Online Appendices

Mixed Motivations

We have compared the two extreme kinds of rewards, purely material or purely psychological. But as Kennedy (1999) aptly puts in his study of "the rumbles of discontent" during the Great Depression, people "can subsist on solely spiritual nourishment little longer than they can live on bread alone" (218).[14] We now analyze movements in which incentives to rebel are a combination of material and psychological rewards. We generalize our payoffs in Figure 1 by adding a parameter $\bar{m} \in [0,1]$ that generates our purely material rewards setting in one extreme ($\overline{m} = 0$) and our purely psychological rewards setting in the other extreme ($\overline{m} = 1$). Figure 4 shows the payoffs—we normalize the population size to 1.

	outcome $m \geq \theta$	$m < \theta$
rebel	$\frac{1}{\overline{m}}(\mathbf{1}_{\{m \leq \overline{m}\}} + \mathbf{1}_{\{m \geq \overline{m}\}} \cdot \frac{\overline{m}}{m}) - c$	$-c$
not rebel	0	0

Figure 4: Payoffs combining material and psychological motivations.

Proposition 6 characterizes the unique equilibrium. Figure 5 illustrates the result.

Proposition 6 *Let θ^* be the equilibrium regime change threshold in the setting with mixed motivations. Then,*

$$\theta^* = \begin{cases} e^{-c} & ; \overline{m} \leq e^{-c} \\ \\ \overline{m}\ (1 - c - \log(\overline{m})) & ; \overline{m} \geq e^{-c}. \end{cases}$$

Moreover, $\theta^m > \theta^*(\overline{m}) > \theta^p$ for $\overline{m} \in (0,1)$, with $\lim_{\overline{m} \to 0} \theta^* = \theta^m$ and $\lim_{\overline{m} \to 1} \theta^* = \theta^p$.

Proof of Proposition 6: The net payoff from rebelling versus not is:

$$\frac{1}{\overline{m}} \left(\mathbf{1}_{\{\theta < m, m \leq \overline{m}\}} + \mathbf{1}_{\{\theta < m, m \geq \overline{m}\}} \cdot \frac{\overline{m}}{m} \right) - c \tag{21}$$

[14] Kennedy, David M. 1999. *Freedom from Fear: The American People in Depression and War, 1929-1945.* New York: Oxford University Press.

As in the pure material rewards setting, this net payoff is non-monotone in the fraction of rebels m. It jumps up at $m = \theta$ (the threshold at which regime change succeeds), but then falls, weakly is some range and strictly in others, as more citizens join the movement.

As before, given a value of θ, the fraction of rebels is $m(\theta) = \Pr(x < x^*|\theta)$, and $\Pr(x_i < x^*|\theta^*) = \theta^*$. Moreover, $m(\theta) < \overline{m}$ if and only if $\theta > \bar{\theta}$, where $\Pr(x_i < x^*|\bar{\theta}) = \overline{m}$. Then, the net expected payoff from rebellion versus not is:

$$\int_{\theta=-\infty}^{\infty} \frac{1}{\overline{m}} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)} \right) \text{pdf}(\theta|x_i) - c. \tag{22}$$

As before, if $c < \min\{1, 1/\overline{m}\} = 1$, we can invoke the Karlin's Theorem to conclude to that the best response to a monotone strategy is also monotone. The indifference condition is:

$$\int_{\theta=-\infty}^{\infty} \left(\mathbf{1}_{\{\theta<\theta^*,\theta\geq\bar{\theta}\}} + \mathbf{1}_{\{\theta<\theta^*,\theta\leq\bar{\theta}\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)} \right) \text{pdf}(\theta|x_i = x^*) = \overline{m}\ c. \tag{23}$$

First, suppose $\bar{\theta} > \theta^*$. Then,

$$\int_{\theta=-\infty}^{\infty} \mathbf{1}_{\{\theta<\theta^*\}} \cdot \frac{\overline{m}}{\Pr(x_i < x^*|\theta)}\ \text{pdf}(\theta|x_i = x^*) = \overline{m}\ c. \tag{24}$$

Thus,

$$\theta^* < \bar{\theta} \ \Rightarrow\ \theta^* = e^{-c}, \tag{25}$$

where we recognize that $\bar{\theta}$ is endogenous and depends on x^*. However, recall that $\Pr(x < x^*|\bar{\theta}) = \overline{m}$ and $\Pr(x < x^*|\theta^*) = \theta^*$. Thus, $\theta^* < \bar{\theta}$ is equivalent to $\theta^* > \overline{m}$. Given (25), $\theta^* > \overline{m}$ is equivalent to: $-c > \log(\overline{m})$.

