$(V^M = 0)$

Consequently, the racial margin of vote can be computed as $\frac{1}{2}|(V^M - V^W)| = \frac{1}{2}|0 - V^W| = \frac{1}{2}|0 - 100| = 50$.

A.4.6 No minority candidate & multiple white candidates

When there are multiple white candidates with no minority candidate, all possible electoral outcomes can be demonstrated as follows:

1st	W
2nd	w
⋮	⋮

Now, this can be thought of as a special case of A.4.2. (one minority candidate & multiple white candidates) where the minority candidate received zero votes.

Following the result of A.4.2, the racial margin of vote is $\frac{1}{2}|(V^M - V^W)| = \frac{1}{2}|(0 - V^W)| = \frac{1}{2}| - V^W|$.

1st	W	(V^W)
2nd	w	
⋮	⋮	
Last	M	$(V^M = 0)$

A.4.7 One minority candidate & no white candidate

When elections are unopposed with a white winner, the only possible pattern is as follows:

1st	M

Similar to A.4.5, this can be considered as a *special case* of A.4.1. (One minority candidate & one white candidate), where the white candidate received zero votes. Thinking this way, the single possible electoral result can be represented as follows:

1st	M	$(V^M = 100)$
2nd	W	$(V^W = 0)$

Consequently, the racial margin of victory can be computed as $\frac{1}{2}|(V^M - V^W)| = \frac{1}{2}|V^M - 0| = \frac{1}{2}|100 - 0| = 50$.

A.4.8 Multiple minority candidates, no white candidate

Finally, when there are multiple white candidates with no minority candidate, all possible electoral outcomes can be demonstrated as follows:

1st	M
2nd	m
⋮	⋮

Similar to A.4.6, this can be thought of as a *special case* of A.4.3. (Multiple minority candidates & one white candidate) where the white candidate received zero votes. Hence,

1st	M	(V^M)
2nd	m	
⋮	⋮	
Last	W	$(V^W = 0)$

As a result, the racial margin of victory is $\frac{1}{2}|(V^M - V^W)| = \frac{1}{2}|V^M - 0| = \frac{1}{2}|V^M|$.

The above results show that $\frac{1}{2}|V^M - V^W|$ (Equation 5) represents the racial margin of victory in any biracial elections under FPTP, regardless of the number of minority and white candidates.

A.5 Computing M under At-Large Election

When applying the logical model to the at-large election, additional consideration will be necessary due to its unique electoral feature. This subsection discusses how to compute the racial margin of victory under this electoral system.

By at-large election, I refer to *plurality at-large block voting* system in which each candidate runs for a specific position (also known as a seat, post, or place), voters in an entire governing body (e.g., city) vote for a single candidate for each seat, and the most vote-earning candidate wins in each position (Davidson and Grofman, 1994, 7). For example, five council members can be elected for five different positions (or seats) in a city-wide (at-large) election. Each position is fought by different candidates and voters in the entire city votes for a single candidate of their choice as in FPTP elections. In other words, the city as an *at-large district* performs five different FPTP elections with the same set of voters.

Suppose that the adjusted racial margin of victory for the five positions in the last election are $M_{1,t-1} = 10$, $M_{2,t-1} = 20$, $M_{3,t-1} = 30$, $M_{4,t-1} = 40$, $M_{5,t-1} = 51$, where $M_{i,t-1}$ denotes the adjusted racial margin of victory for position i at time $t-1$. For the most viable candidate for each position, what would be a reasonable source of information about her future electoral performance? One logical possibility is that the average value of the five racial margin of victories at time $t-1$ can be a reasonable benchmark to consider. To formalize this idea, I first make an additional assumption about the behaviors of minority and white voters.

Assumption B.1 (POSITION IGNORABILITY). *Minority and white voters do not change their voting behavior depending on which position they vote for.*

This assumption rules out the possibility that voters somehow differentiate one position from another and change how they vote (e.g., white voters are more likely to vote for a minority candidate for Position 1 than for Position 3). Importantly, this does not rule out the case that some voters behave differently because of the *types of politicians* who currently represent these positions. For example, some white voters may support a minority incumbent who represents Position 1 and generally vote for white candidates for Positions 2-5. This is not a violation of the above assumption and it holds as long as they do so because of the minority incumbent's attributes and not due to the position *per se*.

