

# Supplementary Appendix for “Internal Opposition Dynamics and Restraints on Authoritarian Control”

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This supplemental appendix contains proofs of the results in the main text, as well as the technical details of Figures 1 and 2.

**Proof of Lemma 3.1** For the opposition leader,  $L$ , to accept an offer from the incumbent,  $I$ , and demobilize, then  $V_L(m = 0, x = \hat{x}) \geq V_L(m = 1)$ .  $L$ 's continuation payoff from mobilizing is the payoff from the current period plus the discounted payoff in the following period, given that the player adheres to the mobilization strategy and survives into the next period. Therefore,  $V_L(m = 1) = z - c_L + \delta[\phi V_L(m = 1)]$ , which simplifies to  $\frac{z - c_L}{1 - \delta\phi}$ . The continuation value from cooptation is calculated the same way, where  $V_L(m = 0, x = \hat{x}) = \hat{x} + \delta[(1 - \phi)V_L(m = 0)]$ . This simplifies to  $\frac{\hat{x}}{1 - \delta + \delta\phi}$ . Therefore, the opposition's optimal concession demand is the value of  $\hat{x}$ , in which  $\frac{\hat{x}}{1 - \delta + \delta\phi} = \frac{z - c_L}{1 - \delta\phi}$ . This results in

$$\hat{x} = \frac{(z - c_L)(1 - \delta + \delta\phi)}{1 - \delta\phi}. \blacksquare \tag{1.1}$$

**Proof of Proposition 3.1** In some cases  $\hat{x} \notin [0, 1]$ . First, when  $\hat{x} < 0$ , then  $L$  accepts any offer, including  $x = 0$ . This occurs when  $0 > \frac{(z - c_L)(1 - \delta + \delta\phi)}{1 - \delta\phi}$ , which simplifies to  $c_L > z$ . Thus,  $L$  unilaterally demobilizes when the costs of conflict exceed the potential benefits, regardless of the strength of the activist base. Additionally, when  $\hat{x} > 1$ , then  $L$  rejects all possible offers from the incumbent. This occurs when  $\frac{(z - c_L)(1 - \delta + \delta\phi)}{1 - \delta\phi} > 1$ , which occurs at large values of  $\phi$ , such that  $\phi > \frac{1 - (z - c_L)(1 - \delta)}{\delta(1 + z - c_L)}$ .

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As established in the Proof of Lemma 3.1, the optimal opposition demand is  $\hat{x} = \frac{(z-c_L)(1-\delta+\delta\phi)}{1-\delta\phi}$ .  $I$  is willing to offer  $\hat{x}$  as long as  $V_I(m=0, x=\hat{x}) \geq V_I(m=1)$ . His continuation value from permitting opposition mobilization is  $V_I(m=1) = 1 - z - c_L + \delta[\phi V_I(m=1) + (1-\phi)\frac{1}{1-\delta}]$ . Since  $I$  monopolizes rents if  $L$ 's base collapses, his continuation value from her collapse is  $\frac{1}{1-\delta}$ . Thus,  $V_I(m=1)$  simplifies to  $\frac{(1-\delta)(1-z-c_L)+\delta(1-\phi)}{(1-\delta\phi)(1-\delta)}$ .

In comparison, the continuation value from striking a deal is  $V_I(m=0, x=\hat{x}) = 1 - \hat{x} + \delta[\phi(\frac{1}{1-\delta}) + (1-\phi)V_I(m=0)]$ , which simplifies to  $\frac{(1-\delta)(1-\hat{x})+\delta\phi}{(1-\delta)(1-\delta+\delta\phi)}$ . Therefore, the incumbent prefers to meet  $L$ 's demand of  $\hat{x}$  when  $\frac{(1-\delta)(1-x)+\delta\phi}{(1-\delta)(1-\delta+\delta\phi)} \geq \frac{(1-\delta)(1-z-c_L)+\delta(1-\phi)}{(1-\delta\phi)(1-\delta)}$ . This results in  $\hat{x} \geq \frac{(z+c_L)(1-\delta+\delta\phi)}{1-\delta\phi}$ . When substituting (1.1) for  $\hat{x}$ , then  $\frac{(z-c_L)(1-\delta+\delta\phi)}{1-\delta\phi} \geq \frac{(z+c_L)(1-\delta+\delta\phi)}{1-\delta\phi}$ , which is always true since  $-c_L$  cannot be larger than  $c_L$ . Hence,  $I$  always offers  $\hat{x}$  as long as  $\hat{x} \in [0, 1]$ .

