[Supplementary material]

Urban form, infrastructure and spatial organisation in the Roman Empire

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In this supplementary online material we derive expected relationships between settlement population and a variety of measurable quantities related to urban infrastructure. These models are based on a view of settlements as social reactors in which individuals on average arrange themselves in space and time so as to balance the costs of movement with the benefits of social interaction. The average cost for a person to interact with others in space is set by the distance L across the area A and is given by $c = \varepsilon L = \varepsilon A^{1/2}$ (where ε is an energy per unit time and distance travelled, such as the metabolic cost of walking or of other forms of transportation, if available). The benefits of the resulting interactions is given by $y = \hat{g} a_0 l m N / A$ (where $\hat{g}$ is the average productivity of an interaction, a_0 is the distance at which interaction occurs, l is the average length of the path taken by an individual across space over some given time interval, and mN/A_m is the population density of the mixing space, denoted by m). By equating costs equal to benefits (spatial equilibrium), $c = y$, one arrives at

$A(N) = (mG/\varepsilon)^{\alpha} N^{\alpha}$, (1) where $G = \hat{g} a_0 l$ and $\alpha = 2/3$. Because $(mG/\varepsilon)^{\alpha}$ is approximately constant in any given context, Equation 1 implies that the total (maximal) area of mixing spaces should grow slower than population, such that mixing space is used more intensively, and becomes increasingly more crowded, as the population increases. The same logic can be used to define an expected relationship between the total built-up area and the total population. This relationship is known as the *amorphous settlement model:*

$A(N) = (G/\varepsilon)^{\alpha} N^{\alpha}$. (2) In this case, the pre-factor $(G/\varepsilon)^{\alpha}$ excludes the fraction m, and the length l incorporated into $G = \hat{g} a_0 l$ refers to the average daily path length of an individual across the settlement as a whole, not just within mixing space.

Equation 2 is just a first approximation, however, because it does not take into account any of the specific general structure of urban spaces, such as streets and buildings, and the constraints created by them. Note that Equation 2 sets an expectation on the maximum area of the settlement, because we have attributed net incomes to transportation, and not to other uses, thus setting the spatial boundary of the settlement by how much it may be worth to travel to any part of it.

As a result, the mixing area of the settlement is a subset of A. Conceptualiing how this area is threaded by communal spaces and street networks is necessary to make sense of our findings for ancient cities. The typical observation, supported by evidence in modern cities, is that area for communal spaces and street networks is gradually developed as the settlement grows in a density dependent way. In such a model, a portion of the overall settlement area is developed as an access network A_n of open spaces and paths as settlements grow.

In modern cities, the detailed assumption that follows is that the average length of streets *per capita* equals the average distance between individuals over A, which is $\sim(A/N)^{1/2}$. This works well in neighbourhoods where the block structure provides street fronts to all buildings. If there are buildings that are not directly connected to streets (as in slums) there will be a deficit of street network space. Thus, whether this assumption holds for ancient cities remains an open question. Following this assumption, the total area of the access network is:

$A_n \sim Nd = A^{1/2}N^{1/2}$. (3) From here, one can substitute aN^{α} for A and simplify, leading to: $A_n \sim N^{(\alpha+1)/2}$. (4) Equation 4 argues that, as settlements grow, movement and interaction become increasingly structured by the access network, and so the area taken up by "networked" (as opposed to amorphous) settlements grows with population more rapidly than in the situation where a constant fraction of the land area is set aside for streets; namely, in accordance with the settlement population to the $(\alpha + 1)/2$ power, which in the case of $\alpha = 2/3$ reduces to $5/6$.

