A Supporting Information for "Reframing the Guardianship Dilemma" by Jack Paine

Appendix A.1 summarizes the critical thresholds that determine optimal actions. Appendix A.2 presents proofs and additional supporting information for the baseline model. Appendices A.3 and A.4 provide formal details on two extensions. Appendix A.5 lists the data sources used for empirical figures in the article.

A.1 Critical Thresholds for Optimal Actions

The following summarizes the threshold values of parameters that determine optimal actions. The expression for each threshold value contains three components. First, a restatement of the parameter for which I am defining the threshold, each of which consists of a Greek letter and a subscript. Second, a symbol above the parameter. For each, "tilde" $\left(\text{e.g., } \tilde{\pi}_{\text{sq}}^{\text{def}}\right)$ refers to thresholds that determine the *competent military*'s preferred action, and "hat" $\left(\text{e.g., } \hat{\pi}_{\text{trans}}^{\text{iso}}\right)$ to the *ruler*'s preferred action. The only critical threshold for the personalist military's preferred action consists of a single parameter, and I omit introducing new notation to express that threshold (see Equation 3). Third, the superscript provides brief descriptive information about the threshold. The following table provides additional elaboration. The symbols are organized by the order in which they are introduced in the article, and the italicized word explains the superscript.

Table A.1: Summary of Critical Threshold Values

Parameter	Defined in	Description
Isolating the outsider threat		
$\tilde{\pi}_{\text{sq}}^{\text{def}}$	Equation 1	Competent military prefers loyalty over *defection* for draws of of π_{sq} above this threshold
$\hat{\pi}_{\text{trans}}^{\text{iso}}$	Equation A.6	When *isolating* defection as the only disloyalty option, a necessary condition for the ruler to choose the competent military is for π_{trans} to not exceed this threshold
$\hat{\theta}_{\text{out}}^{\text{iso}}$	Equation A.7	When *isolating* defection as the only disloyalty option, a necessary condition for the ruler to choose the competent military is for θ_{out} to exceed this threshold
Adding insider threats		
$\tilde{\pi}_{\text{sq}}^{\text{coup}}$	Equation 5	Competent military prefers loyalty over *coup* for draws of π_{sq} above this threshold
$\tilde{\theta}_{\text{out}}^{\text{dis}}$	Equation 6	Competent military prefers the *disloyalty* option of defection over coups for values of θ_{out} above this threshold
$\tilde{\pi}_{\text{trans}}^{\text{coup}}$	Equation A.9	Competent military strictly prefers *coup* to defection for values of π_{trans} lower than this threshold
$\tilde{\pi}_{\text{trans}}^{\text{def}}$	Equation A.10	Competent military strictly prefers *defection* to coup for values of π_{trans} above this threshold
$\hat{\pi}_{\text{trans}}^{\text{dual}}$	Equation A.16	If the military can choose between their *dual* disloyalty options, a necessary condition for the ruler to prefer the competent military is for π_{trans} to not exceed this threshold
$\hat{\theta}_{\text{out}}^{\text{dual}}$	Equations A.17 & A.18	If the military can choose between their *dual* disloyalty options, a necessary condition for the ruler to prefer the competent military is for θ_{out} to exceed this threshold
Reframing the guardianship dilemma		
$\tilde{\pi}_{\text{trans}}^{\text{dis}}$	Equation A.21	Competent military prefers the *disloyalty* option of defection over coups for values of π_{trans} above this threshold
$\hat{\pi}_{\text{trans}}^{\text{def}}$	Equation A.22	Ruler prefers the competent military for values of π_{trans} below this value (fixing *defection* as the preferred disloyalty option)
$\hat{\pi}_{\text{trans}}^{\text{coup}}$	Equation A.23	Ruler prefers the competent military for values of π_{trans} below this value (fixing *coup* as the preferred disloyalty option)

A.2 PROOFS FOR BASELINE MODEL

Throughout, I write f for the pdf of F, the cdf that determines the military's valuation of the incumbent ruler, π_{sq}.

Proof of Proposition 1.

Step 1. Show that increases in θ_{out} strictly raise the dictator's preference for the competent relative to the personalist military. Rearrange Equation 2 to put both terms on the right-hand side, and then define:

$$\Omega_{\text{iso}} \equiv \left[1 - F(\tilde{\pi}_{\text{sq}}^{\text{def}})\right] \cdot p_{\text{comp}} - p_{\text{pers}}. \tag{A.1}$$

We need to determine the sign of:

$$\frac{d\Omega_{\text{iso}}}{d\theta_{\text{out}}} = \left[1 - F(\tilde{\pi}_{\text{sq}}^{\text{def}})\right] \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - f(\tilde{\pi}_{\text{sq}}^{\text{def}}) \cdot \frac{d\tilde{\pi}_{\text{sq}}^{\text{def}}}{d\theta_{\text{out}}} \cdot p_{\text{comp}} - \frac{dp_{\text{pers}}}{d\theta_{\text{out}}}, \tag{A.2}$$

with:

$$\frac{d\tilde{\pi}_{\text{sq}}^{\text{def}}}{d\theta_{\text{out}}} = -\pi_{\text{trans}} \cdot (1-\gamma) \cdot \frac{1}{(p_{\text{comp}})^2} \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}}. \tag{A.3}$$

Combining Equations A.2 and A.3 and simplifying yields:

$$\Bigg[\underbrace{1 - F(\tilde{\pi}_{\text{sq}}^{\text{def}})}_{\text{Direct effect}} + \underbrace{f(\tilde{\pi}_{\text{sq}}^{\text{def}}) \cdot \pi_{\text{trans}} \cdot (1-\gamma) \cdot \frac{1}{p_{\text{comp}}}}_{\text{Indirect effect}}\Bigg] \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - \frac{dp_{\text{pers}}}{d\theta_{\text{out}}}. \tag{A.4}$$

Because $\frac{\partial p}{\partial \theta_{\text{out}}} < 0$, $\frac{\partial^2 p}{\partial \theta_{\text{out}} \partial \theta_{\text{mil}}} < 0$, and $F(\cdot) \leq 1$, the entire expression is strictly positive for any distribution that is sufficiently flat, that is, if $f(\cdot)$ is small enough for all π_{sq}. The uniform distribution imposed in the article satisfies this assumption $\big($by construction, the uniform distribution minimizes the maximum value of $f(\cdot)\big)$, and the entire term in square brackets simplifies after imposing this functional form:

$$\left[1 - \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\max}} \cdot \gamma\right] \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - \frac{dp_{\text{pers}}}{d\theta_{\text{out}}} > 0. \tag{A.5}$$

The sign follows from the partial derivatives on the contest function just stated, and from $\gamma \cdot \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\max}} < 1$.

Step 2. Given Step 1, if the ruler does not prefer the competent military at $\theta_{\text{out}} \to \infty$, then they do not prefer the competent military for any $\theta_{\text{out}} > 0$. Thus, I check whether $\lim_{\theta_{\text{out}} \to \infty} \Omega_{\text{iso}} < 0$ (see Equation A.1). The intermediate value theorem implies that at least one $\hat{\pi}_{\text{trans}}^{\text{iso}} \in \left(0, \pi_{\text{sq}}^{\max}\right)$ exists satisfying $\Omega_{\text{iso}}(\pi_{\text{trans}} = \hat{\pi}_{\text{trans}}^{\text{iso}}, p_{\text{comp}} = p_{\text{comp}}^{\infty}, p_{\text{pers}} = p_{\text{pers}}^{\infty}) = 0$, or:

$$\left[1 - F\left(\hat{\pi}_{\text{trans}}^{\text{iso}} \cdot \left((1-\gamma) \cdot \frac{1}{p_{\text{comp}}^{\infty}} + \gamma\right)\right)\right] \cdot p_{\text{comp}}^{\infty} - p_{\text{pers}}^{\infty} = 0. \tag{A.6}$$