Lastly, to show that  $x = \hat{x}$  and  $m = 0$  iff  $x \geq \hat{x}$  is an equilibrium, this strategy profile must survive one-shot deviation:

$$V_L(m=0, x=\hat{x}) = \frac{x}{1-\delta+\delta\phi} \geq z - c_L + \delta[\phi V_L(m=0, x=\hat{x})]. \quad (1.2)$$

This results in  $\hat{x} \geq \frac{(z-c_L)(1-\delta+\delta\phi)}{1-\delta\phi}$ , which always holds since this is equal to the optimal demand made by  $L$ . Therefore, this stationary strategy profile is a subgame perfect equilibrium. ■

**Proof of Proposition 4.1** The continuation value of the rival leader ( $R$ ) from mounting a challenge,  $V_R(h=1)$ , must exceed 0 for  $R$  to compete for party leadership.  $R$ 's continuation value comprises the costs in the current period of mounting a challenge and the discounted payoffs of mobilizing in all future periods, which he receives with probability  $\phi$ :  $V_R(h=1) = -q + \delta\phi V_R(m=1)$ , where  $V_R(m=1) = z - c_R + \delta\phi V_R(m=1)$ , which simplifies to  $\frac{z-c_R}{1-\delta\phi}$ . Therefore,  $V_R(h=1) = -q + \frac{\delta\phi(z-c_R)}{1-\delta\phi}$  and  $R$  mounts a challenge when  $-q + \frac{\delta\phi(z-c_R)}{1-\delta\phi} \geq 0$ . Thus, for values of  $\phi \geq \phi^*$ , where  $\phi^* = \frac{q}{\delta(q+z-c_R)}$ ,  $R$  chooses  $h=1$ , and  $h=0$  otherwise. Lastly, when  $\phi^* > 1$ , then  $R$  never mounts a challenge since  $\phi$  cannot be sufficiently large to sustain internal competition. This occurs when  $q > \frac{\delta(z-c_R)}{1-\delta}$ .

Consider the case in which  $\phi < \phi^*$  and  $R$  does not mount a challenge.  $L$  accepts an offer from  $I$  and demobilizes when  $V_L(m=0, x=\tilde{x}) \geq V_L(m=1)$ , where  $\tilde{x}$  is  $L$ 's threshold of concessions above which she cooperates and demobilizes. In this case,  $V_L(m=1) = \frac{z-c_L}{1-\delta\phi}$ ,

as in the baseline model. However, the continuation value from cooptation differs, where  $V_L(m = 0, x = \tilde{x}) = x + \delta V_L(m = 0)$ . Since there is no threat of activist defections following cooptation,  $L$  is guaranteed to survive to the next period. Thus,  $V_L(m = 0, x = \tilde{x}) = \frac{x}{1-\delta}$ . Therefore, the opposition cooperates for any value of  $x$  such that  $\frac{x}{1-\delta} \geq \frac{z-c_L}{1-\delta\phi}$ . This results in the threshold of  $\tilde{x} = \frac{(z-c_L)(1-\delta)}{1-\delta\phi}$ . As in the baseline model,  $\tilde{x} < 0$  when  $c_L > z$ , leading to unilateral cooperation.  $\tilde{x} > 1$  when  $\phi > \frac{1-(z-c_L)(1-\delta)}{\delta}$ , which leads  $L$  to reject all offers.