The pre-factor is important in the calculation, however, if the distance along the network between people changes with the size of the settlement. The original calculation demands that the network of streets worked on a number of different levels, from narrow local paths to wider streets crossing the city, to thread this maximal area, so that the total street length, L_n, and total area of communal spaces and street networks, A_n, obey the relationships $L_n(N) = \frac{a}{\ell}N^{\alpha},\ A_n(N) = \frac{s_* a}{\ell}\ N^{(\alpha+1)/2}$, where the total land area per person, which defines the parameter a, is $a \sim N^{\alpha-1}$. In general, this decreases with population, i.e. the settlement densifies. The parameter s_* is the minimal width of the network and tends to not depend on city size; for example, imagine the width of doorways. Finally, the parameter ℓ is defined indirectly by the shortest average distance between people along the street network, which is $\frac{a}{\ell}\ N^{\alpha-1}$. These results give the average *width of streets* in the network as the ratio $\frac{A_n}{L_n}\ (N) = s_*\ N^{(1-\alpha)/2}$, which does not depend on ℓ. Two more fundamental quantities emerge from these calculations: the expected rate of socioeconomic outputs, $Y(N)$; and the costs of movement resulting from dissipation, *W(N)*, a proxy for congestion costs. These quantities are expected to scale with population in the same way so that settlements can exist as spatial equilibria of different sizes. Average socioeconomic outputs can be computed via the expected value of contacts in a city of size N, which is $Y(N) = G\ \frac{N^2}{A_n(N)} = \frac{Gs_*\ell}{a} N^{2-\alpha}$. The calculation of dissipation costs has been elaborated elsewhere, and results in the expression, $W(N) = W_0 N^{\frac{3-\alpha}{2}}$, where the pre-factor $W_0 = \frac{ar}{\ell\ s_*} J_0^2$, where *r* is the energy dissipation (resistance) per unit time and J_0 is the flux of people and goods in the street network (Bettencourt 2013). In general, we expect that if the distance between people along the network, ℓ , changes, J_0 would have to change as well to ensure that on average individuals have their basic needs (food, water, energy) met in similar ways across all settlements. This means that $J_0 {\sim} \ell$, leading to the dependence of ℓ of $W_0 {\sim}\ \ell$, which ensures that the ratio of benefits to costs of settlements of various sizes $Y(N)/W(N)$ is independent of city size *N*. Our empirical results suggest that, unlike in modern cities, $\ell \sim N^{\frac{1}{6}}$, is not independent of city size. As a result, the length of streets scales more slowly than land area, interactions are more frequent than given by the pure density of people in the street network and public spaces (now scaling with an exponent 4/3>1) and congestion costs are commensurately higher too, increasing faster (also with an exponent 4/3) than in contemporary cities. The consequences of these expectations are discussed in the main text.

Table S1. The measures of urban form used in this article

ID	Name	Area (ha)	Population	Forum / agora area (m²)	Street area (m²)	Street length (m)	Street width (m)	Block area (m²)
14	Athenae	225	58 114	25 967				

19	Cassope	18	1989		27 335	4522	7	9996
26	Corinthia	241	63 700	15 905				
110	Alexandria	972	410 535	75 000	1072 216	58 399	21	140 509
172	Carthago	343	102 170	15 000				
194	Sabratha	35	4837	2148	26 676	4057	7	2548
198	Sufetula	46	7037	1265	31 819	3 922	9	11 828
207	Thuburbo Maius	25	3085	1380				
210	Thugga	25	3085	792				
211	Lindum	175	6034		80 869	9995	9	7768
223	Augusta Taurinorum	54	8633		75 709	10 052	8	8992
225	Augusta Praetoria	41	5975	4889	72 451	7379	11	8 404
226	Forum Claudii	22	2655	2134	27 576	2042	15	9758
248	Alabanda	69	11 979	7179				
254	Aphrodisias	84	15 580	14 978	135 308	21 634	7	3507
276	Ephesus	263	71 587	21 090				
284	Hierapolis	77	13 870		58 651	12 073	6	4141
287	Iasus	34	4653	4376				
288	Ilium	25	3085		43 265	6819	7	9742
298	Miletus	97	18 882	29 024	132 666	22 748	7	3751
303	Nysa	44	6567	9143	29 863	5186	6	28 005
305	Pergamum	219	56 053	22 845				
315	Smyrna	194	47 672	20 000				
329	Baelo	11	1030	1046				
336	Corduba	99	19 404		146 177	21 517	8	6133
343	Iluro	10	907		22 497	3215	8	2135
347	Italica	49	7582		83 545	10 204	9	4371
378	Nicaea	114	23 429		111 157	11 847	9	17 310
385	Calleva	45	6767	1807	50 114	7888	7	12 858
386	Camulodunum	48	7 376		61 880	6704	9	14 787

387	Corinium	93	17 849	7483	103 162	7 985	14	27 766
392	Durovernum	59	9718		51 509	6470	8	62 503
394	Glevum	52	8209	1829				
395	Isca	37	5210	2 094	32 926	6056	6	8870
398	Lindum	41	6034	1331				
399	Londinium	160	36 851	9 702	162 663	13 291	13	56 509
404	Ratae	46	6969		40 485	3905	9	36 871
405	Venta Belgarum	58	9 498	2365	60 002	8935	8	17 254
407	Venta Silurum	18	1989	1016	22 724	2887	9	12 058
408	Verulamium	90	17 084	5941	79 307	9309	10	25 437
409	Viroconium	82	15 086	3759				
429	Anazarbus	150	33 807		160 090	12 626	13	11 235
500	Sarmizegetusa	32	4291	1187				
525	Doclea	25	3104	2571				
560	Anderitum	35	4837		42 022	7314	6	5406
567	Burdigala	86	16 187		75 752	12 166	7	24 233
571	Lugdunum Convenarum	49	7582	3296	31 276	6225	5	17 720
576	Vesunna	48	4 327	1259				
580	Augusta Ambianorum	165	38 476		277 294	23 782	13	25 210
587	Colonia Augusta Treverorum	131	28 211	3797	350 425	24 699	17	14 961
597	Agedincum	189	46 171		158 752	20 287	9	33 175
602	Augustodunum	172	40 666		209 238	13 182	17	15 572
613	Forum Segusiavorum	60	9938	2 554				
621	Lutetia	45	6767	6131	51 805	5056	11	25 313
624	Noviodunum	28	3 590		54 510	7274	8	6 216
645	Forum Iulii	49	7 628	4783	97 758	7770	15	5451
647	Glanum	32	4 291	1434				