With this assumption, I then argue that the *average* racial margin of victory in the last election can be represented by a geometric mean of the five racial margin of victories. More generally, consider the logical lower and upper bounds of multiple (at-large) racial margin of victories as follow:

$$\overline{M} = \max(M_{i,t-1}) \tag{18a}$$
$$\underline{M} = \min(M_{i,t-1}), \tag{18b}$$

where the first equation represents the upper bound and the second equation denotes the lower bound of the observed racial margin of victories. Taking the geometric mean of the two logical bounds, the racial margin of victory under at-large election (at time $t-1$) becomes:

$$M = (\underline{M}\overline{M})^{1/2} \tag{19}$$

Substituting M into the logical model yields the probability of minority candidate emergence and electoral success *for any position* in the at-large district via $\Phi\big((MC)^{1/2} - 50\big)$.

Suppose that minority voters consist of 40% and white voters consist of 60% of the electorate. In our running example, since $\overline{M_0} = 51$ and $\underline{M} = 10$, $M = (10 * 51)^{1/2} = (510)^{1/2} = 22.6$. The probability of minority candidate emergence and electoral success *for any position*, then, is $\Phi\big((22.6 * 40)^{1/2} - 50\big) = \Phi\big((904)^{1/2} - 50\big) = \Phi\big(30.1 - 50\big) = \Phi\big(-19.9\big) = 1.044182e - 88$.

A.6 How the Model Accounts for Minority Biracial Coalitions

This subsection discusses how the logical model accounts for minority biracial coalitions. To summarize the conclusion first, the logical model speaks to minority biracial coalitions in two ways: (1) C does not depend upon or "count on" the presence of minority coalitions by assuming that the other minority voters behave as white voters and (2) M reflects the presence and degree of minority electoral coalitions, just as it reflects the presence and degree of white crossover, in the most recent election. Consequently, the logical model systematically incorporates two important coalition behaviors — white crossover and minority coalitions — discussed in the literature.

In the literature on racial and ethnic politics, minority representation, redistricting, voting rights, scholars have paid special attention to the notion of *minority biracial coalitions* (Browning, Marshall and Tabb, 1984; Sonenshein, 1986; Meier and Stewart Jr, 1991; Anderson, 1992; Lublin, 1999; Wright and Middleton IV, 2001; Johnson, 2002; Kaufmann, 2004; Segura and Bowler, 2006; Rocha, 2007; Bullock III, 2010; Meier et al., 2004; Jackson, Gerber and Cain, 1994; Tedin and Murray, 1994). In the context of minority descriptive representation, minority biracial coalitions generally mean that minority voters from multiple racial groups support a minority candidate from either group (or minority voters' candidate of choice) over a white candidate (or white voters' candidate of choice). One of the most important implications of biracial coalitions is that minority voters from a single racial group can elect their candidate of choice *without holding a numerical majority* in their districts (Matsuda, 1990; Anderson, 1992; Delgado, 2002; Johnson, 2002; Lasso, 2005; Grofman, 2006).

In the political science literature, one of the most influential debates over the role of majority-minority districts indeed concerns about the presence of electoral coalition between black and Latino voters and its potential consequences on black electoral success outside majority-minority districts (Cameron, Epstein and O'halloran, 1996; Lublin, 1999; Epstein and O'Halloran, 1999; Casellas, 2009). The courts have also extensively discussed the *potential* minority biracial coalitions districts (often along with white crossover districts in which coalitions between voters of a single minority group and white voters are concerned) (e.g., *Georgia v. Ashcroft* (2003); *Johnson v. DeGrandy* (1994); *Voinovich v. Quilter* (1993); *Texas v. Holder* (2013); *Barlett v. Strickland* (2009); *Perry v. Perez* (2012); *Growe v. Emison* (1993); see also Pildes (2002)).

Despite the importance of the notion of minority biracial coalitions, the main text does not extensively discuss minority coalitions. The reason for the lack of in-depth discussions about minority coalitions is that the logical model proposed in the main text specifically aims to explain and predict minority candidate emergence and electoral success in *biracial districts*, in which voters from a single minority group and white voters compete with each other (**Assumption 1**). Hence, the logical model *in its purest form* does not assume the presence of voters from any other minority group. Empirically, however, it is likely that voters from the "third group" (here I assemble all the other groups into one for conceptual simplicity) consist of a small to moderate proportion of the electorate. To incorporate these more realistic scenarios, I extend the logical model by accounting for potential minority "biracial coalitions" (again here I consider that there are "two" minority groups: the minority group of primary interest and the other groups combined).

