

## Genotypic Distributions of Three and Four loci under Self and Random Mating for the Derivation of Correlations between Pairwise QTL

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For any pair of QTL, they can be located in neighboring or non-neighboring marker intervals. For neighboring case, distribution of three flanking marker loci is needed to obtain the correlation between the two QTL. For non-neighboring case, distributions of four markers is required to obtain the correlation. In general, there are  $2^m$  different gametic genotypes and  $2^{2m-1} + 2^m/2$  zygotic genotypes for  $m$  loci in the populations from two inbred lines. For example, there are 4, 8 and 16 gametic genotypes and 10, 36 and 136 zygotic genotypes for  $m = 2, 3$  and 4. As different populations will undergo various number of meiosis cycle, the distributions of gametic and zygotic genotypes will vary. For selfing, Haldane and Waddington (1931) formulated the transition equations of the ten genotypic frequencies for  $m = 2$ . Kao and Zeng (2009) obtained the transition equations of the 36 genotypic frequencies for  $m = 3$ . The procedures of obtaining transition equations for  $m = 4$  are given below. Let 1 and 0 represent the capital and small-letter alleles, respectively, from  $P_1$  and  $P_2$ , so that the configurations of the 16 gametes can be represented as 1111, 0000, 1110, 0001, 1101, 0010, 1011, 0100, 0111, 1000, 1100, 0011, 1010, 0101, 1001, and 0110. In the  $F_2$  population, these 16 gamete frequencies can be obtained under Haldane map function (using the Markov property), and they are  $P(\underline{1111}) = P(\underline{0000}) = (1 - r_1)(1 - r_2)(1 - r_3)/2$ , where  $r_1, r_2$  and  $r_3$  are the recombination rates between the first and second genes, between the second and third genes, and the third and four genes, respectively. The other frequencies are  $P(\underline{1110}) = P(\underline{0001}) = (1 - r_1)(1 - r_2)r_3/2$ ,  $P(\underline{1101}) = P(\underline{0010}) = (1 - r_1)r_2r_3/2$ ,  $P(\underline{1011}) = P(\underline{0100}) = r_1r_2(1 - r_3)/2$ ,  $P(\underline{0111}) = P(\underline{1000}) = r_1(1 - r_2)(1 - r_3)/2$ ,  $P(\underline{1100}) = P(\underline{0011}) = (1 - r_1)r_2(1 - r_3)/2$ ,  $P(\underline{1010}) = P(\underline{0101}) = r_1r_2r_3/2$ , and  $P(\underline{1001}) = P(\underline{0110}) = r_1(1 - r_2)r_3/2$ , respectively. The random unification of these 16 gametes frequencies will produce the 136 different zygotes in a population. Because of symmetry, the complementary pairs have the same frequencies, there are 72 different genotypic frequencies. Let

the 136 zygotic genotypes occur in the proportions in a population:

$A_1$	$\frac{1111}{1111}$	$\frac{0000}{0000}$	$A_2$	$\frac{1111}{1110}$	$\frac{0000}{0001}$	$A_3$	$\frac{1110}{1110}$	$\frac{0001}{0001}$
$A_4$	$\frac{1111}{1011}$	$\frac{0000}{0100}$	$A_5$	$\frac{1111}{1010}$	$\frac{0000}{0101}$	$A_6$	$\frac{1110}{1011}$	$\frac{0001}{0100}$
$A_7$	$\frac{1110}{1010}$	$\frac{0001}{0101}$	$A_8$	$\frac{1011}{1011}$	$\frac{0100}{0100}$	$A_9$	$\frac{1011}{1010}$	$\frac{0100}{0101}$
$A_{10}$	$\frac{1010}{1010}$	$\frac{0101}{0101}$	$A_{11}$	$\frac{1111}{1101}$	$\frac{0000}{0010}$	$A_{12}$	$\frac{1111}{1100}$	$\frac{0000}{0011}$
$A_{13}$	$\frac{1110}{1101}$	$\frac{0001}{0010}$	$A_{14}$	$\frac{1110}{1100}$	$\frac{0001}{0011}$	$A_{15}$	$\frac{1111}{1001}$	$\frac{0000}{0110}$
$A_{16}$	$\frac{1101}{1011}$	$\frac{0010}{0100}$	$A_{17}$	$\frac{1111}{1000}$	$\frac{0000}{0111}$	$A_{18}$	$\frac{1100}{1011}$	$\frac{0011}{0100}$
$A_{19}$	$\frac{1010}{1101}$	$\frac{0101}{0010}$	$A_{20}$	$\frac{1001}{1110}$	$\frac{0110}{0001}$	$A_{21}$	$\frac{1110}{1000}$	$\frac{0001}{0111}$
$A_{22}$	$\frac{1100}{1010}$	$\frac{0011}{0101}$	$A_{23}$	$\frac{1011}{1001}$	$\frac{0100}{0110}$	$A_{24}$	$\frac{1011}{1000}$	$\frac{0100}{0111}$
$A_{25}$	$\frac{1010}{1001}$	$\frac{0101}{0110}$	$A_{26}$	$\frac{1010}{1000}$	$\frac{0101}{0111}$	$A_{27}$	$\frac{1101}{1101}$	$\frac{0010}{0010}$
$A_{28}$	$\frac{1101}{1100}$	$\frac{0010}{0011}$	$A_{29}$	$\frac{1100}{1100}$	$\frac{0011}{0011}$	$A_{30}$	$\frac{1101}{1001}$	$\frac{0010}{0110}$
$A_{31}$	$\frac{1101}{1000}$	$\frac{0010}{0111}$	$A_{32}$	$\frac{1100}{1001}$	$\frac{0011}{0110}$	$A_{33}$	$\frac{1100}{1000}$	$\frac{0011}{0111}$
$A_{34}$	$\frac{1001}{1001}$	$\frac{0110}{0110}$	$A_{35}$	$\frac{1001}{1000}$	$\frac{0110}{0111}$	$A_{36}$	$\frac{1000}{1000}$	$\frac{0111}{0111}$
$A_{37}$	$\frac{1111}{0111}$	$\frac{0000}{1000}$	$A_{38}$	$\frac{1111}{0110}$	$\frac{0000}{1001}$	$A_{39}$	$\frac{1110}{0111}$	$\frac{0001}{1000}$
$A_{40}$	$\frac{1110}{0110}$	$\frac{0001}{1001}$	$A_{41}$	$\frac{1111}{0011}$	$\frac{0000}{1100}$	$A_{42}$	$\frac{1011}{0111}$	$\frac{0100}{1000}$

$A_{43}$	$\frac{1111}{0010}$	$\frac{0000}{1101}$	$A_{44}$	$\frac{1011}{0110}$	$\frac{0100}{1001}$	$A_{45}$	$\frac{1010}{0111}$	$\frac{0101}{1000}$
$A_{46}$	$\frac{1110}{0011}$	$\frac{0001}{1100}$	$A_{47}$	$\frac{1110}{0010}$	$\frac{0001}{1101}$	$A_{48}$	$\frac{1010}{0110}$	$\frac{0101}{1001}$
$A_{49}$	$\frac{1011}{0011}$	$\frac{0100}{1100}$	$A_{50}$	$\frac{1011}{0010}$	$\frac{0100}{1101}$	$A_{51}$	$\frac{1010}{0011}$	$\frac{0101}{1100}$
$A_{52}$	$\frac{1010}{0010}$	$\frac{0101}{1101}$	$A_{53}$	$\frac{1111}{0101}$	$\frac{0000}{1010}$	$A_{54}$	$\frac{1101}{0111}$	$\frac{0010}{1000}$
$A_{55}$	$\frac{1111}{0100}$	$\frac{0000}{1011}$	$A_{56}$	$\frac{1100}{0111}$	$\frac{0011}{1000}$	$A_{57}$	$\frac{1110}{0101}$	$\frac{0001}{1010}$
$A_{58}$	$\frac{1101}{0110}$	$\frac{0010}{1001}$	$A_{59}$	$\frac{1110}{0100}$	$\frac{0001}{1011}$	$A_{60}$	$\frac{1100}{0110}$	$\frac{0011}{1001}$
$A_{61}$	$\frac{1111}{0001}$	$\frac{0000}{1110}$	$A_{62}$	$\frac{1011}{0101}$	$\frac{0100}{1010}$	$A_{63}$	$\frac{1101}{0011}$	$\frac{0010}{1100}$
$A_{64}$	$\frac{1001}{0111}$	$\frac{0110}{1000}$	$A_{65}$	$\frac{1111}{0000}$		$A_{66}$	$\frac{1000}{0111}$	
$A_{67}$	$\frac{1100}{0011}$		$A_{68}$	$\frac{1010}{0101}$		$A_{69}$	$\frac{1110}{0001}$	
$A_{70}$	$\frac{1011}{0100}$		$A_{71}$	$\frac{1001}{0110}$		$A_{72}$	$\frac{1101}{0010}$	

If random mating persists after  $F_2$  to produce AI populations, the transition equations for the frequencies of the 16 gametic genotypes can be derived using Geiringer's approach (1944), and, in turn, to obtain the 136 genotypic frequencies. If selfing persists after  $F_2$  to generate RI populations, the 72 transition equations for the frequencies of the 136 genotypes are shown below. For  $\frac{1111}{1111}$  genotype, the equation is

$$\begin{aligned}
A'_1 = & A_1 + \frac{1}{4}A_2 + \frac{1}{4}A_4 + \frac{(1-f_2)^2}{4}A_5 + \frac{f_2^2}{4}A_6 + \frac{1}{4}A_{11} + \frac{(1-r_3)^2}{4}A_{12} + \frac{r_3^2}{4}A_{13} + \frac{(1-r_2)^2}{4}A_{15} \\
& + \frac{r_2^2}{4}A_{16} + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{17} + \frac{[r_2(1-r_3)]^2}{4}A_{18} + \frac{(r_2r_3)^2}{4}A_{19} + \frac{[(1-r_2)r_3]^2}{4}A_{20} + \frac{1}{4}A_{37} \\
& + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{4}A_{38} + \frac{[(1-r_1)f_2 + r_1(1-f_2)]^2}{4}A_{39} + \frac{(1-r_1)^2}{4}A_{41} + \frac{r_1^2}{4}A_{42} \\
& + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{43} + \frac{(r_1f_1)^2}{4}A_{44} + \frac{[r_1(1-f_1)]^2}{4}A_{45} + \frac{[(1-r_1)f_1]^2}{4}A_{46} + \frac{(1-f_1)^2}{4}A_{53} \\
& + \frac{f_1^2}{4}A_{54} + \frac{[(1-f_1)(1-r_3)]^2}{4}A_{55} + \frac{[(1-r_3)f_1]^2}{4}A_{56} + \frac{[r_3(1-f_1)]^2}{4}A_{57} + \frac{(r_3f_1)^2}{4}A_{58} \\
& + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{61} + \frac{(r_1r_2)^2}{4}A_{62} + \frac{[(1-r_1)r_2]^2}{4}A_{63} + \frac{[r_1(1-r_2)]^2}{4}A_{64}
\end{aligned}$$

$$\begin{aligned}
& + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{65} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{66} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{67} \\
& + \frac{(r_1 r_2 r_3)^2}{2} A_{68} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{69} + \frac{[r_1 r_2(1-r_3)]^2}{2} A_{70} + \frac{[r_1(1-r_2)r_3]^2}{2} A_{71} \\
& + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{72},
\end{aligned}$$

where  $A'_1$  denotes the frequency of  $\frac{1111}{1111}$  genotype in the next generation,  $f_1 = r_1(1-r_2) + r_2(1-r_1)$  and  $f_2 = r_2(1-r_3) + r_3(1-r_2)$  ( $1-f_1 = (1-r_1)(1-r_2) + r_1 r_2$  and  $1-f_2 = (1-r_2)(1-r_3) + r_2 r_3$ ).

The other 71 equations for the remaining genotypic frequencies are given below.