Next, suppose $\bar{\theta} < \theta^*$, i.e., $\theta^* < \overline{m}$. Then,

$$\begin{aligned} \overline{m}\ c &= \int_{\theta=-\infty}^{\bar{\theta}} \frac{\overline{m}}{\Pr(x_i < x^*|\theta)}\ \text{pdf}(\theta|x_i = x^*)d\theta + \int_{\bar{\theta}}^{\theta^*} \text{pdf}(\theta|x_i = x^*)d\theta \\ &= -\overline{m}\ \log(1 - \Pr(\theta < \bar{\theta}|x_i = x^*)) + \Pr(\theta < \theta^*|x_i = x^*) - \Pr(\theta < \bar{\theta}|x_i = x^*). \end{aligned} \tag{26}$$

Substituting for $\Pr(x_i < x^*|\bar{\theta}) = \overline{m} = 1 - \Pr(\theta < \bar{\theta}|x_i = x^*)$ and $\Pr(\theta < \theta^*|x_i = x^*) = 1 - \theta^*$ yields $-\overline{m}\ \log(\overline{m}) + \overline{m} - \theta^* = \overline{m}\ c$, i.e.,

$$\theta^* = \overline{m}\ (1 - \log(\overline{m})) - \overline{m}\ c. \tag{27}$$

Thus, $\theta^* < \overline{m}$ if and only if $-c < \log(\overline{m})$.

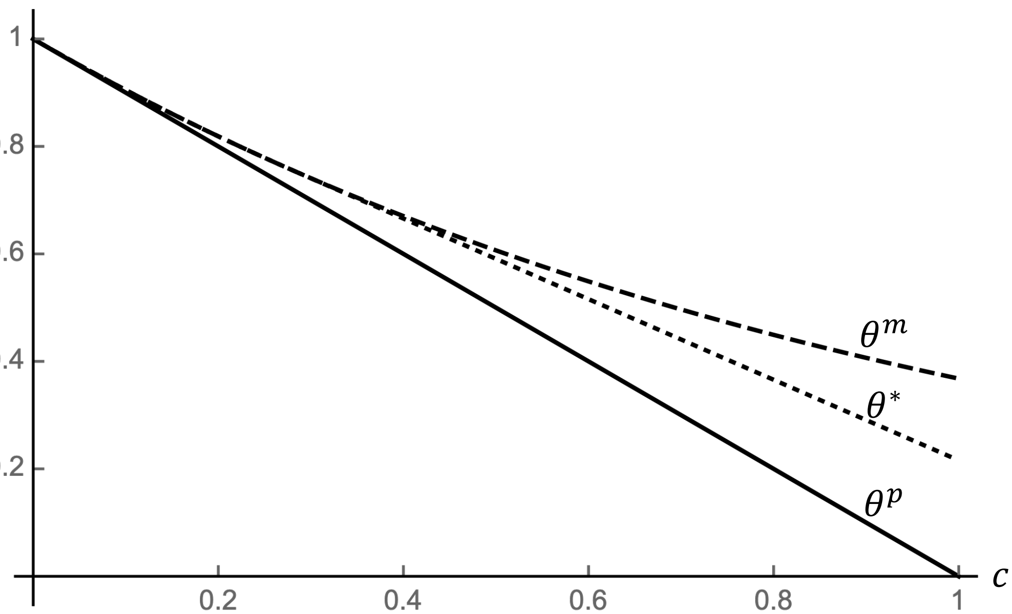

Figure 5: The equilibrium regime change threshold for the settings with pure psychological rewards (solid line, θ^p), pure material rewards (dashed curve, θ^m), and a mix of psychological and material rewards (dotted curve, θ^*). Parameters: $\overline{m} = 0.75$.

Combining this results yield:

$$\theta^* = \begin{cases} e^{-c} & ; c \leq -\log(\overline{m}) \\ \\ \overline{m}\ (1 - c - \log(\overline{m})) & ; c \geq -\log(\overline{m}) \end{cases} \tag{28}$$

We observe that

$$\left.\frac{\theta^*(c)}{dc}\right|_{c=-\log(\overline{m})} = -\overline{m}.$$

□

Proposition 6 and Figure 5 show that when motivations are a mix of psychological and material, the effects of repression and early failure lie in between those effects in the settings with material and psychological rewards analyzed earlier. The key intuition comes from thinking about the extent to which rewards are rival. In the pure material rewards setting, rewards are entirely rival. In the pure psychological rewards setting, rewards are entirely non-rival. In this mixed setting, we can think of some portion of the rewards as being rival and another portion being non-rival.