- At the lower bound $\pi_{\text{trans}} = 0$, we have $\Omega_{\text{iso}}(\pi_{\text{trans}} = 0, p_{\text{comp}} = p^{\infty}_{\text{comp}}, p_{\text{pers}} = p^{\infty}_{\text{pers}}) > 0$. To see why, the term inside the cdf equals 0 which, given the assumption $F \sim U\big(0, \pi^{\text{max}}_{\text{sq}}\big)$, yields $F(0) = 0$. Consequently, Ω_{iso} simplifies to $p^{\infty}_{\text{comp}} - p^{\infty}_{\text{pers}}$, which is strictly positive.
- At the upper bound $\pi_{\text{trans}} = \pi^{\text{max}}_{\text{sq}}$, we have $\Omega_{\text{iso}}(\pi_{\text{trans}} = \pi^{\text{max}}_{\text{sq}}, p_{\text{comp}} = p^{\infty}_{\text{comp}}, p_{\text{pers}} = p^{\infty}_{\text{pers}}) < 0$. To see why, the term inside the cdf equals $\pi^{\text{max}}_{\text{sq}} \cdot \Big[(1-\gamma) \cdot \frac{1}{p^{\infty}_{\text{comp}}} + \gamma\Big]$, which strictly exceeds $\pi^{\text{max}}_{\text{sq}}$ because $p^{\infty}_{\text{comp}} < 1$. Given the assumption $F \sim U\big(0, \pi^{\text{max}}_{\text{sq}}\big)$, $F(x) = 1$ for any $x > \pi^{\text{max}}_{\text{sq}}$. Consequently, Ω_{iso} simplifies to $-p^{\infty}_{\text{pers}} < 0$.
- Continuity follows because the uniformity assumption implies that the cdf is continuous.

The unique threshold claim for $\hat{\pi}^{\text{iso}}_{\text{trans}}$ follows from the (easy-to-prove) fact that $\frac{d\Omega_{\text{iso}}}{d\pi_{\text{trans}}} < 0$.

Step 3. For all $\pi_{\text{trans}} < \hat{\pi}^{\text{iso}}_{\text{trans}}$, the intermediate value theorem implies that at least one $\hat{\theta}^{\text{iso}}_{\text{out}} \in (0, \infty)$ exists that satisfies:

$$\Omega_{\text{iso}}\big(\theta_{\text{out}} = \hat{\theta}^{\text{iso}}_{\text{out}}\big) = 0. \tag{A.7}$$

- At the lower bound $\theta_{\text{out}} = 0$, we have $\Omega_{\text{iso}}(\theta_{\text{out}} = 0) = -F(\pi_{\text{trans}}) < 0$.
- At the upper bound $\theta_{\text{out}} \to \infty$, Step 2 shows that the present assumption of $\pi_{\text{trans}} < \hat{\pi}^{\text{iso}}_{\text{trans}}$ implies $\lim\limits_{\theta_{\text{out}} \to \infty} \Omega_{\text{iso}}(\theta_{\text{out}}) > 0$.
- Continuity follows because the uniformity assumption implies that the cdf is continuous.

The strict positivity of Equation A.5 establishes the unique threshold claim for $\hat{\theta}^{\text{iso}}_{\text{out}}$. ■

Lemma A.1 (Most-preferred disloyalty option for competent military)**.** *Unique threshold values $0 < \tilde{\pi}^{coup}_{trans} < \tilde{\pi}^{def}_{trans} < 1$ exist with the following properties:*

- *If $\pi_{trans} \leq \tilde{\pi}^{coup}_{trans}$, then the competent military prefers coup to defection for all $\theta_{out} > 0$.*
- *If $\pi_{trans} \geq \tilde{\pi}^{def}_{trans}$, then the competent military prefers defection to coup for all $\theta_{out} > 0$.*
- *If $\pi_{trans} \in \big(\tilde{\pi}^{coup}_{trans}, \tilde{\pi}^{def}_{trans}\big)$, then a unique threshold $\tilde{\theta}^{dis}_{out} \in (0, \infty)$ exists such that the competent military prefers coup over defection if and only if $\theta_{out} < \tilde{\theta}^{dis}_{out}$. The implicit characterization of this threshold is Equation 6, which equates the expected utility of each option.*

Proof. Define the difference in the competent military's expected value of the coup and defect options as:

$$\Omega_{\text{dis}}(\theta_{\text{out}}) \equiv \alpha(\theta_{\text{out}}) \cdot p(\theta_{\text{comp}}, \theta_{\text{out}}) + \big[1 - \alpha(\theta_{\text{out}}) \cdot p(\theta_{\text{comp}}, \theta_{\text{out}})\big] \cdot \gamma \cdot \pi_{\text{trans}} - \pi_{\text{trans}}.$$

This function strictly decreases in θ_{out}:

$$\frac{d\Omega_{\text{dis}}}{d\theta_{\text{out}}} = (1 - \gamma \cdot \theta_{\text{out}}) \cdot \left(\alpha \cdot \frac{\partial p_{\text{comp}}}{\partial \theta_{\text{out}}} + p \cdot \frac{d\alpha}{d\theta_{\text{out}}}\right) < 0. \tag{A.8}$$

Therefore, if $\Omega_{\text{dis}}(0) < 0$, then the competent military prefers defection over coup for all $\theta_{\text{out}} > 0$; and if $\lim_{\theta_{\text{out}} \to \infty} \Omega_{\text{dis}}(\theta_{\text{out}}) > 0$, then the opposite is true. This enables defining the two thresholds stated in the lemma:

$$\tilde{\pi}^{\text{coup}}_{\text{trans}} \equiv \frac{p^{\infty}_{\text{in}} \cdot \alpha^{\infty}}{1 - (1 - p^{\infty}_{\text{in}} \cdot \alpha^{\infty}) \cdot \gamma} \tag{A.9}$$

$$\tilde{\pi}^{\text{def}}_{\text{trans}} \equiv \frac{\alpha(0)}{1 - (1 - \alpha(0)) \cdot \gamma}, \tag{A.10}$$

and the assumptions about each parameter ensure that both terms are strictly bounded between 0 and 1.

Finally, if $\pi_{\text{trans}} \in \left(\tilde{\pi}^{\text{coup}}_{\text{trans}}, \tilde{\pi}^{\text{def}}_{\text{trans}}\right)$, then the conditions for the intermediate value theorem hold for establishing the existence of $\tilde{\theta}^{\text{dis}}_{\text{out}} \in (0, \infty)$ such that $\Omega_{\text{dis}}(\tilde{\theta}^{\text{dis}}_{\text{out}}) = 0$, and Equation A.8 establishes uniqueness. ∎

Given Lemma A.1, there are three possible cases for Proposition 2 depending on the value of π_{trans}. I prove the proposition for $\pi_{\text{trans}} \in \left(\tilde{\pi}^{\text{coup}}_{\text{trans}}, \tilde{\pi}^{\text{def}}_{\text{trans}}\right)$. This is the most complicated case (which involves piecewise functions) because the competent military's most-preferred disloyalty option switches from coup to defect for large enough θ_{out}. The proofs for the other two cases follow directly from the proof for this case. The only difference is that for $\pi_{\text{trans}} \leq \tilde{\pi}^{\text{coup}}_{\text{trans}}$, Step 2 needs slight modification. Here we would replace the implicit definition for $\hat{\pi}^{\text{dual}}_{\text{trans}}$ with a term in which coup, rather than defection, is the disloyalty option for the competent military. This would incorporate the term $\Omega_{\text{coup}}(\cdot)$ defined in the proof.

Proof of Proposition 2.

Step 1. Show that increases in θ_{out} strictly raise the dictator's preference for the competent military relative to the personalist military. Unlike Step 1 in the proof for Proposition 1, this step consists of three parts because the competent military's preferred disloyalty option switches for high enough θ_{out}. We need to demonstrate:

(a) Higher θ_{out} strictly raises the dictator's relative preference for the competent military if the competent military's preferred disloyalty option is a coup.

(b) Higher θ_{out} strictly raises the dictator's relative preference for the competent military if the competent military's preferred disloyalty option is to defect.