First, recall that C is an expected racial margin of victory when minority and white voters fully concentrate their ballots on their candidates of choice in a hypothetical election with a single minority and white candidates. I extend C by assuming that voters from the third group behave just like white voters in this limit scenario. In other words, in less strictly biracial elections, C represents an expected racial margin of victory in a hypothetical election in which minority voters exclusively support for their candidate of choice (as in a candidate from the minority group) and white and other voters exclusively support their preferred candidate (as in a white candidate). I extend C in this way because doing so can incorporate additional racial group without changing the *theoretical* meaning of C: an expected racial margin of victory in the most adverse conditions in which all other voters attempt to most effectively exclude representation of the minority group of interest (Rae, Hanby and Loosemore, 1971; Brischetto and Engstrom, 1997; Engstrom, 2010). Substantively, it means that in one *extreme* case, a minority candidate can only expect to receive votes from her racial group and cannot "count on" white voters and voters from any other group.

Second, recall that M is a racial margin of victory or a signed margin of victory of the top minority candidate relative to the top white candidate. As discussed above, M quantifies how many votes the top minority candidate received relative to her white counterpart from the entire electorate in her district, which in turn summarizes the cohesiveness of minority voters and the other (white) voters. To extend the model to less strictly biracial elections, I simply categorize both white voters and voters from the third group as the other voters or voters who are not from the minority group of primary interest without changing any mathematical property of M. To support this generalization, I make two assumptions:

Assumption B.2 (PRIMARY MINORITY VOTERS). *The proportion of the minority group of primary interest (or primary minority voters) in the electorate is strictly larger than the proportion of voters from the third racial group (or secondary minority voters).*

Assumption B.3 (NO VIABLE CANDIDATE FROM THE SECONDARY GROUP). *No viable candidate of the secondary minority group emerges.*

The consequence of these assumptions is that voters from the third group choose between the viable minority or white candidates due to their small relative size in the electorate. They in turn imply that the logical model *cannot* predict the emergence and electoral success of, for example, a Latino candidate in a black-white biracial district where the relative size of Latino voters is less than that of black voters.

Figure 4 and Table 2 both demonstrate that this extension of the logical model to less strictly biracial elections is successful in that the model can accurately predict minority candidate emergence and electoral success even in 80% biracial districts. While this article accounts for minority coalitions in less strictly biracial elections, it does not properly account for them in *multiracial districts* in which more than two sizable minority groups co-reside (e.g., 30% black, 30% Latino, and 40% white districts). To explain and predict minority representation in such districts, both C and M must be further generalized and their theoretical properties must be fully investigated. Extending the model to multiracial districts is beyond the scope of this article, and I leave the challenging task to future research.

A.7 Incorporating Turnout Rates by Race in C

This subsection describes how to make model predictions when minority and white voters turn out at different rates. In short, researchers can directly use the information about turnout rates to compute a turnout-adjusted percentage of minority voters from a raw percentage of minority voters while M already accounts for turnout by race in the most recent election.

In the literature on minority representation, redistricting, and voting rights, it has been well recognized that it is important to take into account the differential levels of turnout rates of minority and white voters when considering the relationship between the "raw" percentage of minority voters in the electorate (e.g., VAP and CVAP) (Grofman, Handley and Lublin, 2001, 1404-1407). For example, when minority turnout is significantly lower than white turnout, creating a majority-minority district with the raw percentage of minority voters via VAP or CVAP may not lead to the election of minority voters even when they fully concentrate their ballot to a single minority candidate because the proportion of minority voters in the population of voters who cast a ballot does not reach the numerical majority (Grofman, Handley and Niemi, 1992, 118-120). Following the seminal work by Fraga (2018), here, I refer to such a difference in turnout rates by race and ethnicity as a *turnout gap*.