652	Nemausus	175	41 539	3 026				
654	Tolosa	91	17 338		46 692	8260	6	15 166
658	Vienna	83	6 767	1 563				
660	Colonia Agrippinensis	75	13 426		109 645	13 056	9	14 963
661	Colonia Ulpia Traiana	80	14 596		101 045	8 543	12	31 359
668	Augusta Raurica	43	6368	2512				
669	Aventicum	36	5022		98 708	7742	16	10 461
671	Lopodunum	42	6171	2568				
674	Noviodunum	6	2290	2504	32 172	4236	9	3923
686	Baetulo	12	1157		21 996	4519	6	2350
691	Caesaraugusta	47	7172	983				
693	Carthago Nova	58	9 498	2 334				
696	Clunia	70	9327	10 627				
700	Emporiae	24	2922	2908				
728	Segobriga	11	1030	1318				
731	Tarraco	70	12 211	3 637				
776	Vesunna	20	7401		83 426	5781	12	53 784
784	Neapolis	82	15 086		134 904	17 728	9	27 399
788	Ostia	154	35 017	3788	177 630	18 475	9	25 082
790	Pompeii	60	9938	4227	46 827	10 173	5	7995
792	Praeneste	44	6567		33 585	7 219	5	7303
803	Surrentum	22	2601		21 731	4198	5	8181
812	Venafrum	27	3469		65 391	7267	10	11 247
831	Herdoniae	19	2138	1635				
845	Venusia	44	6567		35 719	6327	6	11 043
856	Grumentum	33	4471	2797	69 132	12 384	6	12 526
861	Paestum	126	26 781	9071	160 725	20 621	12	77 549
874	Alba Fucentia	33	4471	5218	24 545	4259	6	10 406
906	Asculum	39	5589		105 022	6853	18	7568
917	Pollentia	25	3085		34 670	4285	9	17 099

982	Luca	48	7376		55 279	7552	8	14 789
984	Luna	23	2760	2951	33 426	4811	8	8814
992	Roma	1783	923 406	169 061				
1 009	Ariminium	50	5782		47 252	7540	7	14 160
1 013	Cremona	57	9280		46 988	9417	6	5287
1 021	Placentia	50	7790		105 976	7270	17	25 366
1 025	Veleia	10	907	828				
1 026	Alba Pompeia	35	4916		78 018	7075	13	8729
1 030	Augusta Bagiennorum	16	1 718	2647	32 460	3493	8	14 654
1 037	Libarna	20	2290		22 434	5541	4	6 639
1 045	Aquleia	100	19 667	8707	75 724	12 320	6	23 307
1 048	Brixia	50	7790	4940	57 849	7826	8	6691
1 063	Tridentum	11	1062		14 451	2526	6	6134
1 065	Verona	52	8209	9125	84 421	10 882	9	10 781
1 074	Ticinum	44	6567		73 560	6857	11	9505
1 077	Augusta Emerita	81	14 841		155 080	18 033	10	5961
1 081	Conimbriga	23	2828	1519				
1 097	Antiphellus	6	458	634				
1 113	Cremna	38	5399	1559				
1 128	Perge	63	10 608	2661	117 078	6635	22	23 625
1 133	Sagalassus	20	2601	3250				
1 136	Side	38	5399	3679				
1 145	Xanthus	25	3085	4214				
1 158	Pella	100	29368	31 661	168 567	31 566	7	15 778
1 159	Philippi	51	7999	5156				
1 162	Thessalonica	161	70 860	7381	327 843	18 884	20	17 104
1 171	Cuicul	12	1157	1651				
1 210	Nicopolis	27	3420	1888				
1 213	Oescus	32	4291	3635				
1 237	Virunum	47	7188	5049	119 870	8402	16	12 075
1 246	Hippo Regius	60	9938	2480				

1 258	Thamugadi	50	7790	2224	59 014	9362	7	1618
1 294	Agrigentum	211	53 334		392 022	37 728	12	65 505
1 322	Tyndaris	30	3937		17 475	3233	6	5001
1 326	Apamea	67	11 517		128 777	16 672	10	9774
1 338	Laodicea	220	56 395		146 929	9681	17	36 028
1 340	Palmyra	120	25 091	3815				
1 365	Samaria	76	13 694	7123				

References

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