(c) The probability with which the competent military exhibits loyalty is continuous in θ_{out}.

(a) If the competent military's preferred disloyalty option is a coup, then the top term in Equation 7 is relevant. After rearranging this expression to put both terms on the right-hand side, we can define:

$$\Omega_{\text{coup}} \equiv \big[1 - F(\tilde{\pi}_{\text{sq}}^{\text{coup}})\big] \cdot p_{\text{comp}} - \big[1 - F(\alpha)\big] \cdot p_{\text{pers}}. \tag{A.11}$$

We need to determine the sign of:

$$\frac{d\Omega_{\text{coup}}}{d\theta_{\text{out}}} = \big[1 - F(\pi_{\text{sq}}^{\text{coup}})\big] \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - f(\pi_{\text{sq}}^{\text{coup}}) \cdot \frac{d\alpha}{d\theta_{\text{out}}} \cdot (1 - \gamma \cdot \pi_{\text{trans}}) \cdot p_{\text{comp}} - \bigg[\big[1 - F(\alpha)\big] \cdot \frac{dp_{\text{pers}}}{d\theta_{\text{out}}} - f(\alpha) \cdot \frac{d\alpha}{d\theta_{\text{out}}} \cdot p_{\text{pers}}\bigg].$$

Substituting in the functional form assumption and simplifying yields:

$$\underbrace{\left(1 - \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\max}} \cdot \gamma\right) \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - \frac{dp_{\text{pers}}}{d\theta_{\text{out}}}}_{\text{Equation A.5}} + \frac{\chi_a}{\pi_{\text{sq}}^{\max}} > 0, \tag{A.12}$$

for:

$$\chi_a \equiv \alpha \cdot \left[\frac{dp_{\text{pers}}}{d\theta_{\text{out}}} - \big(1 - \gamma \cdot \pi_{\text{trans}}\big) \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}}\right] + \frac{d\alpha}{d\theta_{\text{out}}} \cdot \Big[p_{\text{pers}} - \big(1 - \gamma \cdot \pi_{\text{trans}}\big) \cdot p_{\text{comp}}\Big].$$

Because the term for Equation A.5 is strictly positive, the imposed assumption that $\pi_{\text{sq}}^{\max}$ is sufficiently large implies that the entire expression is strictly positive.

(b) If the competent military's preferred disloyalty option is to defect, then the bottom term in Equation 7 is relevant. After rearranging this expression to put both terms on the right-hand side, we can define:

$$\Omega_{\text{def}} \equiv \big[1 - F(\tilde{\pi}_{\text{sq}}^{\text{def}})\big] \cdot p_{\text{comp}} - \big[1 - F(\alpha)\big] \cdot p_{\text{pers}}. \tag{A.13}$$

We need to determine the sign of:

$$\frac{d\Omega_{\text{def}}}{d\theta_{\text{out}}} = \big[1 - F(\pi_{\text{sq}}^{\text{def}})\big] \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - f(\pi_{\text{sq}}^{\text{def}}) \cdot \frac{d\tilde{\pi}_{\text{sq}}^{\text{def}}}{d\theta_{\text{out}}} \cdot p_{\text{comp}} - \bigg[\big[1 - F(\alpha)\big] \cdot \frac{dp_{\text{pers}}}{d\theta_{\text{out}}} - f(\alpha) \cdot \frac{d\alpha}{d\theta_{\text{out}}} \cdot p_{\text{pers}}\bigg].$$

Substituting in Equation A.3 and the functional form assumption, and simplifying, yields:

$$\underbrace{\left(1 - \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\max}} \cdot \gamma\right) \cdot \frac{dp_{\text{comp}}}{d\theta_{\text{out}}} - \frac{dp_{\text{pers}}}{d\theta_{\text{out}}}}_{\text{Equation A.5}} + \frac{\chi_b}{\pi_{\text{sq}}^{\max}} > 0, \tag{A.14}$$

for:

$$\chi_b \equiv \alpha \cdot \frac{dp_{\text{pers}}}{d\theta_{\text{out}}} + \frac{d\alpha}{d\theta_{\text{out}}} \cdot p_{\text{pers}}.$$

Because the term for Equation A.5 is strictly positive, the imposed assumption that $\pi_{\text{sq}}^{\text{max}}$ is sufficiently large implies that the entire expression is strictly positive.

(c) It is immediately apparent that the probability with which the competent military exhibits loyalty is continuous in θ_{out} at any value of θ_{out} such that the competent military's preferred disloyalty option does not change. This is true for all θ_{out} except at $\theta_{\text{out}} = \tilde{\theta}_{\text{out}}^{\text{dis}}$, where the preferred disloyalty option switches from coup to defection. Hence, the remaining step to showing that the probability with which the competent military exhibits loyalty is continuous in θ_{out} is to establish:

$$\lim_{\theta_{\text{out}} \to (\tilde{\theta}_{\text{out}}^{\text{dis}})^-} F\big(\pi_{\text{sq}}^{\text{coup}}(\theta_{\text{out}})\big) = \lim_{\theta_{\text{out}} \to (\tilde{\theta}_{\text{out}}^{\text{dis}})^+} F\big(\pi_{\text{sq}}^{\text{def}}(\theta_{\text{out}})\big). \tag{A.15}$$

After imposing the functional form assumption for $F(\cdot)$, this easily reduces to:

$$\alpha(\tilde{\theta}_{\text{out}}^{\text{dis}}) + \big[1 - \alpha(\tilde{\theta}_{\text{out}}^{\text{dis}})\big] \cdot \gamma \cdot \pi_{\text{trans}} = \pi_{\text{trans}} \cdot \left[(1-\gamma) \cdot \frac{1}{p_{\text{comp}}(\tilde{\theta}_{\text{out}}^{\text{dis}})} + \gamma\right].$$

This, in turn, easily reduces to the implicit definition of $\tilde{\theta}_{\text{out}}^{\text{dis}}$ from Equation 6.

Step 2. Given Step 1, if the ruler does not prefer the competent military at $\theta_{\text{out}} \to \infty$, then they do not prefer the competent military for any $\theta_{\text{out}} > 0$. Thus, I check whether $\lim_{\theta_{\text{out}} \to \infty} \Omega_{\text{def}} < 0$ (see Equation A.13). [*NB: because I am proving the case $\pi_{trans} \in \big(\tilde{\pi}_{trans}^{coup}, \tilde{\pi}_{trans}^{def}\big)$, we know that the relevant disloyalty option for the competent military at $\theta_{out} \to \infty$ is defection.*] The intermediate value theorem implies that at least one $\hat{\pi}_{\text{trans}}^{\text{dual}} \in \big(0, \pi_{\text{sq}}^{\text{max}}\big)$ exists satisfying $\Omega_{\text{def}}(\pi_{\text{trans}} = \hat{\pi}_{\text{trans}}^{\text{dual}}, p_{\text{comp}} = p_{\text{comp}}^{\infty}, p_{\text{pers}} = p_{\text{pers}}^{\infty}) = 0$, or:

$$\left[1 - F\left(\hat{\pi}_{\text{trans}}^{\text{dual}} \cdot \left((1-\gamma) \cdot \frac{1}{p_{\text{comp}}^{\infty}} + \gamma\right)\right)\right] \cdot p_{\text{comp}}^{\infty} - \big[1 - F(\alpha^{\infty})\big] \cdot p_{\text{pers}}^{\infty} = 0. \tag{A.16}$$