Under the logical model, it is straightforward to incorporate the impact of the turnout gap into the model predictions. Let C be the *raw* percentage of minority voters in the electoral (i.e., CVAP). Let τ_M and τ_W be the turnout rates for minority and white voters, respectively. Then, the logical model with a baseline turnout adjustment is as follows:

$$P_{\text{run}} = \widehat{P}_{\text{win}} = \Phi\big((MC')^{1/2} - 50\big), \tag{20}$$

where C' is the *turnout-adjusted* percentage of minority voters and computed as:

$$C' = \frac{C\tau_M}{C\tau_M + (100 - C\tau_W)} * 100 \tag{21}$$

In words, C' is the percentage of minority voters in the group of voters who usually turn out in elections. The accompanying software can easily compute C' if researchers include the information about τ_M and τ_W as additional inputs. Researchers can use various approaches to estimate these quantities, such as using exit polls, surveys, registration files, ecological inference models, multilevel regression and poststratification (MRP), and historical analyses, but discussing how to estimate turnout by race is beyond the scope of this section.

A major advantage of this extension is that researchers can predict minority candidate emergence and electoral success with the raw percentage of minority voters (C) while using C' when making the model predictions. To illustrate, Figure A.1 replicates Panel B of Figure 2 with varying levels of τ_M and τ_W. A dashed curve represents districts with $C = 50$.

The left (right) column shows model predictions for districts in which minority voters have a higher (lower) turnout rate than white voters by 10% points. The first, second, and third rows visualize model predictions when the turnout rate ranges from 50-60% to 40-50% to 30-40%. There are two important takeaways from this figure. First, these panels indicate that the turnout gap has a larger impact on the model predictions as C decreases. Second, the figure illustrates that the turnout gap has an increasingly larger impact on the model predictions as the turnout decreases.

Finally, as discussed in the main text, the turnout gap in the most recent election is naturally incorporated by M. When researchers simulate M, however, a similar adjustment applies to M. For this complex option, see Section A.8.

