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A review of hydrofractures in rocks

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Supplementary Material

Supplement to hydro-mechanical model (*Sections 3.b.1-2 and Figs. 4-7*)

The hydro-mechanical model used in this contribution is based on the work of Ghani et al. (2013, 2015) and Koehn et al. (2020). The approach is similar to the approach for poro-elasticity and flow in granular media with a compressible fluid or air as well as fluidization in a fault gouge and instabilities during sedimentation (Flekkøy et al., 2002; Johnsen et al. 2006, 2008a, 2008b; Vinningland et al., 2007a, 2007b, 2010, 2012; Goren et al. 2010, 2011; Niebling et al. 2010a, 2010b). The 2-dimensional modeling scheme is implemented in the modeling environment "Elle" (Bons et al., 2008; Piazolo et al. 2019), with solid and fluid being treated on two different grids. The model represents a vertical cross-section through a crustal domain of 1x1km at a depth of 1km. The bottom and sides of the model are confined (zero strain boundary normal to the boundary), whereas the upper boundary is controlled by gravity (constant stress boundary). Initial fluid pressures are hydrostatic with additional fluid being injected/locally produced in areas with variable geometry. The fluid is described by the fluid pressure P_f in cells of the square fluid grid. Inertia is not considered assuming a low Reynolds number and Darcy's law is used to describe the fluid movement. Continuity equations for solid and fluid on the grain scale are (Ghani et al., 2013; Koehn et al., 2020)

$$\partial_t[(1-\phi)\rho_s] + \nabla \cdot [(1-\phi)\rho_s u_s] = 0$$

$$\partial_t(\phi\rho_f) + \nabla \cdot (\phi\rho_f u_f) = 0$$

where ρ_s and ρ_f are the solid and fluid densities, u_s and u_f are the solid and fluid velocities and ϕ the local porosity of the solid. The Darcy equation can be used to calculate a local velocity $\phi\, u_f$ of the fluid for a pressure change according to the local permeability on a unit area

$$\phi(u_f - u_s) = -\frac{K}{\mu}\nabla P$$

where μ is the fluid viscosity and *P* the fluid pressure. The permeability K is calculated from the local porosity according to the Kozeny-Carman relation (Carman, 1937)

$$K(\phi_{x,y}) = \frac{r^2(\phi_{x,y})^3}{45(1-\phi_{x,y})^2}$$

with *r* the grain radius. The fluid state equation is considered using the fluid compressibility β, as a proportional approximation of the fluid density to pressure variation

$$\rho_f = \rho_0(1 + \beta P)$$

with ρ_0 the fluid density at some reference pressure. When ρ_f and u_f are substituted into equation 5, $\partial_t \phi$ is eliminated and the following diffusion equation for the fluid overpressure is derived (Gidaspow, 1994; Goren et al., 2010, 2011; Ghani et al., 2013; Koehn et al., 2020)

$$\phi\beta\left[\frac{\partial P}{\partial t} + u_s \nabla \cdot P\right] = \nabla \cdot \left[(1+\beta P)\frac{K}{\mu}\nabla \mathrm{P}\right] - (1+\beta P)\,\nabla \cdot u_s$$

with the left hand side of the equation representing the Lagrangian derivative of the fluid pressure, the first term on the right hand side the Darcy diffusion of the fluid pressure and the third term a source term dealing with pressure change as a function of particle movement.

For the solid a triangular lattice is used with elastic springs that have a normal and an angular force. The movement of nodes is a function of the momentum exchange between the solid and fluid in a unit volume cell *dV*, with a solid of mass $dm = \rho_s dV$, and inter-particle force f_e , the fluid force f_p and gravitational loading f_g, so that

$$dm\frac{dV_s}{dt} = f_e + f_p + f_g$$

Once springs have exceeded a breaking threshold (either for extension or for the angular forces) they are thought to break and are removed from the network. However the particles in the network maintain a repulsive force in case fractures

close. Fluid forces that act on each particle in the elastic grid are calculated from the difference of the neighboring fluid pressure cells and then applied on the area that the elastic node represents.

$$f_p = -\sum_k s(r_i - r_k)\left(\frac{\nabla P}{\rho_n}\right)_k \qquad (eq\ 12)$$

with ∇P the fluid pressure gradient, ρ_n the solid fraction, *k* running over the four fluid grid nodes near the particle, $s(r_i - r_k)$ is a smoothing function that satisfies the weighted distribution of particle mass relative to its position (Ghani et al., 2013). A gravitational vertical force is applied on the elastic nodes depending on the depth of the upper boundary (representing the overlying rocks) and gravitational forces from neighbouring nodes. The gravity force on single particles in addition to the load of the overlying sediments is calculated from the particle density φ, the acceleration due to gravity *g*, the real volume of the particle V_R and a scale factor *w (w*=0.74; Sachau & Koehn, 2012)

$$f_g = V_R \varphi g w \qquad (eq\ 13)$$

The model loop includes a) solving the elastic part of the equation with given boundary conditions and the fluid pressure gradients, b) once the elastic model is relaxed the spring with the highest probability will break and the model is relaxed again until no more springs break, c) evolution of the fluid pressure via the diffusion equation as a function of solid movement and porosity. The time scale is mainly given by the fluid pressure diffusion.