- At the lower bound $\theta_{\text{out}} = 0$, we have $\Omega_{\text{def}}(\pi_{\text{trans}} = 0, p_{\text{comp}} = p_{\text{comp}}^{\infty}, p_{\text{pers}} = p_{\text{pers}}^{\infty}) > 0$. To see why, the term inside the cdf equals 0 which, given the assumption $F \sim U\big(0, \pi_{\text{sq}}^{\text{max}}\big)$, yields $F(0) = 0$. Consequently, Ω_{def} simplifies to $p_{\text{comp}}^{\infty} - \big[1 - F(\alpha^{\infty})\big] \cdot p_{\text{pers}}^{\infty}$, which is strictly positive because $p_{\text{comp}}^{\infty} > p_{\text{pers}}^{\infty}$ and $F(\alpha^{\infty}) < 1$.
- At the upper bound $\pi_{\text{trans}} = \pi_{\text{sq}}^{\text{max}}$, we have $\Omega_{\text{def}}(\pi_{\text{trans}} = \pi_{\text{sq}}^{\text{max}}, p_{\text{comp}} = p_{\text{comp}}^{\infty}, p_{\text{pers}} = p_{\text{pers}}^{\infty}) < 0$. To see why, the term inside the cdf equals $\pi_{\text{sq}}^{\text{max}} \cdot \Big[(1-\gamma) \cdot \frac{1}{p_{\text{comp}}^{\infty}} + \gamma\Big]$, which strictly exceeds $\pi_{\text{sq}}^{\text{max}}$ because $p_{\text{comp}}^{\infty} < 1$. Given the assumption $F \sim U\big(0, \pi_{\text{sq}}^{\text{max}}\big)$, $F(x) = 1$ for any $x > \pi_{\text{sq}}^{\text{max}}$. Consequently, Ω_{def} simplifies to $-\big[1 - F(\alpha^{\infty})\big] \cdot p_{\text{pers}}^{\infty} < 0$.
- Continuity follows because the uniformity assumption implies that the cdf is continuous.

Step 3. For all $\pi_{\text{trans}} < \hat{\pi}^{\text{dual}}_{\text{trans}}$, demonstrate that at least one $\hat{\theta}^{\text{dual}}_{\text{out}} \in (0, \infty)$ exists that makes the ruler indifferent between their choice of military. There are two cases to consider because this threshold can be either larger or small than the point at which the competent military's preferred disloyalty option switches, $\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}$. These two cases are summarized intuitively in Table A.2.

(a) Suppose the ruler prefers the competent military at $\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}$, stated formally as $\Omega_{\text{coup}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}) > 0$. Then $\hat{\theta}^{\text{dual}}_{\text{out}} \in \left(0, \tilde{\theta}^{\text{dis}}_{\text{out}}\right)$ and satisfies

$$\Omega_{\text{coup}}\left(\theta_{\text{out}} = \hat{\theta}^{\text{dual}}_{\text{out}}\right) = 0. \tag{A.17}$$

Because the competent military prefers coup over defection for all $\theta_{\text{out}} < \tilde{\theta}^{\text{dis}}_{\text{out}}$, we know that Ω_{coup} is the relevant function. Showing that the conditions for the intermediate value theorem hold establishes existence:

- At the lower bound $\theta_{\text{out}} = 0$, we have $\Omega_{\text{coup}}(\theta_{\text{out}} = 0) < 0$. To see why, $\theta_{\text{out}} = 0$, we have $p_{\text{comp}} = p_{\text{pers}} = 1$. Therefore, it suffices to show $F\big(\alpha(0)\big) < F\big(\tilde{\pi}^{\text{coup}}_{\text{sq}}(0)\big)$. This reduces to $\alpha(0) < \tilde{\pi}^{\text{coup}}_{\text{sq}}(0)$ because $F(\cdot)$ is a strictly increasing function over its support, and then to $\big(1 - \alpha(0)\big) \cdot \gamma \cdot \pi^{\text{out}} > 0$, a true statement because $\alpha < 1$.
- At the upper bound $\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}$, we have $\Omega_{\text{coup}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}) > 0$. This inequality is simply the scope condition for case a.
- Continuity follows because the uniformity assumption implies that the cdf is continuous.

(b) Suppose the ruler prefers the personalist military at $\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}$, stated formally as $\Omega_{\text{def}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}) < 0$. Then $\hat{\theta}^{\text{dual}}_{\text{out}} \in \left(\tilde{\theta}^{\text{dis}}_{\text{out}}, \infty\right)$ and satisfies

$$\Omega_{\text{def}}\left(\theta_{\text{out}} = \hat{\theta}^{\text{dual}}_{\text{out}}\right) = 0. \tag{A.18}$$

Because the competent military prefers defection over coup for all $\theta_{\text{out}} > \tilde{\theta}^{\text{dis}}_{\text{out}}$, we know that Ω_{def} is the relevant function. Showing that the conditions for the intermediate value theorem hold establishes existence:

- At the lower bound $\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}$, we have $\Omega_{\text{def}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}) < 0$. This inequality is equivalent to the scope condition for case b.
- At the upper bound $\theta_{\text{out}} \to \infty$, we have $\lim_{\theta_{\text{out}} \to \infty} \Omega_{\text{def}}(\theta_{\text{out}}) > 0$. This inequality is true because we are assuming in Step 3 that $\pi_{\text{trans}} < \hat{\pi}^{\text{dual}}_{\text{trans}}$.
- Continuity follows because the uniformity assumption implies that the cdf is continuous.

The equations from Step 1 of the proof establish the unique threshold claim for both cases (specifically, Equations A.12, A.14, and A.15). Finally, we need to verify that two cases partition the space, which follows from $\Omega_{\text{def}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}}) = \Omega_{\text{coup}}(\theta_{\text{out}} = \tilde{\theta}^{\text{dis}}_{\text{out}})$. ■

Table A.2: Cases in Step 3 of the Proof for Proposition 2

Case a

	$\theta_{\text{out}} < \hat{\theta}_{\text{out}}^{\text{dual}}$	$\theta_{\text{out}} \in \left(\hat{\theta}_{\text{out}}^{\text{dual}}, \tilde{\theta}_{\text{out}}^{\text{dis}}\right)$	$\theta_{\text{out}} > \tilde{\theta}_{\text{out}}^{\text{dis}}$
Ruler chooses	Personalist military	Competent military	Competent military
Competent military prefers	Coup	Coup	Defection

Case b

	$\theta_{\text{out}} < \tilde{\theta}_{\text{out}}^{\text{dis}}$	$\theta_{\text{out}} \in \left(\tilde{\theta}_{\text{out}}^{\text{dis}}, \hat{\theta}_{\text{out}}^{\text{dual}}\right)$	$\theta_{\text{out}} > \hat{\theta}_{\text{out}}^{\text{dual}}$
Ruler chooses	Personalist military	Personalist military	Component military
Competent military prefers	Coup	Defection	Defection

Note: For parameter values at which the ruler optimally chooses the personalist military, the competent military's preferred disloyalty option is in gray to indicate that this choice occurs off the equilibrium path.

Proof of Proposition 3. Before proving the individual cases, first demonstrate that the partial-equilibrium characterizations of the probability of a coup (derived from Equations 3 and 5) exhibit a smooth and strictly decreasing relationship in θ_{out}:

$$\frac{dF\left(\alpha(\theta_{\text{out}})\right)}{d\theta_{\text{out}}} = f(\alpha) \cdot \frac{d\alpha(\theta_{\text{out}})}{d\theta_{\text{out}}} < 0 \tag{A.19}$$

$$\frac{dF\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}(\theta_{\text{out}})\right)}{d\theta_{\text{out}}} = f(\tilde{\pi}_{\text{sq}}^{\text{coup}}) \cdot (1 - \gamma \cdot \pi_{\text{trans}}) \cdot \frac{d\alpha(\theta_{\text{out}})}{d\theta_{\text{out}}} < 0. \tag{A.20}$$

Unfavorable post-transition fate. Follows from four facts:

1. Ruler chooses the personalist military for all $\theta_{\text{out}} < \hat{\theta}_{\text{out}}^{\text{dual}} \in (0, \infty)$ and the competent military for all $\theta_{\text{out}} \geq \hat{\theta}_{\text{out}}^{\text{dual}}$ (see Proposition 2).
2. Competent military's preferred disloyalty option is coup for all θ_{out} (see Lemma A.1).
3. $F\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}(\theta_{\text{out}})\right) > F\left(\alpha(\theta_{\text{out}})\right)$, which follows from $\gamma > 0$.
4. Equations A.19 and A.20.