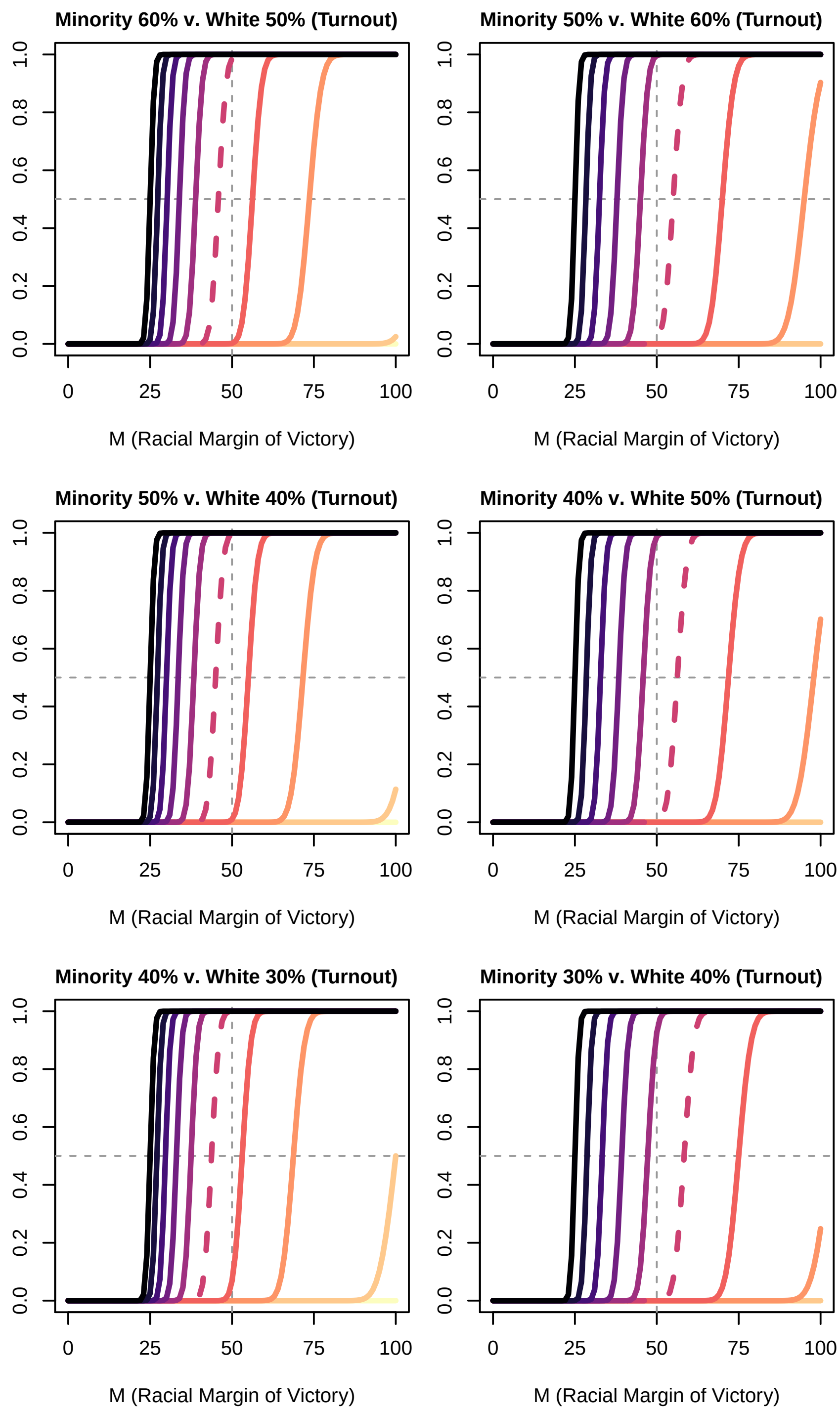

Figure A.1: **Model Predictions Accounting for Turnout Rates**
Note: Each panel plots the probability of minority candidate emergence against M with varying levels of C (raw percentage of minority voters in the electorate) while accounting for turnout rates by race. A dashed curve represents districts with $C = 50$. This figure illustrates that the turnout gap has a increasingly larger impact on the model predictions as C and the turnout decrease, respectively.

A.8 Simulating M with Prior Knowledge

Let C be the raw percentage of minority voters in the electorate. Let β_M and β_W be the proportions of minority and white voters who vote for the most viable minority candidate, respectively. Then, it is straightforward to simulate the vote shares of the top minority and white candidates:

$$V_{\text{sim}}^{M} = C\beta_M + (100 - C)\beta_W \tag{22a}$$

$$V_{\text{sim}}^{W} = C(1 - \beta_M) + (100 - C)(1 - \beta_W), \tag{22b}$$

where the index denotes that they are simulated quantities as opposed to observed vote shares.

Readers may notice that the above quantities look like what I derived for computing C. Indeed, they suggest that Equations 10b and 11b are *special cases* of the above equations, where it is assumed that $\beta_M = 1$ and $\beta_M = 0$. Finally, a simulated version of M can be computed as follows:

$$M_{\text{sim}} = \tfrac{1}{2}(V_{\text{sim}}^{M} - V_{\text{sim}}^{W}) \tag{23}$$

When researchers want to account for the turnout rates of minority and white voters (τ_M and τ_W), respectively, slightly modified simulated vote shares can be used:

$$V_{\text{sim}}^{M} = C'\beta_M + (100 - C')\beta_W \tag{24a}$$

$$V_{\text{sim}}^{W} = C'(1 - \beta_M) + (100 - C')(1 - \beta_W), \tag{24b}$$

where C' is a turnout-adjusted percentage of minority voters as discussed in Section A.7:

$$C' = \frac{C\tau_M}{C\tau_M + (100 - C\tau_W)} * 100$$

B Additional Findings and Discussions

This section summarizes the results of additional analyses that supplement the main analyses reported in the main text. Due to its large size, this section is suppressed here and its full version is available at the APSR Dataverse: https://doi.org/10.7910/DVN/F2OX6O.

C Additional Applications and Motivating Examples

This section offers additional applications, motivating examples, and one complex option (extension) of the logical model. Specifically, it illustrates how to account for turnout rates by race in each type of application. More complex options and different applications are also available, although they are not illustrated in this section. In terms of more complex options, there are at least two possible extensions. First, the model may reflect the "electoral influence" theory developed by Fraga (2018) by making both minority and white turnout rates vary by the value of C. Electoral influence theory suggests that the turnout gap between minority and white voters is a function of the proportion (relative size) of minority voters in the electorate. Second, the model may incorporate "racial threat" or/and "social contact" theories widely discussed in political science, sociology, psychology, and geography research (Key, 1949; Liu, 2001*a*,*b*; Oliver and Wong, 2003; Avery and Fine, 2012; Voss and Miller, 2001; Rocha and Espino, 2009; Weaver and Bagchi-Sen, 2015) by making the level of white crossover depends on the value of C. These more complex options will be incorporated into the corresponding software. Finally, researchers can also explore different applications of the logical model based on a few core functions offered by the software. Indeed, the logical model can be used in countless ways depending on the empirical and/or legal questions of interest.

