[Supplementary material]

The farming-inequality nexus: new insights from ancient western Eurasia

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Modelling labour-limited and land-limited economies

Labour-intensive and land-intensive farming.

Our discussion of labour-limited economies considered four inputs into the production of a farmed output: labour, manure, land and oxen. Manure (a stand-in for any practice that increases the productivity of the land) is termed land-augmenting, and oxen are labour-augmenting, meaning roughly that putting more manure on the land is equivalent to having more land and having the services of a team of oxen is equivalent to having more labour input. Labour and land are complements, meaning that the increased output made possible by having more land (or manure) – the marginal products of these two inputs --will be larger the more labour (or oxen) are devoted to it. Finally, there are diminishing returns to the use of either land or labour.

The two idealised systems of farming are represented in Figure S1 where any point in the space between the axes is a possible combination of labour and land (measured conventionally, say, in hectares cultivated and hours of work time, not effective units), with those in the upper left depicting labour-intensive farming (sometimes referred to as ‘gardens’) while combinations in the lower right are land-intensive farming (‘fields’ or ‘extensive’ farming). Outputs possible in the two farming systems are represented by the curved lines (iso-quants) representing the combination of amounts of land and labour (each with its associated complements, manure and animal traction) sufficient to produce some given amount of output, for example a thousand kilos of the crop in question over a given period of time.

The (negative of the) slope of the isoquants is the ratio of the marginal product of land (using the subscript *T,* for *t*erra) to the marginal product of labour. A steeper isoquant means a higher relative marginal product of land and hence that land is more valuable (scarce). Within each production system having more of one input reduces its relative marginal product (due to diminishing returns): this can be seen in the figure as the flattening of an isoquant as more land and less labour is applied to production.

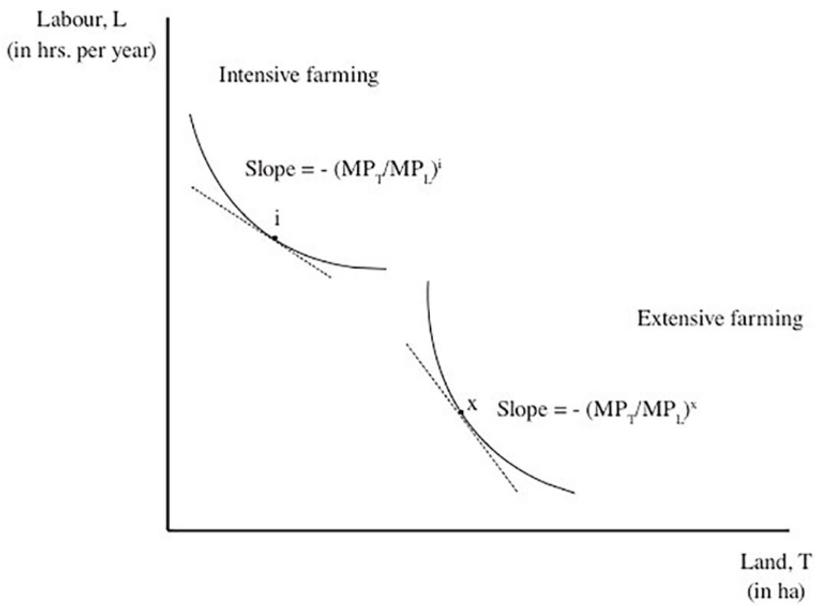

Figure S1. The relative value of land in labour-intensive ('intensive') and land-intensive ('extensive') farming systems.

Despite the diminishing marginal returns to each factor, because of the positive effect of manure on the marginal product of land and the negative effect of animal traction on the marginal product of labour, labour may be more valuable in the labour-intensive farming system and land more valuable in the extensive (that is, land-intensive) system. This is what we assume to be the case, as shown by the slopes of the isoquants flatter at point *i* and steeper at point *x* in the figure. Thus, extensive farming is a material wealth-limited system and intensive farming is labour-limited.