$$\begin{aligned}
A'_2 &= \frac{1}{2} A_2 + \frac{f_2(1-f_2)}{2} A_5 + \frac{f_2(1-f_2)}{2} A_6 + \frac{r_3(1-r_3)}{2} A_{12} + \frac{r_3(1-r_3)}{2} A_{13} \\
&+ \frac{(1-r_2)^2 r_3(1-r_3)}{2} A_{17} + \frac{r_2^2 r_3(1-r_3)}{2} A_{18} + \frac{r_2^2 r_3(1-r_3)}{2} A_{19} + \frac{(1-r_2)^2 r_3(1-r_3)}{2} A_{20} \\
&+ \frac{[(1-r_1)(1-f_2) + r_1 f_2][(1-r_1)f_2 + r_1(1-f_2)]}{2} (A_{38} + A_{39}) \\
&+ \frac{(1-r_1)^2 f_1(1-f_1)}{2} A_{43} + \frac{r_1^2 f_1(1-f_1)}{2} A_{44} + \frac{r_1^2 f_1(1-f_1)}{2} A_{45} + \frac{(1-r_1)^2 f_1(1-f_1)}{2} A_{46} \\
&+ \frac{(1-f_1)^2 r_3(1-r_3)}{2} A_{55} + \frac{f_1^2 r_3(1-r_3)}{2} A_{56} + \frac{(1-f_1)^2 r_3(1-r_3)}{2} A_{57} + \frac{f_1^2 r_3(1-r_3)}{2} A_{58} \\
&+ [(1-r_1)(1-r_2)]^2 r_3(1-r_3) A_{65} + [r_1(1-r_2)]^2 r_3(1-r_3) A_{66} + [(1-r_1)r_2]^2 r_3(1-r_3) A_{67} \\
&+ (r_1 r_2)^2 r_3(1-r_3) A_{68} + [(1-r_1)(1-r_2)]^2 r_3(1-r_3) A_{69} + (r_1 r_2)^2 r_3(1-r_3) A_{70} \\
&+ [r_1(1-r_2)]^2 r_3(1-r_3) A_{71} + [(1-r_1)r_2]^2 r_3(1-r_3) A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_3 &= \frac{1}{4} A_2 + A_3 + \frac{f_2^2}{4} A_5 + \frac{(1-f_2)^2}{4} A_6 + \frac{1}{4} A_7 + \frac{r_3^2}{4} A_{12} + \frac{(1-r_3)^2}{4} A_{13} + \frac{1}{4} A_{14} + \frac{[(1-r_2)r_3]^2}{4} A_{17} \\
&+ \frac{(r_2 r_3)^2}{4} A_{18} + \frac{[r_2(1-r_3)]^2}{4} A_{19} + \frac{[(1-r_2)(1-r_3)]^2}{4} A_{20} + \frac{(1-r_2)^2}{4} A_{21} + \frac{r_2^2}{4} A_{22} \\
&+ \frac{[(1-r_1)f_2 + r_1(1-f_2)]^2}{4} A_{38} + \frac{[(1-r_1)(1-f_2) + r_1 f_2]^2}{4} A_{39} + \frac{1}{4} A_{40} + \frac{[(1-r_1)f_1]^2}{4} A_{43} \\
&+ \frac{[r_1(1-f_1)]^2}{4} A_{44} + \frac{(r_1 f_1)^2}{4} A_{45} + \frac{[(1-r_1)(1-f_1)]^2}{4} A_{46} + \frac{(1-r_1)^2}{4} A_{47} + \frac{r_1^2}{4} A_{48} \\
&+ \frac{[r_3(1-f_1)]^2}{4} A_{55} + \frac{(r_3 f_1)^2}{4} A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{4} A_{57} + \frac{[(1-r_3)f_1]^2}{4} A_{58} + \frac{(1-f_1)^2}{4} A_{59} \\
&+ \frac{f_1^2}{4} A_{60} + \frac{[(1-r_1)(1-r_2)]^2}{4} A_{61} + \frac{(r_1 r_2)^2}{4} A_{62} + \frac{[(1-r_1)r_2]^2}{4} A_{63} + \frac{[r_1(1-r_2)]^2}{4} A_{64} \\
&+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{65} + \frac{[r_1(1-r_2)r_3]^2}{2} A_{66} + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{67} + \frac{[r_1 r_2(1-r_3)]^2}{2} A_{68} \\
&+ \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{69} + \frac{(r_1 r_2 r_3)^2}{2} A_{70} + \frac{r_1(1-r_2)(1-r_3)]^2}{2} A_{71} \\
&+ \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_4 &= \frac{1}{2}A_4 + \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{r_2(1-r_2)}{2}(A_{15} + A_{16}) + \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17} + A_{18}) \\
&+ r_3^2(A_{19} + A_{20})] + \frac{r_1(1-r_1)}{2}(A_{41} + A_{42}) + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) \\
&+ \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
&+ r_1(1-r_1)r_2(1-r_2)[(1-r_3)^2(A_{65} + A_{66} + A_{67} + A_{70}) + r_3^2(A_{68} + A_{69} + A_{71} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_5 &= \frac{(1-f_2)^2}{2}A_5 + \frac{f_2^2}{2}A_6 + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) \\
&+ \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43} + A_{45}) + f_1^2(A_{44} + A_{46})] \\
&+ r_1r_2r_3(1-r_1)(1-r_2)(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_6 &= \frac{f_2^2}{2}A_5 + \frac{(1-f_2)^2}{2}A_6 + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) \\
&+ \frac{r_1(1-r_1)}{2}[f_1^2(A_{43} + A_{45}) + (1-f_1)^2(A_{44} + A_{46})] \\
&+ r_1r_2r_3(1-r_1)(1-r_2)(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_7 &= \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{1}{2}A_7 + \frac{r_2(1-r_2)}{2}[r_3^2(A_{17} + A_{18}) + (1-r_3)^2(A_{19} + A_{20})] \\
&+ \frac{r_2(1-r_2)}{2}(A_{21} + A_{22}) + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) \\
&+ \frac{r_1(1-r_1)}{2}(A_{47} + A_{48}) + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
&+ r_1(1-r_1)r_2(1-r_2)[r_3^2(A_{65} + A_{65} + A_{67} + A_{70}) + (1-r_3)^2(A_{69} + A_{69} + A_{71} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_8 &= \frac{1}{4}A_4 + \frac{f_2^2}{4}A_5 + \frac{(1-f_2)^2}{4}A_6 + A_8 + \frac{1}{4}A_9 + \frac{r_2^2}{4}A_{15} + \frac{(1-r_2)^2}{4}A_{16} + \frac{[r_2(1-r_3)]^2}{4}A_{17} \\
&+ \frac{[(1-r_2)(1-r_3)]^2}{4}A_{18} + \frac{[(1-r_2)r_3]^2}{4}A_{19} + \frac{(r_2r_3)^2}{4}A_{20} + \frac{1}{4}A_{23} + \frac{(1-r_3)^2}{4}A_{24} \\
&+ \frac{r_3^2}{4}A_{25} + \frac{r_1^2}{4}A_{41} + \frac{(1-r_1)^2}{4}A_{42} + \frac{(r_1f_1)^2}{4}A_{43} + \frac{[(1-r_1)(1-f_1)]^2}{4}A_{44} \\
&+ \frac{[(1-r_1)f_1]^2}{4}A_{45} + \frac{[r_1(1-f_1)]^2}{4}A_{46} + \frac{1}{4}A_{49} + \frac{[(1-r_1)(1-f_1) + r_1f_1]^2}{4}A_{50}
\end{aligned}$$

$$\begin{aligned}
& + \frac{[(1-r_1)f_1 + r_1(1-f_1)]^2}{4} A_{51} + \frac{[(1-r_3)(1-f_1)]^2}{4} A_{55} + \frac{[(1-r_3)f_1]^2}{4} A_{56} \\
& + \frac{[r_3(1-f_1)]^2}{4} A_{57} + \frac{(r_3 f_1)^2}{4} A_{58} + \frac{(1-f_1)^2}{4} A_{59} + \frac{f_1^2}{4} A_{60} + \frac{(r_1 r_2)^2}{4} A_{61} \\
& + \frac{[(1-r_1)(1-r_2)]^2}{4} A_{62} + \frac{[r_1(1-r_2)]^2}{4} A_{63} + \frac{[(1-r_1)r_2]^2}{4} A_{64} + \frac{[r_1 r_2(1-r_3)]^2}{2} A_{65} \\
& + \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{66} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{67} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{68} \\
& + \frac{(r_1 r_2 r_3)^2}{2} A_{69} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{70} + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{71} + \frac{[r_1(1-r_2)r_3]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_9 & = \frac{f_2(1-f_2)}{2}(A_5 + A_6) + \frac{1}{2}A_9 + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17} + A_{20}) + (1-r_2)^2(A_{18} + A_{19})] \\
& + \frac{r_3(1-r_3)}{2}(A_{24} + A_{25}) + \frac{f_1(1-f_1)}{2}[r_1^2(A_{43} + A_{46}) + (1-r_1)^2(A_{44} + A_{45})] \\
& + \frac{[(1-r_1)(1-f_1) + r_1 f_1][(1-r_1)f_1 + r_1(1-f_1)]}{2}(A_{50} + A_{51}) \\
& + \frac{r_3(1-r_3)}{2}[(1-f_1)^2(A_{55} + A_{57}) + f_1^2(A_{56} + A_{58})] \\
& + r_3(1-r_3)[r_1^2 r_2^2(A_{65} + A_{69}) + (1-r_1)^2 r_2^2(A_{66} + A_{71}) + r_1^2(1-r_2)^2(A_{67} + A_{72}) \\
& + (1-r_1)^2(1-r_2)^2(A_{68} + A_{70})]
\end{aligned}$$

$$\begin{aligned}
A'_{10} & = \frac{(1-f_2)^2}{4} A_5 + \frac{f_2^2}{4} A_6 + \frac{1}{4} A_7 + \frac{1}{4} A_9 + A_{10} + \frac{(r_2 r_3)^2}{4} A_{17} + \frac{[(1-r_2)r_3]^2}{4} A_{18} \\
& + \frac{[(1-r_2)(1-r_3)]^2}{4} A_{19} + \frac{[r_2(1-r_3)]^2}{4} A_{20} + \frac{r_2^2}{4} A_{21} + \frac{(1-r_2)^2}{4} A_{22} + \frac{r_3^2}{4} A_{24} \\
& + \frac{(1-r_3)^2}{4} A_{25} + \frac{1}{4} A_{26} + \frac{[r_1(1-f_1)]^2}{4} A_{43} + \frac{[(1-r_1)f_1]^2}{4} A_{44} + \frac{[(1-r_1)(1-f_1)]^2}{4} A_{45} \\
& + \frac{(r_1 f_1)^2}{4} A_{46} + \frac{r_1^2}{4} A_{47} + \frac{(1-r_1)^2}{4} A_{48} + \frac{[(1-r_1)f_1 + r_1(1-f_1)]^2}{4} A_{50} \\
& + \frac{[(1-r_1)(1-f_1) + r_1 f_1]^2}{4} A_{51} + \frac{1}{4} A_{52} + \frac{(1-f_1)^2}{4} A_{53} + \frac{f_1^2}{4} A_{54} + \frac{[r_3(1-f_1)]^2}{4} A_{55} \\
& + \frac{(r_3 f_1)^2}{4} A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{4} A_{57} + \frac{[(1-r_3)f_1]^2}{4} A_{58} + \frac{(r_1 r_2)^2}{4} A_{61} + \frac{[(1-r_1)(1-r_2)]^2}{4} A_{62} \\
& + \frac{[r_1(1-r_2)]^2}{4} A_{63} + \frac{[(1-r_1)r_2]^2}{4} A_{64} + \frac{(r_1 r_2 r_3)^2}{2} A_{65} + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{66} + \frac{[r_1(1-r_2)r_3]^2}{2} A_{67} \\
& + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{68} + \frac{[r_1 r_2(1-r_3)]^2}{2} A_{69} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{70} \\
& + \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{71} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{11} &= \frac{1}{2}A_{11} + \frac{r_3(1-r_3)}{2}(A_{12} + A_{13}) + \frac{r_2(1-r_2)}{2}(A_{15} + A_{16}) \\
&+ \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) + \frac{f_1(1-f_1)}{2}(A_{53} + A_{54}) \\
&+ \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
&+ \frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61} + A_{63}) + r_1^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)r_3(1-r_3)[(1-r_1)^2(A_{65} + A_{67} + A_{69} + A_{72}) + r_1^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{12} &= \frac{(1-r_3)^2}{2}A_{12} + \frac{r_3^2}{2}A_{13} + \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17} + A_{18}) + r_3^2(A_{19} + A_{20})] \\
&+ \frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55} + A_{56}) + r_3^2(A_{57} + A_{58})] \\
&+ r_2(1-r_2)[(1-r_1)^2(1-r_3)^2(A_{65} + A_{67}) + r_1^2(1-r_3)^2(A_{66} + A_{70}) + r_1^2r_3^2(A_{68} + A_{71})] \\
&+ (1-r_1)^2r_3^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{13} &= \frac{r_3^2}{2}A_{12} + \frac{(1-r_3)^2}{2}A_{13} + \frac{r_2(1-r_2)}{2}[r_3^2(A_{17} + A_{18}) + (1-r_3)^2(A_{19} + A_{20})] \\
&+ \frac{f_1(1-f_1)}{2}[r_3^2(A_{55} + A_{56}) + (1-r_3)^2(A_{57} + A_{58})] \\
&+ r_2(1-r_2)[(1-r_1)^2r_3^2(A_{65} + A_{67}) + r_1^2r_3^2(A_{66} + A_{70}) + r_1^2(1-r_3)^2(A_{68} + A_{71})] \\
&+ (1-r_1)^2(1-r_3)^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{14} &= \frac{r_3(1-r_3)}{2}(A_{12} + A_{13}) + \frac{1}{2}A_{14} + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) \\
&+ \frac{r_2(1-r_2)}{2}(A_{21} + A_{22}) + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
&+ \frac{f_1(1-f_1)}{2}(A_{59} + A_{60}) + \frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61} + A_{63}) + r_1^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)r_3(1-r_3)[(1-r_1)^2(A_{65} + A_{67} + A_{69} + A_{72}) + r_1^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{15} &= \frac{(1-r_2)^2}{2}A_{15} + \frac{r_2^2}{2}A_{16} + \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17} + A_{20}) + r_2^2(A_{18} + A_{19})] \\
&+ \frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61} + A_{64}) + r_2^2(A_{62} + A_{63})] \\
&+ r_1(1-r_1)r_3(1-r_3)[(1-r_2)^2(A_{65} + A_{66} + A_{69} + A_{71}) + r_2^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{16} &= \frac{r_2^2}{2}A_{15} + \frac{(1-r_2)^2}{2}A_{16} + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17} + A_{20}) + (1-r_2)^2(A_{18} + A_{19})] \\
&+ \frac{r_1(1-r_1)}{2}[r_2^2(A_{61} + A_{64}) + (1-r_2)^2(A_{62} + A_{63})] \\
&+ r_1(1-r_1)r_3(1-r_3)[r_2^2(A_{65} + A_{66} + A_{69} + A_{71}) + (1-r_2)^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{17} &= \frac{[(1-r_2)(1-r_3)]^2}{2}A_{17} + \frac{[r_2(1-r_3)]^2}{2}A_{18} + \frac{(r_2r_3)^2}{2}A_{19} + \frac{[(1-r_2)r_3]^2}{2}A_{20} \\
&+ r_1(1-r_1)[(1-r_2)^2(1-r_3)^2(A_{65} + A_{66}) + r_2^2(1-r_3)^2(A_{67} + A_{70}) + r_2^2r_3^2(A_{68} + A_{72}) \\
&+ (1-r_2)^2r_3^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{18} &= \frac{[r_2(1-r_3)]^2}{2}A_{17} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{18} + \frac{[(1-r_2)r_3]^2}{2}A_{19} + \frac{(r_2r_3)^2}{2}A_{20} \\
&+ r_1(1-r_1)[r_2^2(1-r_3)^2(A_{65} + A_{66}) + (1-r_2)^2(1-r_3)^2(A_{67} + A_{70}) \\
&+ (1-r_2)^2r_3^2(A_{68} + A_{72}) + r_2^2r_3^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{19} &= \frac{(r_2r_3)^2}{2}A_{17} + \frac{[(1-r_2)r_3]^2}{2}A_{18} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{19} + \frac{[r_2(1-r_3)]^2}{2}A_{20} \\
&+ r_1(1-r_1)[r_2^2r_3^2(A_{65} + A_{66}) + (1-r_2)^2r_3^2(A_{67} + A_{70}) + (1-r_2)^2(1-r_3)^2(A_{68} + A_{72}) \\
&+ r_2^2(1-r_3)^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{20} &= \frac{[(1-r_2)r_3]^2}{2}A_{17} + \frac{(r_2r_3)^2}{2}A_{18} + \frac{[r_2(1-r_3)]^2}{2}A_{19} + \frac{[(1-r_2)(1-r_3)]^2}{2}A_{20} \\
&+ r_1(1-r_1)[(1-r_2)^2r_3^2(A_{65} + A_{66}) + r_2^2r_3^2(A_{67} + A_{70}) + r_2^2(1-r_3)^2(A_{68} + A_{72}) \\
&+ (1-r_2)^2(1-r_3)^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{21} &= \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17} + A_{20}) + r_2^2(A_{18} + A_{19})] + \frac{(1-r_2)^2}{2}A_{21} + \frac{r_2^2}{2}A_{22} \\
&+ \frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61} + A_{64}) + r_2^2(A_{62} + A_{63})] \\
&+ r_1(1-r_1)r_3(1-r_3)[(1-r_2)^2(A_{65} + A_{66} + A_{69} + A_{71}) + r_2^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{22} &= \frac{r_3(1-r_3)}{2}[r_2^2(A_{17} + A_{20}) + (1-r_2)^2(A_{18} + A_{19})] + \frac{r_2^2}{2}A_{21} + \frac{(1-r_2)^2}{2}A_{22} \\
&+ \frac{r_1(1-r_1)}{2}[r_2^2(A_{61} + A_{64}) + (1-r_2)^2(A_{62} + A_{63})] \\
&+ r_1(1-r_1)r_3(1-r_3)[r_2^2(A_{65} + A_{66} + A_{69} + A_{71}) + (1-r_2)^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{23} &= \frac{r_2(1-r_2)}{2}(A_{15} + A_{16}) + \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) + \frac{1}{2}A_{23} \\
&+ \frac{r_3(1-r_3)}{2}(A_{24} + A_{25}) + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
&+ \frac{f_1(1-f_1)}{2}(A_{59} + A_{60}) + \frac{r_2(1-r_2)}{2}[r_1^2(A_{61} + A_{63}) + (1-r_1)^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)r_3(1-r_3)[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{24} &= \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17} + A_{18}) + r_3^2(A_{19} + A_{20})] + \frac{(1-r_3)^2}{2}A_{24} + \frac{r_3^2}{2}A_{25} \\
&+ \frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55} + A_{56}) + r_3^2(A_{57} + A_{58})] \\
&+ r_2(1-r_2)[r_1^2(1-r_3)^2(A_{65} + A_{67}) + (1-r_1)^2(1-r_3)^2(A_{66} + A_{70})] \\
&+ (1-r_1)^2r_3^2(A_{68} + A_{71}) + r_1^2r_3^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{25} &= \frac{r_2(1-r_2)}{2}[r_3^2(A_{17} + A_{18}) + (1-r_3)^2(A_{19} + A_{20})] + \frac{r_3^2}{2}A_{24} + \frac{(1-r_3)^2}{2}A_{25} \\
&+ \frac{f_1(1-f_1)}{2}[r_3^2(A_{55} + A_{56}) + (1-r_3)^2(A_{57} + A_{58})] \\
&+ r_2(1-r_2)[r_1^2r_3^2(A_{65} + A_{67}) + (1-r_1)^2r_3^2(A_{66} + A_{70})] \\
&+ (1-r_1)^2(1-r_3)^2(A_{68} + A_{71}) + r_1^2(1-r_3)^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{26} &= \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) + \frac{r_2(1-r_2)}{2}(A_{21} + A_{22}) \\
&+ \frac{r_3(1-r_3)}{2}(A_{24} + A_{25}) + \frac{1}{2}A_{26} + \frac{f_1(1-f_1)}{2}(A_{53} + A_{54}) \\
&+ \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
&+ \frac{r_2(1-r_2)}{2}[r_1^2(A_{61} + A_{63}) + (1-r_1)^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)r_3(1-r_3)[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{27} = & \frac{1}{4}A_{11} + \frac{r_3^2}{4}A_{12} + \frac{(1-r_3)^2}{4}A_{13} + \frac{r_2^2}{4}A_{15} + \frac{(1-r_2)^2}{4}A_{16} + \frac{(r_2r_3)^2}{4}A_{17} + \frac{[(1-r_2)r_3]^2}{4}A_{18} \\
& + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{19} + \frac{[r_2(1-r_3)]^2}{4}A_{20} + A_{27} + \frac{1}{4}A_{28} + \frac{1}{4}A_{30} + \frac{(1-f_2)^2}{4}A_{31} + \frac{f_2^2}{4}A_{32} \\
& + \frac{[(1-r_1)(1-f_2)]^2}{4}A_{43} + \frac{(r_1f_2)^2}{4}A_{44} + \frac{[r_1(1-f_2)]^2}{4}A_{45} + \frac{[(1-r_1)f_2]^2}{4}A_{46} \\
& + \frac{(1-r_1)^2}{4}A_{47} + \frac{r_1^2}{4}A_{48} + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{4}A_{50} + \frac{[(1-r_1)f_2 + r_1(1-f_2)]^2}{4}A_{51} \\
& + \frac{1}{4}A_{52} + \frac{f_1^2}{4}A_{53} + \frac{(1-f_1)^2}{4}A_{54} + \frac{(r_3f_1)^2}{4}A_{55} + \frac{[r_3(1-f_1)]^2}{4}A_{56} + \frac{[(1-r_3)f_1]^2}{4}A_{57} \\
& + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{58} + \frac{[(1-r_1)r_2]^2}{4}A_{61} + \frac{[r_1(1-r_2)]^2}{4}A_{62} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{63} \\
& + \frac{(r_1r_2)^2}{4}A_{64} + \frac{[(1-r_1)r_2r_3]^2}{2}A_{65} + \frac{(r_1r_2r_3)^2}{2}A_{66} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{67} \\
& + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{68} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{69} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{70} \\
& + \frac{[r_1r_2(1-r_3)]^2}{2}A_{71} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{28} = & \frac{r_3(1-r_3)}{2}(A_{12} + A_{13}) + \frac{r_3(1-r_3)}{2}[r_2^2(A_{17} + A_{20}) + (1-r_2)^2(A_{18} + A_{19})] + \frac{1}{2}A_{28} \\
& + \frac{f_2(1-f_2)}{2}(A_{31} + A_{32}) + \frac{f_2(1-f_2)}{2}[(1-r_1)^2(A_{43} + A_{46}) + r_1^2(A_{44} + A_{45})] \\
& + \frac{[(1-r_1)(1-f_2) + r_1f_2][r_1(1-f_2) + (1-r_1)f_2]}{2}(A_{50} + A_{51}) \\
& + \frac{r_3(1-r_3)}{2}[f_1^2(A_{55} + A_{57}) + (1-f_1)^2(A_{56} + A_{58})] + r_3(1-r_3)[(1-r_1)^2r_2^2(A_{65} + A_{69}) \\
& + r_1^2r_2^2(A_{66} + A_{71}) + (1-r_1)^2(1-r_2)^2(A_{67} + A_{72}) + r_1^2(1-r_2)^2(A_{68} + A_{70})]
\end{aligned}$$

