Electoral Competition, Control and Learning *

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APPENDIX

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Appendix A: Robustness to Changes in our Model Primitives.

Qualitatively similar results emerge with other subtle changes to our model. Consider the case where the prior probability of being competent in the second period task differs between the incumbent and the opponent. Assuming that any revelation of information overrides these initial differences, then the analysis of these cases is analogous to that of equilibrium selection discussed above: when the incumbent has higher priors than the opponent, x is equal to 1; when the opponent has higher priors than the incumbent, x is equal to 0.

We have assumed that the prior probability of executive competence when implementing the period specific tasks coincides with that of successfully running a risky campaign. Of course these qualities may be very different. A simple extension relaxes this assumption by considering, for example, a situation where the prior probability that the opponent is successful in running a risky campaign is large relative to that of the incumbent successfully implementing reform. It is then very likely that the opponent will run a successful campaign. Anticipating this induces higher first period reform by the incumbent; indeed the case is similar to one where $x = 0$. Similarly when the situation is reversed so that the prior that the opponent successfully implements a risky campaign is relatively low, the opponent will avoid risky campaigns; this is similar to the case where campaigns are non-informative.

A further extension involves our informational environment. As we have seen, in the optimal equilibrium both with and without an informative campaign the voter conditions her choice of x on knowledge of p and r. Relaxing the assumption of full knowledge of these parameters implies that the voter must make a choice of equilibrium that involves a trade-off between the cost of over-investment in reform (due to gambling on success) and under-investment (due to fear of failure). The optimal choice of x depends on the likelihood of these different scenarios.

Moving to the timing of our game we consider whether our central findings are robust to a different sequence of moves. In particular, suppose that the order were reversed so that the opponent campaigns *before* the incumbent chooses policy. Then it is straightforward to prove that we will still observe under and over-investment for low and high values of p, respectively.[1]

We have not explored a situation where the politician is privately informed about his competence and so may use policy to signal his information. This is not a straightforward extension as it would

[1]Imagine that there exists an equilibrium (with $x \in [0, 1]$) where the incumbent plays safe for high enough p. If $x > 0$, the opponent has an incentive to run a risky campaign for high enough p as this increases his chance of being elected. This, in turn, implies that it is not optimal for him to play safe. When $x = 0$ the incumbent has incentives to run risky a risky campaign for high p.

involve a recasting of our model. Nevertheless, there are reasons to believe that our central finding that competitive elections with asymmetric learning technologies induce inefficiently high levels of reform would be robust to asymmetric information. In a separating equilibrium an informed competent politician would still over-invest in risky reform anticipating that a sequentially rational voter will reward him when doing so. Relatedly, Chen (2015) shows that private knowledge of ability leads to over-investment in risky projects where the agents payoff depends on the realized value of the project and his reputation.

Appendix B: Proofs

Before presenting proofs of our results, we provide microfoundations for our informational structure: $p^L < p^l < p < p^h < p^H$. Consider two random variables θ_1 and θ_2 describing the incumbent's competence when implementing risky policies in periods 1 and 2, respectively. They take value 1 when the incumbent is competent, and 0 otherwise. The joint distribution of these two Bernouilli random variables is described by the following matrix

	$\theta_2 = 0$	$\theta_2 = 1$
$\theta_1 = 0$	p_{00}	p_{01}
$\theta_1 = 1$	p_{10}	p_{11}

such that $p_{00} + p_{01} + p_{10} + p_{11} = 1$. Both variables are positively correlated if and only if $\frac{p_{11}}{p_{10}} > \frac{p_{01}}{p_{00}}$. We assume that the prior probability of being competent in any of the two periods is in both cases equal to p, it follows that $p_{10} = p_{01}$. We can thus rewrite the joint distribution as follows:

	$\theta_2 = 0$	$\theta_2 = 1$
$\theta_1 = 0$	$1 - p(2 - p^H)$	$p \cdot (1 - p^H)$
$\theta_1 = 1$	$p \cdot (1 - p^H)$	$p \cdot p^H$

where p and p^H are numbers between 0 and 1. Under this new notation, both variables are positively correlated if and only if $p^H > p$. Finally note that p^H is the conditional probability of a high realization in θ_2 given a high realization in θ_1 ($\Pr\{\theta_2 = 1 \mid \theta_1 = 1\} = \frac{p_{11}}{p_{10}+p_{11}} = p_H$). It is easy to show that the conditional realization of a high realization in θ_2 given a low realization in θ_1 is smaller than p when both random variables are positively correlated ($\Pr\{\theta_2 = 1 \mid \theta_1 = 0\} = \frac{p_{01}}{p_{00}+p_{10}} < p$).[2] We now consider a different variable θ_1^c that captures the competence at running a risky campaign in period 1. The joint distribution of these two variables can be summarized by the following matrix:

[2] This last conditional probability is our p^L.

	$\theta_2 = 0$	$\theta_2 = 1$
$\theta_1^c = 0$	t_{00}	t_{01}
$\theta_1^c = 1$	t_{10}	t_{11}

Assuming once again that the prior probability of a high realization of this new variable is p, and that it is positively correlated with θ_2, we can write the joint distribution of θ_1^C and θ_2 as:

	$\theta_2 = 0$	$\theta_2 = 1$
$\theta_1 = 0$	$1 - p(2 - p^h)$	$p \cdot (1 - p^h)$
$\theta_1 = 1$	$p \cdot (1 - p^h)$	$p \cdot p^h$

Finally, assuming a smaller positive correlation between these two variables than among the variables we analysed earlier is equivalent to stating that: $p_{11}p_{00} - p_{10}p_{01} > t_{11}t_{00} - t_{10}t_{01} > 0$. From this last inequality, with straightforward algebra we obtain $p^L < p^l < p < p^h < p^H$ where $p^h = \Pr\{\theta_2 = 1 \mid \theta_1^c = 1\}$ and $p^l = \Pr\{\theta_2 = 1 \mid \theta_1 = 0\}$.

Having provided micro-foundations for our information structure we can now prove our main results.

Proof of Lemma 1. When $p^H r < 1$ equation (1) is never satisfied and so $p^H r > 1$ is a necessary condition for the voter to desire risk taking in period 1. When $p^H r > 1$, by rearranging equation (1) we observe that risk taking occurs only when $pr > \frac{1+p}{1+p^H}$. $\square$

Proof of Proposition 1. The voter would like the incumbent to choose the risky policy in the first period whenever

$$p\left(r + \max\{p^H r, 1\}\right) + (1-p) > 2 \Leftrightarrow p\left(r + \max\{p^H r, 1\}\right) > 1 + p \tag{1}$$

which establishes the efficient level of risk. In equilibrium, if the voter reelects the incumbent with probability $x \in (0,1)$ when indifferent, the incumbent implements the risky policy in the first period whenever

$$p\left(r + \max\{p^H r, 1\}\right) > 1 + x. \tag{2}$$

Note that for any $p < x$, the RHS of 2 is smaller than the RHS of 1 which implies that, in equilibrium, risk taking will be lower than the efficient level ("fear of failure"). Instead, for any $p > x$ there risk taking will be higher than the efficient level ("gambling on success"). $\square$

Proofs of Propositions 2 and 3. We first note that, in the absence of any strategic effect on the incumbent's choice, the voter weakly prefers that the opponent chooses the risky campaign. When campaigns are informative the efficient level of risk involves the incumbent choosing risky whenever

$$p\left(r+\max\{p^H r,1\}\right)+(1-p)\Big(p\cdot\max\{p^h r,1\}+(1-p)\Big)>1+\Big(p\cdot\max\{p^h r,1\}+(1-p)\Big).$$

This inequality can be rewritten as:

$$p\left(r+\max\{p^H r,1\}\right)>1+p\Big(p\cdot\max\{p^h r,1\}+(1-p)\Big). \tag{3}$$

When $p^h r<1$ the opponent's action has no effect on the efficient level of risk-taking (the voter does not benefit from information about his competence). Note that the RHS in inequality (3) is greater than the RHS in inequality (1) when $p^h r>1$. Therefore, when campaigns are informative the voter desires less risk taking in period 1 as claimed in the first part of proposition 3.