A production function summarises in mathematical form how inputs are related to outputs. The equation below illustrates the production function we have in mind, where the following notation is used: Q = quantity of output produced; m=amount of manure applied to the land; T= amount of land cultivated; x= a measure of ox team services and L = hours of labour services applied to cultivation. We assume that T and L are positive, while in one technology or the other oxen or manure may be not used at all (in our simplified model). A is a positive constant indicating how productive the technology is, while the land exponent, $\beta < 1$, is a measure of the importance of land (possibly augmented by manure) in the production process that is equal to the percentage increase in output associated with a one percent increase in land (1- β analogously is the relative importance of labour (possibly augmented by animal traction)).

$$Q = A(m+T)^{\beta}(x+L)^{1-\beta} \qquad (1)$$

In the equation (m+T) represents "effective land cultivated" recognising the land-augmenting nature of manure, while (x+L) is "effective labour devoted to cultivation" possibly augmented by animal traction. (We have defined the units in which manure is measured so that one unit of manure is equivalent to one hectare of land, and analogously for the use of an ox team and labour.) Because the exponents of effective land and effective labour sum to one, this production function describes a technology with constant returns to scale (doubling all inputs, for example, will double output), which we believe is empirically plausible for the crops and periods in question.

Table S1 clarifies our use of the terms extensive and intensive farming. We term intensive farming labour-limited because the value of an additional unit of labour (the marginal product of labour) is higher relative to the marginal product of land in that farming practice than in extensive farming. In the text, the isoquants are 'steep' (meaning that labour is more valued and hence more constraining than land) because the intensive farming methods (manuring in our model) are land augmenting, making land effectively abundant. This can be seen using the production function above by differentiating total output with respect to labour and then land to determine the marginal productivity of each or

$$Q_T = \beta A(m+T)^{\beta-1}(x+L)^{1-\beta} = \text{marginal product of land} \qquad (2)$$

$$Q_L = (1-\beta)A(m+T)^{\beta}(x+L)^{-\beta} = \text{marginal product of labour} \qquad (3)$$

and then expressing their ratio as

$$\frac{Q_T}{Q_L} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{x+L}{m+T}\right) = \text{scarcity of land relative to labour} \qquad (4)$$

This latter expression (with a minus sign in front) is the slope of the isoquants in Figure 1. Land is less valuable in the labour-intensive farming system because the use of manure makes land effectively abundant and the absence of animal traction ($x=0$) makes labour effectively scarce. It seems likely that labour-intensive farming that we describe would have been associated with a lower value of *β*, but we are not able to establish its value from archaeological data. See however the evidence below on societies in the historical and ethnographic record.

Table S1. Labour-intensive and extensive farming in our model. AP and MP respectively are average and marginal productivity of the entity in parentheses.

	Labour	*Manure*	*Land*	*Oxen*	*AP(land)/ AP(labour)*	*MP(land)/ MP(labour)*	*β*
Labour-intensive	Much	Much	Little	None	High	Low	Low
Extensive	Little	None	Much	Much	Low	High	High

Ethnographic evidence

A group of economists and ethnographers (including one of the current authors) studying small-scale societies in the ethnographic record wanted to know how important material wealth was to a person's livelihood compared to two other forms of wealth, which they termed 'embodied' (referring to an individual's health, strength and other individual capacities) and relational or network wealth referring to the individual's social ties (Borgerhoff-Mulder *et al*. 2009). They solicited ethnographers' judgments (for each of the three wealth classes in the population they studied) of the percentage difference in household well-being associated with a one percent difference in amount of a given wealth class, holding other wealth classes constant at the average for that population and requiring these percentage effects to sum to one. The ethnographers' assessment of the importance of material wealth in this thought experiment is a measure of *β*. Their estimates are shown in Figure S2.