$$\begin{aligned}
A'_{29} = & \frac{(1-r_3)^2}{4}A_{12} + \frac{r_3^2}{4}A_{13} + \frac{1}{4}A_{14} + \frac{[r_2(1-r_3)]^2}{4}A_{17} + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{18} + \frac{[(1-r_2)r_3]^2}{4}A_{19} \\
& + \frac{(r_2r_3)^2}{4}A_{20} + \frac{r_2^2}{4}A_{21} + \frac{(1-r_2)^2}{4}A_{22} + \frac{1}{4}A_{28} + A_{29} + \frac{f_2^2}{4}A_{31} + \frac{(1-f_2)^2}{4}A_{32} + \frac{1}{4}A_{33} \\
& + \frac{(1-r_1)^2}{4}A_{41} + \frac{r_1^2}{4}A_{42} + \frac{[(1-r_1)f_2]^2}{4}A_{43} + \frac{[r_1(1-f_2)]^2}{4}A_{44} + \frac{(r_1f_2)^2}{4}A_{45} \\
& + \frac{[(1-r_1)(1-f_2)]^2}{4}A_{46} + \frac{1}{4}A_{49} + \frac{[r_1(1-f_2) + (1-r_1)f_2]^2}{4}A_{50} + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{4}A_{51} \\
& + \frac{[(1-r_3)f_1]^2}{4}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{56} + \frac{(r_3f_1)^2}{4}A_{57} + \frac{[r_3(1-f_1)]^2}{4}A_{58} + \frac{f_1^2}{4}A_{59} \\
& + \frac{(1-f_1)^2}{4}A_{60} + \frac{[(1-r_1)r_2]^2}{4}A_{61} + \frac{[r_1(1-r_2)]^2}{4}A_{62} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{63} + \frac{(r_1r_2)^2}{4}A_{64}
\end{aligned}$$

$$\begin{aligned}
& + \frac{[(1-r_1)r_2(1-r_3)]^2}{2} A_{65} + \frac{[r_1 r_2 (1-r_3)]^2}{2} A_{66} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2} A_{67} \\
& + \frac{[r_1(1-r_2)r_3]^2}{2} A_{68} + \frac{[(1-r_1)r_2 r_3]^2}{2} A_{69} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2} A_{70} + \frac{(r_1 r_2 r_3)^2}{2} A_{71} \\
& + \frac{[(1-r_1)(1-r_2)r_3]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{30} &= \frac{r_2(1-r_2)}{2}(A_{15} + A_{16}) + \frac{r_2(1-r_2)}{2}[r_3^2(A_{17} + A_{18}) + (1-r_3)^2(A_{19} + A_{20})] + \frac{1}{2}A_{30} \\
& + \frac{f_2(1-f_2)}{2}(A_{31} + A_{32}) + \frac{r_1(1-r_1)f_2(1-f_2)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) \\
& + \frac{r_1(1-r_1)}{2}(A_{47} + A_{48}) + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
& + r_1(1-r_1)r_2(1-r_2)[r_3^2(A_{65} + A_{66} + A_{67} + A_{70}) + (1-r_3)^2(A_{68} + A_{69} + A_{71} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{31} &= \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) + \frac{(1-f_2)^2}{2}A_{31} + \frac{f_2^2}{2}A_{32} \\
& + \frac{r_1(1-r_1)}{2}[(1-f_2)^2(A_{43} + A_{45}) + f_2^2(A_{44} + A_{46})] \\
& + r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_{32} &= \frac{r_2(1-r_2)r_3(1-r_3)}{2}(A_{17} + A_{18} + A_{19} + A_{20}) + \frac{f_2^2}{2}A_{31} + \frac{(1-f_2)^2}{2}A_{32} \\
& + \frac{r_1(1-r_1)}{2}[f_2^2(A_{43} + A_{45}) + (1-f_2)^2(A_{44} + A_{46})] \\
& + r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_{33} &= \frac{r_2(1-r_2)}{2}[(1-r_3)^2(A_{17} + A_{18}) + r_3^2(A_{19} + A_{20})] + \frac{r_2(1-r_2)}{2}(A_{21} + A_{22}) \\
& + \frac{f_2(1-f_2)}{2}(A_{31} + A_{32}) + \frac{1}{2}A_{33} + \frac{r_1(1-r_1)}{2}(A_{41} + A_{42}) \\
& + \frac{r_1(1-r_1)f_2(1-f_2)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) + \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61} + A_{62} + A_{63} + A_{64}) \\
& + r_1(1-r_1)r_2(1-r_2)[(1-r_3)^2(A_{65} + A_{66} + A_{67} + A_{70}) + r_3^2(A_{68} + A_{69} + A_{71} + A_{72})]
\end{aligned}$$

$$A'_{34} = \frac{(1-r_2)^2}{4}A_{15} + \frac{r_2^2}{4}A_{16} + \frac{[(1-r_2)r_3]^2}{4}A_{17} + \frac{(r_2 r_3)^2}{4}A_{18} + \frac{[r_2(1-r_3)]^2}{4}A_{19}$$

$$\begin{aligned}
& + \frac{[(1-r_2)(1-r_3)]^2}{4}A_{20} + \frac{1}{4}A_{23} + \frac{r_3^2}{4}A_{24} + \frac{(1-r_3)^2}{4}A_{25} + \frac{1}{4}A_{30} + \frac{f_2^2}{4}A_{31} + \frac{(1-f_2)^2}{4}A_{32} \\
& + A_{34} + \frac{1}{4}A_{35} + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{4}A_{38} + \frac{[r_1(1-f_2) + (1-r_1)f_2]^2}{4}A_{39} + \frac{1}{4}A_{40} \\
& + \frac{(r_1f_2)^2}{4}A_{43} + \frac{[(1-r_1)(1-f_2)]^2}{4}A_{44} + \frac{[(1-r_1)f_2]^2}{4}A_{45} + \frac{[r_1(1-f_2)]^2}{4}A_{46} \\
& + \frac{r_1^2}{4}A_{47} + \frac{(1-r_1)^2}{4}A_{48} + \frac{(r_3f_1)^2}{4}A_{55} + \frac{[r_3(1-f_1)]^2}{4}A_{56} + \frac{[(1-r_3)f_1]^2}{4}A_{57} \\
& + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{58} + \frac{f_1^2}{4}A_{59} + \frac{(1-f_1)^2}{4}A_{60} + \frac{[r_1(1-r_2)]^2}{4}A_{61} + \frac{[(1-r_1)r_2]^2}{4}A_{62} \\
& + \frac{(r_1r_2)^2}{4}A_{63} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{64} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{65} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{66} \\
& + \frac{(r_1r_2r_3)^2}{2}A_{67} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{68} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{69} + \frac{[(1-r_1)r_2r_3]^2}{2}A_{70} \\
& + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{71} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{35} & = \frac{r_3(1-r_3)}{2}[(1-r_2)^2(A_{17} + A_{20}) + r_2^2(A_{18} + A_{19})] + \frac{r_3(1-r_3)}{2}(A_{24} + A_{25}) \\
& + \frac{f_2(1-f_2)}{2}(A_{31} + A_{32}) + \frac{1}{2}A_{35} + \frac{[(1-r_1)(1-f_2) + r_1f_2][r_1(1-f_2) + (1-r_1)f_2]}{2}(A_{38} + A_{39}) \\
& + \frac{f_2(1-f_2)}{2}[r_1^2(A_{43} + A_{46}) + (1-r_1)^2(A_{44} + A_{45})] \\
& + \frac{r_3(1-r_3)}{2}[f_1^2(A_{55} + A_{57}) + (1-f_1)^2(A_{56} + A_{58})] \\
& + r_3(1-r_3)[r_1^2(1-r_2)^2(A_{65} + A_{69}) + (1-r_1)^2(1-r_2)^2(A_{66} + A_{71}) + r_1^2r_2^2(A_{67} + A_{72}) \\
& + (1-r_1)^2r_2^2(A_{68} + A_{70})]
\end{aligned}$$