Next, and in order to complete the proof of Proposition 2 and provide that of Proposition 3, we fully characterise the equilibrium actions of both incumbent and opponent when campaigns are informative.

We first consider the case when $p^h r<1$ where the opponent has no effect on the efficient level of risk taking. When the incumbent plays safe, the opposition's best response is safe when $p<1-x$. When anticipating safe play by the opponent, the incumbent adopts the risky policy in period 1 if $p\left(r+\max\{1,p^H r\}\right)>1+x$. The condition is the same as that obtained in the absence of an opposition, so the introduction of informative campaigns does not change the level of risk taking in period 1 in this case. Given that the efficient level of risk taking also remains unchanged when $p^h r<1$ with the introduction of informative campaigns, Proposition 1 applies: risk taking will be lower than the efficient level (fear of failure) for small values of p and higher than the efficient level (gambling on success) for large values of p. When the opposition's best response to safe is risky ($p>1-x$), the incumbent adopts the risky policy in period 1 if $p\left(r+\max\{1,p^H r\}\right)>2-p)$ which, upon inspection of equation 2, implies that the introduction of informative campaigns increases risk taking by the incumbent. Comparing this inequality with that in (1), that describes the efficient adoption of the risky policy, we can conclude that, when $p^h r<1$, investment in the risky policy is too low for small values of p and too high at larger values of p.

Turning to the case where $p^h r>1$, the efficient level of risk satisfies the following condition:

$$p\left(r+p^H r\right)>1+p(p\cdot p^h r+(1-p)). \tag{4}$$

As above, we consider two cases according to the best response of the opponent. When $p \cdot p^h r < (1-x)$ the opposition runs a safe campaign when the incumbent implements the safe policy. In this case, the incumbent implements a risky policy when $p\left(r + p^H r\right) > 1 + x$, which is the same condition as obtained in the absence of an opposition. Comparing the RHS of this inequality with that in (4) and noting that (since $x < 1$ and $p^h r > 1$) the RHS is smaller than that in equation (4) we conclude that the level of risk will be inefficiently low. When $p \cdot p^h r > \frac{1}{2}$ the opposition runs a risky campaign when the incumbent implements the safe policy. This implies that the incumbent implements a risky policy when $p(r + p^H r > 1 + (1-p)$. The RHS of this last inequality crosses once and from above the RHS of equation (4). Therefore for low values of p (p close to $\frac{1}{2 \cdot p^h r}$) the level of risk will be too low but for large p it will again be too high.

We can therefore confirm the claim that when campaigns are informative investment in the risky policy is too low (fear of failure) for low p and too high (gambling on success) for large p. □

Proof of Proposition 4. First, we note that introducing informative campaigns cannot harm voter welfare if it does not modify policy in the first period. If this were the case the voter is (weakly) better off by having more information without a change in the incumbent's actions. As shown in Proposition 3, introducing informative campaigns (weakly) increases risk-taking in the first period. Therefore, a necessary condition for welfare to decrease with their introduction is that there is no risk-taking when they are absent (this is so when $p\left(r + \max\{p^H r, 1\}\right) < 1 + x$) and risk-taking when they are present (this is so when $p\left(r + \max\{p^H r, 1\}\right) > 1 + (1-p)$).[3] The relevant curves are depicted in the figure below: in the absence of informative campaigns the risky action is taken for values of p and $1/r$ to the right of B; in their presence they are taken for values to the right of A; therefore be tween these curves is a region where the introduction of informative campaigns may harm welfare.

In this region, the absence of informative campaigns means the voter obtains a payoff of 2 as the safe policy is implemented in both periods. In their presence she instead obtains $p\left(r + \max\{p^H r, 1\}\right) + (1-p)\left(p \cdot \max\{p^h r, 1\} + (1-p)\right)$. This provides a range of values for r for which the voter might be worse off. Below we show that at the lowest possible value of r, the voter is *strictly* worse off (continuity of the payoff function ensures there is a non empty range of parameters for which the introduction of informative campaigns harms the voter).

[3]Note that the last inequality takes into account that the opponent runs a risky campaign after the incumbent plays safe in period 1. If this wasn't the case, the incumbent would have no incentives to modify his actions in the presence of informative campaigns. A necessary condition for this to hold true is that $p \cdot \max\{p^h r, 1\} > 1 - x$.