Intermediate post-transition fate. Follows from three facts:

1. Facts 1, 3, and 4 from the previous case.
2. Competent military's preferred disloyalty option switches from defection to coup at $\theta_{\text{out}} = \tilde{\theta}_{\text{out}}^{\text{dis}} \in (0, \infty)$ (see Lemma A.1).
3. $\hat{\theta}_{\text{out}}^{\text{dual}} < \tilde{\theta}_{\text{out}}^{\text{dis}}$ follows from step 3 of the proof for Proposition 2.

Favorable post-transition fate. Follows from three facts:

1. Ruler prefers the personalist military for all $\theta_{\text{out}} > 0$ (see Proposition 2).
2. Equation A.19.
3. Competent military's preferred disloyalty option is defection for all $\theta_{\text{out}} > 0$ (see Lemma A.1). ■

Before providing a formal statement to correspond with the intuition highlighted in Figure 7, we need to define additional threshold values of π_{trans}. First, the value $\tilde{\pi}^{\text{dis}}_{\text{trans}}$ at which the competent military is indifferent between their disloyalty options of coup and defection:

$$\alpha \cdot p_{\text{comp}} + (1 - \alpha \cdot p_{\text{comp}}) \cdot \gamma \cdot \tilde{\pi}^{\text{dis}}_{\text{trans}} = \tilde{\pi}^{\text{dis}}_{\text{trans}}. \tag{A.21}$$

Second, the value $\hat{\pi}^{\text{def}}_{\text{trans}}$ at which the ruler is indifferent between the competent and personalist militaries, fixing defection as the preferred disloyalty option for the competent military:

$$\left[1 - F\left(\tilde{\pi}^{\text{def}}_{\text{sq}}(\hat{\pi}^{\text{def}}_{\text{trans}})\right)\right] \cdot p_{\text{comp}} = \left[1 - F(\alpha)\right] \cdot p_{\text{pers}}. \tag{A.22}$$

Third, the value $\hat{\pi}^{\text{coup}}_{\text{trans}}$ at which the ruler is indifferent between the competent and personalist militaries, fixing coup as the preferred disloyalty option for the competent military:

$$\left[1 - F\left(\tilde{\pi}^{\text{coup}}_{\text{sq}}(\hat{\pi}^{\text{coup}}_{\text{trans}})\right)\right] \cdot p_{\text{comp}} = \left[1 - F(\alpha)\right] \cdot p_{\text{pers}}. \tag{A.23}$$

The following statement presents two distinct cases, the first of which corresponds with the parameter values assumed for Figure 7. Table A.3 provides an intuitive summary of the distinction between the two cases.

Proposition A.1 (How post-transition fate influences equilibrium outcomes)**.**

- ***Case 1.*** *Suppose $\tilde{\pi}^{dis}_{trans} < \hat{\pi}^{def}_{trans}$.*
  - *The equilibrium probability of regime survival weakly decreases in π_{trans}, and this relationship is strict for $\pi_{trans} < \hat{\pi}^{def}_{trans}$.*
  - *$Pr(coup^*)$ is non-monotonic in π_{trans}: positive and strictly increasing for $\pi_{trans} < \tilde{\pi}^{dis}_{trans}$, a discrete decrease to 0 at $\pi_{trans} = \tilde{\pi}^{dis}_{trans}$, and a discrete and permanent increase to $F(\alpha) > 0$ at $\pi_{trans} = \hat{\pi}^{def}_{trans}$.*
- ***Case 2.*** *Suppose $\tilde{\pi}^{dis}_{trans} > \hat{\pi}^{def}_{trans}$.*
  - *The equilibrium probability of regime survival weakly decreases in π_{trans}, and this relationship is strict for $\pi_{trans} < \hat{\pi}^{coup}_{trans}$.*
  - *$Pr(coup^*)$ is non-monotonic in π_{trans}: positive and strictly increasing for $\pi_{trans} < \hat{\pi}^{coup}_{trans}$, and a discrete and permanent decrease to $F(\alpha) > 0$ at $\pi_{trans} = \hat{\pi}^{def}_{trans}$.*

Proof. In Step 1, I evaluate outcomes at low values of π_{trans}. In Step 2, I evaluate outcomes at high values of π_{trans}. In Step 3, I evaluate outcomes at intermediate values of π_{trans}. The two cases in the proposition are distinct only in Step 3.

Step 1. At $\pi_{\text{trans}} = 0$:

- The competent military prefers coup to defection. We know that $\tilde{\pi}_{\text{trans}}^{\text{dis}} > 0$ because $\alpha \cdot p_{\text{comp}} > 0$.
- The ruler chooses the competent military. To see why, at $\pi_{\text{trans}} = 0$, the competent military's preferred disloyalty option is a coup and their probability of exhibiting loyalty is $F(\alpha)$. This is identical to the corresponding probability for the personalist military, hence the claim follows from $p_{\text{comp}} > p_{\text{pers}}$.
- Given continuity in π_{trans}, for low enough π_{trans}, the following two derivatives imply, respectively, that the equilibrium probability of survival strictly decreases and the equilibrium probability of a coup strictly increases in π_{trans}:

$$\frac{d}{d\pi_{\text{trans}}}\left[\left[1 - F\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}\right)\right] \cdot p_{\text{comp}}\right] = -f\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}\right) \cdot p_{\text{comp}} \cdot (1-\alpha) \cdot \gamma < 0, \tag{A.24}$$

$$\frac{d}{d\pi_{\text{trans}}} F\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}\right) = f\left(\tilde{\pi}_{\text{sq}}^{\text{coup}}\right) \cdot (1-\alpha) \cdot \gamma > 0. \tag{A.25}$$

Step 2. At $\pi_{\text{trans}} = \pi_{\text{sq}}^{\text{max}}$, the ruler chooses the personalist military because the probability that the competent military exhibits loyalty is 0. To see this, the competent military's utility to defection is a lower bound for their payoff. At $\pi_{\text{trans}} = \pi_{\text{sq}}^{\text{max}}$, this disloyalty option strictly exceeds their expected utility to loyalty for any draw of π_{sq}. Continuity in π_{trans} implies that, for large enough π_{trans}, neither survival nor coups are a function of π_{trans} because the ruler chooses the personalist military. The equilibrium probability of survival equals $\left[1 - F(\alpha)\right] \cdot p_{\text{pers}}$ and the equilibrium probability of a coup equals $F(\alpha)$.

Step 3. We know that the competent military switches their preference from coup to defection at $\pi_{\text{trans}} = \tilde{\pi}_{\text{trans}}^{\text{dis}}$ and that the ruler switches their choice from the competent to the personalist military at $\pi_{\text{trans}} = \hat{\pi}_{\text{trans}}^{\text{def}}$. Because these thresholds are not strictly ordered, we need to consider two cases. These are stated in the proposition, and summarized more intuitively in Table A.3.

In Case 1, for $\pi_{\text{trans}} \in \left(\tilde{\pi}_{\text{trans}}^{\text{dis}}, \hat{\pi}_{\text{trans}}^{\text{def}}\right)$, the ruler chooses the competent military, whose preferred disloyalty option is defection. Hence $\Pr\left(\text{coup}^*\right) = 0$ in this range. The strictly decreasing relationship for survival follows from:

$$\frac{d}{d\pi_{\text{trans}}}\left[\left[1 - F\left(\tilde{\pi}_{\text{sq}}^{\text{def}}\right)\right] \cdot p_{\text{comp}}\right] = -f\left(\tilde{\pi}_{\text{sq}}^{\text{def}}\right) \cdot \left[1 - \gamma \cdot (1 - p_{\text{comp}})\right] < 0.$$

At $\pi_{\text{trans}} = \hat{\pi}_{\text{trans}}^{\text{def}}$, $\Pr\left(\text{coup}^*\right)$ discretely increases to $F(\alpha) > 0$ because the ruler switches from the competent to personalist military, and the probability of survival is constant in π_{trans}.