Supplement to Dynamics of hydrofracture flow on a crustal scale (*Sections 3.b.4 and Figs. 9-10*)

The modelling results that are presented in section 4.b.4, are also part of the study of de Riese et al. (2020). The simulations are developed using a two-dimensional cellular automaton, based on the sandpile modelling approach by Bak et al. (1988), which was the first model of a dynamical system that displayed self-organised criticality (SOC). The falling sand grains of the sandpile model are replaced in our approach by increments of fluid pressure, until the failure criterion is reached. At this point the system experiences a sudden and transient discharge through hydrofractures, which propagate in avalanches. Miller and Nur (2000) used a similar

model to simulate the development of hydrofractures, while Bons and van Milligen (2001) simulated the production, accumulation and the transport of melt within the crust also with a similar cellular automaton in one dimension only. More recently, Wangen et al. (2022) modelled the expulsion of brine by hydraulic fracturing from tight rocks undergoing compaction using used the model of Miller and Nur (2000). The approach used in the present contribution (and de Riese et al., 2020) allows the investigation of the transition from systems dominated by Darcian flow to scenarios where hydrofractures control fluid flow though the crust.

1. Diffusional Pressure Dissipation

A rigid matrix model with a constant porosity (ϕ) is assumed (see de Riese et al. (2020) for details). Fluid pressure and the amount of fluid in the pore space are related through the compressibility K (in Pa^{-1}) of the fluid:

$$dP = \frac{dV}{KV_0}$$

V_0 is the reference volume of fluid at ΔP=0 (hydrostatic pressure), dV additional volume of fluid added to the pore space. The fluid flux (J_f) is then related to a change in pressure in time (e.g. Barenblatt et al., 1989):

$$\frac{dP}{dt} = \frac{1}{\phi\alpha}\frac{dV}{dt} = \frac{-1}{\phi\alpha}\frac{dJ_f}{dz} = \frac{\kappa}{\eta\phi\alpha}\frac{d^2P}{dz^2}$$

where κ describes the permeability (in m^2) and z is the depth (in m). This equation is similar to Fick's second law for diffusion. Following this law, pressure gradients are able to be dissipated in a diffusional way with an effective diffusivity $D = \kappa/\eta\phi\alpha$ (in m^2/s).

2. Model

A two-dimensional matrix with 100 x 100 elements with an element size of 100 m represents a 10 km vertical section through the Earth's crust. A fixed fluid flux of J =

10^{-11} m/s is used, which is typical for metamorphic fluid flux in the crust at a depth of 10-15 km (Ingebritsen and Manning, 1999). Fluid enters at the base of the model and is transported towards the top. The model is laterally wrapping, *i.e.* fluid leaving the model on one side enters again on the other side. We track the overpressure for each element, where ΔP is defined as the difference between the actual fluid pressure (P_f) and the hydrostatic fluid pressure P_{hydro}. At the top row $P_{hydro.}$ is set to zero, and P_{hydro} increases with 10^4 Pa/m with depth z. The flow of fluid is implicitly modelled by tracking the evolution of ΔP using the pressure diffusion equation (S2).

The loop used in the simulations works as follows: Every time step each element in the bottom row is selected in random order and fluid pressure is added. After the fluid production causes an increase in fluid pressure, the possible initiation of a hydrofracture is evaluated for every element in the model. If the fluid pressure reaches lithostatic pressure a hydrofracture forms, which is simulated by "breaking" the element where this happens and at least one of its neighbours. If a fracture is initiated, a propagation loop starts. As only one hydrofracture can exist at a time step, all elements in the matrix which are "broken" form one connected cluster. The first step of the fracture propagation subloop is an equalization of the pressure within the cluster, which is the average of the individual pressures of all cluster elements. Next, the element on the edge of the cluster with highest pressure is selected. If the failure criterion ($P - P_{lith} > 0$) is reached, one randomly selected neighbour element that has not yet failed is added to the cluster. This subloop is repeated, until either (1) none of the elements in the cluster reaches the failure criterion, or (2) the cluster reaches the surface. In this case, the pressure in all cluster elements is set to zero, which means that fluid pressure is reduced to hydrostatic and the excess fluid is released at the surface. When fracture propagation ends, all elements in the cluster are reset to "unbroken", which implies an instantaneous healing of the fracture. At the end, once pressures are increased in all elements of the bottom row and all resulting hydrofractures are dealt with, Darcian flow is simulated using an explicit, forward finite difference scheme.

In this model, fractures heal after one calculation step. Healing is therefore effectively instantaneous relative to the diffusional flow. Equation (2) indicates that the effective pressure diffusion coefficient D is a function of porosity, fluid viscosity,

compressibility, and permeability. The pressure diffusion coefficient is varied in our simulations, which implies a variation in permeability (all other variables are kept constant). This approach allows the investigation of the transition from hydrofracture (low *D*) to Darcian-flow dominated (high *D*) behaviour. The variables in this model are scaled. A detailed description of the scaling can be found in de Riese et al. (2020).

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