Interestingly, this points to a different interpretation of this version of the model, where we interpret the rewards as material, but imperfectly divisible, such as promises to hold future government office. Suppose there are a total of $\overline{m}$ offices available. If the rebellion is small, $m < \overline{m}$, and succeeds, each participant in the rebellion gets an office. But there are too many offices for the rebels to fill all of them. So some offices must be left in the hands of their previous holders. (Think of a small rebel group not fully purging the bureaucracy after taking control of the state.) However, if the rebellion is large, $m > \overline{m}$, there are not enough offices to go around and the congestion externality returns. So, if the number of participants is smaller than the number of offices, an increase in participation in the rebellion does not diminish the rewards an individual enjoys should they success. For example, if 1000 offices are available, whether 600 or 800 citizens rebel, there are enough offices for each to get one. This feature shares the non-rival aspect of the psychological rewards setting. However, if the number of participants exceed 1000, further increases in the number of participants reduces the chances that each rebel receives an office upon success because there will not be enough government offices to go around. This feature shares the rival aspect of the material rewards setting.

The real world, of course, is not so clear cut. More offices can be created and responsibilities may be shared. However, the insight that government offices tend to be more discrete than, for example, cash, diamonds, or land remains true. As such, in settings where such offices are the main reward of victory, the effect of repression on the rebel movement falls between the effects in settings with pure (continuous) material rewards and settings with psychological rewards.

Relaxing Informational Assumptions

In the text, we focused on the case in which players share a prior that θ is distributed uniformly on $\mathbb{R}$ (improper prior). With a smooth (proper) prior, the same results obtain in the limit when the information content of the prior becomes vanishingly small, e.g., $\theta \sim N(\mu, \sigma_0)$ when σ_0 becomes unboundedly large. Here, we show that the same results also obtain for any smooth prior in the limit when the noise becomes vanishingly small ($\sigma \to 0$). We then provide numerical examples for a standard normal prior for both the case of a uniform distribution of noise and a standard normal distribution of noise. Finally, we provide additional numerical examples for the effect of a public signal about the strength of the regime (θ) in both settings with psychological and material rewards.

Consider the setting in the text, but suppose $\theta \sim G$, where $G(\cdot)$ is smooth and $G(\theta) \in (0,1)$ for all $\theta \in \mathbb{R}$. Let $g(\cdot)$ be the corresponding pdf. We begin by proving that $\theta^m > \theta^p$. From the belief consistency condition, $x^j = \theta^j + \sigma F^{-1}(\theta^j/a)$, $j \in \{p,m\}$. Because the right hand side is increasing in θ^j, it is invertible. Define $\Omega(\cdot)$, so that $\theta^j = \Omega(x^j)$. Thus, the indifference conditions can be written as:

$$c = \int_{-\infty}^{\Omega(x^m)} \frac{\mathrm{pdf}(\theta|x^m)}{F\left(\frac{x^m-\theta}{\sigma}\right)} d\theta = \int_{-\infty}^{\Omega(x^p)} \mathrm{pdf}(\theta|x^p) d\theta. \tag{29}$$

Because $F(\cdot)$ in the denominator is less that 1, we have:

$$c < \int_{-\infty}^{\Omega(x^p)} \frac{\mathrm{pdf}(\theta|x^p)}{F\left(\frac{x^p-\theta}{\sigma}\right)} d\theta.$$

Thus, $x^m = x^p$ (and hence $\theta^m = \theta^p$) cannot be part of the equilibrium in the material rewards setting. x^m (and hence θ^m) must adjust to restore the equilibrium. In the stable equilibrium, they must increase, so that higher costs c imply higher likelihoods of regime change. Thus, we have:

Proposition 7 *In a stable equilibrium of the material rewards setting, $\theta^m > \theta^p$.*

To further characterize the equilibrium regime change thresholds, we provide analytical results when the noise is very small ($\sigma \to 0$) and numeral results when noise is larger.

Analytical Results for Vanishingly Small Noise

Lemma 3 *θ^j, $j \in \{p,m\}$, is an equilibrium regime change threshold if it satisfies the following equation:*

$$c = \Gamma_j(\theta^j;\sigma) \equiv \frac{\int_{F^{-1}(\theta^j/a)}^{\infty} f(z)\ g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))\ \frac{1}{\boldsymbol{1}_{\{j=m\}}F(z)+\boldsymbol{1}_{\{j=p\}}} dz}{\int_{-\infty}^{\infty} f(z)\ g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))\ dz},$$

where $\boldsymbol{1}_{\{\cdot\}}$ is the indicator function.

Proof of Lemma 3: The belief consistency condition is:

$$\frac{\theta^j}{a} = F\left(\frac{x^j - \theta^j}{\sigma}\right). \tag{30}$$

The indifference condition is:

$$\begin{aligned}
c &= \int_{\theta=-\infty}^{\theta^j} \mathrm{pdf}(\theta|x^j)\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(\frac{x^m-\theta}{\sigma}\right) + \mathbf{1}_{\{j=p\}}}\ d\theta \\
&= \int_{\theta=-\infty}^{\theta^j} \frac{f\left(\frac{x^j-\theta}{\sigma}\right) g(\theta)}{\int_{\theta=-\infty}^{\infty} f\left(\frac{x^j-\theta}{\sigma}\right) g(\theta) d\theta}\ \frac{1}{\mathbf{1}_{\{j=m\}}F\left(\frac{x^m-\theta}{\sigma}\right) + \mathbf{1}_{\{j=p\}}}\ d\theta \\
&= \int_{z=z^j}^{\infty} \frac{f(z)\, g(x^j - \sigma z)}{\int_{z=-\infty}^{\infty} f(z)\, g(x^j - \sigma z) dz}\ \frac{1}{\mathbf{1}_{\{j=m\}}F(z) + \mathbf{1}_{\{j=p\}}}\ dz \\
&= \int_{z=F^{-1}(\theta^j/a)}^{\infty} \frac{f(z)\, g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))}{\int_{z=-\infty}^{\infty} f(z)\, g(\theta^j + \sigma(F^{-1}(\theta^j/a) - z))dz}\ \frac{1}{\mathbf{1}_{\{j=m\}}F(z) + \mathbf{1}_{\{j=p\}}}\ dz \quad \text{(from (30))},
\end{aligned}$$

where, in the third equality, we did a change of variables from θ to $z = \frac{x^j-\theta}{\sigma}$, with $z^j = \frac{x^j-\theta^j}{\sigma}$. $\square$

In the limit when $\sigma \to 0$, the terms involving $g(\cdot)$ in Lemma 3 will cancel, and θ^j simplifies to those in Proposition 2 in the text with improper uniform prior.

Proposition 8 *In the limit when the noise become vanishingly small ($\sigma \to 0$) we have:*

$$\lim_{\sigma \to 0} \Gamma_j(\theta^j;\sigma) = \begin{cases} 1 - \theta^p/a & ; j = p \\ -\log(\theta^m/a) & ; j = m, \end{cases}$$

so that $\theta^p = a(1-c)$ and $\theta^m = ae^{-c}$.

Proof of Proposition 8: From Lemma 3,

$$\lim_{\sigma \to 0} \Gamma_j(\theta^j;\sigma) = \int_{F^{-1}(\theta^j/a)}^{\infty} f(z)\ \frac{1}{\mathbf{1}_{\{j=m\}}F(z) + \mathbf{1}_{\{j=p\}}} dz = \begin{cases} 1 - F(F^{-1}(\theta^p/a)) & ; j = p \\ \log(1) - \log(F(F^{-1}(\theta^m/a))) & ; j = m. \end{cases}$$

$\square$

An Example: Uniform Noise

We next provide a simple example to demonstrate a special case of this general result. Suppose $F = U[-1, 1]$. Then,

$$\mathrm{pdf}(\theta|x_i) = \frac{\mathrm{pdf}(x_i|\theta)g(\theta)}{\int_{-\infty}^{\infty}\mathrm{pdf}(x_i|\theta)g(\theta)d\theta} = \begin{cases} \frac{\frac{1}{2\sigma}g(\theta)}{\int_{x_i-\sigma}^{x_i+\sigma}\frac{1}{2\sigma}g(\theta)d\theta} = \frac{g(\theta)}{G(x_i+\sigma)-G(x_i-\sigma)} & ;\theta-\sigma \le x_i \le \theta+\sigma \\ 0 & ;otherwise. \end{cases} \tag{31}$$

Thus, for a given $\hat{\theta}$ and $\hat{x}$,

$$\Pr(\theta < \hat{\theta}|x_i = \hat{x}) = \begin{cases} 0 & ;\hat{\theta} \le \hat{x}-\sigma \\ \frac{G(\hat{\theta})-G(\hat{x}-\sigma)}{G(\hat{x}+\sigma)-G(\hat{x}-\sigma)} & ;\hat{x}-\sigma \le \hat{\theta} \le \hat{x}+\sigma \\ 1 & ;\hat{x}+\sigma \le \hat{\theta}. \end{cases} \tag{32}$$

Similarly,

$$\Pr(x_i < \hat{x}|\theta = \hat{\theta}) = \begin{cases} 1 & ;\hat{\theta} \le \hat{x}-\sigma \\ \frac{\hat{x}-(\hat{\theta}-\sigma)}{2\sigma} & ;\hat{x}-\sigma \le \hat{\theta} \le \hat{x}+\sigma \\ 0 & ;\hat{x}+\sigma \le \hat{\theta}. \end{cases} \tag{33}$$

Lemma 4 *For any $\hat{x}$ and $\hat{\theta}$, we have:*

$$\lim_{\sigma\to 0}\Pr(\theta < \hat{\theta}|x_i = \hat{x}) = 1 - \lim_{\sigma\to 0}\Pr(x_i < \hat{x}|\theta = \hat{\theta}).$$