$$\begin{aligned}
A'_{36} & = \frac{[(1-r_2)(1-r_3)]^2}{4}A_{17} + \frac{[r_2(1-r_3)]^2}{4}A_{18} + \frac{(r_2r_3)^2}{4}A_{19} + \frac{[(1-r_2)r_3]^2}{4}A_{20} \\
& + \frac{(1-r_2)^2}{4}A_{21} + \frac{r_2^2}{4}A_{22} + \frac{(1-r_3)^2}{4}A_{24} + \frac{r_3^2}{4}A_{25} + \frac{1}{4}A_{26} + \frac{(1-f_2)^2}{4}A_{31} + \frac{f_2^2}{4}A_{32} + \frac{1}{4}A_{33} \\
& + \frac{1}{4}A_{35} + A_{36} + \frac{1}{4}A_{37} + \frac{[r_1(1-f_2) + (1-r_1)f_2]^2}{4}A_{38} + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{4}A_{39} \\
& + \frac{r_1^2}{4}A_{41} + \frac{(1-r_1)^2}{4}A_{42} + \frac{[r_1(1-f_2)]^2}{4}A_{43} + \frac{[(1-r_1)f_2]^2}{4}A_{44} + \frac{[(1-r_1)(1-f_2)]^2}{4}A_{45} \\
& + \frac{(r_1f_2)^2}{4}A_{46} + \frac{f_1^2}{4}A_{53} + \frac{(1-f_1)^2}{4}A_{54} + \frac{[(1-r_3)f_1]^2}{4}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{4}A_{56} + \frac{(r_3f_1)^2}{4}A_{57} \\
& + \frac{[r_3(1-f_1)]^2}{4}A_{58} + \frac{[r_1(1-r_2)]^2}{4}A_{61} + \frac{[(1-r_1)r_2]^2}{4}A_{62} + \frac{(r_1r_2)^2}{4}A_{63} + \frac{[(1-r_1)(1-r_2)]^2}{4}A_{64} \\
& + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{65} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{66} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{67}
\end{aligned}$$

$$\begin{aligned}
& + \frac{[(1-r_1)r_2r_3]^2}{2}A_{68} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{69} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{70} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{71} \\
& + \frac{(r_1r_2r_3)^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{37} &= \frac{1}{2}A_{37} + \frac{[(1-r_1)(1-f_2) + r_1f_2][r_1(1-f_2) + (1-r_1)f_2]}{2}(A_{38} + A_{39}) + \frac{r_1(1-r_1)}{2}(A_{41} + A_{42}) \\
& + \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43} + A_{45}) + f_1^2(A_{44} + A_{46})] + \frac{f_1(1-f_1)}{2}(A_{53} + A_{54}) \\
& + \frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55} + A_{56}) + r_3^2(A_{57} + A_{58})] \\
& + \frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61} + A_{64}) + r_2^2(A_{62} + A_{63})] \\
& + r_1(1-r_1)[(1-r_2)^2(1-r_3)^2(A_{65} + A_{66}) + r_2^2(1-r_3)^2(A_{67} + A_{70}) + r_2^2r_3^2(A_{68} + A_{72}) \\
& + (1-r_2)^2r_3^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{38} &= \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{2}A_{38} + \frac{[r_1(1-f_2) + (1-r_1)f_2]^2}{2}A_{39} \\
& + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
& + r_1(1-r_1)r_3(1-r_3)[(1-r_2)^2(A_{65} + A_{66} + A_{69} + A_{71}) + r_2^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{39} &= \frac{[r_1(1-f_2) + (1-r_1)f_2]^2}{2}A_{38} + \frac{[(1-r_1)(1-f_2) + r_1f_2]^2}{2}A_{39} \\
& + \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43} + A_{44} + A_{45} + A_{46}) + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55} + A_{56} + A_{57} + A_{58}) \\
& + r_1(1-r_1)r_3(1-r_3)[(1-r_2)^2(A_{65} + A_{66} + A_{69} + A_{71}) + r_2^2(A_{67} + A_{68} + A_{70} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{40} &= \frac{[(1-r_1)(1-f_2) + r_1f_2][r_1(1-f_2) + (1-r_1)f_2]}{2}(A_{38} + A_{39}) + \frac{1}{2}A_{40} \\
& + \frac{r_1(1-r_1)}{2}[f_1^2(A_{43} + A_{45}) + (1-f_1)^2(A_{44} + A_{46})] + \frac{r_1(1-r_1)}{2}(A_{47} + A_{48}) \\
& + \frac{f_1(1-f_1)}{2}[r_3^2(A_{55} + A_{56}) + (1-r_3)^2(A_{57} + A_{58})] + \frac{f_1(1-f_1)}{2}(A_{59} + A_{60}) \\
& + \frac{r_1(1-r_1)}{2}[(1-r_2)^2(A_{61} + A_{64}) + r_2^2(A_{62} + A_{63})] \\
& + r_1(1-r_1)[(1-r_2)^2r_3^2(A_{65} + A_{66}) + r_2^2r_3^2(A_{67} + A_{70}) + r_2^2(1-r_3)^2(A_{68} + A_{72}) \\
& + (1-r_2)^2(1-r_3)^2(A_{69} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{41} &= \frac{(1-r_1)^2}{2}A_{41} + \frac{r_1^2}{2}A_{42} + \frac{f_1(1-f_1)}{2}[(1-r_1)^2(A_{43} + A_{46}) + r_1^2(A_{44} + A_{45})] \\
&+ \frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61} + A_{63}) + r_1^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)[(1-r_1)^2(1-r_3)^2(A_{65} + A_{67}) + r_1^2(1-r_3)^2(A_{66} + A_{70})] \\
&+ r_1^2r_3^2(A_{68} + A_{71}) + (1-r_1)^2r_3^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{42} &= \frac{r_1^2}{2}A_{41} + \frac{(1-r_1)^2}{2}A_{42} + \frac{f_1(1-f_1)}{2}[r_1^2(A_{43} + A_{46}) + (1-r_1)^2(A_{44} + A_{45})] \\
&+ \frac{r_2(1-r_2)}{2}[r_1^2(A_{61} + A_{63}) + (1-r_1)^2(A_{62} + A_{64})] \\
&+ r_2(1-r_2)[r_1^2(1-r_3)^2(A_{65} + A_{67}) + (1-r_1)^2(1-r_3)^2(A_{66} + A_{70})] \\
&+ (1-r_1)^2r_3^2(A_{68} + A_{71}) + r_1^2r_3^2(A_{69} + A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{43} &= \frac{[(1-r_1)(1-f_1)]^2}{2}A_{43} + \frac{(r_1f_1)^2}{2}A_{44} + \frac{[r_1(1-r_1)]^2}{2}A_{45} + \frac{[(1-r_1)f_1]^2}{2}A_{46} \\
&+ r_2(1-r_2)r_3(1-r_3)[(1-r_1)^2(A_{65} + A_{67} + A_{69} + A_{72}) + r_1^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{44} &= \frac{(r_1f_1)^2}{2}A_{43} + \frac{[(1-r_1)(1-f_1)]^2}{2}A_{44} + \frac{[(1-r_1)f_1]^2}{2}A_{45} + \frac{[r_1(1-f_1)]^2}{2}A_{46} \\
&+ r_2(1-r_2)r_3(1-r_3)[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{45} &= \frac{[r_1(1-f_1)]^2}{2}A_{43} + \frac{[(1-r_1)f_1]^2}{2}A_{44} + \frac{[(1-r_1)(1-f_1)]^2}{2}A_{45} + \frac{(r_1f_1)^2}{2}A_{46} \\
&+ r_2(1-r_2)r_3(1-r_3)[r_1^2(A_{65} + A_{67} + A_{69} + A_{72}) + (1-r_1)^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{46} &= \frac{[(1-r_1)f_1]^2}{2}A_{43} + \frac{[r_1(1-f_1)]^2}{2}A_{44} + \frac{(r_1f_1)^2}{2}A_{45} + \frac{[(1-r_1)(1-f_1)]^2}{2}A_{46} \\
&+ r_2(1-r_2)r_3(1-r_3)[(1-r_1)^2(A_{65} + A_{67} + A_{69} + A_{72}) + r_1^2(A_{66} + A_{68} + A_{70} + A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{47} &= \frac{f_1(1-f_1)}{2}[(1-r_1)^2(A_{43}+A_{46})+r_1^2(A_{44}+A_{45})] + \frac{(1-r_1)^2}{2}A_{47} + \frac{r_1^2}{2}A_{48} \\
&+ \frac{r_2(1-r_2)}{2}[(1-r_1)^2(A_{61}+A_{63})+r_1^2(A_{62}+A_{64})] \\
&+ r_2(1-r_2)[(1-r_1)^2r_3^2(A_{65}+A_{67})+r_1^2r_3^2(A_{66}+A_{70})+r_1^2(1-r_3)^2(A_{68}+A_{71}) \\
&+(1-r_1)^2(1-r_3)^2(A_{69}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{48} &= \frac{f_1(1-f_1)}{2}[r_1^2(A_{43}+A_{46})+(1-r_1)^2(A_{44}+A_{45})] + \frac{r_1^2}{2}A_{47} + \frac{(1-r_1)^2}{2}A_{48} \\
&+ \frac{r_2(1-r_2)}{2}[r_1^2(A_{61}+A_{63})+(1-r_1)^2(A_{62}+A_{64})] \\
&+ r_2(1-r_2)[r_1^2r_3^2(A_{65}+A_{67})+(1-r_1)^2r_3^2(A_{66}+A_{70})+(1-r_1)^2(1-r_3)^2(A_{68}+A_{71}) \\
&+ r_1^2(1-r_3)^2(A_{69}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{49} &= \frac{r_1(1-r_1)}{2}(A_{41}+A_{42}) + \frac{r_1(1-r_1)}{2}[f_1^2(A_{43}+A_{45})+(1-f_1)^2(A_{44}+A_{46})] \\
&+ \frac{1}{2}A_{49} + \frac{[(1-r_1)(1-f_1)+r_1f_1][(1-r_1)f_1+r_1(1-f_1)]}{2}(A_{50}+A_{51}) \\
&+ \frac{f_1(1-f_1)}{2}[(1-r_3)^2(A_{55}+A_{56})+r_3^2(A_{57}+A_{58})] + \frac{f_1(1-f_1)}{2}(A_{59}+A_{60}) \\
&+ \frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64})+(1-r_2)^2(A_{62}+A_{63})] \\
&+ r_1(1-r_1)[r_2^2(1-r_3)^2(A_{65}+A_{66})+(1-r_2)^2(1-r_3)^2(A_{67}+A_{70}) \\
&+(1-r_2)^2r_3^2(A_{68}+A_{72})+r_2^2r_3^2(A_{69}+A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{50} &= \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46}) + \frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{2}A_{50} \\
&+ \frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{2}A_{51} + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58}) \\
&+ r_1(1-r_1)r_3(1-r_3)[r_2^2(A_{65}+A_{66}+A_{69}+A_{71})+(1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{51} &= \frac{r_1(1-r_1)f_1(1-f_1)}{2}(A_{43}+A_{44}+A_{45}+A_{46}) + \frac{[(1-r_1)f_1+r_1(1-f_1)]^2}{2}A_{50} \\
&+ \frac{[(1-r_1)(1-f_1)+r_1f_1]^2}{2}A_{51} + \frac{r_3(1-r_3)f_1(1-f_1)}{2}(A_{55}+A_{56}+A_{57}+A_{58}) \\
&+ r_1(1-r_1)r_3(1-r_3)[r_2^2(A_{65}+A_{66}+A_{69}+A_{71})+(1-r_2)^2(A_{67}+A_{68}+A_{70}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{52} &= \frac{r_1(1-r_1)}{2}[(1-f_1)^2(A_{43}+A_{45})+f_1^2(A_{44}+A_{46})] + \frac{r_1(1-r_1)}{2}(A_{47}+A_{48}) \\
&+ \frac{[(1-r_1)(1-f_1)+r_1f_1][(1-r_1)f_1+r_1(1-f_1)]}{2}(A_{50}+A_{51}) + \frac{1}{2}A_{52} \\
&+ \frac{f_1(1-f_1)}{2}(A_{53}+A_{54}) + \frac{f_1(1-f_1)}{2}[r_3^2(A_{55}+A_{56})+(1-r_3)^2(A_{57}+A_{58})] \\
&+ \frac{r_1(1-r_1)}{2}[r_2^2(A_{61}+A_{64})+(1-r_2)^2(A_{62}+A_{63})] + r_1(1-r_1)[r_2^2r_3^2(A_{65}+A_{66}) \\
&+(1-r_2)^2r_3^2(A_{67}+A_{70})+(1-r_2)^2(1-r_3^2)(A_{68}+A_{72})+r_2^2(1-r_3)^2(A_{69}+A_{71})]
\end{aligned}$$

$$\begin{aligned}
A'_{53} &= \frac{(1-f_1)^2}{2}A_{53} + \frac{f_1^2}{2}A_{54} + \frac{r_3(1-r_3)}{2}[(1-f_1)^2(A_{55}+A_{57})+f_1^2(A_{56}+A_{58})] \\
&+ \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64}) \\
&+ r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_{54} &= \frac{f_1^2}{2}A_{53} + \frac{(1-f_1)^2}{2}A_{54} + \frac{r_3(1-r_3)}{2}[f_1^2(A_{55}+A_{57})+(1-f_1)^2(A_{56}+A_{58})] \\
&+ \frac{r_1(1-r_1)r_2(1-r_2)}{2}(A_{61}+A_{62}+A_{63}+A_{64}) \\
&+ r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65}+A_{66}+A_{67}+A_{68}+A_{69}+A_{70}+A_{71}+A_{72})
\end{aligned}$$

$$\begin{aligned}
A'_{55} &= \frac{[(1-r_3)(1-f_1)]^2}{2}A_{55} + \frac{[(1-r_3)f_1]^2}{2}A_{56} + \frac{[r_3(1-f_1)]^2}{2}A_{57} + \frac{(r_3f_1)^2}{2}A_{58} \\
&+ r_1(1-r_1)r_2(1-r_2)[(1-r_3)^2(A_{65}+A_{66}+A_{67}+A_{70})+r_3^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{56} &= \frac{[(1-r_3)f_1]^2}{2}A_{55} + \frac{[(1-r_3)(1-f_1)]^2}{2}A_{56} + \frac{(r_3f_1)^2}{2}A_{57} + \frac{[r_3(1-f_1)]^2}{2}A_{58} \\
&+ r_1(1-r_1)r_2(1-r_2)[(1-r_3)^2(A_{65}+A_{66}+A_{67}+A_{70})+r_3^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}$$

$$\begin{aligned}
A'_{57} &= \frac{[r_3(1-f_1)]^2}{2}A_{55} + \frac{(r_3f_1)^2}{2}A_{56} + \frac{[(1-r_3)(1-f_1)]^2}{2}A_{57} + \frac{[(1-r_3)f_1]^2}{2}A_{58} \\
&+ r_1(1-r_1)r_2(1-r_2)[r_3^2(A_{65}+A_{66}+A_{67}+A_{70})+(1-r_3)^2(A_{68}+A_{69}+A_{71}+A_{72})]
\end{aligned}$$

$$A'_{58} = \frac{(r_3 f_1)^2}{2} A_{55} + \frac{[r_3(1-f_1)]^2}{2} A_{56} + \frac{[(1-r_3)f_1]^2}{2} A_{57} + \frac{[(1-r_3)(1-f_1)]^2}{2} A_{58} \\ + r_1(1-r_1)r_2(1-r_2)[r_3^2(A_{65} + A_{66} + A_{67} + A_{70}) + (1-r_3)^2(A_{68} + A_{69} + A_{71} + A_{72})]$$