In Case 2, the competent military's preferred disloyalty option is coup for all values of π_{trans} at which the ruler chooses the competent military. For $\pi_{\text{trans}} < \hat{\pi}^{\text{coup}}_{\text{trans}}$, Equations A.24 and A.25 establish the results for survival and probability of a coup. For $\pi_{\text{trans}} > \hat{\pi}^{\text{coup}}_{\text{trans}}$, the probability of survival is constant in π_{trans}. At $\pi_{\text{trans}} = \hat{\pi}^{\text{coup}}_{\text{trans}}$, $\Pr(\text{coup}^*)$ discretely decreases from $F(\tilde{\pi}^{\text{coup}}_{\text{sq}})$ to $F(\alpha)$. We know that the former term is larger than the latter because $\gamma > 0$. ■

Table A.3: Cases in Proposition A.1

Case 1

	$\pi_{\text{trans}} < \tilde{\pi}^{\text{dis}}_{\text{trans}}$	$\pi_{\text{trans}} \in (\tilde{\pi}^{\text{dis}}_{\text{trans}}, \hat{\pi}^{\text{def}}_{\text{trans}})$	$\pi_{\text{trans}} > \hat{\pi}^{\text{def}}_{\text{trans}}$
Ruler chooses	Competent military	Competent military	Personalist military
Competent military prefers	Coup	Defection	Defection

Case 2

	$\pi_{\text{trans}} < \hat{\pi}^{\text{def}}_{\text{trans}}$	$\pi_{\text{trans}} \in (\hat{\pi}^{\text{def}}_{\text{trans}}, \tilde{\pi}^{\text{dis}}_{\text{trans}})$	$\pi_{\text{trans}} > \tilde{\pi}^{\text{dis}}_{\text{trans}}$
Ruler chooses	Competent military	Personalist military	Personalist military
Competent military prefers	Coup	Coup	Defection

Note: For parameter values at which the ruler optimally chooses the personalist military, the competent military's preferred disloyalty option is in gray to indicate that this choice occurs off the equilibrium path.

A.3 Extension: Multiple Coercive Units

In the baseline model, the ruler can perfectly assess the composition of the future outsider threat they will face. Yet in reality, dictators cannot anticipate the *exact* nature of future outsider threats. One common strategy for hedging bets is to counterbalance a more professionally organized and competent conventional force with a personalist paramilitary (Geddes et al. 2018; De Bruin 2020).

Here I formally extend the model to incorporate this consideration. The ruler makes a continuous choice over how to allocate a budget of size B between two distinct coercive units: a more competent conventional military and a personalist paramilitary. Resources dedicated to the competent unit more effectively translate into coercive capacity, but this coercive force also anticipates a better post-transition fate. The ruler knows the distribution of possible outsider threats when allocating funds, but is uncertain about the exact outsider movement that will arise. After observing Nature draws for θ_{out} and π_{trans}, the ruler deploys either the conventional military or personalist paramilitary, who in turn chooses between loyalty and defection.

The option to empower a counterbalancing unit yields a similar fundamental tradeoff as in the baseline model. Any additional soldier for or dollar of spending on the personalist paramilitary creates an opportunity cost by weakening the more competent conventional forces. Thus, rulers may indeed hedge their bets when organizing their coercive apparatus, but this does not obviate the main point that they trade off between bolstering coercive capabilities and worsening the post-transition fate. Furthermore, when the ruler can precisely assess which outsider threat they will confront, they dedicate all resources to one unit or the other. This recovers the assumed binary structure of the baseline model.

One new result is that robust fiscal health, i.e., high B, mollifies the main tradeoff by enabling the ruler to allocate more funds to each coercive unit. Thus, a looser budget constraint enables the ruler to come closer to maximizing the strength of each, given diminishing marginal returns for the contest functions. Following the formal analysis, I discuss the case of Iraq in this context.

Setup. Consider the following sequence of moves:

1. *Organizing coercion.* Ruler chooses $N_{\text{comp}} \geq 0$ meritocratic officers for a competent unit and $N_{\text{pers}} \geq 0$ sycophant officers for a personalist unit, subject to a budget constraint $N_{\text{comp}} + N_{\text{pers}} \leq B$, with $B > 0$.

2. *Outsider threat realized.* Nature determines the attributes of the mass outsider threat from a Bernoulli distribution:

$$(\theta_{\text{out}}, \pi_{\text{trans}}) = \begin{cases} (\theta_{\text{out}}{}', \pi_{\text{trans}}{}') & \text{with Pr= } q \in [0,1] \\ (\theta_{\text{out}}{}'', \pi_{\text{trans}}{}'') & \text{with Pr= } 1-q \end{cases}$$

   Below, I impose assumptions that make the ruler inclined toward the competent unit under the first draw, and the personalist unit under the second draw.

3. *Deploying the coercive unit.* Upon observing the Nature draw, the ruler decides which coercive unit to deploy (with the resources for each fixed at the levels chosen in Step 1) to repress the mass actor.

4. *Military's valuation of incumbent realized.* Nature draws π_{sq} from the same distribution as in the baseline model.

5. *Strategic loyalty choice.* The chosen coercive unit decides between loyalty and defection.

   [I omit the coup option because it does not affect the main mechanism of interest for this extension. Because the assumption $\gamma > 0$ yields informative results only when coups are a strategic option, I also set $\gamma = 0$ to simplify the expressions.]

6. *Outcomes and payoffs.* Unchanged from the baseline model.

Because this extension entails the ruler making a continuous choice, to close out the model, I impose two additional (standard) assumptions for the contest function: diminishing marginal returns, $\frac{\partial^2 p}{\partial {\theta_{\text{mil}}}^2} < 0$, and an Inada condition for the bounds, $\lim\limits_{\theta_{\text{mil}} \to \infty} \frac{\partial p(\theta_{\text{mil}}, \theta_{\text{out}})}{\partial \theta_{\text{mil}}} = 0$.

Analysis. If the ruler deploys the competent unit and they choose to act loyally, then the ruler survives with probability $p(N_{\text{comp}}, \theta_{\text{out}})$. The equivalent term for the personalist unit is $p(\delta \cdot N_{\text{pers}}, \theta_{\text{out}})$. Assuming $\delta \in (0, 1)$ expresses the weaker coercive capabilities of members of the personalist unit. The personalist unit always acts loyally, and the competent unit acts loyally with probability $1 - \frac{1}{p(N_{\text{comp}}, \theta_{\text{out}})} \cdot \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\text{max}}}$. These results and expressions follow from terms in the baseline model and from assuming $\gamma = 0$. Consequently, the probability of survival is $p(\delta \cdot N_{\text{pers}}, \theta_{\text{out}})$ if the ruler deploys the personalist unit and $p(N_{\text{comp}}, \theta_{\text{out}}) - \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\text{max}}}$ if the ruler deploys the competent unit. The full optimization problem is:

$$\max_{N_{\text{comp}}, N_{\text{pers}}, \lambda_{\text{comp}}, \lambda_{\text{pers}}, \lambda_B} \quad q \cdot S(N_{\text{comp}}, N_{\text{pers}}; {\theta_{\text{out}}}', {\pi_{\text{trans}}}') + (1 - q) \cdot S(N_{\text{comp}}, N_{\text{pers}}; {\theta_{\text{out}}}'', {\pi_{\text{trans}}}'')$$

$$+ \lambda_{\text{comp}} \cdot N_{\text{comp}} + \lambda_{\text{pers}} \cdot N_{\text{pers}} + \lambda_B \cdot (B - N_{\text{comp}} - N_{\text{pers}}),$$

with the probability of survival S equaling:

$$S(N_{\text{comp}}, N_{\text{pers}}; \theta_{\text{out}}, \pi_{\text{trans}}) = \max \left\{ p(N_{\text{comp}}, \theta_{\text{out}}) - \frac{\pi_{\text{trans}}}{\pi_{\text{sq}}^{\text{max}}}, \;\; p(\delta \cdot N_{\text{pers}}, \theta_{\text{out}}) \right\}.$$

The probability-of-survival function incorporates the ruler's best response after observing the Nature draw for the type of outsider threat: they deploy whichever coercive unit maximizes the probability of regime survival. This depends both on the Nature draw (exogenous parameters) *and* on how many resources the ruler allocated to each unit at an earlier information set in the game (an endogenous choice).