Proof of Lemma 4: From equations (32) and (33), the result is immediate for the cases of $\hat{\theta} \le \hat{x}-\sigma$ and $\hat{x}+\sigma \le \hat{\theta}$. For completeness, consider $\hat{x}-\sigma \le \hat{\theta} \le \hat{x}+\sigma$ and equation (32). Using a Taylor's expansion, in the limit $\sigma \to 0$, we have:

$$\begin{aligned} G(\hat{\theta}) - G(\hat{x}-\sigma) &= G(\hat{x}+(\hat{\theta}-\hat{x})) - G(\hat{x}-\sigma) = G(\hat{x}) + g(\hat{x})(\hat{\theta}-\hat{x}) - (G(\hat{x}) - g(\hat{x})\sigma) \\ &= g(\hat{x})(\hat{\theta}-\hat{x}+\sigma). \end{aligned} \tag{34}$$

Similarly,

$$G(\hat{x}+\sigma) - G(\hat{x}-\sigma) = G(\hat{x}) + g(\hat{x})\sigma - (G(\hat{x}) - g(\hat{x})\sigma) = g(\hat{x})2\sigma. \tag{35}$$

Combining equations (34) and (35), for $\hat{x}-\sigma \le \hat{\theta} \le \hat{x}+\sigma$, we have:

$$\lim_{\sigma\to 0}\frac{G(\hat{\theta})-G(\hat{x}-\sigma)}{G(\hat{x}+\sigma)-G(\hat{x}-\sigma)} = \frac{g(\hat{x})(\hat{\theta}-\hat{x}+\sigma)}{g(\hat{x})2\sigma} = 1 - \frac{\hat{x}-(\hat{\theta}-\sigma)}{2\sigma} = 1 - \Pr(x_i < \hat{x}|\theta = \hat{\theta}).$$

$\square$

As shown in Shadmehr (2019a,b), Lemma 4 is the statistical property that delivers the uniform beliefs property, which, in turn, delivers the result in Proposition 2 in the text.

Numerical Simulations for Larger Noise

We have established the results analytically in the asymptotic cases of vanishingly small noise and no prior information (about θ). The results also hold when the noise is sufficiently small, or there is sufficiently little common knowledge, e.g., the prior is $N(\mu, \sigma_0)$ and σ_0 is sufficiently large. We now provide numerical simulations to show that our results are not limited to the cases of very small noise or very little common knowledge, leaving to future research a fuller characterization of the interactions between information and motivation in contentious settings.

First, we continue the example above by providing a numerical example for the case of standard normal prior distribution and uniform noise distribution: $G = N(0,1)$, $F = U[-1,1]$, and $\sigma = 1$. From equation (33), the belief consistency condition can be written as:

$$\frac{\theta^j}{a} = \min\{1, \max\{0, \frac{1}{2} + \frac{x^j - \theta^j}{2\sigma}\}\},\ j \in \{p, m\}.$$

Because $\theta^j/a \in (0,1)$, we have:

$$\theta^j = \frac{x^j + \sigma}{\frac{2\sigma}{a} + 1},\ j \in \{p, m\}. \tag{36}$$

From equation (31), the indifference condition can be written as:

$$\begin{aligned} c &= \int_{-\infty}^{\theta^j} \frac{g(\theta) \cdot \mathbf{1}_{\{x^j - \sigma \leq \theta \leq x^j + \sigma\}}}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \Pr(x_i \leq x^j | \theta) + \mathbf{1}_{\{j=p\}}} d\theta \\ &= \int_{x^j - \sigma}^{\theta^j} \frac{g(\theta)}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \left(\frac{1}{2} + \frac{x^m - \theta}{2\sigma}\right) + \mathbf{1}_{\{j=p\}}} d\theta,\ j \in \{p, m\}. \end{aligned} \tag{37}$$

Substituting from (36) into (37) yields:

$$c = R^j(x^j) \equiv \int_{x^j - \sigma}^{\frac{x^j + \sigma}{\frac{2\sigma}{a} + 1}} \frac{g(\theta)}{G(x^j + \sigma) - G(x^j - \sigma)} \frac{1}{\mathbf{1}_{\{j=m\}} \left(\frac{1}{2} + \frac{x^m - \theta}{2\sigma}\right) + \mathbf{1}_{\{j=p\}}} d\theta,\ j \in \{p, m\}. \tag{38}$$

To demonstrate, suppose $G = N(0,1)$, and $\sigma = a = 1$, so that $\theta^j = \frac{x^j + 1}{3}$. Figure 6 shows $R^j(x^j)$, $j \in \{p, m\}$. Both $R^m(x)$ and $R^p(x)$ are decreasing, so that raising the costs (c) reduces the equilibrium threshold. Moreover, when c approaches 0, both x^m and x^p approach 2, implying that θ^m and θ^p approach 1. When, instead, c approaches 1, x^p approaches -1, so that