$$A'_{59} = \frac{r_3(1-r_3)}{2} [(1-f_1)^2(A_{55} + A_{57}) + f_1^2(A_{56} + A_{58})] + \frac{(1-f_1)^2}{2} A_{59} + \frac{f_1^2}{2} A_{60} \\ + \frac{r_1(1-r_1)r_2(1-r_2)}{2} (A_{61} + A_{62} + A_{63} + A_{64}) \\ + r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})$$

$$A'_{60} = \frac{r_3(1-r_3)}{2} [f_1^2(A_{55} + A_{57}) + (1-f_1)^2(A_{56} + A_{58})] + \frac{f_1^2}{2} A_{59} + \frac{(1-f_1)^2}{2} A_{60} \\ + \frac{r_1(1-r_1)r_2(1-r_2)}{2} (A_{61} + A_{62} + A_{63} + A_{64}) \\ + r_1(1-r_1)r_2(1-r_2)r_3(1-r_3)(A_{65} + A_{66} + A_{67} + A_{68} + A_{69} + A_{70} + A_{71} + A_{72})$$

$$A'_{61} = \frac{[(1-r_1)(1-r_2)]^2}{2} A_{61} + \frac{(r_1 r_2)^2}{2} A_{62} + \frac{[(1-r_1)r_2]^2}{2} A_{63} + \frac{[r_1(1-r_2)]^2}{2} A_{64} \\ + r_3(1-r_3)[(1-r_1)^2(1-r_2)^2(A_{65} + A_{69}) + r_1^2(1-r_2)^2(A_{66} + A_{71}) + r_1^2 r_2^2(A_{68} + A_{70}) \\ + (1-r_1)^2 r_2^2(A_{67} + A_{72})]$$

$$A'_{62} = \frac{(r_1 r_2)^2}{2} A_{61} + \frac{[(1-r_1)(1-r_2)]^2}{2} A_{62} + \frac{[r_1(1-r_2)]^2}{2} A_{63} + \frac{[(1-r_1)r_2]^2}{2} A_{64} \\ + r_3(1-r_3)[r_1^2 r_2^2(A_{65} + A_{69}) + (1-r_1)^2 r_2^2(A_{66} + A_{71}) + r_1^2(1-r_2)^2(A_{67} + A_{72}) \\ + (1-r_1)^2(1-r_2)^2(A_{68} + A_{70})]$$

$$A'_{63} = \frac{[(1-r_1)r_2]^2}{2} A_{61} + \frac{[r_1(1-r_2)]^2}{2} A_{62} + \frac{[(1-r_1)(1-r_2)]^2}{2} A_{63} + \frac{(r_1 r_2)^2}{2} A_{64} \\ + r_3(1-r_3)[(1-r_1)^2 r_2^2(A_{65} + A_{69}) + r_1^2 r_2^2(A_{66} + A_{71}) + (1-r_1)^2(1-r_2)^2(A_{67} + A_{72}) \\ + r_1^2(1-r_2)^2(A_{68} + A_{70})]$$

$$\begin{aligned}
A'_{64} &= \frac{[r_1(1-r_2)]^2}{2}A_{61} + \frac{[(1-r_1)r_2]^2}{2}A_{62} + \frac{(r_1r_2)^2}{2}A_{63} + \frac{[(1-r_1)(1-r_2)]^2}{2}A_{64} \\
&+ r_3(1-r_3)[r_1^2(1-r_2)^2(A_{65} + A_{69}) + (1-r_1)^2(1-r_2)^2(A_{66} + A_{71}) + r_1^2r_2^2(A_{67} + A_{72}) \\
&+ (1-r_1)^2r_2^2(A_{68} + A_{70})]
\end{aligned}$$

$$\begin{aligned}
A'_{65} &= \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{65} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{66} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{67} \\
&+ \frac{(r_1r_2r_3)^2}{2}A_{68} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{69} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{70} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{71} \\
&+ \frac{[(1-r_1)r_2r_3]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{66} &= \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{65} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{66} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{67} \\
&+ \frac{[(1-r_1)r_2r_3]^2}{2}A_{68} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{69} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{70} \\
&+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{71} + \frac{(r_1r_2r_3)^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{67} &= \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{65} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{66} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{67} \\
&+ \frac{[r_1(1-r_2)r_3]^2}{2}A_{68} + \frac{[(1-r_1)r_2r_3]^2}{2}A_{69} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{70} + \frac{(r_1r_2r_3)^2}{2}A_{71} \\
&+ \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{68} &= \frac{(r_1r_2r_3)^2}{2}A_{65} + \frac{[(1-r_1)r_2r_3]^2}{2}A_{66} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{67} + \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{68} \\
&+ \frac{[r_1r_2(1-r_3)]^2}{2}A_{69} + \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{70} + \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{71} \\
&+ \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{69} &= \frac{[(1-r_1)(1-r_2)r_3]^2}{2}A_{65} + \frac{[r_1(1-r_2)r_3]^2}{2}A_{66} + \frac{[(1-r_1)r_2r_3]^2}{2}A_{67} + \frac{[r_1r_2(1-r_3)]^2}{2}A_{68} \\
&+ \frac{[(1-r_1)(1-r_2)(1-r_3)]^2}{2}A_{69} + \frac{(r_1r_2r_3)^2}{2}A_{70} + \frac{[r_1(1-r_2)(1-r_3)]^2}{2}A_{71} \\
&+ \frac{[(1-r_1)r_2(1-r_3)]^2}{2}A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{70} &= \frac{[r_1 r_2 (1 - r_3)]^2}{2} A_{65} + \frac{[(1 - r_1) r_2 (1 - r_3)]^2}{2} A_{66} + \frac{[r_1 (1 - r_2) (1 - r_3)]^2}{2} A_{67} \\
&+ \frac{[(1 - r_1) (1 - r_2) r_3]^2}{2} A_{68} + \frac{(r_1 r_2 r_3)^2}{2} A_{69} + \frac{[(1 - r_1) (1 - r_2) (1 - r_3)]^2}{2} A_{70} \\
&+ \frac{[(1 - r_1) r_2 r_3]^2}{2} A_{71} + \frac{[r_1 (1 - r_2) r_3]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{71} &= \frac{[r_1 (1 - r_2) r_3]^2}{2} A_{65} + \frac{[(1 - r_1) (1 - r_2) r_3]^2}{2} A_{66} + \frac{(r_1 r_2 r_3)^2}{2} A_{67} + \frac{[(1 - r_1) r_2 (1 - r_3)]^2}{2} A_{68} \\
&+ \frac{[r_1 (1 - r_2) (1 - r_3)]^2}{2} A_{69} + \frac{[(1 - r_1) r_2 r_3]^2}{2} A_{70} + \frac{[(1 - r_1) (1 - r_2) (1 - r_3)]^2}{2} A_{71} \\
&+ \frac{[r_1 r_2 (1 - r_3)]^2}{2} A_{72}
\end{aligned}$$

$$\begin{aligned}
A'_{72} &= \frac{[(1 - r_1) r_2 r_3]^2}{2} A_{65} + \frac{(r_1 r_2 r_3)^2}{2} A_{66} + \frac{[(1 - r_1) (1 - r_2) r_3]^2}{2} A_{67} + \frac{[r_1 (1 - r_2) (1 - r_3)]^2}{2} A_{68} \\
&+ \frac{[(1 - r_1) r_2 (1 - r_3)]^2}{2} A_{69} + \frac{[r_1 (1 - r_2) r_3]^2}{2} A_{70} + \frac{[r_1 r_2 (1 - r_3)]^2}{2} A_{71} \\
&+ \frac{[(1 - r_1) (1 - r_2) (1 - r_3)]^2}{2} A_{72}
\end{aligned}$$

## The R code of the 72 transition equations for deriving the 136 genotypic frequencies

Miao-Hei Zeng, Hsiang-An Ho and Chen-Hung Kao

```
te<-function(r1,r2,r3,n){
k<-n-1
freq<-rep(0,72)
freq[65]<-1
for(i in 1:k){
freq<-tem(r1,r2,r3)%*%freq}
freq
}

tem<-function(r1,r2,r3){
f1<-r1*(1-r2)+r2*(1-r1)
f2<-r2*(1-r3)+r3*(1-r2)
tm<-matrix(rep(0,72^2),nrow=72)
tm[1,]<-c(4,1,0,1,(1-f2)^2,f2^2,rep(0,4),1,(1-r3)^2,r3^2,0,(1-r2)^2,r2^2,((1-r2)*(1-r3))^2,(r2*(1-r3))^2,(r2*r3)^2,((1-r2)*r3)^2,rep(0,16),1,((1-r1)*(1-f2)+r1*f2)^2,((1-r1)*f2+r1*(1-f2))^2,0,(1-r1)^2,r1^2,((1-r1)*(1-f1))^2,(r1*f1)^2,(r1*(1-f1))^2,((1-r1)*f1)^2,rep(0,6),(1-f1)^2,f1^2,((1-f1)*(1-r3))^2,(f1*(1-r3))^2,((1-f1)*r3)^2,(f1*r3)^2,0,0,((1-r1)*(1-r2))^2,(r1*r2)^2,((1-r1)*r2)^2,(r1*(1-r2))^2,2*((1-r1)*(1-r2)*(1-r3))^2,2*(r1*(1-r2)*(1-r3))^2,2*((1-r1)*r2*(1-r3))^2,2*(r1*r2*r3)^2,2*((1-r1)*(1-r2)*r3)^2,2*(r1*r2*(1-r3))^2,2*(r1*(1-r2)*r3)^2,2*((1-r1)*r2*r3)^2)/4
tm[2,]<-c(0,1,0,0,(1-f2)*f2,(1-f2)*f2,rep(0,5),(1-r3)*r3,(1-r3)*r3,0,0,0,(1-r2)^2*(1-r3)*r3,r2^2*(1-r3)*r3,r2^2*(1-r3)*r3,rep(0,17),((1-r1)*(1-f2)+r1*f2)*((1-r1)*f2+r1*(1-f2)),((1-r1)*(1-f2)+r1*f2)*((1-r1)*f2+r1*(1-f2)),0,0,0,(1-r1)^2*f1*(1-f1),r1^2*f1*(1-f1),r1^2*f1*(1-f1),(1-r1)^2*f1*(1-f1),rep(0,8),(1-f1)^2*r3*(1-r3),f1^2*r3*(1-r3),f1^2*r3*(1-r3),rep(0,6),2*(1-r1)^2*(1-r2)^2*r3*(1-r3),2*r1^2*(1-r2)^2*r3*(1-
```

$r3), 2*(1-r1)^2*r2^2*r3*(1-r3)$   
 $, 2*r1^2*r2^2*r3*(1-r3), 2*(1-r1)^2*(1-r2)^2*r3*(1-r3), 2*r1^2*r2^2*r3*(1-r3), 2*r1^2*(1-r2)^2*r3$   
 $*(1-r3)$   
 $, 2*(1-r1)^2*r2^2*r3*(1-r3))/2$   
 $tm[3], <-c(0, 1, 4, 0, f2^2, (1-f2)^2, 1, 0, 0, 0, 0, r3^2, (1-r3)^2, 1, 0, 0, ((1-r2)*r3)^2, (r2*r3)^2, (r2*(1-r3))^2, ($   
 $(1-r2)*(1-r3))^2$   
 $, ($   
 $, ((1-r1)*(1-f1))^2, (1-r1)^2, r1^2, rep(0, 6), (r3*(1-f1))^2, (r3*f1)^2, ((1-r3)*(1-f1))^2, ((1-r3)*f1)^2, (1-$   
 $f1)^2, f1^2$   
 $, ((1-r1)*(1-r2))^2, (r1*r2)^2, ((1-r1)*r2)^2, (r1*(1-r2))^2, 2*((1-r1)*(1-r2)*r3)^2, 2*(r1*(1-r2)*r3)^2,$   
 $2*((1-r1)*r2*r3)^2$   
 $, 2*(r1*r2*(1-r3))^2, 2*((1-r1)*(1-r2)*(1-r3))^2, 2*(r1*r2*r3)^2, 2*(r1*(1-r2)*(1-r3))^2, 2*((1-r1)*r2$   
 $*(1-r3))^2)/4$   
 $tm[4], <-c(0, 0, 0, 1, rep((1-f2)*f2, 2), rep(0, 8), r2*(1-r2), r2*(1-r2), r2*(1-r2)*c((1-r3)^2, (1-r3)^2, r3^2, r3$   
 $^2), rep(0, 20), r1*(1-r1)$   
 $, r1*(1-r1), rep(r1*(1-r1)*f1*(1-f1), 4), rep(0, 14), rep(r1*(1-r1)*r2*(1-r2), 4)$   
 $, 2*r1*(1-r1)*r2*(1-r2)*c((1-r3)^2, (1-r3)^2, (1-r3)^2, r3^2, r3^2, (1-r3)^2, r3^2, r3^2))/2$   
 $tm[5], <-c(0, 0, 0, 0, (1-f2)^2, f2^2, rep(0, 10), rep(r2*(1-r2)*r3*(1-r3), 4), rep(0, 22), r1*(1-r1)*(1-f1)^2, r1$   
 $*(1-r1)*f1^2$   
 $, r1*(1-r1)*(1-f1)^2, r1*(1-r1)*f1^2, rep(0, 18), rep(2*r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8))/2$   
 $tm[6], <-c(0, 0, 0, 0, f2^2, (1-f2)^2, rep(0, 10), rep(r2*(1-r2)*r3*(1-r3), 4), rep(0, 22), r1*(1-r1)*f1^2, r1*(1-$   
 $-r1)*(1-f1)^2$   
 $, r1*(1-r1)*f1^2, r1*(1-r1)*(1-f1)^2, rep(0, 18), rep(2*r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8))/2$   
 $tm[7], <-c(0, 0, 0, 0, f2*(1-f2), f2*(1-f2), 1, rep(0, 9), r2*(1-r2)*r3^2, r2*(1-r2)*r3^2, r2*(1-r2)*(1-r3)^2, r$   
 $2*(1-r2)*(1-r3)^2$   
 $, r2*(1-r2), r2*(1-r2), rep(0, 20), rep(r1*(1-r1)*f1*(1-f1), 4), r1*(1-r1), r1*(1-r1), rep(0, 12), rep(r1*(1-r1)$   
 $*r2*(1-r2), 4)$   
 $, 2*r1*(1-r1)*r2*(1-r2)*c(r3^2, r3^2, r3^2, (1-r3)^2, (1-r3)^2, r3^2, (1-r3)^2, (1-r3)^2))/2$   
 $tm[8], <-c(0, 0, 0, 1, f2^2, (1-f2)^2, 0, 4, 1, rep(0, 5), r2^2, (1-r2)^2, (r2*(1-r3))^2, ((1-r2)*(1-r3))^2, ((1-r2)*$   
 $r3)^2, (r2*r3)^2$   
 $, 0, 0, 1, (1-r3)^2, r3^2, rep(0, 15), r1^2, (1-r1)^2, (r1*f1)^2, ((1-r1)*(1-f1))^2, ((1-r1)*f1)^2, (r1*(1-f1))^2,$   
 $0, 0, 1$   
 $, ((1-r1)*(1-f1)+r1*f1)^2, ((1-r1)*f1+r1*(1-f1))^2, 0, 0, 0, ((1-r3)*(1-f1))^2, ((1-r3)*f1)^2, (r3*(1-f1))^2$   
 $, (r3*f1)^2$   
 $, (1-f1)^2, f1^2, (r1*r2)^2, ((1-r1)*(1-r2))^2, (r1*(1-r2))^2, ((1-r1)*r2)^2, 2*(r1*r2*(1-r3))^2, 2*((1-r1)$   
 $*r2*(1-r3))^2$   
 $, 2*(r1*(1-r2)*(1-r3))^2, 2*((1-r1)*(1-r2)*r3)^2, 2*(r1*r2*r3)^2, 2*((1-r1)*(1-r2)*(1-r3))^2, 2*((1-r1)$   
 $*r2*r3)^2$