C.1 Predicting the Probability of Minority Electoral Success

In *Heyes v. Louisiana* (1992), one of the main controversies was about the empirical validity of the claim that minority voters can influence electoral results (to elect minority candidates) in districts with about 20% minority voters. While the plaintiffs maintained that such districts can be minority influence districts, the state contended that "there was no evidence" to support such a theory given a strong racially polarized voting pattern (Engstrom and Kirksey, 1998, 250). The logical model offers one answer to this debate: the probability of minority candidate emergence in districts with 20% minority voters with a strong racially polarized voting pattern is almost 0.

Other claims have been that 35% to 45% (*Heyes v. Louisiana* (1994)) (Engstrom and Kirksey, 1998, 258) minority voters are sufficient to provide minority voters with a realistic chance to elect a candidate of their choice. To examine this claim, one can predict the probability of minority electoral success by setting $C = 40$ (to simplify the problem). Figure C.1 shows the prediction while accounting for different turnout rates by race. The most left curve represents the prediction when minority turnout is 50% and white turnout is 40%. The middle curve shows the prediction when there is no difference in turnout rates between groups. The right curve denotes the prediction when minority turnout is 40% and white turnout is 50%. I then simulate the value of M by assuming that all minority voters support the minority candidate and one-third of white voters cast crossover votes. The simulated value (M'=58) is represented by a vertical line in the plot. The result suggests that there is a very low probability that the minority candidate can win in this hypothetical district *unless* minority voters turn out at a significantly higher rate than white voters (0.5 and 0.4 specifically).

C.2 Simulating the Impact of Redistricting on Minority Representation

Concerning the effectiveness of the plan for a new majority-minority District 4 supported by the Senate in the 1990 round of redistricting, "Sherman Copelin, the African-American representative who sponsored the [alternative] plan, complained that the new minority district in the plan passed by the Senate did not contain enough African-American voters to ensure that African-Americans would elect a candidate of their choice.. . .The percentage of African-Americans among the registered voters in this district was 63.2, almost 4 percentage points higher than the second minority district in the other version" (Engstrom and Kirksey,

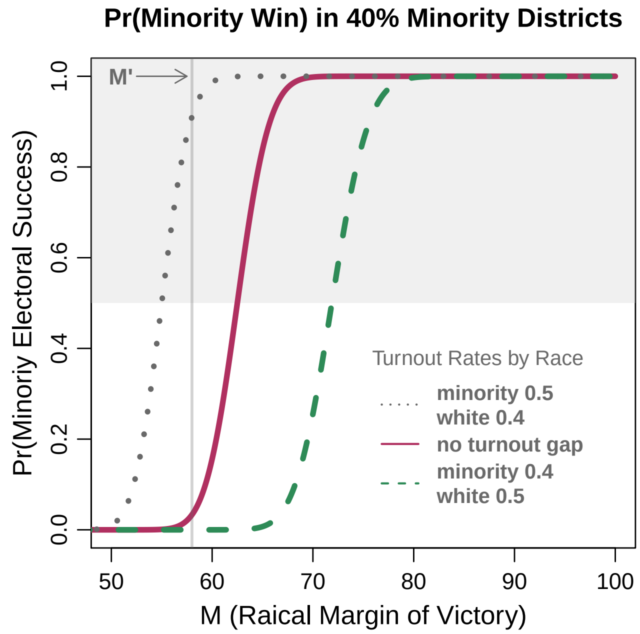

Figure C.1: **Probability of Minority Electoral Success in 40% Minority Districts**
Note: This figure visualizes the probability of minority candidate emergence and electoral success in districts where minority voters consist of 40% of the electorate with different turnout rates by race.

1998, 245).

To examine Copelin's claim and whether the additional 4 percentage points made a difference, one can use the logical model to predict the impact of increasing C from 59.2 to 63.2 on the probability of minority electoral success. To make predictions, I assume that there is an extreme racial polarization (perfect minority bloc voting and no white crossover). I also assume that minority expected turnout is 0.5, while white expected turnout is 0.6 (high white mobilization). Figure C.2 visualizes the impact of increasing C from 59.2 to 63.2, suggesting that *under these assumptions* the Senate Plan already *assured* (with probability 1) minority electoral success and the additional 4 percentage points did not change the probability of minority victory at all.