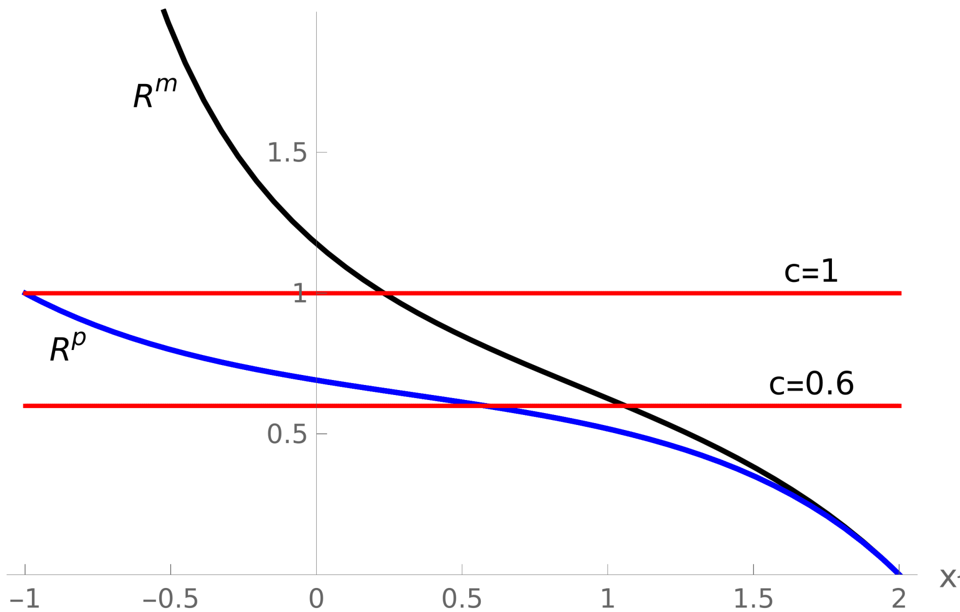

Figure 6: An example with $G = N(0,1)$, $F = U[-1,1]$, and $\sigma = a = 1$. From equation (38), the equilibrium threshold satisfies $R^j(x^j) = c$, $j \in \{p, m\}$. Note that an increase in c causes a sharper reduction in x^p than in x^m.

θ^p approaches 0. In contrast, x^m and hence θ^m both remain positive as in our model with no prior information about θ. Critically, $R^m(x)$ changes faster with x, so that as increase in costs c causes a smaller reduction in x^m than in x^p. Equation (38) shows the additional term $\frac{1}{2} + \frac{x^m - \theta}{2\sigma}$ in the denominator for the material rewards settings. When c increases, in any stable equilibrium, the equilibrium threshold x^j must fall so restore the indifference condition—citizens become less likely to revolt. In our example, the presence of $\frac{1}{2} + \frac{x^m - \theta}{2\sigma}$ in the denominator causes x^m to fall by less. That is, the same reduction in x^j has a larger effect in restoring the indifference condition and the equilibrium in the material rewards setting.

Next, we provide a numerical example when both the prior and the noise have the standard normal distribution: $G = N(0, \sigma_0)$, $F = N(0,1)$, and $\sigma_0 = \sigma = a = 1$. The belief consistency condition is:

$$\theta^j = \Phi\left(\frac{x^j - \theta^j}{\sigma}\right), \text{ so that } x^j = \theta^j + \sigma\Phi^{-1}(\theta^j). \tag{39}$$

The indifference conditions are:

$$c = \Phi\left(\frac{\theta^p - bx^p}{\sqrt{b\sigma^2}}\right) = \int_{-\infty}^{\theta^m} \frac{\frac{1}{\sqrt{b\sigma^2}}\phi\left(\frac{\theta - bx^m}{\sqrt{b\sigma^2}}\right)}{\Phi\left(\frac{x^m - \theta}{\sigma}\right)} \, d\theta, \text{ where } b = \frac{\sigma_0^2}{\sigma^2 + \sigma_0^2}. \tag{40}$$