$,2*(r1*(1-r2)*r3)^2/4$   
 $tm[9,]<-c(0,0,0,0,f2*(1-f2),f2*(1-f2),0,0,1,rep(0,7),r3*(1-r3)*r2^2,r3*(1-r3)*(1-r2)^2,r3*(1-r3)*(1-r2)^2,r3*(1-r3)*r2^2$   
 $,0,0,0,r3*(1-r3),r3*(1-r3),rep(0,17),f1*(1-f1)*r1^2,f1*(1-f1)*(1-r1)^2,f1*(1-f1)*(1-r1)^2,f1*(1-f1)*r1^2,0,0,0$   
 $,rep(((1-r1)*(1-f1)+r1*f1)*((1-r1)*f1+r1*(1-f1)),2),0,0,0,r3*(1-r3)*(1-f1)^2,r3*(1-r3)*f1^2,r3*(1-r3)*(1-f1)^2$   
 $,r3*(1-r3)*f1^2,rep(0,6),2*r3*(1-r3)*c(r1^2*r2^2,(1-r1)^2*r2^2,r1^2*(1-r2)^2,(1-r1)^2*(1-r2)^2,r1^2*r2^2,(1-r1)^2*(1-r2)^2$   
 $,((1-r1)^2*r2^2,r1^2*(1-r2)^2))/2$   
 $tm[10,]<-c(0,0,0,0,(1-f2)^2,f2^2,1,0,1,4,rep(0,6),r2^2*r3^2,(1-r2)^2*r3^2,(1-r2)^2*(1-r3)^2,r2^2*(1-r3)^2,r2^2,(1-r2)^2$   
 $,0,r3^2,(1-r3)^2,1,rep(0,16),(r1*(1-f1))^2,((1-r1)*f1)^2,((1-r1)*(1-f1))^2,(r1*f1)^2,r1^2,(1-r1)^2,0$   
 $,((1-r1)*f1+r1*(1-f1))^2,((1-r1)*(1-f1)+r1*f1)^2,1,(1-f1)^2,f1^2,(r3*(1-f1))^2,(r3*f1)^2,((1-r3)*(1-f1))^2,((1-r3)*f1)^2$   
 $,0,0,(r1*r2)^2,((1-r1)*(1-r2))^2,(r1*(1-r2))^2,((1-r1)*r2)^2,2*(r1*r2*r3)^2,2*((1-r1)*r2*r3)^2,2*(r1*(1-r2)*r3)^2$   
 $,2*((1-r1)*(1-r2)*(1-r3))^2,2*(r1*r2*(1-r3))^2,2*((1-r1)*(1-r2)*r3)^2,2*((1-r1)*r2*(1-r3))^2,2*(r1*(1-r2)*(1-r3))^2/4$   
 $tm[11,]<-c(rep(0,10),1,r3*(1-r3),r3*(1-r3),0,r2*(1-r2),r2*(1-r2),rep(r2*(1-r2)*r3*(1-r3),4),rep(0,32),(1-f1)*f1,(1-f1)*f1$   
 $,rep(r3*(1-r3)*(1-f1)*f1,4),0,0,r2*(1-r2)*c((1-r1)^2,r1^2,(1-r1)^2,r1^2)$   
 $,2*r2*r3*(1-r2)*(1-r3)*c((1-r1)^2,r1^2,(1-r1)^2,r1^2,(1-r1)^2,r1^2,r1^2,(1-r1)^2))/2$   
 $tm[12,]<-c(rep(0,11),(1-r3)^2,r3^2,0,0,0,r2*(1-r2)*c((1-r3)^2,(1-r3)^2,r3^2,r3^2),rep(0,34)$   
 $,f1*(1-f1)*c((1-r3)^2,(1-r3)^2,r3^2,r3^2),rep(0,6)$   
 $,2*r2*(1-r2)*c(((1-r1)*(1-r3))^2,(r1*(1-r3))^2,((1-r1)*(1-r3))^2,(r1*r3)^2,((1-r1)*r3)^2,(r1*(1-r3))^2,(r1*r3)^2,$   
 $((1-r1)*r3)^2))/2$   
 $tm[13,]<-c(rep(0,11),r3^2,(1-r3)^2,0,0,0,r2*(1-r2)*c(r3^2,r3^2,(1-r3)^2,(1-r3)^2),rep(0,34)$   
 $,f1*(1-f1)*c(r3^2,r3^2,(1-r3)^2,(1-r3)^2),rep(0,6)$   
 $,2*r2*(1-r2)*c(((1-r1)*r3)^2,(r1*r3)^2,((1-r1)*r3)^2,(r1*(1-r3))^2,((1-r1)*(1-r3))^2,(r1*r3)^2,(r1*(1-r3))^2,$   
 $((1-r1)*(1-r3))^2))/2$   
 $tm[14,]<-c(rep(0,11),r3*(1-r3),r3*(1-r3),1,0,0,rep(r2*(1-r2)*r3*(1-r3),4),r2*(1-r2),r2*(1-r2),rep(0,32)$   
 $,rep(r3*(1-r3)*f1*(1-f1),4),f1*(1-f1),f1*(1-f1),r2*(1-r2)*c((1-r1)^2,r1^2,(1-r1)^2,r1^2)$   
 $,2*r2*(1-r2)*r3*(1-r3)*c((1-r1)^2,r1^2,(1-r1)^2,r1^2,(1-r1)^2,r1^2,r1^2,(1-r1)^2))/2$   
 $tm[15,]<-c(rep(0,14),(1-r2)^2,r2^2,r3*(1-r3)*c((1-r2)^2,r2^2,r2^2,(1-r2)^2),rep(0,40)$

$$\frac{r_1(1-r_1)c((1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2) + 2r_1(1-r_1)r_3(1-r_3)c((1-r_2)^2, (1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2, r_2^2, (1-r_2)^2, r_2^2)}{2}$$

$$\text{tm}[16,] <- c(\text{rep}(0, 14), r_2^2, (1-r_2)^2, r_3(1-r_3)c(r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2), \text{rep}(0, 40))$$

$$\frac{r_1(1-r_1)c(r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2) + 2r_1(1-r_1)r_3(1-r_3)c(r_2^2, r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2, (1-r_2)^2, r_2^2, (1-r_2)^2)}{2}$$

$$\text{tm}[17,] <- c(\text{rep}(0, 16), ((1-r_2)(1-r_3))^2, (r_2(1-r_3))^2, (r_2r_3)^2, ((1-r_2)r_3)^2, \text{rep}(0, 44))$$

$$\frac{2r_1(1-r_1)c(((1-r_2)(1-r_3))^2, ((1-r_2)(1-r_3))^2, (r_2(1-r_3))^2, (r_2r_3)^2, ((1-r_2)r_3)^2, (r_2(1-r_3))^2, ((1-r_2)r_3)^2)}{(r_2r_3)^2)}{2}$$

$$\text{tm}[18,] <- c(\text{rep}(0, 16), (r_2(1-r_3))^2, ((1-r_2)(1-r_3))^2, ((1-r_2)r_3)^2, (r_2r_3)^2, \text{rep}(0, 44))$$

$$\frac{2r_1(1-r_1)c((r_2(1-r_3))^2, (r_2(1-r_3))^2, ((1-r_2)(1-r_3))^2, ((1-r_2)r_3)^2, (r_2r_3)^2, ((1-r_2)(1-r_3))^2, (r_2r_3)^2)}{((1-r_2)r_3)^2)}{2}$$

$$\text{tm}[19,] <- c(\text{rep}(0, 16), (r_2r_3)^2, ((1-r_2)r_3)^2, ((1-r_2)(1-r_3))^2, (r_2(1-r_3))^2, \text{rep}(0, 44))$$

$$\frac{2r_1(1-r_1)c((r_2r_3)^2, (r_2r_3)^2, ((1-r_2)r_3)^2, ((1-r_2)(1-r_3))^2, (r_2(1-r_3))^2, ((1-r_2)r_3)^2, (r_2(1-r_3))^2)}{((1-r_2)(1-r_3))^2)}{2}$$

$$\text{tm}[20,] <- c(\text{rep}(0, 16), ((1-r_2)r_3)^2, (r_2r_3)^2, (r_2(1-r_3))^2, ((1-r_2)(1-r_3))^2, \text{rep}(0, 44))$$

$$\frac{2r_1(1-r_1)c(((1-r_2)r_3)^2, ((1-r_2)r_3)^2, (r_2r_3)^2, (r_2(1-r_3))^2, ((1-r_2)(1-r_3))^2, (r_2r_3)^2, ((1-r_2)r_3)^2)}{(r_2(1-r_3))^2)}{2}$$

$$\text{tm}[21,] <- c(\text{rep}(0, 16), r_3(1-r_3)c((1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2), (1-r_2)^2, r_2^2, \text{rep}(0, 38))$$

$$\frac{r_1(1-r_1)c((1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2) + 2r_1(1-r_1)r_3(1-r_3)c((1-r_2)^2, (1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2, r_2^2, (1-r_2)^2, r_2^2)}{2}$$

$$\text{tm}[22,] <- c(\text{rep}(0, 16), r_3(1-r_3)c(r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2), r_2^2, (1-r_2)^2, \text{rep}(0, 38))$$

$$\frac{r_1(1-r_1)c(r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2) + 2r_1(1-r_1)r_3(1-r_3)c(r_2^2, r_2^2, (1-r_2)^2, (1-r_2)^2, r_2^2, (1-r_2)^2, r_2^2, (1-r_2)^2)}{2}$$

$$\text{tm}[23,] <- c(\text{rep}(0, 14), r_2(1-r_2), r_2(1-r_2), \text{rep}(r_2(1-r_2)r_3(1-r_3), 4), 0, 0, 1, r_3(1-r_3), r_3(1-r_3), \text{rep}(0, 29))$$

$$\frac{\text{rep}(r_3(1-r_3)f_1(1-f_1), 4), f_1(1-f_1), f_1(1-f_1), r_2(1-r_2)c(r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2) + 2r_2(1-r_2)r_3(1-r_3)c(r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2, (1-r_1)^2, r_1^2)}{2}$$

$$\text{tm}[24,] <- c(\text{rep}(0, 16), r_2(1-r_2)c((1-r_3)^2, (1-r_3)^2, r_3^2, r_3^2), 0, 0, 0, (1-r_3)^2, r_3^2, \text{rep}(0, 29))$$

$$\frac{f_1(1-f_1)c((1-r_3)^2, (1-r_3)^2, r_3^2, r_3^2), \text{rep}(0, 6) + 2r_2(1-r_2)c((r_1(1-r_3))^2, ((1-r_1)(1-r_3))^2, (r_1(1-r_3))^2, ((1-r_1)r_3)^2, (r_1r_3)^2, ((1-r_1)(1-r_3))^2, ((1-r_1)r_3)^2)}{(r_1r_3)^2)}{2}$$