C.3 Discovering Sufficient Percentage of Minority Voters and the Electoral Sweet Spot

Researchers of racial redistricting have been interested in whether a particular percentage of minority voters is sufficient to give minority voters a viable chance of electing the candidate of their choice (Lublin et al., 2019). Some empirical studies show that majority-minority districts are not necessary (and 47.3% minority districts in the Northeast are sufficient) to provide an "equal opportunity" for minority voters to elect minority candidates (Cameron, Epstein and O'halloran, 1996; Epstein and O'Halloran, 1999), while other research considers majority-minority districts (more than 55%) are essential to "assure" minority descriptive representation (Lublin, 1997, 1999). While both camps seem to disagree with each other, in the context of the logical model, they may not necessarily offer conflicting ideas. To illustrate, Figure C.3 shows two electoral "sweet spots" (Lublin et al., 2019) — the minimum percentages of minority voters that are sufficient to give minority voters an equal opportunity (with probability 0.5) and guarantee (with probability 1) to

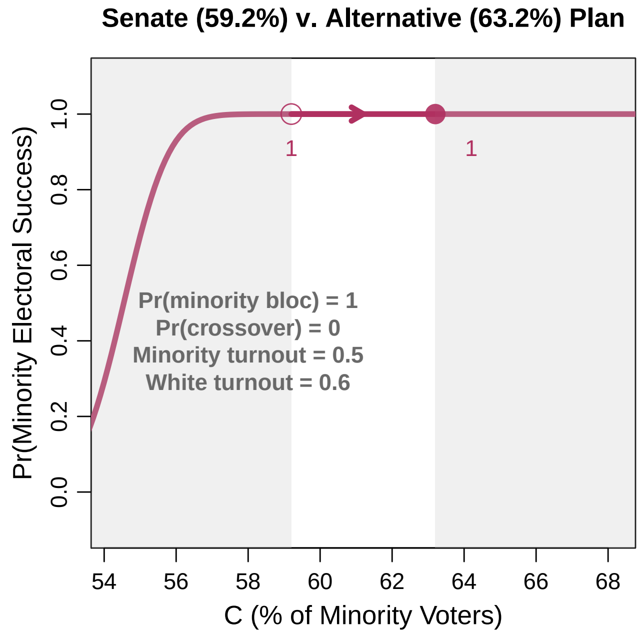

Figure C.2: **Simulated Impact of Redistricting on Minority Representation**
Note: This figure visualizes the probability of minority candidate emergence and electoral success when the percentage of minority voters changed from 59.2 to 63.2. The simulation assumes that there is the strongest minority bloc voting and no white crossover voting.

elect minority candidates, respectively. Here, I use 0.5 following Cameron, Epstein and O'halloran (1996) and Epstein and O'Halloran (1999), while I use 1 by modifying from Lublin (1997, 1999) who used 0.86. Additionally, I assume that minority voters fully engage in bloc voting, and one-third of white voters cast crossover votes to the minority candidate. Also, I assume that minority and white voters turn out at the rates of 0.4 and 0,5, respectively.

The figure shows that to secure an equal opportunity of electing minority candidates 47.6% is minimally sufficient, whereas it takes 57% to assure minority descriptive representation. These results are remarkably close to the original claims made by these authors. This example reemphasizes the importance of clearly defined thresholds when discussing the sufficient number of minority voters and electoral sweet spots. In other words, researchers must operationalize what a "viable change" means *before* predicting the probability of minority electoral success (just as one needs to pre-specify a threshold of statistical significance before conducting a hypothesis test).

C.4 Predicting the Number of Minority Officeholders

Using the logical model, it is straightforward to predict the number of minority officeholders in an entire governing body, including (but not limited to) city, county, and state (see also Section **??**). One simple strategy is to perform many Monte Carlo simulations in which we repeatedly sample the *number of minority officeholders* (i.e., number of districts that minority candidates win) from a Binomial distribution whose success probability vector is a vector of predicted probabilities based on the logical model. After repeating the simulations, we can evaluate the results by simply drawing a histogram of the sampled values.