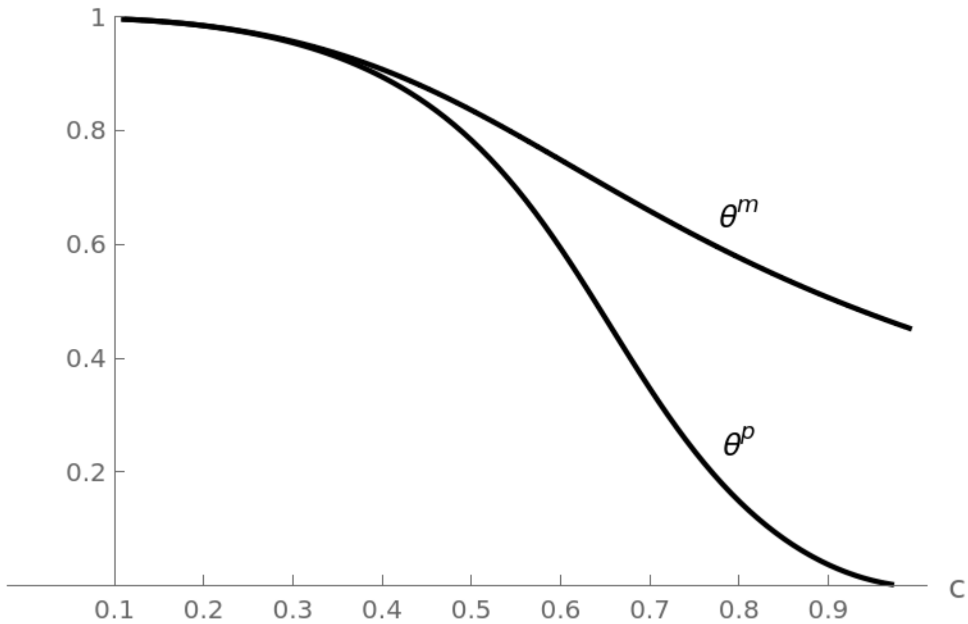

Figure 7: The equilibrium regime change threshold for the psychological rewards setting (θ^p), and material rewards setting (θ^m) when $\theta \sim N(0,1)$, $\epsilon_i \sim iidN(0,1)$, and $\sigma = a = 1$.

Substituting from (39) into (40) yields:

$$c = \Phi\left(\frac{(1-b)\theta^p - b\sigma\Phi^{-1}(\theta^p)}{\sqrt{b\sigma^2}}\right) = \int_{-\infty}^{\theta^m} \frac{\frac{1}{\sqrt{b\sigma^2}}\phi\left(\frac{\theta - b\theta^m - b\sigma\Phi^{-1}(\theta^m)}{\sqrt{b\sigma^2}}\right)}{\Phi\left(\frac{\theta^m + \sigma\Phi^{-1}(\theta^m) - \theta}{\sigma}\right)}\, d\theta. \tag{41}$$

Based on (41), Figure 7 demonstrates the equilibrium regime change thresholds θ^p and θ^m as functions of costs c when $\sigma_0 = \sigma = a = 1$.

Public Signal

To further highlight the logic behind our results, we also compare the effect of public signals about the regime's strength on the equilibrium regime change threshold in the psychological and material rewards settings. There is a link between this analysis and our discussion of a general prior. Suppose players share an improper uniform prior about θ as in the paper, but receive a noisy public signal p about θ in the form of $p = \theta + \sigma_p \nu$, where $\nu \sim H$. This setting is equivalent to players having a (proper) prior with mean p. For example, if $\nu \sim N(0,1)$ and $p = 1$, players will share a prior that $\theta \sim N(1, \sigma_p)$. In particular, beginning from no prior information about θ, the public signal will generates some common knowledge about θ. We now investigate the effect of a higher public signal in both settings. A higher public signal generates

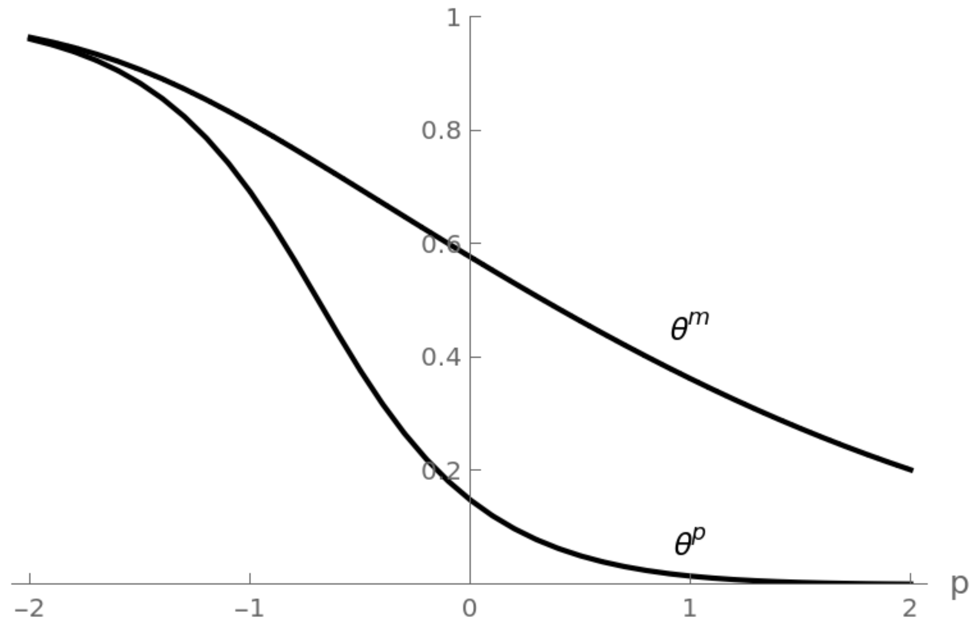

Figure 8: The equilibrium regime change threshold for the psychological rewards setting (θ^p), and material rewards setting (θ^m) as a function of a public signal $p = \theta + \nu$. Parameters: $G = N(0, 1)$, $\nu \sim N(0, 1)$, $F = N(0, 1)$, $\sigma = a = 1$.