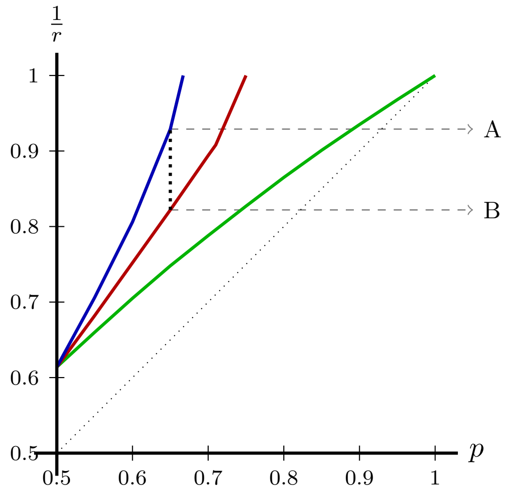

FIGURE 1. Illustration of the Proof for Proposition 4

We consider the lowest bound of r that satisfies the equation $p\left(r + \max\{p^H r, 1\}\right) > 1 + (1-p)$: $r = \frac{2}{p} - 1 - \max\{p^H r, 1\}$. We use these equalities and the fact that $p^h < p^H$ to obtain an upper bound for the voter's payoff with informative campaigns which is $1 + (1-p)(4 - 2p - pr)$. Finally note that this last expression is strictly smaller than 2 when p is large enough. $\square$

Proof of Proposition 5. We prove the first claim with respect to noninformative campaigns by comparing the equations for efficient and equilibrium levels of reform (see equations (1) and (2)) and observing that they are equivalent when $x = p$. As a consequence the voter can give the correct incentives so that the incumbent implements the efficient level of reform in period 1.

Next we prove that when campaigns are informative, the voter can avoid fear of failure by simply setting $x = 0$ (any $x \leq p$ works). In this case, the opponent runs a safe campaign when the incumbent implements the safe policy: by running a safe campaign the opponent is elected with probability 1; instead, the expected payoff of running a risky campaign is $p\max\{p^h r, 1\}$ which is always strictly smaller than 1. In turn, this implies that the incumbent implements the risky policy when $p(r+\max\{p^H r, 1\}) > 1$ –this equality is always satisfied when the efficient level of risk involves the incumbent choosing risky (see equation(3)).

Finally, we prove that when campaigns are informative the voter cannot avoid gambling on success. We prove this result by separately analysing both possible opponent's responses when the the incumbent chooses safe.

First, we consider the case when the opponent runs a safe campaign after the incumbent plays safe, i.e. $p \cdot \max\{1, p^h r\} < 1 - x$. The voter desires the adoption of the safe policy in period 1. when

$p\left(r+\max\{p^H r,1\}\right) < 1+p(p\cdot\max\{1,p^h r\}+(1-p))$. Yet, the incumbent implements the safe policy only when $p\left(r+\max\{p^H r,1\}\right) < 1+x$. Note that the minimal x that ensures the incumbent playing safe when it is optimal to do so is $x^* = p(p\cdot\max\{1,p^h r\}+(1-p))$. However, this re-election rule is too benign with the incumbent and makes the opponent not willing to run a safe campaign after the incumbent plays safe. Specifically, $p\cdot\max\{1,p^h r\} < 1-x = 1-p(p\cdot\max\{1,p^h r\}+(1-p))$ is not satisfied for p large enough.

Second, we consider the case when the opponent runs a risky campaign after the incumbent plays safe, i.e. $p\cdot\max\{1,p^h r\} > 1-x$. The incumbent chooses safe in period 1 only when $p(r+\max\{p^H r,1\}) < 1+(1-p)$. For large values of p the right hand side of the last inequality is smaller than that in equation (3). In other words, for large values of p, the voter desires a safe policy yet in equilibrium he cannot avoid the incumbent gambling on success.$\square$

References

Chen, Y. (2015): "Career Concerns and Excessive Risk-Taking," *Journal of Economics and Management Strategy*, 24(1), 110–130.