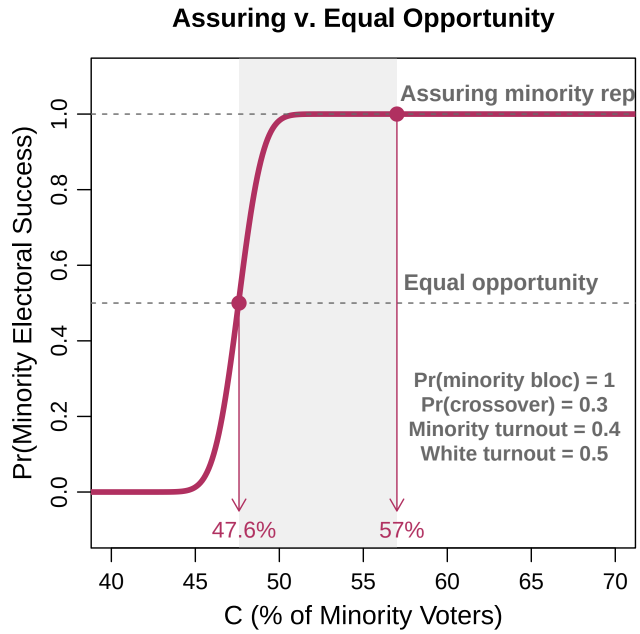

Figure C.3: **Sufficient Percentages of Minority Voters and Sweet Spot of Redistricting to Elect Minority Candidates**
Note: This figure visualizes the minimum percentages of minority voters that are sufficient to give minority voters an equal opportunity (with probability 0.5) and guarantee (with probability 1) to elect minority candidates, respectively.

Building on this application, one can predict the impact of converting from at-large election to single-member district on the number of minority officeholders at the jurisdiction level. Suppose that a jurisdiction, which uses at-large election, has six seats and the jurisdiction-wide percentage of minority voters is 47.5. Suppose also that the jurisdiction can draw six single-member districts with the percentages of minority voters 50, 40, 60, 30, 50, and 80. Finally, suppose that for each seat M is 30, 40, 50, 65, 70, and 30, respectively. To simplify the problem, assume that each representative of the seat under at-large election is from each district.

First, following Section A.5, I compute the average M for at-large election by taking a geometric mean of all Ms, obtaining 45.8. Next, I predict the probability of minority electoral success for each seat. This returns 0.0004 for all seat under at-large election and 0.000, 0.000, 1.000, 0.000, 1.000, and 0.1562 for each seat under single-member district. Finally, I perform Monte Carlo simulations in which I repeatedly draw samples (number of successes) from a Binomial distribution with success probabilities being the above-mentioned predicted values. Figure C.4 summarizes the results for the two electoral systems: the most likely number of minority officeholders is 0, followed by 1 in at-large elections, while the most likely number is 2, followed by 3 in single-member district elections.

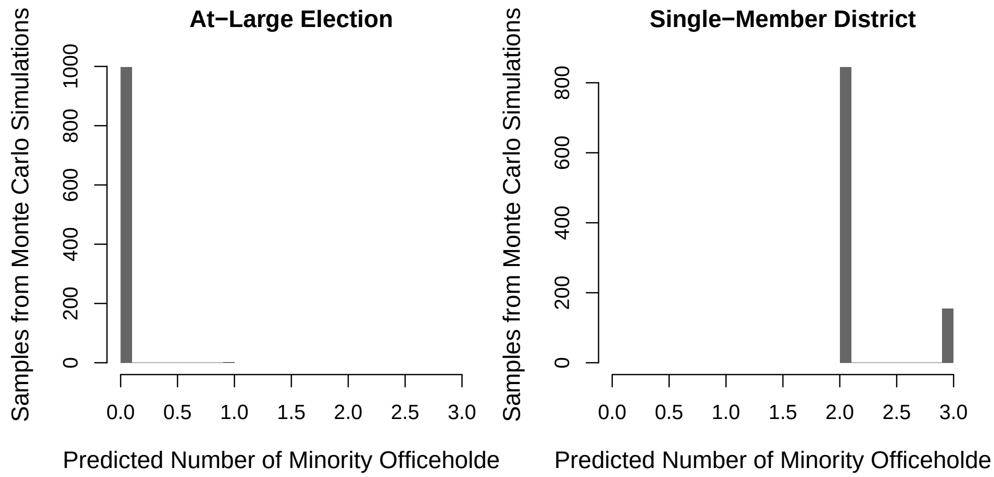

Figure C.4: **Predicted Number of Minority Officeholders by Electoral Systems**
Note: This figure visualizes the predicted number of minority officeholders from 1000 Monte Carlo Simulations.

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