common knowledge that the regime is stronger, and hence less likely to collapse. As a result, both θ^p and θ^m and the likelihood of regime change fall. Figure 8 illustrates the equilibrium regime change thresholds for different values of p for the came of Normal noise: $\nu \sim N(0, 1)$, $F = N(0, 1)$, $\sigma = a = 1$. As expected $\theta^m > \theta^p$. Moreover, as long as θ^p/θ^m is not too small, the marginal effect of a higher p is lower in the material rewards settings. The logic is the same as before: all else equal, a higher p reduces the citizens' incentives to revolt, and hence the likelihood of regime change. But, in the material rewards setting, a smaller number of revolutionaries makes the rewards of a successful regime change larger, thereby partially canceling the first effect. All else equal is important. An p increases, the probably of regime change in the psychological rewards setting falls to almost 0. Beginning from such a low probability, the marginal effect of a higher p then becomes very small. Thus, for the right comparison, one must compare the slopes of θ^p and θ^m when the levels are about the same ($\theta^p \approx \theta^m$). Now, it is clear that the marginal effect of a higher p is much smaller at in the material rewards setting.

Normalization of Material Rewards

In the text, we normalized material rewards to $\frac{a}{m}$, so that if the rebellion succeeds, the total available reward in both settings is a. We now explore the robustness of our results by using more general payoffs. In particular, we assume that material rewards are $\frac{k\times a}{m}$, for some $k>0$. Our analysis in the text corresponds to $k=1$. Given the payoff structure represented in Figure 1, this change is equivalent to normalizing the costs in the material rewards setting from $c\in(0,1)$ to $c/k\in(0,1)$. Then, the equilibrium regime change thresholds in Proposition 2 become: $\theta^p=1-c$ (as before) and $\theta^m=e^{-c/k}$ (new). Thus, $\frac{d\theta^p}{dc}=-1$ and $\frac{d\theta^m}{dc}=-\frac{1}{k}e^{-c/k}$, so that $\frac{d\theta^p}{dc}<\frac{d\theta^m}{dc}$ if and only if $e^{-c/k}<k$, i.e., $-k\log(k)<c$. When $k\geq 1$, the left hand side is non-positive, and the inequality holds for all $c>0$. When $k<1$, this inequality holds at the upper bound of $c=k<1$ if and only if $-k\log(k)<k$, i.e., $k>1/e\approx 0.37$. Then, there will be a threshold $\hat{c}\in(0,k)$ such that $-k\log(k)<c$ if and only if $c>\hat{c}(k)$. To summarize:

Proposition 9 *$\frac{d\theta^p}{dc}<\frac{d\theta^m}{dc}$ if and only if either $k\geq 1$, or $k>1/e$ and $c>\hat{c}(k)$, where $\hat{c}\in(0,k)$.*

Proposition 5 has a similar analogue. In the proof of Proposition 5, observe that changing c to c/k in the material rewards setting will change (20) to:

$$\Delta^m=\lim_{\sigma\to 0}\max\{\theta_2^m(\sigma)\}-\theta_1^m=e^{-ac/k}-e^{-c/k}\quad\text{and}\quad\Delta^p=\lim_{\sigma\to 0}\max\{\theta_2^p(\sigma)\}-\theta_1^p=(1-a)c.$$

Thus, $\Delta^m<\Delta^p$ if and only if $\frac{e^{-c/k}-e^{-ac/k}}{c-ac}>-1$, i.e., $\frac{e^{-c/k}-e^{-ac/k}}{c/k-ac/k}>-k$. Now, let $d=c/k\in(0,1)$ and observe that d can change independently of k. Thus, $\Delta^m<\Delta^p$ if and only if $\frac{e^{-d}-e^{-ad}}{d-ad}>-k$, for $d\in(0,1)$. Because e^{-x} is strictly decreasing and convex with $\frac{de^{-x}}{dx}\big|_{x=0}=-1$, this inequality holds for all $k\geq 1$. When $k<1$, as long as $k>1/e$, there exists $a,c/k\in(0,1)$ such that $\Delta^m<\Delta^p$. To see this, observe that $\frac{de^{-x}}{dx}\big|_{x=1}=-1/e$. Thus, we have:

Proposition 10 *Suppose the noise in the second period's private signals becomes vanishingly small, and we focus on the largest equilibrium. Conditional on failure in the first period, the chances of success is higher in the psychological rewards setting than in the material rewards setting if (i) $k\geq 1$, or (ii) $k>1/e$ and a and c are sufficiently large.*