To make the problem strategically interesting, I assume that the ruler is inherently inclined toward the competent unit if Nature draws the first type of threat, and inherently inclined toward the personalist unit if Nature draws the second type of threat. By *inherently inclined*, I mean that the ruler prefers a particular security unit when comparing both *at full strength*. Formally:

$$\underbrace{p(B, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\max}}}_{\text{Competent}} > \underbrace{p(\delta \cdot B, \theta_{\text{out}}{}')}_{\text{Personalist}} \text{ and } \underbrace{p(B, \theta_{\text{out}}{}'') - \frac{\pi_{\text{trans}}{}''}{\pi_{\text{sq}}^{\max}}}_{\text{Competent}} < \underbrace{p(\delta \cdot B, \theta_{\text{out}}{}'')}_{\text{Personalist}} \tag{A.26}$$

The equilibrium allocation depends on the likelihood of each type of outsider threat, parameterized by q. The following demonstrates the existence of unique thresholds $\underline{q} \in (0,1)$ and $\overline{q} \in (0,1)$ such that:

1. If $q < \underline{q}$, then the ruler sets $N_{\text{comp}} = 0$ and $N_{\text{pers}} = B$.
2. If $q > \overline{q}$, then the ruler sets $N_{\text{comp}} = B$ and $N_{\text{pers}} = 0$.
3. If $\underline{q} < \overline{q}$ and $q \in (\underline{q}, \overline{q})$, then the ruler chooses interior optimal solutions $N_{\text{comp}} = N^*_{\text{comp}}$ and $N_{\text{pers}} = N^*_{\text{pers}}$, which I define shortly.

In each solution, the budget constraint binds. Given the assumption about inherent inclinations, it follows directly that if the ruler knows for sure what type of threat they will face, i.e., $q \in \{0,1\}$, then they will devote all their resources to only one coercive unit. If $q = 1$, then $N_{\text{comp}} = B$ and $N_{\text{pers}} = 0$; and if $q = 0$, then $N_{\text{comp}} = 0$ and $N_{\text{pers}} = B$. Given assumed continuity in the objective functions, this also implies that the ruler will dedicate all resources to one unit if q is "close" to either 0 or 1, and I formalize these thresholds below as $\underline{q}$ and $\overline{q}$.

If the optimization problem has an interior solution, then it takes the form:

$$\max_{N_{\text{comp}}, N_{\text{pers}}, \lambda} \; q \cdot \left[p(N_{\text{comp}}, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\max}} \right] + (1-q) \cdot p(\delta \cdot N_{\text{pers}}, \theta_{\text{out}}{}'') + \lambda \cdot (B - N_{\text{comp}} - N_{\text{pers}}). \tag{A.27}$$

Slightly rearranging the first-order conditions yields a system of implicit solutions for the optimal choices N^*_{comp} and N^*_{pers}. The imposed assumption of diminishing marginal returns implies that the solutions are maxima.

$$q \cdot \frac{\partial}{\partial \theta_{\text{mil}}} p(N^*_{\text{comp}}, \theta'_{\text{out}}) = (1-q) \cdot \delta \cdot \frac{\partial}{\partial \theta_{\text{mil}}} p(\delta \cdot N^*_{\text{pers}}, \theta''_{\text{out}}) \tag{A.28}$$

$$N^*_{\text{comp}} + N^*_{\text{pers}} = B \tag{A.29}$$

The ruler devotes all resources to the competent unit if and only if:

$$q \cdot \left[p(B, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\max}} \right] + (1-q) \cdot \left[p(B, \theta_{\text{out}}{}'') - \frac{\pi_{\text{trans}}{}''}{\pi_{\text{sq}}^{\max}} \right] \geq$$

$$q \cdot \left[p(N^*_{\text{comp}}, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\max}} \right] + (1-q) \cdot p(\delta \cdot N^*_{\text{pers}}, \theta_{\text{out}}{}'').$$

The left-hand side of the inequality expresses the ruler's utility to devoting all resources to the competent unit. In this case, the ruler deploys the competent unit regardless of which outsider threat materializes. The right-hand side expresses the ruler's utility to choosing the interior-optimal allocation. In this case, the ruler deploys the competent unit if the first type of outsider threat materializes, and the personalist unit if the second type.

Deriving this inequality with respect to q shows that it is strictly more likely to hold for higher q (note that the envelope theorem applies to the term on the right-hand side). Combining this with the boundary conditions in Equation A.26 enables implicitly defining a unique threshold $\overline{q} \in (0,1)$ such that:

$$\overline{q} \cdot \left[p(B, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-\overline{q}) \cdot \left[p(B, \theta_{\text{out}}{}'') - \frac{\pi_{\text{trans}}{}''}{\pi_{\text{sq}}^{\text{max}}} \right] =$$

$$\overline{q} \cdot \left[p\big(N^*_{\text{comp}}(\overline{q}), \theta_{\text{out}}{}'\big) - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-\overline{q}) \cdot p\big(\delta \cdot N^*_{\text{pers}}(\overline{q}), \theta_{\text{out}}{}''\big).$$

The mechanics for characterizing the unique $\underline{q} \in (0,1)$ threshold are identical:

$$\underline{q} \cdot p(\delta \cdot B, \theta_{\text{out}}{}') + (1-\underline{q}) \cdot p(\delta \cdot B, \theta_{\text{out}}{}'') = \underline{q} \cdot \left[p\big(N^*_{\text{comp}}(\underline{q}), \theta_{\text{out}}{}'\big) - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-\underline{q}) \cdot p\big(\delta \cdot N^*_{\text{pers}}(\underline{q}), \theta_{\text{out}}{}''\big).$$

Thus, we can characterize the ruler's equilibrium probability of survival as a function of q:

$$q \cdot \left[p(B, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-q) \cdot \left[p(B, \theta_{\text{out}}{}'') - \frac{\pi_{\text{trans}}{}''}{\pi_{\text{sq}}^{\text{max}}} \right] \text{ if } q \geq \overline{q}.$$

$$q \cdot p(\delta \cdot B, \theta_{\text{out}}{}') + (1-q) \cdot p(\delta \cdot B, \theta_{\text{out}}{}'') \text{ if } q \leq \underline{q}.$$

$$q \cdot \left[p(N^*_{\text{comp}}, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-q) \cdot p(\delta \cdot N^*_{\text{pers}}, \theta_{\text{out}}{}'') \text{ if } q \in \big(\underline{q}, \overline{q}\big). \tag{A.30}$$

Accurate threat assessment recovers binary choice. The analysis shows that if the ruler is certain (or nearly so) about the type of threat they will confront, then optimal allocation collapses to the simple binary structure assumed in the baseline model—either all resources to the competent unit, or all to the personalist unit.