$I$  is willing to offer  $\tilde{x}$  as long as  $V_I(m = 0, x = \tilde{x}) \geq V_I(m = 1)$ . The continuation value from permitting opposition mobilization is the same as in the baseline model:  $V_I(m = 1) = \frac{(1-\delta)(1-z-c_I)+\delta(1-\delta)}{(1-\delta\phi)(1-\delta)}$ . Yet,  $V_I(m = 0, x = \tilde{x}) = 1 - \tilde{x} + \delta V_I(m = 0)$ , which simplifies to  $\frac{1-\tilde{x}}{1-\delta}$ . Therefore,  $I$  offers  $\tilde{x}$  when  $\frac{1-\tilde{x}}{1-\delta} \geq \frac{(1-\delta)(1-z-c_I)+\delta(1-\delta)}{(1-\delta\phi)(1-\delta)}$ . This results in  $\tilde{x} \leq \frac{(z+c_I)(1-\delta)}{1-\delta\phi}$ . When substituting  $\frac{(z-c_L)(1-\delta)}{1-\delta\phi}$  for  $\tilde{x}$ , then  $\frac{(z-c_L)(1-\delta)}{1-\delta\phi} \leq \frac{(z+c_I)(1-\delta)}{1-\delta\phi}$ , which is always true since  $-c_L$  cannot be larger than  $c_I$ . Hence,  $I$  always offers  $\tilde{x}$  as long as  $\tilde{x} \in [0, 1]$ .

Lastly, to show that  $x = \tilde{x}$  and  $m = 0$  iff  $x \geq \tilde{x}$  is an equilibrium when  $\phi < \phi^*$ , this strategy profile must survive one-shot deviation:

$$V_L(m = 0, x = \tilde{x}) = \frac{x}{1-\delta} \geq z - c_L + \delta[\phi V_L(m = 0, x = \tilde{x})]. \quad (1.3)$$

This results in  $\tilde{x} \geq \frac{(z-c_L)(1-\delta)}{1-\delta\phi}$ , which always holds since this is equal to the optimal demand made by  $L$ . Therefore, this stationary strategy profile is a subgame perfect equilibrium. ■

**Proof of Proposition 4.2** As shown in the Proof of Proposition 4.1, the rival leader mounts a challenge when  $\phi \geq \phi^*$ . In this case, the leader's continuation value from cooptation is  $V_L(m = 0, x = \tilde{x}) = (1 - \phi)(x + \delta V_L(m = 0))$ . Since there is a chance that  $L$  is removed in the current period, then she only receives the current *and* future payoffs of cooptation with probability  $1 - \phi$ . Thus,  $V_L(m = 0, x = \tilde{x}) = \frac{x(1-\phi)}{1-\delta+\delta\phi}$ . Additionally,  $V_L(m = 1) = \frac{z-c_L}{1-\delta\phi}$ , as in the baseline model. Therefore, the opposition cooperates when  $\frac{x(1-\phi)}{1-\delta+\delta\phi} \geq \frac{z-c_L}{1-\delta\phi}$ . This inequality is true for any value of  $x \geq \tilde{x}$ , where  $\tilde{x} = \frac{(z-c_L)(1-\delta+\delta\phi)}{(1-\delta\phi)(1-\phi)}$ . As in the baseline model,  $\tilde{x} < 0$  when  $c_L > z$ , leading to unilateral cooperation.  $\tilde{x} > 1$  when  $z > \frac{(1-\delta\phi)(1-\phi)}{1-\delta+\delta\phi} + c_L$ , which leads  $L$  to reject all offers.