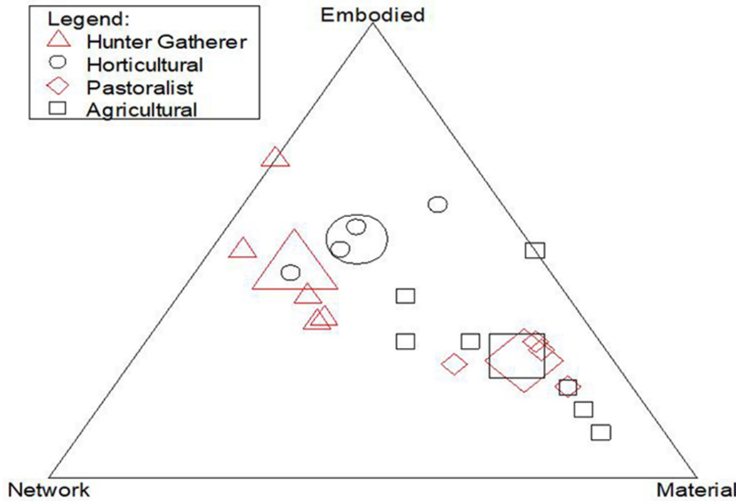

Figure S2. Labour and material wealth as limits on livelihoods. The location of the points indicates the relative importance of the forms of wealth at the three vertices. Large shapes are averages. Coordinates sum to one; the importance of the form of wealth named at each of the nodes is indicated by the distance between the point in the simplex and the edge opposite the node. Thus, a point at the node for material wealth would indicate that in that society the only form of wealth that mattered is material wealth (the coordinate for that dimension would be 1, and the other two, 0). The small triangle on the left edge means, for example, that according to the ethnographer of that hunting-and-gathering economy (the Hadza of Tanzania), material wealth was unimportant, while among the two other components, relational (network) wealth was 43 per cent and embodied wealth 57 per cent.

Consistent with descriptive ethnographies of these and other populations, embodied and relational wealth are relatively important for hunter-gatherers and horticulturalists, while material wealth is key in pastoral and agricultural populations.

Statistical estimates

Econometrically estimated values of β—the importance of material wealth—are available from diverse populations including two horticultural, two pastoral and eight small-scale agricultural economies (Borgerhoff-Mulder *et al*. 2009). These estimates are remarkably close to the ethnographers' estimates. Table S2 summarises these results.

Table S2. The importance of material wealth in horticultural, pastoral and agricultural economies. Sources given in column 6 and (with the exception of the first line) reported in (Borgerhoff–Mulder *et al*. 2009). The exponent for The Gambia was estimated by the current authors from data in the source. The numbers in column 5 are the averages for the mode of productions shown from the above source, except that agriculture (all cereal) is the average of the non-rice agriculture sites in that source and the entry for rice agriculture is the average of the Khasi and Bengali sites. The entry for The Gambia was estimated from data provided in the cited work.

Population (1)	Date (2)	Mode of production (3)	Material wealth (4)	Estimated β (Ethnographic estimate) (5)	Source of econometric estimate (7)
Yamaguchi Prefecture, China	1840s	Agriculture (rice|)	Land and fertilizer	0.45 (0.42)	(Nishikawa 1978)
Nyaturu TZ	1950s	Agro pastoralist	Cattle and land	0.76 (0.61)	(Massell 1963)
India	1950s	Agriculture (all cereal)	Land	0.68 (0.66)	(Bardhan 1973)
India	1950s	Agriculture (rice)	Land	0.33 (0.42)	(Bardhan 1973)
Borara Ethiopia	2000s	Pastoral	Livestock	0.84 (0.61)	(Berhanu *et al*. 2007)
Borara Ethiopia	2000s	Horticultural	Land livestock	0.23 (0.21)	(Berhanu *et al*. 2007)
Gambia	1940s	Horticultural	Land	0.11 (0.21)	(Haswell 1953)

Finally, we present evidence from the contemporary and historical record of the statistical relationship between a measure of inequality in material wealth and the importance of material wealth as measured by the ethnographers' estimates of β. The data in Figure S3 are consistent with our suggestion that greater importance of land and other forms of material wealth is associated with elevated levels of material wealth inequality.