$$\begin{aligned}
& \text{tm}[25,] \leftarrow -c(\text{rep}(0,16), r^2(1-r^2) * c(r^3^2, r^3^2, (1-r^3)^2, (1-r^3)^2), 0, 0, 0, r^3^2, (1-r^3)^2, \text{rep}(0,29) \\
& , f_1(1-f_1) * c(r^3^2, r^3^2, (1-r^3)^2, (1-r^3)^2), \text{rep}(0,6) \\
& , 2 * r^2(1-r^2) * c((r_1 * r_3)^2, ((1-r_1) * r_3)^2, (r_1 * r_3)^2, ((1-r_1) * (1-r_3))^2, (r_1 * (1-r_3))^2, ((1-r_1) * r_3)^2, ((1-r_1) * (1-r_3))^2 \\
& , (r_1 * (1-r_3))^2) / 2 \\
& \text{tm}[26,] \leftarrow -c(\text{rep}(0,16), \text{rep}(r^2(1-r^2) * r^3(1-r^3), 4), r^2(1-r^2), r^2(1-r^2), 0, r^3(1-r^3), r^3(1-r^3), 1, \text{rep}(0,26) \\
& ), f_1(1-f_1), f_1(1-f_1) \\
& , \text{rep}(r^3(1-r^3) * f_1(1-f_1), 4), 0, 0, r^2(1-r^2) * c(r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2) \\
& , 2 * r^2(1-r^2) * r^3(1-r^3) * c(r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2, r_1^2, (1-r_1)^2, (1-r_1)^2, r_1^2) / 2 \\
& \text{tm}[27,] \leftarrow -c(\text{rep}(0,10), 1, r^3^2, (1-r^3)^2, 0, r^2^2, (1-r^2)^2, (r^2 * r^3)^2, ((1-r^2) * r^3)^2, ((1-r^2) * (1-r^3))^2, (r^2 * (1-r^3))^2, \text{rep}(0,6) \\
& , 4, 1, 0, 1, (1-f_2)^2, f_2^2, \text{rep}(0,10), ((1-r_1) * (1-f_2))^2, (r_1 * f_2)^2, (r_1 * (1-f_2))^2, ((1-r_1) * f_2)^2 \\
& , (1-r_1)^2, r_1^2, 0, ((1-r_1) * (1-f_2) + r_1 * f_2)^2, (r_1 * (1-f_2) + (1-r_1) * f_2)^2, 1, f_1^2, (1-f_1)^2 \\
& , (r_3 * f_1)^2, (r_3 * (1-f_1))^2, ((1-r_3) * f_1)^2, ((1-r_3) * (1-f_1))^2, 0, 0, ((1-r_1) * r_2)^2, (r_1 * (1-r_2))^2, ((1-r_1) * (1-r_2))^2, (r_1 * r_2)^2 \\
& , 2 * ((1-r_1) * r_2 * r_3)^2, 2 * (r_1 * r_2 * r_3)^2, 2 * ((1-r_1) * (1-r_2) * r_3)^2, 2 * (r_1 * (1-r_2) * (1-r_3))^2, 2 * ((1-r_1) * r_2 * (1-r_3))^2 \\
& , 2 * (r_1 * (1-r_2) * r_3)^2, 2 * (r_1 * r_2 * (1-r_3))^2, 2 * ((1-r_1) * (1-r_2) * (1-r_3))^2) / 4 \\
& \text{tm}[28,] \leftarrow -c(\text{rep}(0,11), r^3(1-r^3), r^3(1-r^3), 0, 0, 0, r^3(1-r^3) * c(r^2^2, (1-r^2)^2, (1-r^2)^2, r^2^2), \text{rep}(0,7), 1, 0 \\
& , 0, (1-f_2) * f_2, (1-f_2) * f_2, \text{rep}(0,10) \\
& , f_2(1-f_2) * c((1-r_1)^2, r_1^2, r_1^2, (1-r_1)^2), 0, 0, 0, \text{rep}(((1-r_1) * (1-f_2) + r_1 * f_2) * (r_1 * (1-f_2) + (1-r_1) * f_2), 2), \\
& 0, 0, 0 \\
& , r^3(1-r^3) * c(f_1^2, (1-f_1)^2, f_1^2, (1-f_1)^2), \text{rep}(0,6) \\
& , 2 * r^3(1-r^3) * c(((1-r_1) * r_2)^2, (r_1 * r_2)^2, ((1-r_1) * (1-r_2))^2, (r_1 * (1-r_2))^2, ((1-r_1) * r_2)^2, (r_1 * (1-r_2))^2, (r_1 * r_2)^2 \\
& , ((1-r_1) * (1-r_2))^2) / 2 \\
& \text{tm}[29,] \leftarrow -c(\text{rep}(0,11), (1-r^3)^2, r^3^2, 1, 0, 0, (r^2(1-r^3))^2, ((1-r^2) * (1-r^3))^2, ((1-r^2) * r^3)^2, (r^2 * r^3)^2, r^2^2, (1-r^2)^2, \text{rep}(0,5) \\
& , 1, 4, 0, f_2^2, (1-f_2)^2, 1, \text{rep}(0,7), (1-r_1)^2, r_1^2, ((1-r_1) * f_2)^2, (r_1 * (1-f_2))^2, (r_1 * f_2)^2, ((1-r_1) * (1-f_2))^2 \\
& , 0, 0, 1 \\
& , (r_1 * (1-f_2) + (1-r_1) * f_2)^2, ((1-r_1) * (1-f_2) + r_1 * f_2)^2, 0, 0, 0, ((1-r_3) * f_1)^2, ((1-r_3) * (1-f_1))^2, (r_3 * f_1)^2, (r_3 * (1-f_1))^2, f_1^2 \\
& , (1-f_1)^2, ((1-r_1) * r_2)^2, (r_1 * (1-r_2))^2, ((1-r_1) * (1-r_2))^2, (r_1 * r_2)^2, 2 * ((1-r_1) * r_2 * (1-r_3))^2, 2 * (r_1 * r_2 * (1-r_3))^2 \\
& , 2 * ((1-r_1) * (1-r_2) * (1-r_3))^2, 2 * (r_1 * (1-r_2) * r_3)^2, 2 * ((1-r_1) * r_2 * r_3)^2, 2 * (r_1 * (1-r_2) * (1-r_3))^2, 2 * (r_1 * r_2 * r_3)^2 \\
& , 2 * ((1-r_1) * (1-r_2) * r_3)^2) / 4 \\
& \text{tm}[30,] \leftarrow -c(\text{rep}(0,14), r^2(1-r^2), r^2(1-r^2), r^2(1-r^2) * c(r^3^2, r^3^2, (1-r^3)^2, (1-r^3)^2), \text{rep}(0,9), 1, f_2(1-f_2)
\end{aligned}$$

$2), f_2*(1-f_2), \text{rep}(0, 10)$   
 $, \text{rep}(r_1*(1-r_1)*f_2*(1-f_2), 4), r_1*(1-r_1), r_1*(1-r_1), \text{rep}(0, 12), \text{rep}(r_1*(1-r_1)*r_2*(1-r_2), 4)$   
 $, 2*r_1*(1-r_1)*r_2*(1-r_2)*c(r_3^2, r_3^2, r_3^2, (1-r_3)^2, (1-r_3)^2, r_3^2, (1-r_3)^2, (1-r_3)^2))/2$   
 $\text{tm}[31,] <- c(\text{rep}(0, 16), \text{rep}(r_2*(1-r_2)*r_3*(1-r_3), 4), \text{rep}(0, 10), (1-f_2)^2, f_2^2, \text{rep}(0, 10), r_1*(1-r_1)*c((1-f_2)$   
 $)^2, f_2^2, (1-f_2)^2, f_2^2)$   
 $, \text{rep}(0, 18), \text{rep}(2*r_1*(1-r_1)*r_2*(1-r_2)*r_3*(1-r_3), 8))/2$   
 $\text{tm}[32,] <- c(\text{rep}(0, 16), \text{rep}(r_2*(1-r_2)*r_3*(1-r_3), 4), \text{rep}(0, 10), f_2^2, (1-f_2)^2, \text{rep}(0, 10), r_1*(1-r_1)*c(f_2^2,$   
 $(1-f_2)^2, f_2^2, (1-f_2)^2)$   
 $, \text{rep}(0, 18), \text{rep}(2*r_1*(1-r_1)*r_2*(1-r_2)*r_3*(1-r_3), 8))/2$   
 $\text{tm}[33,] <- c(\text{rep}(0, 16), r_2*(1-r_2)*c((1-r_3)^2, (1-r_3)^2, r_3^2, r_3^2), r_2*(1-r_2), r_2*(1-r_2), \text{rep}(0, 8), f_2*(1-f_2)$   
 $), f_2*(1-f_2), 1, \text{rep}(0, 7)$   
 $, r_1*(1-r_1), r_1*(1-r_1), \text{rep}(r_1*(1-r_1)*f_2*(1-f_2), 4), \text{rep}(0, 14), \text{rep}(r_1*(1-r_1)*r_2*(1-r_2), 4)$   
 $, 2*r_1*(1-r_1)*r_2*(1-r_2)*c((1-r_3)^2, (1-r_3)^2, (1-r_3)^2, r_3^2, r_3^2, (1-r_3)^2, r_3^2, r_3^2))/2$   
 $\text{tm}[34,] <- c(\text{rep}(0, 14), (1-r_2)^2, r_2^2, ((1-r_2)*r_3)^2, (r_2*r_3)^2, ((1-r_3)*r_2)^2, ((1-r_2)*(1-r_3))^2, 0, 0, 1, r_3^2,$   
 $2, (1-r_3)^2, 0, 0, 0, 0, 1$   
 $, f_2^2, (1-f_2)^2, 0, 4, 1, 0, 0, ((1-r_1)*(1-f_2)+r_1*f_2)^2, ((1-r_1)*f_2+r_1*(1-f_2))^2, 1, 0, 0, (r_1*f_2)^2, ((1-r_1)*(1-f_2))^2,$   
 $((1-r_1)*f_2)^2)$   
 $, (r_1*(1-f_2))^2, r_1^2, (1-r_1)^2, \text{rep}(0, 6), (r_3*f_1)^2, (r_3*(1-f_1))^2, ((1-r_3)*f_1)^2, ((1-r_3)*(1-f_1))^2, f_1^2, (1-f_1)^2,$   
 $(r_1*(1-r_2))^2$   
 $, ((1-r_1)*r_2)^2, (r_1*r_2)^2, ((1-r_1)*(1-r_2))^2, 2*(r_1*(1-r_2)*r_3)^2, 2*((1-r_1)*(1-r_2)*r_3)^2, 2*(r_1*r_2*r_3)^2,$   
 $2*((1-r_1)*r_2*(1-r_3))^2$   
 $, 2*(r_1*(1-r_2)*(1-r_3))^2, 2*((1-r_1)*r_2*r_3)^2, 2*((1-r_1)*(1-r_2)*(1-r_3))^2, 2*(r_1*r_2*(1-r_3))^2)/4$   
 $\text{tm}[35,] <- c(\text{rep}(0, 16), r_3*(1-r_3)*c((1-r_2)^2, r_2^2, r_2^2, (1-r_2)^2, 0, 0, 0, 1, 1), \text{rep}(0, 5), f_2*(1-f_2), f_2*(1-f_2)$   
 $, 0, 0, 1, 0, 0$   
 $, \text{rep}(((1-r_1)*(1-f_2)+r_1*f_2)*((1-r_1)*f_2+r_1*(1-f_2)), 2), 0, 0, 0, f_2*(1-f_2)*c(r_1^2, (1-r_1)^2, (1-r_1)^2, r_1^2), \text{rep}(0, 8)$   
 $, r_3*(1-r_3)*c(f_1^2, (1-f_1)^2, f_1^2, (1-f_1)^2), \text{rep}(0, 6)$   
 $, 2*r_3*(1-r_3)*c((r_1*(1-r_2))^2, ((1-r_1)*(1-r_2))^2, (r_1*r_2)^2, ((1-r_1)*r_2)^2, (r_1*(1-r_2))^2, ((1-r_1)*r_2)^2, ((1-r_1)*(1-r_2))^2,$   
 $(r_1*r_2)^2))/2$   
 $\text{tm}[36,] <- c(\text{rep}(0, 16), ((1-r_2)*(1-r_3))^2, (r_2*(1-r_3))^2, (r_2*r_3)^2, ((1-r_2)*r_3)^2, (1-r_2)^2, r_2^2, 0, (1-r_3)^2,$   
 $2, r_3^2, 1, 0, 0, 0, 0$   
 $, (1-f_2)^2, f_2^2, 1, 0, 1, 4, 1, ((1-r_1)*f_2+r_1*(1-f_2))^2, ((1-r_1)*(1-f_2)+r_1*f_2)^2, 0, r_1^2, (1-r_1)^2, (r_1*(1-f_2))^2,$   
 $((1-r_1)*f_2)^2)$   
 $, ((1-r_1)*(1-f_2))^2, (r_1*f_2)^2, \text{rep}(0, 6), f_1^2, (1-f_1)^2, ((1-r_3)*f_1)^2, ((1-r_3)*(1-f_1))^2, (r_3*f_1)^2, (r_3*(1-f_1))^2,$   
 $0, 0, (r_1*(1-r_2))^2, (r_2*(1-r_1))^2, (r_1*r_2)^2, ((1-r_1)*(1-r_2))^2, 2*(r_1*(1-r_2)*(1-r_3))^2, 2*((1-r_1)*(1-r_2)*(1-r_3))^2,$   
 $(1-r_3)^2)$

$$,2*(r1*r2*(1-r3))^2,2*((1-r1)*r2*r3)^2,2*(r1*(1-r2)*r3)^2,2*((1-r1)*r2*(1-r3))^2,2*((1-r1)*(1-r2)*r3)^2,2*(r1*r2*r3)^2)/4$$

$$tm[37,]<-c(rep(0,36),1,rep(((1-r1)*(1-f2)+r1*f2)*((1-r1)*f2+r1*(1-f2)),2),0,r1*(1-r1)*c(1,1,(1-f1)^2,f1^2,(1-f1)^2,f1^2),rep(0,6),f1*(1-f1)*c(1,1,(1-r3)^2,(1-r3)^2,r3^2,r3^2),0,0,r1*(1-r1)*c((1-r2)^2,r2^2,r2^2,(1-r2)^2),2*r1*(1-r1)*c(((1-r2)*(1-r3))^2,((1-r2)*(1-r3))^2,(r2*(1-r3))^2,(r2*r3)^2,((1-r2)*r3)^2,(r2*(1-r3))^2,((1-r2)*r3)^2),(r2*r3^2)/2$$

$$tm[38,]<-c(rep(0,37),((1-r1)*(1-f2)+r1*f2)^2,((1-r1)*f2+r1*(1-f2))^2,0,0,0,rep(r1*(1-r1)*f1*(1-f1),4),rep(0,8),rep(r3*(1-r3)*f1*(1-f1),4),rep(0,6),2*r1*(1-r1)*r3*(1-r3)*c((1-r2)^2,(1-r2)^2,r2^2,r2^2,(1-r2)^2,r2^2,(1-r2)^2,r2^2)/2$$

$$tm[39,]<-c(rep(0,37),((1-r1)*f2+r1*(1-f2))^2,((1-r1)*(1-f2)+r1*f2)^2,0,0,0,rep(r1*(1-r1)*f1*(1-f1),4),rep(0,8),rep(r3*(1-r3)*f1*(1-f1),4),rep(0,6),2*r1*(1-r1)*r3*(1-r3)*c((1-r2)^2,(1-r2)^2,r2^2,r2^2,(1-r2)^2,r2^2,(1-r2)^2,r2^2)/2$$

$$tm[40,]<-c(rep(0,37),rep(((1-r1)*(1-f2)+r1*f2)*((1-r1)*f2+r1*(1-f2)),2),1,0,0,r1*(1-r1)*c(f1^2,(1-f1)^2,f1^2,(1-f1)^2,1,1),rep(0,6),f1*(1-f1)*c(r3^2,r3^2,(1-r3)^2,(1-r3)^2,1,1),r1*(1-r1)*c((1-r2)^2,r2^2,r2^2,(1-r2)^2),2*r1*(1-r1)*c(((1-r2)*r3)^2,((1-r2)*r3)^2,(r2*r3)^2,(r2*(1-r3))^2,((1-r2)*(1-r3))^2,(r2*r3)^2,((1-r2)*(1-r3))^2),(r2*(1-r3)^2)/2$$

$$tm[41,]<-c(rep(0,40),(1-r1)^2,r1^2,f1*(1-f1)*c((1-r1)^2,r1^2,r1^2,(1-r1)^2),rep(0,14),r2*(1-r2)*c((1-r1)^2,r1^2,(1-r1)^2,r1^2),2*r2*(1-r2)*c(((1-r1)*(1-r3))^2,(r1*(1-r3))^2,((1-r1)*(1-r3))^2,(r1*r3)^2,((1-r1)*r3)^2,(r1*(1-r3))^2,(r1*r3)^2),((1-r1)*r3^2)/2$$

$$tm[42,]<-c(rep(0,40),r1^2,(1-r1)^2,f1*(1-f1)*c(r1^2,(1-r1)^2,(1-r1)^2,r1^2),rep(0,14),r2*(1-r2)*c(r1^2,(1-r1)^2,r1^2,(1-r1)^2),2*r2*(1-r2)*c((r1*(1-r3))^2,((1-r1)*(1-r3))^2,(r1*(1-r3))^2,((1-r1)*r3)^2,(r1*r3)^2,((1-r1)*(1-r3))^2,((1-r1)*r3)^2),(r1*r3^2)/2$$