Loosening the budget constraint. Robust fiscal health mollifies the main tradeoff faced by the ruler by enabling them to allocate more funds to each coercive unit. A benchmark is the ruler's equilibrium probability of survival if they can spend the entire budget B on *each* coercive unit. In this case, they deploy the competent unit in response to the first type of outsider threat, and the personalist unit in response to the second:

$$q \cdot \left[p(B, \theta_{\text{out}}{}') - \frac{\pi_{\text{trans}}{}'}{\pi_{\text{sq}}^{\text{max}}} \right] + (1-q) \cdot p(\delta \cdot B, \theta_{\text{out}}{}''). \tag{A.31}$$

To show that an arbitrarily large budget mitigates the allocation problem, I show that the difference in the probability of survival between Equation A.31 and the interior-optimal allocation in Equation A.30 goes to 0 as the budget diverges to infinity:

$$\lim_{B\to\infty}\left\{q\cdot\left[p(B,\theta_{\text{out}}{}')-p(N^*_{\text{comp}},\theta_{\text{out}}{}')\right]+(1-q)\cdot\left[p(\delta\cdot B,\theta_{\text{out}}{}'')-p(\delta\cdot N^*_{\text{pers}},\theta_{\text{out}}{}'')\right]\right\}$$

It suffices to show that $\lim_{B\to\infty} N^*_{\text{comp}} \to \infty$ and $\lim_{B\to\infty} N^*_{\text{pers}} \to \infty$. The following establishes the first claim, and the proof for the second is identical. Using Equations A.28 and A.29 enables restating the implicit definition of N^*_{comp} as:

$$\frac{\frac{\partial}{\partial\theta_{\text{mil}}}p(N^*_{\text{comp}},\theta'_{\text{out}})}{\frac{\partial}{\partial\theta_{\text{mil}}}p(\delta\cdot(B-N^*_{\text{comp}}),\theta''_{\text{out}})}=\frac{1-q}{q}\cdot\delta. \tag{A.32}$$

The right-hand side is bounded, which implies the left-hand side must be as well. Given this, we can prove the claim by contradiction. Suppose $\lim_{B\to\infty} N^*_{\text{comp}} < \infty$. Then $\lim_{B\to\infty}(B-N^*_{\text{comp}})=\infty$. Given the Inada assumption introduced above, this implies that the denominator converges to 0 and hence the left-hand side is unbounded, yielding a contradiction.

Empirical example. In empirical cases of robust fiscal health, rulers often lavish personalist paramilitary units with lucrative pay and weapons, while still having considerable revenues left over to spend on a more professional and socially inclusive conventional military. By contrast, cash-strapped regimes lack this luxury. In Iraq under Saddam Hussein, Blaydes (2018, 269-73) connects the general decline in state fiscal resources between the 1970s–90s to a major restructuring of the military from a more socially inclusive force with formidable counterbalancing units to an unambiguously personalist and socially exclusive military. This case helps to isolate the budget mechanism because the state's fiscal position changed over time. Long-standing, unchanging factors do not provide a compelling alternative explanation for a single country's shift over time in military organization.

Amid an oil boom during the 1970s–80s, the Iraqi army grew enormously, from roughly 50,000 personnel in 1968 to almost 1 million in 1988. Alongside this buildup of the conventional army, the Ba'th Party created and expanded paramilitary units such as the Republican Guard and Popular Army. Thus, they combined competent and personalist units within the overall security apparatus.

Later, following war with Iran throughout the 1980s, deteriorating finances made maintaining a large and socially inclusive standing army "beyond the economic capability of the regime," and the army risked becoming an "'uncontrolled leviathan' at its full mobilization capacity" (Blaydes 2018, 271). This fear manifested in 1991. Following the failed the invasion of Kuwait, retreating soldiers mutinied and participated in major uprisings that almost toppled the regime. Ultimately, personalist Republican Guard units put down the insurrections. They remained loyal because they feared a transition: "Hussein's fall would be a tremendous loss for them as well" (272).

Reforms to the military after 1991, amid a period of fiscal austerity because of UN sanctions, completed the transition to a personalist military. Recruitment to the officer corps became increasingly geographically narrow and favored individuals from in and near Saddam Hussein's home area of Tikrit. This "privileged loyalty over competence, hurting Iraq's military readiness" (273).

A.4 Extension: Preventive Repression

In the baseline model, the military can only *react* to mass movements that have already formed. Yet real-life rulers also use repression to *prevent* mass threats from arising. Secret police and other intelligence agencies engage in activities such as surveillance, low-profile harassment, denial of benefits such as public employment, and prosecuting political opponents (hence, in this extension, I refer to a general "coercive agent" rather than "the military" specifically). These tactics seek to deter and undermine mass anti-regime movements (Levitsky and Way 2010; Greitens 2016; Dragu and Przeworski 2019). Power-sharing arrangements serve a similar preventive purpose, although I do not explicitly model this non-coercive strategy. For example, sharing influential positions in the central government with members of other ethnic groups can help to prevent civil wars. In regions where residents are represented in the central government, the state has denser brokerage networks that facilitate better intelligence collection about nascent anti-regime movements (Roessler 2016; Blaydes 2018).

The strategic calculus is identical when the goal is prevention rather than reaction. To see why, consider an extension identical to the baseline model until the information sets following the choice of loyalty/defect/coup choice by the coercive agent. Following this move, now suppose that a strategic masses actor decides whether to mobilize (a choice which itself follows a new Nature move described below). Mobilization by the masses establishes outsider rule for sure, and governance yields for them a benefit of $b > 0$. The masses also pay a cost to mobilizing that depends on the action the coercive agent took:

- If either type of coercive agent defected, then the cost is 0.
- If the competent agent acted loyally, then the cost is $c_{\text{comp}} \equiv c(\theta_{\text{comp}}, \theta_{\text{out}})$.
- If the competent agent staged a coup, then the cost is $\alpha \cdot c_{\text{comp}}$.
- If the competent agent acted loyally, then the cost is $c_{\text{pers}} \equiv c(\theta_{\text{pers}}, \theta_{\text{out}})$.
- If the competent agent staged a coup, then the cost is $\alpha \cdot c_{\text{pers}}$.

For any cost-of-mobilization amount c faced by the masses, they will mobilize if $b > c$.

To align this extension with the idea of using coercion to prevent rather than to react to mass threats, I assume that the coercive agent uncertain as to how the masses will respond to repression. Specifically, following the move by the coercive agent but before the move by the masses, Nature draws b from a distribution $G(\cdot)$ that satisfies standard properties and has strictly positive support. Thus, for an action that imposes a cost c for the masses to mobilize, the military knows that the probability of non-mobilization equals $G(c)$. Appropriate assumptions about how the θ terms affect the cost of mobilization recovers probability-of-survival terms isomorphic to those in the baseline model, p_{comp} and p_{pers}. Thus, even if repression is used to prevent rather than react to endogenous outsider threats, the strategic interaction between the ruler and their coercive agent is equivalent.

A.5 DATA SOURCES

- Replication data available at Paine (2022).
- The data in Figure 8 (and mentioned in the first paragraph of the article) incorporate a global sample of authoritarian regimes between 1946 and 2015. All the following sources cover all these years unless otherwise noted. I exclude separatist movements, which do not directly imperil the survival of the incumbent regime.
- To identify authoritarian country-years, I used the updated version of the dataset from Boix et al. (2013). They include broad coverage of countries, although I include only countries with complete data on all variables in Figure 8. This ensures that the numbers are comparable across rows in the figure, and in practice excludes a handful of small island nations.
- Data on center-seeking rebels and ethnic rebels from Vogt et al. (2015). To calculate the figure for the number of *armed insurgencies that aimed to seize the capital city* in the first paragraph of the article, I summed the number of onsets of center-seeking wars. This is a binary variable, and hence years with multiple new wars are counted only as a single onset.
- Data on Marxist rebels from Kalyvas and Balcells (2010). Their data end in 2006. The only Marxist rebellion in their dataset that was ongoing in 2006, FARC in Colombia, is coded by Correlates of War (Dixon and Sarkees 2015) as lasting through 2015. Hence, I count one Marxist rebellion from 2007–15.
- Data on Islamist rebels from Gleditsch and Rudolfsen (2016). Their data end in 2014.
- Data on non-violent movements from Chenoweth and Lewis (2013). To calculate the figure for the number of *non-violent movements that sought regime change* in the first paragraph of the article, I summed the number of onsets of non-violent movements. This is a binary variable, and hence years with multiple new movements are counted as a single onset.
- In Figure 9, data on military personalism from Geddes et al. (2018). Their data end in 2010. Their country coverage is broad, although somewhat more restrictive than that in Boix et al. (2013). Each variable is a component of their personalism index (see pgs. 79-85). These three (of their eight) components most directly pertain to the concept of a personalist military: dictator's personal *control* of the security apparatus, creation of loyalist *paramilitary* forces, and military *promotions* based primarily on loyalty to the regime leader or ascriptive ties rather than merit and seniority.

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