$I$  is willing to offer  $\tilde{x}$  as long as  $V_I(m = 0, x = \tilde{x}) \geq V_I(m = 1)$ . The continuation value

from permitting opposition mobilization is the same as in the baseline model:  $V_I(m = 1) = \frac{(1-\delta)(1-z-c_I)+\delta(1-\delta)}{(1-\delta\phi)(1-\delta)}$ . Yet,  $V_I(m = 0, x = \tilde{x}) = \phi[1+\delta V_I(m = 1)]+(1-\phi)[1-\tilde{x}+\delta V_I(m = 0)]$ . When the leader faces a challenge, the deal is temporarily undermined, leading to one period of disorganization (a payoff of 1 for  $I$ ) followed by permanent mobilization, with probability  $\phi$ . The deal remains intact and cooptation occurs with probability  $1 - \phi$ . This payoff simplifies to  $\frac{\delta\phi V_I(m=1)+1-\tilde{x}(1-\phi)}{1-\delta+\delta\phi}$ . Thus,  $V_I(m = 0, x = \tilde{x}) \geq V_I(m = 1)$  results in  $\tilde{x} \leq \frac{(z+c_I)(1-\delta)}{(1-\phi)(1-\delta\phi)}$ .

When substituting  $\frac{(z-c_L)(1-\delta+\delta\phi)}{(1-\delta\phi)(1-\phi)}$  for  $\tilde{x}$ , then  $\frac{(z-c_L)(1-\delta+\delta\phi)}{(1-\delta\phi)(1-\phi)} \leq \frac{(z+c_I)(1-\delta)}{(1-\phi)(1-\delta\phi)}$ , which is only true when  $\phi \leq \frac{(c_I-c_L)(1-\delta)}{\delta(z-c_L)}$ . Therefore, there are some values for which  $I$  chooses to permit mobilization, unlike in the baseline model.

Lastly, to show that  $x = \tilde{x}$  and  $m = 0$  iff  $x \geq \tilde{x}$  is an equilibrium when  $\phi \geq \phi^*$ , this strategy profile must survive one-shot deviation:

$$V_L(m = 0, x = \tilde{x}) = \frac{(1-\phi)x}{1-\delta+\delta\phi} \geq z - c_L + \delta[\phi V_L(m = 0, x = \tilde{x})]. \quad (1.4)$$

This results in  $\tilde{x} \geq \frac{(z-c_L)(1-\delta+\delta\phi)}{(1-\phi)(1-\delta\phi)}$ , which always holds since this is equal to the optimal demand made by  $L$ . Therefore, this stationary strategy profile is a subgame perfect equilibrium. ■

## Comparative Statics for Figures 1 and 2

The left panel of Figure 1 graphs the optimal demand of concessions—the concessions threshold  $\hat{x}$ —for the opposition leader, as defined in Lemma 3.1, for various levels of regime vulnerability  $z$ . Each line is plotted with the following values held constant:  $c_L = 0.1$  and  $\delta = 1$  (i.e. there is no discounting), while the top line is set at  $\phi = 0.75$  and the bottom line is set at  $\phi = 0.25$  as labeled. However, the dashed line represents the demanded concessions when there is no activist base that influences the opposition leader's political survival. In this case  $V_L(m = 0, x = \hat{x}) = \frac{x}{1-\delta}$  and  $V_L(m = 1) = \frac{z-c_L}{1-\delta}$ . Thus,  $\hat{x} = z - c_L$  and this line is plotted with  $c_L$  also held at 0.1.

The right panel of Figure 1 graphs the equilibrium concessions accepted by  $L$  as a function of  $\phi$ . Both curves are graphed with  $c_L = 0.1$  and  $\delta = 0.5$ . At high levels of  $\phi$  for the curve where  $z = 0.75$ ,  $\hat{x} > 1$ . Therefore  $L$  rejects all offers and  $x = 0$ .

Figure 2 graphs the threshold of concessions above which the opposition leader cooperates under unified leadership, as defined in Lemma 3.1, and divided leadership, as defined in (1.2) and (1.3). Both curves are graphed with the values  $z = .35$ ,  $c_L = 0.1$  and  $\delta = 0.5$ .