$$tm[43,]<-c(rep(0,42),((1-r1)*(1-f1))^2,(r1*f1)^2,(r1*(1-f1))^2,((1-r1)*f1)^2,rep(0,18),2*r2*(1-r2)*r3*(1-r3)*((1-r1)^2*c(1,0,1,0,1,0,0,1)+r1^2*c(0,1,0,1,0,1,1,0))/2$$

$$tm[44,]<-c(rep(0,42),(r1*f1)^2,((1-r1)*(1-f1))^2,((1-r1)*f1)^2,(r1*(1-f1))^2,rep(0,18),2*r2*(1-r2)*r3*(1-r3)*(r1^2*c(1,0,1,0,1,0,0,1)+(1-r1)^2*c(0,1,0,1,0,1,1,0))/2$$

$$tm[45,]<-c(rep(0,42),(r1*(1-f1))^2,((1-r1)*f1)^2,((1-r1)*(1-f1))^2,(r1*f1)^2,rep(0,18),2*r2*(1-r2)*r3*(1-r3)*(r1^2*c(1,0,1,0,1,0,0,1)+(1-r1)^2*c(0,1,0,1,0,1,1,0))/2$$

$$\text{tm}[46,] \leftarrow -c(\text{rep}(0,42), ((1-r1)*f1)^2, (r1*(1-f1))^2, (r1*f1)^2, ((1-r1)*(1-f1))^2, \text{rep}(0,18),$$

$$, 2*r2*(1-r2)*r3*(1-r3)*((1-r1)^2*c(1,0,1,0,1,0,0,1) + r1^2*c(0,1,0,1,0,1,1,0))) / 2$$

$$\text{tm}[47,] \leftarrow -c(\text{rep}(0,42), f1*(1-f1)*c((1-r1)^2, r1^2, r1^2, (1-r1)^2), (1-r1)^2, r1^2, \text{rep}(0,12),$$

$$, r2*(1-r2)*c((1-r1)^2, r1^2, (1-r1)^2, r1^2)$$

$$, 2*r2*(1-r2)*c(((1-r1)*r3)^2, (r1*r3)^2, ((1-r1)*r3)^2, (r1*(1-r3))^2, ((1-r1)*(1-r3))^2, (r1*r3)^2, (r1*(1-r3))^2$$

$$, ((1-r1)*(1-r3))^2) / 2$$

$$\text{tm}[48,] \leftarrow -c(\text{rep}(0,42), f1*(1-f1)*c(r1^2, (1-r1)^2, (1-r1)^2, r1^2), r1^2, (1-r1)^2, \text{rep}(0,12),$$

$$, r2*(1-r2)*c(r1^2, (1-r1)^2, r1^2, (1-r1)^2)$$

$$, 2*r2*(1-r2)*c((r1*r3)^2, ((1-r1)*r3)^2, (r1*r3)^2, ((1-r1)*(1-r3))^2, (r1*(1-r3))^2, ((1-r1)*r3)^2, ((1-r1)*(1-r3))^2$$

$$, (r1*(1-r3))^2) / 2$$

$$\text{tm}[49,] \leftarrow -c(\text{rep}(0,40), r1*(1-r1)*c(1,1, f1^2, (1-f1)^2, f1^2, (1-f1)^2), 0,0,1, \text{rep}(((1-r1)*(1-f1) + r1*f1)*(1-r1)*f1 + r1*(1-f1)), 2)$$

$$, \text{rep}(0,3), f1*(1-f1)*c((1-r3)^2, (1-r3)^2, r3^2, r3^2, 1,1), r1*(1-r1)*c(r2^2, (1-r2)^2, (1-r2)^2, r2^2),$$

$$2*r1*(1-r1)*c((r2*(1-r3))^2, (r2*(1-r3))^2, ((1-r2)*(1-r3))^2, ((1-r2)*r3)^2, (r2*r3)^2, ((1-r2)*(1-r3))^2$$

$$, (r2*r3)^2$$

$$, ((1-r2)*r3)^2) / 2$$

$$\text{tm}[50,] \leftarrow -c(\text{rep}(0,42), \text{rep}(r1*(1-r1)*f1*(1-f1), 4), 0,0,0, ((1-r1)*(1-f1) + r1*f1)^2, ((1-r1)*f1 + r1*(1-f1))^2, 0,0,0,$$

$$, \text{rep}(r3*(1-r3)*f1*(1-f1), 4), \text{rep}(0,6), 2*r1*(1-r1)*r3*(1-r3)*(r2^2*c(1,1,0,0,1,0,1,0) + (1-r2)^2*c(0,0,1,1,0,1,0,1))) / 2$$

$$\text{tm}[51,] \leftarrow -c(\text{rep}(0,42), \text{rep}(r1*(1-r1)*f1*(1-f1), 4), 0,0,0, ((1-r1)*f1 + r1*(1-f1))^2, ((1-r1)*(1-f1) + r1*f1)^2, 0,0,0,0,$$

$$, \text{rep}(r3*(1-r3)*f1*(1-f1), 4), \text{rep}(0,6), 2*r1*(1-r1)*r3*(1-r3)*(r2^2*c(1,1,0,0,1,0,1,0) + (1-r2)^2*c(0,0,1,1,0,1,0,1))) / 2$$

$$\text{tm}[52,] \leftarrow -c(\text{rep}(0,42), r1*(1-r1)*c((1-f1)^2, f1^2, (1-f1)^2, f1^2, 1,1), 0, \text{rep}(((1-r1)*(1-f1) + r1*f1)*(1-r1)*f1 + r1*(1-f1)), 2)$$

$$, 1, f1*(1-f1)*c(1,1, r3^2, r3^2, (1-r3)^2, (1-r3)^2), 0,0, r1*(1-r1)*c(r2^2, (1-r2)^2, (1-r2)^2, r2^2)$$

$$, 2*r1*(1-r1)*c((r2*r3)^2, (r2*r3)^2, ((1-r2)*r3)^2, ((1-r2)*(1-r3))^2, (r2*(1-r3))^2, ((1-r2)*r3)^2, (r2*(1-r3))^2$$

$$, ((1-r2)*(1-r3))^2) / 2$$

$$\text{tm}[53,] \leftarrow -c(\text{rep}(0,52), (1-f1)^2, f1^2, r3*(1-r3)*c((1-f1)^2, f1^2, (1-f1)^2, f1^2), 0,0, \text{rep}(r1*(1-r1)*r2*(1-r2), 4)$$

$$, 2*\text{rep}(r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8)) / 2$$

$$\text{tm}[54,] \leftarrow -c(\text{rep}(0,52), f1^2, (1-f1)^2, r3*(1-r3)*c(f1^2, (1-f1)^2, f1^2, (1-f1)^2), 0,0, \text{rep}(r1*(1-r1)*r2*(1-r2), 4)$$

$$, 2*\text{rep}(r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8)) / 2$$

$$\text{tm}[55,] \leftarrow -c(\text{rep}(0,54), ((1-r3)*(1-f1))^2, ((1-r3)*f1)^2, (r3*(1-f1))^2, (r3*f1)^2, \text{rep}(0,6),$$

$$, 2*r1*(1-r1)*r2*(1-r2)*((1-r3)^2*c(1,1,1,0,0,1,0,0) + r3^2*c(0,0,0,1,1,0,1,1))) / 2$$

$$\text{tm}[56,] \leftarrow -c(\text{rep}(0,54), ((1-r3)*f1)^2, ((1-r3)*(1-f1))^2, (r3*f1)^2, (r3*(1-f1))^2, \text{rep}(0,6),$$

$$, 2*r1*(1-r1)*r2*(1-r2)*((1-r3)^2*c(1,1,1,0,0,1,0,0) + r3^2*c(0,0,0,1,1,0,1,1))) / 2$$

$$\text{tm}[57,] \leftarrow -c(\text{rep}(0,54), (r3*(1-f1))^2, (r3*f1)^2, ((1-r3)*(1-f1))^2, ((1-r3)*f1)^2, \text{rep}(0,6),$$

$$, 2*r1*(1-r1)*r2*(1-r2)*(r3^2*c(1,1,1,0,0,1,0,0) + (1-r3)^2*c(0,0,0,1,1,0,1,1))) / 2$$

$$\text{tm}[58,] \leftarrow -c(\text{rep}(0,54), (r3*f1)^2, (r3*(1-f1))^2, ((1-r3)*f1)^2, ((1-r3)*(1-f1))^2, \text{rep}(0,6),$$

$$, 2*r1*(1-r1)*r2*(1-r2)*(r3^2*c(1,1,1,0,0,1,0,0) + (1-r3)^2*c(0,0,0,1,1,0,1,1))) / 2$$

$$\text{tm}[59,] \leftarrow -c(\text{rep}(0,54), r3*(1-r3)*c((1-f1)^2, f1^2, (1-f1)^2, f1^2), (1-f1)^2, f1^2, \text{rep}(r1*(1-r1)*r2*(1-r2), 4),$$

$$, 2*\text{rep}(r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8)) / 2$$

$$\text{tm}[60,] \leftarrow -c(\text{rep}(0,54), r3*(1-r3)*c(f1^2, (1-f1)^2, f1^2, (1-f1)^2), f1^2, (1-f1)^2, \text{rep}(r1*(1-r1)*r2*(1-r2), 4),$$

$$, 2*\text{rep}(r1*(1-r1)*r2*(1-r2)*r3*(1-r3), 8)) / 2$$

$$\text{tm}[61,] \leftarrow -c(\text{rep}(0,60), ((1-r1)*(1-r2))^2, (r1*r2)^2, ((1-r1)*r2)^2, (r1*(1-r2))^2,$$

$$, 2*r3*(1-r3)*c(((1-r1)*(1-r2))^2, (r1*(1-r2))^2, ((1-r1)*r2)^2, (r1*r2)^2, ((1-r1)*(1-r2))^2, (r1*r2)^2, (r1*(1-r2))^2,$$

$$, ((1-r1)*r2)^2)) / 2$$

$$\text{tm}[62,] \leftarrow -c(\text{rep}(0,60), (r1*r2)^2, ((1-r1)*(1-r2))^2, (r1*(1-r2))^2, ((1-r1)*r2)^2,$$

$$, 2*r3*(1-r3)*c((r1*r2)^2, (r2*(1-r1))^2, ((1-r2)*r1)^2, ((1-r1)*(1-r2))^2, (r1*r2)^2, ((1-r1)*(1-r2))^2, (1-r1)*r2)^2,$$

$$, (r1*(1-r2))^2)) / 2$$

$$\text{tm}[63,] \leftarrow -c(\text{rep}(0,60), ((1-r1)*r2)^2, (r1*(1-r2))^2, ((1-r1)*(1-r2))^2, (r1*r2)^2,$$

$$, 2*r3*(1-r3)*c(((1-r1)*r2)^2, (r1*r2)^2, ((1-r1)*(1-r2))^2, (r1*(1-r2))^2, ((1-r1)*r2)^2, (r1*(1-r2))^2, (r1*r2)^2,$$

$$, ((1-r1)*(1-r2))^2)) / 2$$

$$\text{tm}[64,] \leftarrow -c(\text{rep}(0,60), (r1*(1-r2))^2, ((1-r1)*r2)^2, (r1*r2)^2, ((1-r1)*(1-r2))^2,$$

$$, 2*r3*(1-r3)*c((r1*(1-r2))^2, ((1-r1)*(1-r2))^2, (r1*r2)^2, ((1-r1)*r2)^2, (r1*(1-r2))^2, ((1-r1)*r2)^2, (1-r1)*(1-r2))^2,$$

$$, (r1*r2)^2)) / 2$$

$$\text{tm}[65,] \leftarrow -c(\text{rep}(0,64), ((1-r1)*(1-r2)*(1-r3))^2, (r1*(1-r2)*(1-r3))^2, ((1-r1)*r2*(1-r3))^2, (r1*r2*r3)^2,$$

$$, (r1*r2*(1-r3))^2, (r1*(1-r2)*r3)^2, ((1-r1)*r2*r3)^2) / 2$$

$$\text{tm}[66,] \leftarrow -c(\text{rep}(0,64), (r1*(1-r2)*(1-r3))^2, ((1-r1)*(1-r2)*(1-r3))^2, (r1*r2*(1-r3))^2, ((1-r1)*r2*r3)^2,$$

$$, (r1*(1-r2)*r3)^2,$$

$$, ((1-r1)*r2*(1-r3))^2, ((1-r1)*(1-r2)*r3)^2, (r1*r2*r3)^2) / 2$$

$$\text{tm}[67,] \leftarrow -c(\text{rep}(0,64), ((1-r1)*r2*(1-r3))^2, (r1*r2*(1-r3))^2, ((1-r1)*(1-r2)*(1-r3))^2, (r1*(1-r2)*r3)^2,$$

$$, ((1-r1)*r2*r3)^2$$

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,(r1*(1-r2)*(1-r3))^2,(r1*r2*r3)^2,((1-r1)*(1-r2)*r3)^2)/2
tm[68,]<-c(rep(0,64),(r1*r2*r3)^2,((1-r1)*r2*r3)^2,(r1*(1-r2)*r3)^2,((1-r1)*(1-r2)*(1-r3))^2,(r1*r
2*(1-r3))^2
,((1-r1)*(1-r2)*r3)^2,((1-r1)*r2*(1-r3))^2,(r1*(1-r2)*(1-r3))^2)/2
tm[69,]<-c(rep(0,64),((1-r1)*(1-r2)*r3)^2,(r1*(1-r2)*r3)^2,((1-r1)*r2*r3)^2,(r1*r2*(1-r3))^2,((1-r1
)*(1-r2)*(1-r3))^2
,(r1*r2*r3)^2,(r1*(1-r2)*(1-r3))^2,((1-r1)*r2*(1-r3))^2)/2
tm[70,]<-c(rep(0,64),(r1*r2*(1-r3))^2,((1-r1)*r2*(1-r3))^2,(r1*(1-r2)*(1-r3))^2,((1-r1)*(1-r2)*r3)^
2,(r1*r2*r3)^2
,((1-r1)*(1-r2)*(1-r3))^2,((1-r1)*r2*r3)^2,(r1*(1-r2)*r3)^2)/2
tm[71,]<-c(rep(0,64),(r1*(1-r2)*r3)^2,((1-r1)*(1-r2)*r3)^2,(r1*r2*r3)^2,((1-r1)*r2*(1-r3))^2,(r1*(
1-r2)*(1-r3))^2
,((1-r1)*r2*r3)^2,((1-r1)*(1-r2)*(1-r3))^2,(r1*r2*(1-r3))^2)/2
tm[72,]<-c(rep(0,64),((1-r1)*r2*r3)^2,(r1*r2*r3)^2,((1-r1)*(1-r2)*r3)^2,(r1*(1-r2)*(1-r3))^2,((1-r1
)*r2*(1-r3))^2
,(r1*(1-r2)*r3)^2,(r1*r2*(1-r3))^2,((1-r1)*(1-r2)*(1-r3))^2)/2
tm}

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