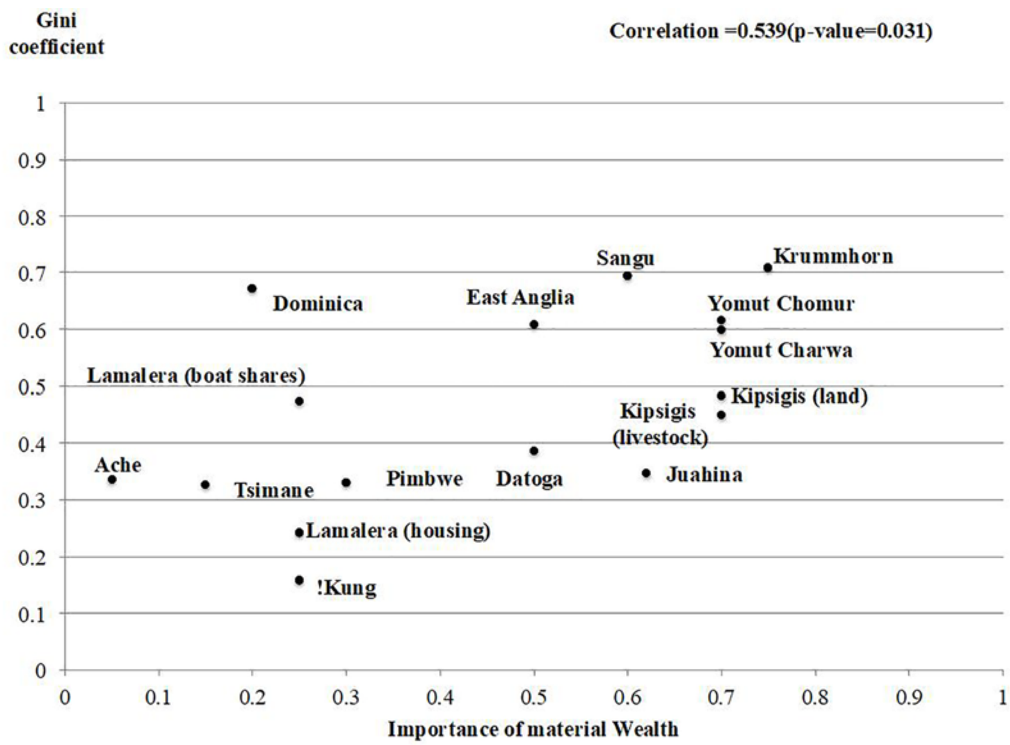

Figure S3. The importance of material wealth and material wealth inequality in small-scale societies. Data are from (Borgerhoff-Mulder et al*. 2009) except Pimbwe wealth inequality (a data correction) and !Kung wealth inequality from (Fochesato & Bowles 2017); the exponent is for Jo'hansi (a !Kung community) reported in the first source above, and Ache wealth inequality (data supplied by Kim Hill, with the anthropologists' employees eliminated from the sample).*

Analysis of difference in means

Table S3 shows the difference in means between the Gini coefficients of labour- and land-limited cases used in our analysis confirming that the means of the Gini coefficients of the two groups are substantially and significantly different in the whole period covered by the dataset, when including or not a time trend (respectively, column (2) and (3) of Table S3), in the period in which the two groups overlap (column (4)) and when using unadjusted Gini coefficients (column 5).

Table S3. Mean Gini coefficients for labour- and land-limited economies. In the whole period (9000 BC–AD 79) there are there are observations from 71 labour-limited and 19 land-limited economies. In the period of the overlap (5200–3900 BC) there are observations from 64 labour-limited and seven land-limited economies. Column 5 shows the difference in means between the two groups when the unadjusted Gini coefficients are used.

(1)	Unconditional mean – whole period (9000 BC–AD 79) (2)	Mean conditional on time trend – whole period (5200 BC– AD 79) (3)	Unconditional mean – period of overlap (5200–3900 BC) (4)	Unconditional mean – whole period Unadjusted Gini coefficients (5)
Labour-limited	0.230	0.313	0.220	0.223
Land-limited	0.546	0.591	0.443	0.619
t-stat diff in means	-10.247	-8.559	-4.957	-9.879

Labour-limited and land- limited cases

For the dataset for the labour-limited and land-limited cases, see Fochesato *et al.* (2019: OSM_Dataset).

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